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How the human mind affects its related brain?

A mechanism for this influence, using an extended Bohmian QM in Avicenna's monotheistic perspective

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Abstract

In quantum approaches to consciousness, the authors try to propose a model and mechanism for the mind-brain interaction using modern physics and some quantum concepts which do not exist in the classical physics. The independent effect of mind on the brain has been one of the challenging issues in the history of science and philosophy. In some recent mind-brain interaction models, the direct influence of mind on matter is either not accepted (as in Stapp's model) or not clear, and there have not been any clear mechanism for it (as in Penrose-Hameroff's model or in Eccles's model). In this manuscript we propose a model and mechanism for mind's effect on the matter using an extended Bohmian quantum mechanics and Avicenna's ideas. We show that mind and mental states can affect brain's activity without any violation of physical laws. This is a mathematical and descriptive model which shows the possibility of providing a causal model for mind's effect on matter. It is shown that this model guarantees the realistic philosophical constraints and respects the laws of nature. In addition, it is shown that it is in agreement with the Libet style experimental results and parapsychological data.

To propose this model, we obtained a modified (non-unitary) Schrödinger equation via second quantization method which affects the particle through a modified quantum potential and a new term in the continuity equation. At the second quantized level, which is equivalent to quantum field theory level (QFT), we can use the path integral formalism of Feynman. We show that there are three methods to extend Bohmian QM via path integral formalism, which has different interpretations. By numerical simulation of trajectories in the two-slits experiment, we show their differences and choose one of these methods for our mind-brain model which can be the basis for explaining some phenomena which are not possible to explain in the standard Bohmian QM.

Keywords: mind brain interaction, quantum consciousness, Avicenna, Bohmian quantum mechanics, modified Schrödinger equation, quantum field theory

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Introduction

Mind and brain issues are one of the most challenging issues in the history of science and philosophy [1]. Mind is very different from the matter due to its specific properties, such as unity, integrity and irreducibility to components, and the presence of some special laws and concepts such as perception, specific quality of mental states, thinking, creativity, self-awareness, consciousness, etc. In contrast, matter, especially in classical physics, is known to have properties such as locality in time and space, reducibility to components, atomistic individuality, etc. In this perspective, these two concepts (mind and brain) are so far apart, that it is very difficult to connect them and to explain the modality of their interaction [2, 3]. The developments in physics in the last century, especially in gravity and fundamental particle physics, have led to a new understanding of natural concepts and laws via theories of quantum mechanics and relativity. Relativity changed our understanding of time and the relation between spacetime and matter. Quantum mechanics introduced some concepts such as non-reductionism to components, integrity, and wholeness of quantum systems, nonlocality due to quantum entanglement of a system's components, etc. These concepts are very close to the mind concept which is considered to be a nonphysical concept or an illusion concept. Therefore, because of this similarity and our more advanced understanding of nature, the research on the mind-brain interaction and consciousness has begun in physics, by using our modern physical theories. In addition, some attitudes such as top-down causality developed by physicists like Ellis [4-7], some empirical neuroscience results about consciousness and brain activity pattern², relation between a subject's report and brain activity record, as in Libet's experiment [9-12], and the mathematical and philosophical arguments about non-computational aspects of mind as in Penrose argument [13, 14], has strengthened the quantum approach to mind-brain interaction and consciousness.

Quantum approach to consciousness has a wide-spread scope[15], but only some researches have tried to connect the concepts of mind and brain through quantum aspects, and can explain part of mind's features through quantum properties. This has Specially occurred in the models of Penrose-Hameroff [16-18], Stapp [3], Eccles-Beck [19, 20], and Bohm [21]. However, there is no complete and consistent model of mind-brain interaction in which mind is considered as an effective and independent identity. The Orch-OR theory describes some features of mental states in terms of the properties of quantum computing which are claimed to occur in the microtubules of neurons, and it tries to explain some of the experiments in neuroscience and time non-locality of mind effects, in terms of the objective reduction of wave function via quantum gravity effects [16, 18]. But the mechanism and modality of mind-brain interaction is obscure in this theory. In Stapp's model, mental states are recognized as aspects of the wave function reduction process- This process has two aspects; one is the actualization of a pattern of the neural activity at the brain; the other is the creation of a mental state [3]. In this model, the freedom and free will have no place and have no causal effect on the actualization of reality. The free will is not an illusion³, but it only affects

² Such as [8] W.J. Freeman, B. Baird, Relation of olfactory EEG to behavior: spatial analysis, Behavioral neuroscience, 101 (1987) 393.

³ The person who considers that the free will is an illusion phenomenon must be answer about how we perceive the freedom of choice between some possibilities and how we perceive the agency of our soul in choosing between various possibilities, if our free will is an illusion? In fact, where is the source of the relation between the

the state representation before the nature's choice of actualization (wave reduction) and the direct mindagency on the brain activity is denied [22]. In Eccles's model, mind has an important role in the determination of brain activity, but there is no mechanism for its effect on the neurons' connection and the brain pattern [23]. In all of these models, a realistic physical theory and a philosophical or psychological theory of mind define a model of mind-brain interaction. In this manuscript, we try to show the possibility of the presence of a causal description of mind's effect on the brain through a mathematical mechanism for the causal effect of mind, as an incorporeal essence on the brain activity. At first, we chose a consistent and appropriate philosophical and psychological framework, which is in coordination with the viewpoint of the famous Muslim philosopher and physician "Avicenna", and try to describe his model in terms of our modern understanding [24]. To preserve human's free will, we need both causality, (which related to self-agency and refers to "will"), and mind's freedom of choice (which refers to "free"). In physics, we chose the Bohmian quantum mechanics [25, 26], due to its causal description of events. But to achieve our aims, we extended it to quantum field theory and used the path integral formalism to describe it. In the following, after a review on the Avicenna's model and modified Bohmian quantum mechanics, we describe our model of mind-matter relation, using the laws of modern physics.

Recent empirical development in the foundations of quantum mechanics, especially delayed choice style experiments [27-32], appear to indicate the non-local feature of nature in time, in addition to its non-locality in space. This intrinsic property of nature is consistent with incorporeity of mind, which leads to its non-temporality. In addition, psychology and parapsychology evidences, especially precognition experiments (e.g. see [33, 34]) and neuroscience experiments, such as Libet style experiments, show the complicated nature of mind-brain interaction, and are in agreement with time non-locality in physics. Therefore, we try to explain the mechanism of the non-locality in the delayed choice experiments by the use of a modified Bohmian quantum mechanics, obtained via the quantization of Schrödinger wave function and using the path integral formalism of QFT. Then, by using this extension and a structure similar to the delayed choice experiments, we try to present a causal model for the mind-brain interaction.

A quantum delayed choice eraser experiment says that the design of the experimental setup at the final point affects a particle's evolution in the past. More precisely, the choice and design of the experimental setup leads to getting some information about the wave or particle nature of a quantum system. Therefore, due to the entanglement of the system's components, this information, that causes the appearance of particle or wave behavior, affects the determination of the interference pattern, occurred in the past at the other side of system. To get more explanation, one can refer to [35]. Specifically, in the delayed choice experiment proposed by Dopfer [36] or in Scully's version [28], there are two path entangled particles, in which the destruction of interference pattern due to the special design of mirror and lens location on one side, leads to getting information about the path of the other particle, due to the nature of entanglement. Because of the quantum nature of system, this information ensures that there is no interference pattern on the other side (second screen) at any time. Therefore, since the detection of second particle has occurred in the past, there is an effect of the future setup on the past result. This can

self and an action we which perceive in voluntary action, i.e. where did it come from? In fact, the deniers of free-will suppose that there is no actual relation between mind-agency and action and there has not been any previous phenomenon concerning mind brain relation. Therefore, this illusion has no prior history that leads to this illusion!

be interpreted as a kind of complicated coordination between the future and the past, which is not like the classical deterministic case. This is due to the nature of quantum mechanics. Although there is no signaling from future to past, but there is a nonlocal influence in time between entangled components, which is the same as space nonlocality in Bell-style experiments. There is the coordination between the future and the past that creates a whole unity. It is a kind of ultimate goal, a kind of novel force, due to the evolution of the whole system that affects the system components in the space-time. It is our Bohmian description, after its extension to QFT via path integral method, which leads to a non-unitary effect on the pilot wave.

Avicenna's model of mind-brain interaction

As we mentioned in our recent article [24], Avicenna, based on his proficiency in physics, medicine, and brain anatomy, proposed an interesting model of mind-brain interaction. In his model, mind as an incorporeal substance which affects the matter through an intermediate substance, is called the form of matter. Here are some possibilities because of the complexity level and properties of matter. The choice of one of these possibilities and its actualization needs a cause. The mind can be considered as the efficient cause of this actualization. In fact, the mind by imagining what he wants, affects the choice of actuality of matter by determining its form. The matter can potentially accept many different actualities and forms. In Avicenna's model, mind can have independent effects on the brain dynamics. The possibility of this interaction is based on the proximity of the quality of mind to the formal substance which both are produced by the same origins and basics. The mental properties are very close to the formal properties; for example, non-reducibility to parts, integrity, informational aspects of it, etc.

In physics, the quantum possibilities in matter states, either in quantum mechanics or in quantum field theory, are similar to the formal level (form of matter) in Avicenna's model. We showed how the mental state of imagination can have an influence on the matter via the quantum level of possibilities.

Additionally, due to the incorporeity of mind, mind doesn't have space-time properties, such as material properties. In Avicenna's perspective, mind has an important property which is its non-temporality and space-time independency. Therefore, the effects of mind on matter cannot be at a special space-time point, as the mental states are at a higher level beyond that of the matter, and its effects are on the whole space-time pack of motion and evolution. This demand is accomplished, for example, in the Penrose-Hameroff model of consciousness [17, 18]. But in our investigation, we took a different approach, which has been discussed in the following. It must be noticed that in the Islamic philosophy we do not see any violation in natural laws, and if a higher level affects a system, it occurs according to some specific laws of nature [37].

Extended Bohmian quantum mechanics

As Stapp noticed [3], the presentation of any quantum consciousness model needs a realistic quantum theory and its interpretation. He chose Heisenberg's model of QM. Penrose chose a realistic reduction of wave function due to gravitation [16], which is expected by some scholars to be explained by

quantum-gravity, etc. Our model of mind-brain interaction needs a causal quantum mechanics theory because our aim is to explain the causal effect of mind on the brain. Presently, there is a realistic causal model of quantum mechanics, due to Bohm. In this model, the wave function acts as a pilot wave on the particle trajectory, via an extra quantum potential in the equation of motion [26]. Both particle and wave exist in nature. The wave as a real object represents the multiple possibilities of the quantum state and guides the particle in its path. This standard Bohmian QM cannot guarantee our request for mind influence on the brain. Although the properties of this pilot wave are similar to the properties of philosophical form, but it is not known abaut how the mind affects this wave function. In physics, the change in the wave function can occur in the following ways: change in Hamiltonian (particle potential), and change in the boundary conditions or system constraints. There is no method and mechanism for mind to act via these ways in physics. Philosophically, these methods are at the level of matter for the change of wave function. Therefore, by considering the mind to affect through these levels, we are considering the mind as a cause acting at the level of material causes, whereas we are considering mind to be an efficient cause, at a higher level beyond matter.

There is another method to create a change in the pilot wave. It is the change in the dynamic equation. In the standard physics, this can be accomplished at the quantum field level. The extension of Bohmian quantum mechanics to the quantum field theory leads to a modified dynamic equation, where it has an extra term [38, 39]. This additional term contains all QFT effects on the dynamics of the wave function. As we have shown in the appendix.1, the extension of Bohmian QM via second quantization leads to an extra term in the Schrödinger equation, which affects the pilot wave dynamics and leads to a new quantum potential in the particle dynamics, in addition to the standard Bohmian quantum potential. The modified Schrödinger equation is equal to:

$$i\hbar \frac{\partial}{\partial t} \psi(x,t) = \left[-\frac{\hbar^2}{2m} \nabla^2 + U(x,t) \right] \psi(x,t) + \frac{\delta}{\delta \psi^*} \mathcal{Q} \Big|_{\psi(x,t)}$$
(1)

where:

$$Q = U \frac{\left[\frac{\delta}{\delta \psi} \frac{\delta}{\delta \psi^*} \mathcal{R}(\psi, t)\right]}{\mathcal{R}}$$
 (2)

and ψ is the wave function, and \mathcal{R} satisfies the following functional equation in which Ψ is a functional of " ψ " and time. The " Ψ " is a functional which is obtained through quantization of Schrödinger equation (appendix.1):

$$\Psi(\psi, t) = \mathcal{R}(\psi, t)e^{iS(\psi, t)} \tag{3}$$

$$i\hbar \frac{\partial}{\partial t} \Psi(\psi, t) = \int d^3x \left[-\hbar^2 \frac{\delta}{\delta \psi} U \frac{\delta}{\delta \psi^*} - \nabla \psi^* \nabla \psi \right] \Psi(\psi, t) \tag{4}$$

The quantum force on the matter in this extension is equal to:

$$F = m\ddot{x} = -\nabla(U + \mathbb{Q}) \tag{5}$$

$$\mathbb{Q} = -\frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} + \frac{1}{R} \frac{\delta}{\delta R} \mathcal{Q} ; \mathcal{Q} = \frac{\frac{\delta}{\delta \psi} U \frac{\delta}{\delta \psi^*} \mathcal{R}}{\mathcal{R}}$$
 (6)

where "R" is the amplitude of the wave function:

$$\psi(x,t) = R(x,t)e^{\frac{i}{\hbar}S(x,t)} \tag{7}$$

If we continue this quantization to third and further quantization by following the idea of Bohm in his mind-matter model [21], we obtain some time-nonlocal properties, as is explained in the following. In the Lagrangian of a system (particle, wave function, functional, etc.), if there is a square of time derivative of a dynamic quantity, the Euler-Lagrange equation leads to a second derivative of time in the equation of motion⁴. Then, the quantization of related Hamiltonian, in the Bohmian approach leads to the guidance equation, in which the time derivative of the quantity is proportional to the gradient of the phase of the wave in quantized level with respect to that quantity. For example, in standard Bohmian, the guidance equation is equal to:

$$m\frac{\partial}{\partial t}\vec{x} = \vec{\nabla}S(x,t) \tag{8}$$

where "S" is the phase part of the wave function (Eq.7). Then to have the second time derivative of the particle position (accelerate) we need to have the time derivative of the wave function (its phase). This can be found in the guidance equation of the second quantization. By quantization of Klein-Gordon equation, the guidance equation is equal to [38, 39]:

$$\frac{\partial}{\partial t}\varphi = \frac{\delta}{\delta\varphi}\mathcal{S}(\varphi, t) \tag{9}$$

where "S" is the phase part of the functional at the quantized level in the Schrödinger picture [38].

Therefore, if we can write the Lagrangian, which contains the square of time derivative of the system state, we have the nth time derivative of the particle position at any level of quantization (n-quantization). Then, if the position function (x(t)) is assumed to be an analytical function, by knowing its position at a specific time according to the Taylor expansion, we can determine the function for all times. In fact, in the above picture we have infinite number of quantizations. We use the first quantization to get particle's velocity from the time derivative of its position (standard Bohmian). In the second quantization, we get acceleration from the second time derivative of the position (QFT Bohmian in Schrödinger picture). The third quantization gives us jerk from the third time derivative of the position, etc. Therefore, we have the position function at all times. This can be interpreted as a kind of non-locality in time. To reach this non-locality, we assume that the path integral method can be used alternatively.

⁴ To grantee this condition in the equation of motion at each level of the quantized field, we can use the equation " $-\frac{\partial^2}{\partial t^2}\Psi=\widehat{H}^2\Psi$ ", instead of the Schrödinger equation " $i\frac{\partial}{\partial t}\Psi=H\Psi$ ", because its solutions are more general than Schrödinger's equation, so, some solutions are not physical.

Feynman picture

We can use the path integral formalism of quantum field theory to obtain the functional " Ψ " [40]. It can be seen that the path integral approach is more suitable for our purpose in this paper. In this approach, the probability of finding the system in the state ψ_f , if it were in the state ψ_i , is equal to the squares of the propagator between these two states [41, 42]:

$$\langle \psi_f | \psi_i \rangle = K(\psi_i, t_i; \psi_f, t_f) = \int_{b.c:\psi_i}^{\psi_f} D\psi e^{\frac{i}{\hbar} \int_{t_i}^{t_f} d^4 x \mathcal{L}(\psi, \partial_\mu \psi; t)}$$
(10)

where "b.c" refers to the boundary condition, which contains the initial state (condition) " ψ_i " at the initial time " t_i " and the final state " ψ_f " at the final time " t_f ". This integral is the summation of all possible paths from the initial state to the final state.

Due to the equivalence of the Schrödinger and Feynman's picture, we can obtain the wave functional in the form:

$$\Psi(\psi,t) = \mathcal{R}e^{i\mathcal{S}} = \int_{-\infty}^{\infty} d\psi_i \Psi_0(\psi_i,t_i) \langle \psi | \psi_i \rangle = \int_{-\infty}^{\infty} d\psi_i \Psi_0(\psi_i,t_i) \int_{b.c:\psi_i}^{\psi} D\psi e^{\frac{i}{\hbar} \int_{t_i}^t d^4x \mathcal{L}(\psi,\partial_\mu \psi;t)}$$
(11)

According to the appendix.2, we can rewrite this as:

$$\Psi(\psi, t) = \mathcal{R}e^{i\mathcal{S}} = \int_{-\infty}^{\infty} d\psi_i \Psi_0(\psi_i, t_i) \int d\psi_f K(\psi_i, t_i; \psi_f, t_f) \langle \psi | \psi_f \rangle$$
 (12)

In fact, a realistic interpretation of path integral formalism and its use in Bohmian QM has some difficulties. As we showed in appendix.2, there are three mathematical methods to obtain Bohmian version of Feynman path integral QM. One of them is irrational. Another one is the same as the standard Bohmian QM. For Eq.12 we used the third extension. We simulated the electron trajectories in the double slits experiment by the use of these extensions, in appendix.2. Our simulation for third one in addition to the description of experimental interference results shows that the final state can affect the trajectory details, especially before than the electron reaches to the slits (Fig. 1).

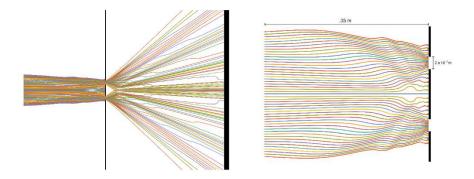


Figure 1. The trajectories of electron in two slits experiment before and after the slits (appendix. 2)

Similarly, this choice of path integral extension (Eq.12), according to Eq.2, leads to an extra term in the Schrödinger equation which forces the wave function to reach one of the possible final states " ψ_f ". This picture of Bohmian QM provides a background which can explain phenomena such as delayed choice experiment. In fact, the experimental setup preparing the final condition affects the final distribution of the wave functions, i.e. it permits some of them and bans others. For example, in the delayed choice experiment, the design of the location of mirrors and lenses leads to specific external observable states, among the possible states. This choice of experimental setup can be due to an observer's free will, i.e. because of its volition. Therefore, the change in the final experimental setup causes the change of the distribution of wave functions (Ψ_f). According to the Eq.12, the effect of changing the final distribution at the functional level, leads to an extra term in Schrödinger equation, which is interpreted as a Bohmian force at the QM level and guides the wave function to reach the final state in the domain of possibilities⁵. Although the final setup is related to the final time, but its result at the quantum level affects the evolution path of the wave function from the initial point to final one. In addition, it leads to a new Bohmian potential which guides the particle to the possible final location (Eq.6). It contains all of the QFT effects on the particle dynamics. This force is not relevant to a specific time. It exists throughout the path and is not determined only by the past. According to Eq.12, it is also a function of future and final constraints and setup.

Although in the delayed choice experiment, it is shown that the nature of quantum world does not allow a signaling to the past, but experimental data in the delayed choice experiments and in our modified Bohmian QM show the harmony and coordination between future and the past. It is due to the mentioned space-time nonlocal effects in the extra term in the Eq.1. In some delayed choice experiments, such as [28, 43], there are no defined trajectory before determining the experimental setup. For example in Scully version [28]. The selection of the particle behavior (D1&D2 detectors) or the wave behavior (D3&D4 detectors) for idler photon, affects the preparation the state function and then the signal photon dynamics⁶.

In summary, the extension of Bohmian QM to the quantum field theory and its representation in the path integral formalism leads to a quantum force which has two main properties: 1. Its effect is due to the possibilities level of wave functions (QFT level), and 2. It is a space-time nonlocal effect.

Mechanism of causal effects of mind on the matter

In the hierarchical structure of nature, one of the effects of higher levels on the lower levels is the insertion of some higher level constraints on the lower level, which have some causal effects on a lower level [45]. For example, although the movement of a piston in a cylindrical chamber of gas is a phenomenon at the

⁵ This is a non-unitary evolution of pilot wave. In this approach the induced extra term in the Schrödinger equation, leads to an effective non-unitary Hamiltonian, which guarantees the possibility of the effect of soul agency on the material dynamics.

⁶ In discussion with prof. Ruth E. Kastner, we reached the conclusion that in Bohmian explanation of some delayed choice experiments [44] B. Hiley, R. Callaghan, What is erased in the quantum erasure?, Foundations of Physics, 36 (2006) 1869-1883., they are considering a specific mixture state of initial quantum preparation. The question of why we should take a mixture state and why this specific preparation is unanswered in the Bohmian context.

thermodynamic level, but it can be considered as a constraint/condition which affects the dynamics of gas molecules at the microscopic level. Another example is the movement of the walls in the case of a quantum particle in a box which is a macroscopic constraint but has causal effects on the particle dynamics in the box. Similarly, in the mind-brain case, a mind can affect the brain via the creation of constraints which are not necessarily present at the material level.

In Avicenna's model, the mind by imagination affects the form of the matter. For example, the imagination of raising one's hand, activates the neural pattern which is related to this function. In our proposed model, the mind by the imagination of a possible state⁷ that wants to reach, prohibits an ensemble of possibilities. This prohibition affects the distribution function in the possibilities world. This constraint, according the Eq.12 and Eq.4, leads to a space-time nonlocal effect on the physical system (Eq.6). In fact, this constraint on the possibilities distribution which is caused by mind imagination causes to the creation of a quantum force that guides the physical system to the intended state. This mechanism needs a special design in a matter where this quantum force, in addition to guiding the system to a special state, causes the production of some constraints in the final condition that corresponds to the mind intended possibilities. In fact, this causal force, thanks to the intelligent design in the brain and life, leads to some physical constraints at the right time, which are in correspondence with the mind intended state. In this proposed mechanism, mind without direct involvement at the matter level, and through the mentioned intermediate force, creates some constraints at the brain's matter level. These constraints lead to the selection of especial patterns at the neurons level, and finally lead to the occurrence of some special act or state.

We try to illustrate this mechanism with a simple quantum setup. Consider the following picture which is taken from Dopfer's delayed choice experiment⁸ [35, 36].

According to what we have mentioned, mind by requesting the state 1 (or 2) related to time t_2 , creates a causal force throughout the particle path, via effectiveness on the final probability distribution at the possibilities space. This causal force causes to the destruction of the interference pattern at time t_1 . This change in the interference pattern, by activating the embedded sensor, causes a deterministic effect on the location of the screen at time t_2 , leading it to location (1). This means selecting some specific output states, which contains the information of particle path (which-path information of particle). Therefore, due to this selection (information) and according to the quantum prediction we don't have interference pattern at time t_1 . This example shows how mind through a coordinated design and without any philosophical contradiction can affect the whole system, by using of quantum mechanical laws, to reach its aim.

⁷ This imagination has two features. The first one is related to the limitations that is generated in mind upon the effect of matter/brain on mind and the mind capability. Secondly, this imagination has to be in the acting domain of the brain and the corresponding features which might happen at the level of matter.

⁸ For simplicity, we used the Dopfer's case, our explanation can be extended to explicated cases such as [28] Y.-H. Kim, R. Yu, S.P. Kulik, Y. Shih, M.O. Scully, Delayed "choice" quantum eraser, Physical Review Letters, 84 (2000) 1.

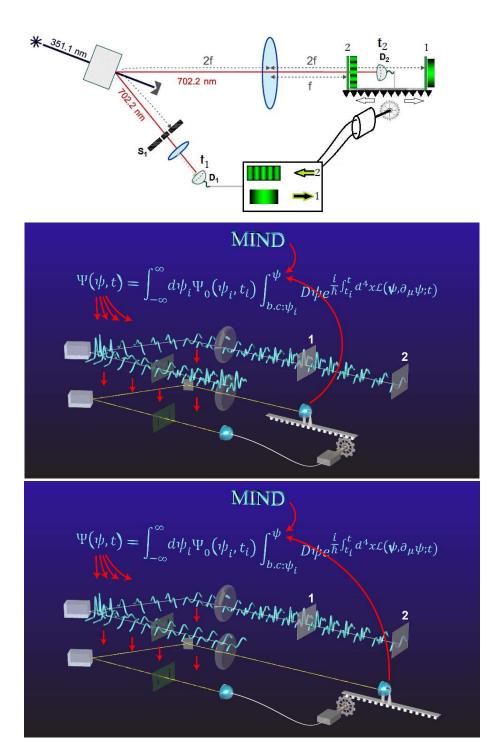


Figure 2. Schematic of mind effect on the mater. A. The states 1 and 2 are classically deferent states, state 1 represents the interference pattern (wave behavior) and the state 2 contains the particle behavior. The mind by imagination of an intended state causes the change of possibilities distribution of final state (highest red arrow). This leads to a nonlocal effect on the wave function (pilot wave) via determining of functional (Eq.12). This pilot wave affects the particle dynamics (classical system dynamics such as neurons) via the quantum potential (Eq.6). B. It guides the system to lead to inactivation of the sensor and then positioning the detector and screen on the position2. Now the experimental setup, which determines the final mind intended possibilities, is actualized. This is a material cause (down-top) which guarantee the final possible states to determine the functional (Eq.12). C. the choice of the situation and state1 causes the quantum force which guides the system to lead to the activation of sensor and then actualization of state 1.

There are two possibilities in the aforementioned design:

- 1. The sensors are activated at time t₁ (without any interference pattern), and the screen is located in position 1.
- 2. The sensors are not activated at time t₁ (with an interference pattern), and the screen is located in position2.

Both states are consistent and possible. Observation of one of them in the laboratory does not have any contradiction with physical laws, and are completely consistent. It is an important point that these states are the only possible states, and although there is no opposite physical argument against them and they are internally consistent, observing one of them raises a question: "why didn't the other possible state occur and why was this state was selected to actualize?

In the above situation, if an observer records the state 1 for the brain activity, and we ask him "why the screen is located in position 1?" He says because of the absence of interference pattern at time t_1 . But, if we ask "why didn't interference pattern occurred at t_1 ?" He says that it is due to the location of the screen at t_2 , that the information of the particle path is accessible and determined. Therefore, the particle feature appears and the interference pattern cannot be created at t_1 . In fact, all are consistent at the material level, but one needs to select one of the states at the philosophical level. Indeed, it needs a cause to actualize one of the possibilities. In our proposed model, this cause is a mind, which through the mentioned mechanism affects the appearance of states.

A more detailed explanation is that due to Avicenna's suggestion in which one considers a possibilities space which is in agreement with the functional space of the quantum field theory (analogous to possibilities space in Heisenberg quantum interpretation in the Stapp's mind-brain model). Mind can be effective on the particle/matter dynamics through an effect on the final possibility distribution. The material necessity to cause a change in the possible states is the existence of special constraints and special experimental setup. Therefore, there must be a perfect design in the brain so that the mind, having an effect on possibilities distribution (according to the modified Bohmian quantum mechanics), leads to a causal quantum force on the whole space-time of the particle evolution. and causes it to create some constraints and an appropriate material setup which are necessary for the desired distribution (this setup prohibits occurrence of other possible states).

The special properties of the proposed model include the following.

- 1. The effect of mind is a type of creation of material and physical possible states, which have causal influence on the brain matter, caused by free will and volition.
- 2. This model contains a definite physical mechanism which describes the mind effect on the brain in accordance with modern physics. It introduces neither self as a physical or material identity, nor disrespects the laws of physics at the material level.
- 3. The effect of mind is the influence on the whole of space-time of a system evolution (due to the modified Bohmian quantum potential) and does not refer to a specific point in space-time. This feature is consistent with philosophical argument, and with the incorporeity of mind, which leads to the non-temporality of mind.
- 4. Although the imagination and the will of mind correspond to time t₂, its effect via quantum force to create correspondence material setup is at time t₁, and it is consistent with empirical data in Libet-

- like experiments. Some other models which describe this data, accept the illusion of free will (i.e. they consider the free will as an illusion phenomenon).
- 5. As we mentioned, for the effectiveness of mind on the matter, there is a need for a perfect design which provides necessary material basics. Therefore, any manipulation on the matter of the brain may lead to some constraints on the mental states and mind choices by a change in this design or in possible states.

The purpose of our proposal is to prove the possibility of presenting a causal model of mind's effect on the matter (brain), but needs further research to provide a neural structure appropriate to it. However, generally speaking, the existence of a large number of inhibitory neurons and their important role in the brain activity to create the possible patterns, are in agreement with our model. Activating these neurons at t_1 causes the inactivation of some of the next neurons. it is similar to the role of sensors in our proposed quantum design. In addition, due to Stapp's model and according to the quantum connection between neurons in Eccles' and Beck's formalism, the possible patterns of neural activity are the quantum states. By noticing the role of inhibitory neurons in the formation of these patterns, it seems that our model of mind-brain is not far from reality, although it needs further investigation.

Agreement with natural laws: Libet experiment, Parapsychological data, and quantum delayed choice experiments

The best support for our model is the results of Libet-type experiments. The experiment conducted in 1983 by Libet and his colleagues at the University of California at San Francisco, describes why there is a delay between the time of specific neural activity in motor cortex (i.e. the readiness potential), which leads to the voluntary muscle movement, and the time of conscious will [10, 46]. In fact, this experiment shows that the rise in readiness potential is visible about 350 ms before the subject became consciously aware of his decision [46]. Apparently, the Libet's experiment is contrary to free-will. Some scholars think that based on the Libet's results, the conscious will and free-will are illusion phenomena [47-51], but some others interpreted this experiment through other ways [52, 53]. Libet himself suggests a kind of mind ability to consciously veto before this unconscious process is led to action and muscle movement [9], which was recently confirmed experimentally [54]. The illusion interpretation of Libet's results is a hasty conclusion. Especially, the result of Libet's experiment on the perception and relation between neural activity and sensing awareness shows the complexity of mind-brain interaction [55, 9], which is an evidence for the presence of complex physical laws at its background. Another type of Libet's experiment shows that the awareness sensing for which time is related to the evoked potential (EP) spike in the somatosensory cortex, depends on the ongoing cortical activity up to 500 ms. In the usual sensing, the conscious awareness of it occurs in 30 ms, concurrent with (at the same time of) the EP, but ongoing weak cortical activity continues about 500 ms after EP. Although the stimulation of medial lemniscuses of the thalamus in the sensory pathway, with the period less than 500 ms, causes an EP and a brief ongoing cortical activity, it does not lead to any conscious sensing. If it continues until 500 ms, the subject reports the sensing it about 30 ms after the beginning of the stimulation.

The first kind of Libet's experiment has two assumptions: 1. a kind of free will and 2. time ordering (chronological ordering) and classical determinism. If the results of Libet's experiment are correct, then

at least one of these assumptions has to be false. Some scholars abandon the idea of freedom, but according to recent results of the delayed choice experiments, which represent the space-time nonlocal nature of the quantum world, rational judgment is to have a revision of classical determinism and time ordering. Our model, by preserving causality and free will, clarifies and describe the Libet's results. In fact, according to the nature of the modified Bohmian quantum mechanics and due to its QFT extension, the nonlocal quantum force transfers the mind effect to the brain as a top down causation which does not need to take free will in the time ordering, at the brain-activity level.

The second kind of Libet's experiment is consistent with the nature of the quantum nonlocality in time. This is confirmed by the delayed choice experiments and is explained by our model of modified Bohmian quantum mechanics. In addition, for a realistic interpretation of QM, there are two models which explain such phenomena: the transactional QM which was developed by Cramer and Kastner [35, 56, 57], and the realistic wave reduction due to quantum gravity, proposed in Penros-Hameroff "Orch OR" theory [16]. According to our model, the whole evolution and neural activity affects the mind perception [24]. In fact, according to the nature of path integral formalism of QFT, and due to the extended Bohmian QM, the constraints and the structure of neural connection and their activity on the whole of the process determines the possible states in the possibilities space, of which the mind can be informed. A certain perception of a sensory detection occurs when one possible state is determined. But, due to the modified Bohmian QM, it is determined when all constraints, from the beginning to the final state, are determined. Therefore, a certain mind perception needs the determination of the whole neural process. In addition, according to our model, there can be some feedback effects of the possibilities space on the neural activity, via the quantum force. The actualized activity is due to both effects of the top level (possibilities space) and the down level (sensory input).

The time related to mind perception is associated with the ongoing mind effects on the brain due to this perception, which can even be a kind of feedback on the neural activity in the somatosensory cortex. Therefore, its time can be between 0 to 500 ms of neural activity. In addition to these neuroscience experiments, there are some experiments in psychology and parapsychology which confirm the nature of time-nonlocality in the mind-brain interaction. These experiments that are called precognition or premonition of a future event shows a person's awareness of the event or phenomenon in the future [33, 58]. For example, as Hameroff mentioned in [18], the results of Ben in [33] shows the kind of picture, which appears in the future time and have effects on a person's guesses and his decision to predict the location of the image appearance among some possibilities.

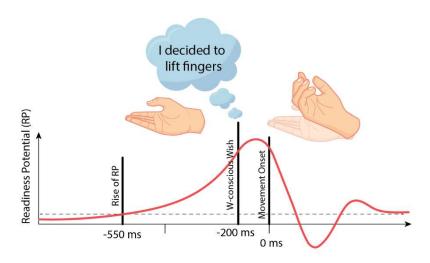


Figure 3. Schematic of readiness potential in terms of time in the Libet's experiment about the time of free-will, and time of deferent events on the Libet's experiment. The rise of readiness potential occurs 350 ms before the conscious intention (feeling of free-will).

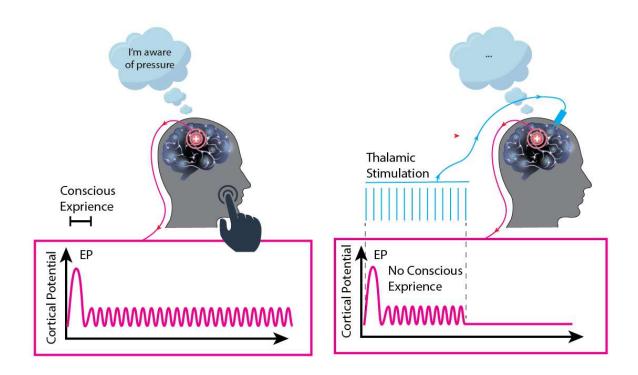


Figure 4. Schematic of cortical potential at the somatosensory area in term of time in the second type of Libet's experiment. A. In sensing experiment after the occurrence of an EP with ongoing cortical activity up to 500 ms, the patient reports a conscious experience at the same time of the EP (30 ms). B. If via thalamic stimulation, an EP be created but the ongoing activity doesn't continue until 500 ms, there is no conscious experiment report.

Conclusion

In this article, we proposed a causal model and a mechanism for the mind's influence on the brain (matter). It is consistent with the free-will. In this model, freedom is not an illusion, but is due to an upper level, and the mind is the cause of the effect via this higher level. This model is in agreement with the view of some philosophical schools, which believe in the incorporeity of mind, which leads to non-temporal properties of mind, i.e. it cannot affect a special space-time point of mater. In fact, in our model, mind affects the whole activity of the brain from the beginning to end, as a space-time nonlocal effect. These effects apply to the brain through a quantum force which is a function of all constraints and matter setup (complexity, design) in the future and the past of its motion (evolution); the past is its history and the future is its possibilities. This functionality is derived from a modified Bohmian quantum mechanics due to its extension to quantum field theory, via a path integral method. In fact, mind affects the possibilities' distribution (in the final state through its dependency of space-time) and leads to the creation of the quantum forces that guide the brain activity to what mind wants. At the final point, the matter constraints and neural setup of connections must guarantee the possibilities distribution which mind has imagined. Therefore, the created quantum force due to the special design of brain must lead to these necessary constraints at the final state. Every matter has a quantum behavior in its nature, but the mentioned special matter design is a special complexity which explains the difference between spiritual matter⁹ and normal matter.

In fact, we have proved the possibility of the presence of a causal description of the incorporeal mind's influence on the brain. We assumed the presence of the top-down causality and showed it in the mind-brain interaction through considering the quantum filed theory as a high level of possibilities. We assumed that this higher level has two aspects: one is affected by mind's imagination and other is affected by matter's constraints. In the free-will phenomenon, the mind affects these possibilities at first, and due to proposed mathematical formalism, the created force on the matter causes some constraints which are in agreement with these possibilities in the final state. In fact, everything is consistent at the matter level. However, the occurrence of a certain state among multiple possibilities needs an efficient cause, and this is mind.

In our modeling, we took advantage of Avicenna's philosophy and his model of mind-brain interaction. In addition, we used modern physical laws to obtain a mechanism to describe the mind's effect on the brain. Because of our belief in causality and the presence of free-will, we chose extended Bohmian quantum mechanics. The introduction of special neural connection setup can be appropriate for our description, and this needs some more investigation.

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⁹ The matter which is capable of accepting soul; a specific biological matter.

Appendix

A. Modified Bohmian quantum potential due to the second quantization of Schrödinger equation

1. Introduction

During the last decade, there have been worthy efforts to extend Bohmian quantum mechanics to a quantum field theory [59-64]. Specifically, this has been done for the case of bosonic particles obeying Klein-Gordon equation [60, 61]: When we extend Bohmian mechanics to a quantum field theory, a new term appears in the Klein-Gordon equation [26, 38]. This new term is called quantum potential.

In the first quantization, a quantum potential appears that has all effects of the quantum level on the classical level and the path of particles. Bohm considers higher levels as well [21]. Higher levels can have effects on this quantum level through a new quantum potential. This higher level can be reached through a functional approach to quantum field theory, which can be called second quantization. Since the laboratory setups are arranged for detecting particles, we expect that all observable effects of the quantum field theory to be seen in this generalized quantum potential.

In this article, we started with the Schrödinger equation and quantized it to obtain the quantum potential at the level of the Schrödinger equation. This modified Schrödinger equation can generate a new quantum potential in the Hamilton-Jacobi equation, which has effects on the path of the particles. The new continuity equation also includes a new term, which is due to a second quantization and can explain phenomena like creation and annihilation of the particles, and the lack of conservation of probability at the level of parts, although the total probability is conserved.

2. Schrödinger picture of QFT and a modified Schrödinger equation in QM

To obtain the Schrödinger equation of the standard QM, we can take the following Lagrangian density (assuming that $m = \hbar = 1$):

$$\mathcal{L} = i[\psi \dot{\psi}^* - \psi^* \dot{\psi}] + 2\psi^* U \psi + \nabla \psi \nabla \psi^*$$
 (1)

Where "U = U(x, t)" is the classical potential. Using the action principle, the Euler- Lagrange equation is obtained:

$$\frac{\delta \mathcal{L}}{\delta \psi^*} - \frac{\partial}{\partial x^{\mu}} \frac{\delta \mathcal{L}}{\delta \psi_{,\mu}^*} = 0$$

$$\frac{\delta \mathcal{L}}{\delta \psi} - \frac{\partial}{\partial x^{\mu}} \frac{\delta \mathcal{L}}{\delta \psi_{,\mu}} = 0$$
(2)

Which leads to the Schrödinger equation:

$$-i\dot{\psi} + U\psi - \frac{\nabla^2 \psi}{2} = 0 \tag{3}$$

By definition. The corresponding (conjugate) momenta of ψ^* and ψ are, respectively, equal to:

$$\Pi = \frac{\delta \mathcal{L}}{\delta \dot{\psi}^*} = i\psi \tag{4}$$

$$\Pi^* = \frac{\delta \mathcal{L}}{\delta \dot{\psi}} = -i\psi^* \tag{5}$$

Now, we can obtain the Hamiltonian density:

$$\mathcal{H} = \Pi \dot{\psi}^* + \Pi^* \dot{\psi} - \mathcal{L} = -2\psi^* U \psi - \nabla \psi \nabla \psi^* \tag{6}$$

$$H = \int d^3x \mathcal{H} = \int d^3x (-2\psi^* U\psi - \nabla\psi \nabla\psi^*) \tag{7}$$

and calculate the Hamilton-Jacobi equation by using the identity of " $H + \dot{S} = 0$ " and relations (4) and (5):

$$\dot{S} - \int d^3x (2\Pi^* U\Pi + \nabla \psi \nabla \psi^*) = 0 \tag{8}$$

Now, using the identities:

$$\frac{\delta S}{\delta \psi^*} \equiv \Pi \ , \qquad \frac{\delta S}{\delta \psi} \equiv \Pi^* \tag{9}$$

The Hamilton-Jacobi equation is obtained:

$$\dot{S} - \int d^3x \left(2 \frac{\delta S}{\delta \psi} U \frac{\delta S}{\delta \psi^*} + \nabla \psi \nabla \psi^* \right) = 0$$
 (10)

So far, everything has been classical. Now, according to the canonical method of second quantization [40], we can obtain the time evolution of the functional field. The canonical method includes the time evolution principle and the change of momenta to the field differential operators:

$$i\frac{\partial}{\partial t}\Psi = H\Psi; \ \Pi \to \frac{i}{\hbar}\frac{\delta}{\delta\psi^*}, \Pi^* \to \frac{i}{\hbar}\frac{\delta}{\delta\psi}$$
 (11)

Now, by using equation (7) for the Hamiltonian, we have:

$$H = \int d^3x (-2\Pi^* U\Pi - \nabla \psi \nabla \psi^*) = \int d^3x \left(2\frac{\delta}{\delta \psi} U \frac{\delta}{\delta \psi^*} - \nabla \psi \nabla \psi^* \right)$$
 (12)

So, according to relation (11), the time evolution of functional field is equal to:

$$i\frac{\partial}{\partial t}\Psi = \left[\int d^3x \left(2\frac{\delta}{\delta\psi}U\frac{\delta}{\delta\psi^*} - \nabla\psi\nabla\psi^*\right)\right]\Psi\tag{13}$$

Now, as in Bohmian approach, we can write the wave functional " Ψ " in the polar form in term of two real functional fields. Then, by separating the real and imaginary parts of equation (13), we obtain the following equations:

$$\Psi(\psi, t) = \mathcal{R}(\psi, t)e^{i\mathcal{S}(\psi, t)} \tag{14}$$

$$-\dot{S} = \int d^3x \left(-\nabla \psi \nabla \psi^* - 2 \frac{\delta S}{\delta \psi} U \frac{\delta S}{\delta \psi^*} + \frac{2}{\mathcal{R}} \frac{\delta}{\delta \psi} U \frac{\delta}{\delta \psi^*} \mathcal{R} \right)$$
(15)

$$\frac{\dot{\mathcal{R}}}{\mathcal{R}} = \int d^3x \left[\frac{2U}{\mathcal{R}} \left(\frac{\delta \mathcal{S}}{\delta \psi} \frac{\delta \mathcal{R}}{\delta \psi^*} + \frac{\delta \mathcal{R}}{\delta \psi} \frac{\delta \mathcal{S}}{\delta \psi^*} \right) + 2 \frac{\delta}{\delta \psi} U \frac{\delta}{\delta \psi^*} \mathcal{S} \right]$$
(16)

Assuming the following identities:

$$\Pi \equiv \frac{\delta S}{\delta \psi^*} , \qquad \Pi^* \equiv \frac{\delta S}{\delta \psi}$$
 (17)

And inserting them in Eq. (15), we get:

$$-\dot{S} = \int d^3x \left(-\nabla \psi \nabla \psi^* - 2\Pi^* U \Pi + \frac{2}{\mathcal{R}} \frac{\delta}{\delta \psi} U \frac{\delta}{\delta \psi^*} \mathcal{R} \right)$$
 (18)

Now, by comparing this identity with the Hamilton-Jacobi equation of the classical field, Eq. (8), a modified Hamilton-Jacobi equation is obtained, through the second quantization:

$$\dot{S} = \int d^3x (2\Pi^* U\Pi + \nabla \psi \nabla \psi^* - 2Q) \tag{19}$$

$$Q = \frac{\left[\frac{\delta}{\delta\psi}U\frac{\delta}{\delta\psi^*}\mathcal{R}(\psi,t)\right]}{\mathcal{R}}$$
(20)

Then, by using the identity " $H + \dot{S} = 0$ ", we get the modified Hamiltonian in the form:

$$H = \int d^3x (-2\Pi^* U\Pi - \nabla \psi \nabla \psi^* + 2Q)$$
 (21)

$$\mathcal{H} = -2\Pi^* U \Pi - \nabla \psi \nabla \psi^* + \frac{2}{\mathcal{R}} \left[\frac{\delta}{\delta \psi} U \frac{\delta}{\delta \psi^*} \mathcal{R}(\psi, t) \right]$$
 (22)

Therefore, by using the relation between Hamiltonian and Lagrangian, Eq. (6), the new Lagrangian is equal to:

$$\mathcal{L} = \Pi \dot{\psi}^* + \Pi^* \dot{\psi} - \mathcal{H} = i \left[\psi \dot{\psi}^* - \psi^* \dot{\psi} \right] + 2 \psi^* U \psi + \nabla \psi \nabla \psi^* - \frac{2}{\mathcal{R}} \left[\frac{\delta}{\delta \psi} U \frac{\delta}{\delta \psi^*} \mathcal{R}(\psi, t) \right]$$
(23)

By the variation of this new Lagrangian in terms of ψ or ψ^* , and using the Euler- Lagrange equation, the modified Schrödinger equation is obtained at the QM level:

$$i\frac{\partial}{\partial t}\psi(x,t) = \left[-\frac{\nabla^2}{2} + U(x,t)\right]\psi(x,t) + \frac{\delta}{\delta\psi^*}Q\Big|_{\psi(x,t)}$$
(24)

$$Q = \frac{\left[\frac{\delta}{\delta\psi}U\frac{\delta}{\delta\psi^*}\mathcal{R}(\psi,t)\right]}{\mathcal{R}}$$
(25)

3. Modified Bohmian quantum potential due to second quantization

We have obtained a modified Schrödinger equation at the QM level, which has an extra term " $\frac{\delta}{\delta \psi^*} Q$ " relative to the normal Schrödinger equation. To obtain the second quantized effects on the dynamic and evolution of a particle, Eq. (24) can be rewritten as:

$$i\hbar \frac{\partial}{\partial t} \psi(x,t) = \left[-\frac{\hbar^2}{2m} \nabla^2 + U(x,t) + \frac{1}{\psi} \frac{\delta}{\delta \psi^*} Q \right] \psi(x,t)$$
 (26)

As in the Bohmian approach [26], we write the wave function in the polar form:

$$\psi(x,t) = R(x,t)e^{\frac{i}{\hbar}S(x,t)} \tag{27}$$

where R and S are real functions. Then, using the following identity in the Eq. (26):

$$\frac{1}{\psi} \frac{\delta}{\delta \psi^*} = \frac{1}{R} \frac{\delta}{\delta R} + \frac{i\hbar}{R^2} \frac{\delta}{\delta S}$$
 (28)

We obtain the following two real functions, which are equivalent to the modified Schrödinger equation (Eq. (26)):

$$-\frac{\partial}{\partial t}S = \frac{(\nabla S)^2}{2m} + U - \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} + \frac{1}{R} \frac{\delta}{\delta R} Q \Big|_{RS}$$
 (29)

$$\frac{\partial}{\partial t}R^2 + \nabla \left(R^2 \frac{\nabla S}{m}\right) = -2 \frac{\delta}{\delta S} \mathcal{Q}\Big|_{RS}$$
(30)

Assuming that particle's speed is equal to " $\frac{\sqrt{S}}{m}$ ", the first equation (29) has the same form as in the classical Hamilton-Jacobi equation, with two extra terms:

$$-\frac{\partial}{\partial t}S = \frac{(\nabla S)^2}{2m} + U + \mathbb{Q} \tag{31}$$

$$\mathbb{Q} = -\frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} + \frac{1}{R} \frac{\delta}{\delta R} \mathcal{Q} \Big|_{RS} \tag{32}$$

The first term is the same as the normal Bohmian quantum potential, but the second term is a new Bohmian potential which is due to the second quantization. In fact, potential " \mathbb{Q} " includes all effects of QFT level and QM level on the particle trajectory (particle dynamics level).

4. Continuity equation and evolutional effects

Considering Eq. (30), we see that it is the same as the normal Bohmian continuity equation [26] with an extra term, which is due to QFT effects and second quantization:

$$\frac{\partial}{\partial t}R^2 + \nabla \left(R^2 \frac{\nabla S}{m}\right) = -2 \frac{\delta}{\delta S} Q \tag{33}$$

In the standard QM, the term of " R^2 " is interpreted as the probability of the particle detection, after observation, and in the statistical many particle Bohmian QM, it can be interpreted as a particle density distribution (or probability distribution of particle's location). As it is seen in Eq. (33), the extra term " $2\frac{\delta}{\delta S}Q$ ", apparently causes the none conservation of probability. However, if we repeat the above calculations for " ψ^* " field, another continuity equation is obtained, which is:

$$\frac{\partial}{\partial t}\bar{R}^2 + \nabla\left(\bar{R}^2 \frac{\nabla S}{m}\right) = -2\frac{\delta}{\delta S}\bar{Q} \tag{34}$$

where the $\bar{\mathcal{Q}}$ is equal to:

$$\bar{Q} = \frac{\left[\frac{\delta}{\delta \psi^*} U \frac{\delta}{\delta \psi} \mathcal{R}(\psi, t)\right]}{\mathcal{R}}$$
(35)

If we consider the anti-commutator relation between Π^* and Π , the result is:

$$\left[\frac{\delta}{\delta \psi^*}, \frac{\delta}{\delta \psi}\right]_{\perp} = 0 \tag{36}$$

$$\bar{Q} = -Q \tag{37}$$

Therefore, due to Eq.34 and Eq.37, the continuity equation for " ψ^* " field, is equal to:

$$\frac{\partial}{\partial t}\bar{R}^2 + \nabla\left(\bar{R}^2 \frac{\nabla S}{m}\right) = +2\frac{\delta}{\delta S}Q\tag{38}$$

Comparison with Eq.33 and Eq.38 can be interpreted as the survival probability that the whole system (particles and antiparticles) will preserve. This is because the dynamical operations at the QFT level are unitary.

In Bohmian QM, this extra term can be taken as a basis for the causal description of creation and annihilation phenomena, and in the Bohmian interpretation, this term affects the magnitude of the pilot wave (which is interpreted as active information [39, 65, 66]). But, we know that the quantum potential

which is its result, is independent of wave magnitude. However, the effect of quantum potential is valid as far as the wave magnitude is not zero. When due to the Eq. (33), "R" becomes zero, then there is no pilot wave or active information. So why must we consider a particle with no wave function? This situation can be interpreted as an annihilation effect. And the reverse process for " ψ *" can be interpreted as a creation effect.

We can see this effect in the Eq. (26), where the new extra term in the modified Schrödinger equation is complex. This term is same as an effective potential term (and effective non-unitary Hamiltonian) and we know that mathematically an imaginary potential can destroy the conservation of probability in the case of Schrödinger equation [67], and in many parts of physics, it can be considered as a dissipative effect. We propose that because of this extra term, which leads to a nonlinear modified Schrödinger equation, there may be a possibility for the solution of measurement problem in QM (due to its nonlinearity and non-unitary evolution). In the Bohmian interpretation of mind and matter relation, which was proposed in [21], the effect of higher level possibilities on the lower level dynamics, can be tied to the causal transformation of active information to inactive information in a Bohmian QM, and also, the reduction of the wave function in the standard QM. This is achieved through the change of regular Schrödinger equation to a nonlinear modified Schrödinger equation, thanks to QFT effects.

For bosonic particle (or real field), to preserve the conservation of probability, the extra term of " $\frac{\delta}{\delta s}Q$ " must be zero, therefore "Q" must be invariant from phase transition, and it causes a kind of charge conservation due to Noether's theorem.

5. Conclusion

In Bohmian mechanics, the effects of the level of possibilities, described by the wave function, on particle trajectories are summarized in the quantum potential. By the quantization of the Schrödinger equation, we obtain a functional field, which shows the possibilities of the wave function. We have shown that this generalization to quantum field theory leads to an extra potential-like term in the Schrödinger equation. We call this new equation, the modified Schrödinger equation. (Eq. (24), (25)).

It was shown that this modified Schrödinger equation causes two effects on the particle evolution. One is via the modified quantum potential (Eq. (32)) which has a new extra term with aspect to the normal Bohmian potential. It determines the effect of second quantization on the particle trajectory. Another is an extra term in the continuity equation which can provide a basis for a causal explanation of QFT level effects on the particle evolution, such as creation and annihilation phenomena. We propose that this dissipative extra term maybe considered as a solution for the measurement problem in the standard QM, because of its effect on the nonlinearity of modified Schrödinger equation, and also providing a base for producing a mechanism of transformation of active information to inactive one in the Bohmian interpretation and its relation to the mind effect on matter.

B. Extension of Bohmian quantum mechanics via path integral method

1. Introduction

In the standard quantum mechanics, we deal with probabilities, for example, the probability of detecting or finding a system in a well-defined state, such as a specific location or a specific range of energy [67]. There are two mathematical formalisms to calculate this probability: the Schrödinger wave equation and the Feynman path integral method [68]. In the first one, the wave-function solution of Schrödinger's equation contains, mathematically, all of the information about the quantum system that can be essentially known. It is related to the observational results via the probability density which is equal to the square modulus of the wave-function. To solve the Schrödinger equation, in addition to the system Hamiltonian operator (\hat{H}) , we need the initial wave-function and all of the constraints in the present and the past.

$$i\hbar \frac{\partial}{\partial t} \psi(x, t) = \widehat{H} \psi(x, t) \tag{1}$$

In the Feynman path integral method of QM, the probability amplitude of finding the system in a specific final state is calculated by integration of the weighted function over all of the possible paths between the initial and final state. The probability is equal to the square modulus of this probability amplitude, which is called propagator. Each path has a weight function " $e^{\frac{i}{\hbar}S}$ " where "S" is the action of the path. The propagator between the initial and final states is equal to:

$$K(x_i, t_i; x_f, t_f) = \int_{x_i}^{x_f} Dx e^{\frac{i}{\hbar} \int_{t_i}^{t_f} \mathcal{L}dt}$$
(2)

Where " $\int Dx$ " is the path integral over all possible trajectories, and " \mathcal{L} " is the Lagrangian density of the system [69].

This propagator is related to the wave function via the following identity:

$$\psi(x,t) = \int_{-\infty}^{\infty} dx_i \psi_0(x_i, t_i) K(x_i, t_i; x, t) = \int_{-\infty}^{\infty} dx_i \psi_0(x_i, t_i) \int_{x_i}^{x} Dx e^{\frac{i}{\hbar} \int_{t_i}^{t} \mathcal{L} dt}$$
(3)

where " ψ_0 " is the initial wave function. It can be shown that at this level, these two formalisms are equivalent, and it has been shown that the propagator is a function which satisfies the Schrödinger equation [68].

In an ontological and realistic perspective of the quantum world, it is difficult to use the path integral method and yet have a realistic interpretation of it. This is true especially in the Bohmian interpretation, which is developed by considering the wave function of the Schrödinger formalism as a pilot wave. Here, an ontological view of the path integral method has some difficulties.

In this article, we show that the extension of Bohmian QM via path integral formalism can be done by three different approaches. These extensions lead to different ontological interpretations. Furthermore, each of them can have a different observational aspect. In addition, we show these differences numerically by finding the trajectories of a particle in the two slits experiment after and before the slits.

2. Extension of Bohmian QM via the path integral method

In Bohmian QM, there is both a point particle and a wave function, in which the latter satisfies the Schrödinger equation. The wave function or pilot wave, can be written in the polar form with two real functions, and it leads to two real equations, when inserted in the Schrödinger equation. We get a Hamilton-Jacobi like equation and a continuity equation [26, 70].

$$\psi(x,t) = R(x,t)e^{iS(x,t)} \tag{4}$$

$$-\frac{\partial}{\partial t}S = \frac{(\nabla S)^2}{2m} + U - \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R}$$
 (5)

$$\frac{\partial}{\partial t}R^2 + \nabla \left(R^2 \frac{\nabla S}{m}\right) = 0 \tag{6}$$

By assuming of the following identity:

$$\vec{v} = \vec{\nabla}S(x, t) \tag{7}$$

Eq. (5) is the same as the classical Hamilton-Jacobi equation, with an extra term " $-\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}$ ", which is called quantum potential. Here " \vec{v} " is the particle velocity.

To use the path integral method for a Bohmian calculation, there are three methods. In the path integral formalism, the propagator has an essential role. As shown in Eq. (2) there are two free variables: the initial particle position (x_i) and its final position (x_f) . In the Bohmian picture, the particle has a definite trajectory, and as a result, it has a definite position at any time. Hence we can take the present position of a Bohmian particle as the final free variable or the initial's or between of them in path integral method.

If we consider the position of the particle as the final constraint in the propagator, the wave function can be obtained in the following way:

$$\psi_{H}(x,t) = \int_{-\infty}^{\infty} dx_{i} \psi_{0}(x_{i}, t_{i}) \langle x | x_{i} \rangle ; \langle x | x_{i} \rangle \equiv K(x_{i}, t_{i}; x, t)$$

$$\psi(x,t) = R(x,t) e^{iS(x,t)}$$

$$\vec{v} = \vec{\nabla} S(x,t)$$
(8)

which is the same as in the standard Bohmian QM. It has been shown that the pilot wave (ψ_H) has all information of the particle's history via two factors: initial wave function " ψ_0 ", and all possibilities which

it could reach to the final position "K". In fact, the particle velocity is determined only by the system history, and the wave function can be interpreted as the memory of the particle.

On the other hand, one can consider the initial state of the propagator as the present position of the particle. Therefore, the initial wave function is equal to the Dirac delta function, and the wave function can be obtained in the following way:

$$\psi(x,t) = \int_{-\infty}^{\infty} dx_i \delta(x - x_i, t - t_i) K(x_i, t_i; x_f, t_f) = K(x, t; x_f, t_f) = \int_{x}^{x_f} Dx e^{\frac{i}{\hbar} \int_{t}^{t_f} \mathcal{L} dt}$$

$$\psi(x,t) = R(x,t) e^{iS(x,t)}$$

$$\vec{v} = \vec{\nabla} S(x,t)$$
(9)

where, " δ " is the Dirac delta function. In this extension, particle velocity is only affected by all of the possible paths that could lead to the final state. The wave function is the function of all possibilities that may occur in the future and the final state of the system. This path integral Bohmian formalism can be a candidate for the explanation of delayed choice experiment [28, 29, 32], which claims that the future observed state or future experimental setup can affect the quantum behavior of the particle. Unfortunately, it cannot explain the particle behavior and its surreal trajectory after the slits, in the two slits experiment [71]. This is because of the pilot wave's independence of its history.

The third type of Bohmian path integral extension is to suppose that the particle position is between the initial and final states. By using the Identity operator in Dirac picture, we have:

$$\langle x|x_i\rangle = \int dx_f \, \langle x|x_f\rangle \langle x_f|x_i\rangle = \int dx_f \, K(x_i;x_f)\langle x|x_f\rangle \tag{10}$$

and by inserting this relation in Eq. (8), the wave function becomes:

$$\psi_U(x,t) = \int_{-\infty}^{\infty} dx_i \psi_0(x_i, t_i) \int dx_f \, K(x_i, t_i; x_f, t_f) \langle x | x_f \rangle$$
 (11)

Apparently, in this picture, the wave function is affected by the initial state (ψ_0) and the past and future possibilities $(K(x_i;x_f))$. But we know that this representation doesn't have any concept in addition to Eq. (8) and the first interpretation. Since the second integral in Eq. (11) satisfies the Eq. (10), which in " $\langle x|x_i\rangle$ " has no future effect.

Because Hamiltonian is a unitary operator, we can use the Identity operator in Eq. (10). But if it has some constraints due to the experimental setup in the measurement, which leads to a nonlinear evolution of the wave function, the identity (10) is invalid. This is because of the change of the Hilbert space due to the final setup. In these situations, although Eq. (10) is invalid, but if we keep the validity of the Eq. (11) as a definition of the wave function (ψ_U), then we have a new extension with a novel property. Now Eq. (11) is different from Eq. (8) and has both the initial condition and the future possibilities. In summary, due to this fact that " $K(x_i; x_f)$ " is interpreted as a probability amplitude of future possibilities, if we consider it as an independent parameter which is caused by the observer action and the experimental setup, in

addition to the unitary evolution of Schrödinger equation, it is a new extension of Bohmian QM via path integral method. We can rewrite the Eq. (11) in the following form:

$$\psi_U(x,t) = \int dx_f \langle x | x_f \rangle \int_{-\infty}^{\infty} dx_i \psi_0(x_i, t_i) K(x_i, t_i; x_f, t_f) = \int dx_f \langle x | x_f \rangle \psi_f(x_f, t_f)$$
(12)

where " ψ_f " is the probability amplitude or wave function corresponding to the final situation, which can be affected by the final experimental conditions. By using Eq. (8), the wave function of this new extension is equal to:

$$\psi(x,t) = \frac{1}{2}(\psi_H + \psi_U) = \frac{1}{2} \int_{-\infty}^{\infty} dx_i \psi_0(x_i, t_i) \langle x | x_i \rangle + \frac{1}{2} \int_{-\infty}^{\infty} dx_f \langle x | x_f \rangle \psi_f(x_f, t_f)$$

$$\psi(x,t) = R(x,t) e^{iS(x,t)}$$

$$\vec{v} = \vec{\nabla} S(x,t)$$
(13)

If there is no additional effect for the determination of the final possibility distribution, in addition of standard Schrödinger evolution with no limiting constraints, this definition is returned to the previous one and the standard definition of the wave function. intrinsically, the particle velocity at any time is affected by its history (ψ_0) and the future possibilities (ψ_f) . This can be interpreted as an ultimate goal of nature.

3. Numerical analysis of a particle trajectory in a two slits experiment

Numerical simulation of a particle trajectory in the two slits setup was done by Hiley and his colleagues in 1979 for the first time [39, 72]. They obtained these trajectories for an electron, especially for its behavior after the slits, for a setup which was the same as the experimental setup of the electron interference in [73]. In their calculation, it was supposed that the distribution of electron position in the slits is a Gaussian function of their centers. The same simulations have been done and repeated several times for the two slits interference experiments [74-77]. Especially in [75], the particle trajectories are simulated before and after the slits and without the aforementioned Gaussian assumption. Our simulation confirms these results for the electron (Fig. 5).

If the second and third extension of the Bohmian QM, via path integral method (Eq. (12)), is considered, the final condition and experimental setup can affect the trajectory of the particle. As was mentioned in the previous section, the simulation of the second extension (Eq. (9)) cannot produce the interference fringes of the two slits. In addition, for the electron trajectories, before they reach to slits, the future setup, which allows the electron to pass only at slits, forces all of the trajectories to end of slits as complex paths (Fig. 6). In fact, these constraints lead to this behavior.

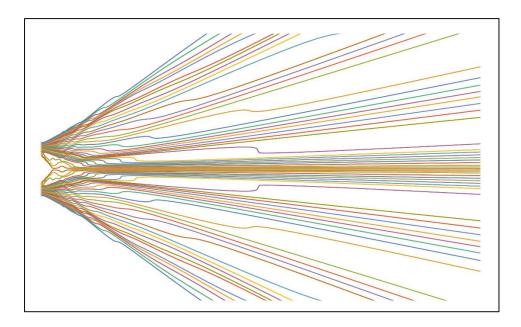


Figure 5. Electron trajectories in the double slits experiment through the first extension of Bohmian QM via path integral method. These trajectories are the same as the results of [75], but have some initial distortion after the slits.

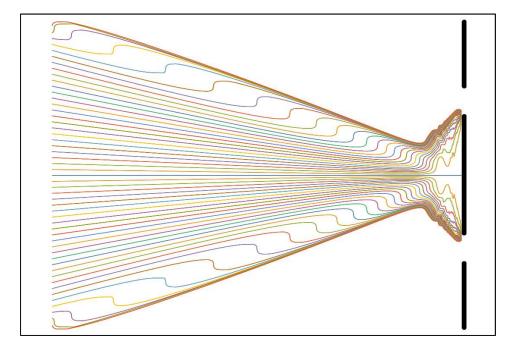


Figure 6. Electron trajectories before they reach the slits as derived through the second extension of Bohmian QM via path integral formalism. The future setup (i.e. double slits plate), forces each of the trajectories to end up in one of the slits

In the trajectory simulation via the third extension of Bohmian QM (Eq. (13)), which is shown in Fig. 7, we observe two differences compared to the prediction of the standard Bohmian QM. For the paths before the slits, it seems that near the slit the electron propensity is to avoid the slit. In fact, the Bohmian quantum potential in this picture guides the electron to the slit and it seems that electron senses the future possibilities and the surrounding constraints. On the other hand, without any assumption of Gaussian distribution of the electron position at the slits, for the paths after the slits, the trajectories are the same as the first simulation of Ref [72]. These trajectories don't have any initial distortion of Bohmian standard simulation [75]. It is because of this fact that in the standard simulation, which was done before, they used the free particle propagator formula, i.e.:

$$K(x_i, t_i; x, t) = \left(\frac{m}{2\pi i \hbar (t - t_i)}\right)^{\frac{1}{2}} e^{im(x - x_i)^2 / 2\hbar (t - t_i)}$$
(14)

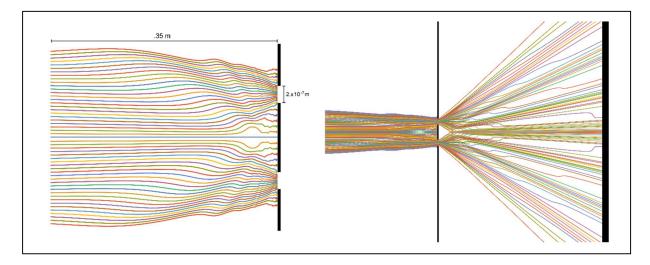


Figure 7. The electron trajectories computed through the third extension of Bohmian QM via the path integral method. Left: the trajectories before they reach to the slits. Right: the trajectories in double slits experiment by the initial Gaussian distribution with the variance of " $10^{-6} \mu m$ ".

This formula is invalid because in the two slits experiment we have some constraints, such as the presence of the final screen and the double slits plate, which prohibit the presence of some trajectories (Fig. 8). Therefore, independent of any change of conditions in the future, to determine Bohmian paths through path integral formalism, our proposal (Eq. (13)) is more suitable, as it takes in to account our constraints.

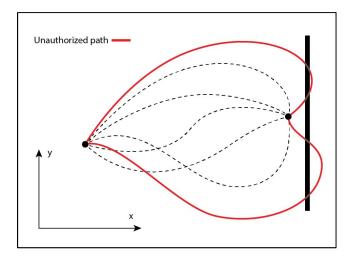


Figure 8. Schematics of some authorized and unauthorized trajectories at the presence of some constraints.

4. Details of the numerical simulation:

For simulation, we use the Wolfram Mathematica software. The conditions (Fig. 9) are the same as the experimental conditions used in ref. [73] and are the same as the previous simulations [72]. For the second simulation (Fig. 6 and Eq. (9)) the final state was considered to contain all possible final position at two slits with the uniform distribution. Therefore, the following identity was used to calculate particle trajectories (Fig. 6).

$$\psi(x,t) = \int_{-\infty}^{\infty} dx_f P(x_f) \int_{-\infty}^{\infty} dx_i \delta(x - x_i, t - t_i) K(x_i, t_i; x_f, t_f) = \int_{-\infty}^{\infty} dx_f P(x_f) K(x, t; x_f, t_f)$$
(15)

For the third simulation, we used Eq. (13) (Fig. 9). The initial condition of the electron distribution was Gaussian, with the variance of $1 \mu m$, and the final wave function was calculated according to Eq. (12).

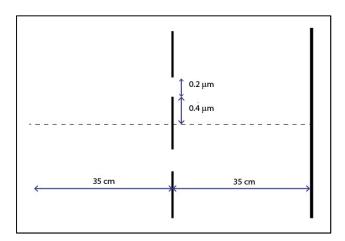


Figure 9. The setup conditions of double slits experiment which used in [73].

5. Conclusion

Bohmian quantum mechanics is a deterministic model, where the wave function, which is satisfying the Schrödinger equation, guides the particle in its trajectory via a quantum potential. In this model, the particle velocity is influenced by the past and present conditions. But in the path integral formalism, we deal with the initial and final states without any mechanism and dynamics of the particle evolution between these points. We use the mathematical formalism of path integral formalism to extend Bohmian quantum mechanics. It can be done via three approaches: one of them is the same as the standard Bohmian QM and is an alternative to it. The second is invalid due to the result of simulation that cannot explain the experimental data. The final extension, through some new assumption, can produce the experimental results of the double slits interference fringes. In addition, the last approach, has some extra predictions about the electron trajectory, before it reaches the slits, which can be experimentally tested. The authors propose that this simple extension has some advantages: for example, because it is often reduced to the standard wave function definition, and for the trajectory simulation via path integral method it considers the constraints more clearly, so, it is useful for these aims. The other advantage is that it can be the basis for explaining some phenomena, such as the delayed choice entanglement experiment or for the situations where the observer action is important in the experimental results. However, it seems that there is a need for an extension to QFT, as Hiley mentioned in [78]. The effect of future state and possibilities on the particle trajectories in our extension can be interpreted as some sort of awareness of the particle. Perhaps it can be interpreted as some sort of ultimate purpose for the particle.

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