The impact of nuclear spin and isospin of Dirac particles on the mass spectrum of leptons

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Summary
A calculation is presented for the mass relationships between electron, muon and the tau particle. It is shown that charged leptons with mass levels beyond the one of the tau particle cannot exist because of insufficient binding energy. The underlying mechanism of the relationship between the leptons is the isospin feature of Dirac particles, which appears being a substantial attribute next to nuclear angular spin. It is further shown that the energy of the elementary electric charge, such as embodied by the mass of an electron, is due to the interaction between the isospins of the pion's quark and antiquark.

Keywords: Dirac particle; isospin; muon; tau particle; electron mass

Introduction

In his classic paper on electrons, Dirac [1] has derived expressions for two elementary dipole momenta. The first one is due to the well known elementary angular momentum, eventually dubbed as spin $S=\hbar/2$, which manifests itself physically as a magnetic dipole moment, with a magnitude linearly proportionate to S. The second one is an elementary dipole moment with magnitude $\hbar/2c$, which is less known. It has been waived away by Dirac, because it showed up as an imaginary quantity for which he could not find a physical justification. If it would have a physical justification, one might expect that it would show up as an elementary electrical dipole moment. This view has been adopted by Hestenes [2,3] in his studies on the jitter ("zitterbewegung") of electrons. In present day quantum physics, quite some studies are going on, aiming to establish values for the electrical dipole moment of electrons if it would exist [4,5,6,7]. The existence of it an electric moment of electrons is put into doubt, because it would violate the CPT pillar of the Standard Model of particle physics [8]. This would be the case indeed if the electrical moment would be the result of a possible distribution of electric charge in a spatial structure. Curiously, in those studies usually no reference is given to Dirac's paper, who took the pointlike format of an electron as an axiom.

It is my aim to show in this article that, in spite of Dirac's conclusion, the second elementary dipole moment of a Dirac particle, which we shall indicate for short as isospin, is not an imaginary quantity, but a real one. I wish to show the impact of this isospin on the structure of nuclear particles that can be composed by quarks, in particular on the structure of a pion before decay and after its decay into a muon. The primary aim is showing the relationship between the masses of the electron, the muon and the tau particle and revealing the reason why no charged leptons exist with a mass beyond the tau particle's one. Doing so, various axioms of present day quantum physics will be discussed and put into a new light. Among these are the Higgs field, the isospin of quarks and nucleons, the large amount of elementary particles and the hadrionic mass spectrum.

The article falls apart into two parts. The analysis of spin and isospin of Dirac particles, although a basic ingredient of the article, will be described in the Appendix. The results of it, will be invoked in the main text that describes how the masses of charged leptons depend on these attributes.

The energy field of quarks and electrons

The Standard Model of particle physics heavily relies upon the concept of an omnipresent energetic nuclear background field, dubbed as the Higgs field. In its most simple representation, this field Φ is characterized by its Lagrangian density $U_H(\Phi)$. This density is heuristically defined as [9],

$$U_H(\Phi) = \mu_N^2 \frac{\Phi^2}{2} - \lambda_N^2 \frac{\Phi^4}{4}, \tag{1}$$

where μ_N and λ_N are characteristic real constants. The justification for this format is the simple fact that many predictions from the theoretical elaboration of this underlying axiom of the Standard Model are in agreement with experimental evidence. It is instructive to compare this heuristically conceived energetic background field with the background field around an electric pointlike charge in an ionized plasma [10], where the background field has the format

$$U_{DB} = \lambda_{DB}^2 \frac{\Phi^2}{2}. \tag{2}$$

The energetic field Φ flowing from the pointlike charge is influenced by this background field and can be derived by the overall Lagrangian density L with the generic format

$$\mathrm{L} = -\frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi + U(\Phi) + \rho\Phi, \tag{3}$$

where $U(\Phi)$ is the potential energy of the background field and where $\rho\Phi$ is the source term. By application of the Euler-Lagrange equation, the potential Φ of the pointlike source of this (Debije) field can be derived as,

$$\Phi_{\mathrm{DB}} = \Phi_0 \frac{\exp(-\lambda_{DB} r)}{\lambda_{DB} r}, \tag{4}$$

with $\Phi_0 = Q\lambda / 4\pi\varepsilon_0$, where Q is the electric charge and ε_0 the vacuum electric permittivity.

Supposing that a quark is an energetic pointlike source, we may try to establish its potential, thereby expecting it being influenced by the energetic background field. Unfortunately, the format of the Higgs background field prevents obtaining straightforwardly an analytical expression. However, by profiling a solution, a numerical procedure may serve for establishing a good fit. To make a long story short, documented in previous work [11], the result is a quark field with the format,

$$\Phi(r) = \Phi_0 \exp(-\lambda r)\{\frac{1}{\lambda^2 r^2} - g_m \frac{1}{\lambda r}\}, \tag{5}$$

where the parameter λ determines the spatial range of the field. Its value depends on the Higgs parameters μ_N and λ_N. The numerical value of the dimensionless factor g_m can be established from curve fitting as g_m = 2. It will be shown later in this text that it can be conceived as a gyrometric factor that is related with the gyromagnetic constant of quantum mechanical electromagnetism. This field can be conceived as the sum of a near field and a far field such that,

$\Phi(r) = \Phi_N(r) + \Phi_F(r)$, with

$$\Phi_N(r) = \Phi_0 \frac{\exp(-\lambda r)}{(\lambda r)^2} \quad \text{and} \quad \Phi_F(r) = g_m \Phi_0 \frac{\exp(-\lambda r)}{\lambda r} \tag{7}$$

The latter one, i.e. $\Phi_N(r)$, has the same format as an electric field if $\lambda r \to 0$. It has the characteristics of a Debije field, where the free energetic flow from the source is suppressed by a surrounding background field. The near field $\Phi_N(r)$ shows the characteristics of a dipole field, or, more precisely, the characteristics of a field from an linear dipole moment aligned along the dipole axis.

The far field behaves similarly as the Debije field, i.e. as a field where the free energetic flow is suppressed by an energetic background field. Each of two quarks in a meson are coupled to the field of the other with a coupling factor g. Hence, the combined field from two quarks in a meson aligned along the $x-$axis, spaced at a $2d$ distance, can be expanded as,

$$V(x) = \Phi(d+x) + \Phi(d-x) = g\Phi_0(k_0 + k_2\lambda^2 x^2 + ...), \tag{8}$$

where $k_0$and k_2 are dimensionless coefficients with magnitudes that depend on the spacing d. The interesting feature of the meson configuration now is, that its center of mass is subject to a potential that (almost) depends on the square of x. Hence, the meson shows the characteristics of a quantum mechanical oscillator. It is therefore subject to excitation, which is the underlying mechanism for the systematic characteristic of the mass spectrum of mesons [12]. The two quarks in the meson align themselves in the condition of minimum energy, such that they are spaced at $d\lambda = d'_{\min}$ ($\approx$0.856).

There is one nasty thing though. Where the far field can be explained in conventional field theory by considering it as the description of a repelling force from one quark to another, like in the electromagnetic case, the gluing near field is not clear. Therefore, the open question is, how to explain the attracting force that keeps two forces in equilibrium toward a quasi stable configuration. In particle physics, the problem is settled by defining a force of unknown origin, dubbed as color force, which make mesons and baryons quasi stable. This

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color force is conceived in a frame work of mathematical formalism, but, in fact, a true physical justification is lacking. Nevertheless, Quantum Chromatic Dynamics (QCD) is one of the pillars of present day quantum physics. Except this, the justification of the Higgs field is not very convincing, because it is mainly based upon analogies [9], from which it is not clear why those should apply to all space. If, on the other hand, an explanation would be found for the field as described by the profiled solution (5), the Higgs field (1) would be explained as well. In previous works, I have adopted the quark field profile (5) by simply assuming that, next to a vectorial bosonic far field, a scalar near field exists with the desired properties. It is my aim in this work to show that this assumption can be justified and improved because of two recent obtained results. The first of these is the awareness of Dirac's second dipole moment of electrons. The second of these is the awareness of an omnipresent cosmological energetic background field, which shows up as the vacuum solution of Einstein's Field Equation under adoption of a non-zero value for the Cosmological Constant[13].The combined effect from the two novel views result in a theoretically based model for the quark potential similar to (5). Such without the need for adopting the spontaneous symmetry breaking (SSB) mechanism, which so far has been proposed to justify the Higgs field format (1). The crux is the adoption of the Dirac particle description for quarks, including its potential to eject an energetic flow like electrons do, and the validation of isospin for Dirac particles next to spin. The interaction between the isospins, i.e., the second dipole moments of quarks, is the reason that they attract. Because dipole fields are decaying rapidly, the attracting near field range show a r^{-2} dependency of the potential, while the range of the repelling far field from the energetic flow, shows the regular r^{-1} dependency. The additional exponential decay is due to an energetic background field with a format as shown by the Debije effect.

The second dipole moment will change the scalar field of an electron as well. There is some difference with the nuclear field of a quark, though, because the field of an electron in free space is not subject to the shield of the Higgs field. Hence, the scalar field of a pointlike electric charge is made up as,

$$\Phi_{\mathrm{e}}(r) = \Phi_0 \{\frac{1}{\lambda^2 r^2} - g_m \frac{1}{\lambda r}\} \ . \qquad (9)$$

As just explained, a quark and an antiquark in conjunction, compose a meson. The archetype one is the pion.

The mass of the electron

Apart from the electron itself, the smallest observable particle showing an electric charge is the pion. The simple pion model, as discussed in this text so far, is inadequate for explaining the origin of electric charge. It can be understood from the influence of the spins of the quark and the antiquark on the rest mass of the pion. As explained in this text, a quark has an angular moment, spin for short, and a second dipole moment, which is non-angular in its origin, in this text dubbed as isospin for short. It is well known that the spin has a major impact on the rest mass of a meson. If the spins are parallel, the meson is of the pseudoscalar type. If the spins are anti-parallel, the meson is of the vector type. A pion is a pseudoscalar meson. It has a rest mass of about 140 MeV. Its vector type counter part is the rho meson, which has a rest mass of about 780 MeV. This marks a major impact of the

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angular spin on the meson's mass. The mass difference is due to the interaction between the spins of the quark and the antiquark. The energy of the spin interaction is the result of the in-product of the spin vectors. Hence, it has to be expected that also the isospin vectors of will have an impact on the mass of the mesons, on those of the vector type as well on those of the pseudoscalar type. The difference becomes manifest in the mass difference between the charged pions and the neutral pion. That means that the origin of electric charge can be traced back to the second dipole moment of the quark. It is not by accident that the ratio of the difference between the mass of the charged pion and the neutral pion over the difference between the mass the pseudoscalar pion and its vector-type counter part is just equal to the electromagnetic fine constant 1/137. This is clear from

(139.57-135)/(780-140) $\approx$1/137.

This suggests that we may consider the electron-positron bond as the electromagnetic version of the pion's quark-antiquark bond. Hence, it may be expected that the ratio of the massive energy of this electron-positron bond over the rest mass of the charged pion is equal to 1/137 as well. The mass of the electron is just half this value. The result of the calculation shows (139.57/137)/2 = 0.509 MeV. This corresponds rather accurately with the known energy 0.511 MeV massive energy as established from experiments reported by the Particle Data Group (PDG), [14]. Obviously, the pion hides the origin of electron charge. As shown in Appendix B, the electric energy is due to the potential energy of the second dipole moment of the quark with respect to the second moment of the antiquark. Hence, the second dipole moment of the quark effectively is an electric dipole moment. The electric energy, though, is a holistic result of the quark-antiquark conjunction.

The masses of the muon and the tau particle

Let us now proceed by considering a muon as the boson-fermion transformation (decay) of a pion. Where a pion can be conceived as a quark spaced from an antiquark under equilibrium of an attractive field component and a repulsive field component, a muon can be hypothetically conceived as two equally signed electric charges $q/2$ under equilibrium by the two field components shown by (7). This, of course, is a conjecture that should be supported later with experimental evidence. Under this hypothesis, the two muon components spaced $2d$ apart, compose an anharmonic oscillator, described by a wave equation for its wave function ψ that results from from (9) as,

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+V(x)\psi=E\psi\text{, with}$$

$$V(x)=U(d+x)+U(d-x);\;\; U(x)=\frac{q}{2}\Phi_0\left(\frac{1}{\lambda^2x^2}-g_m\frac{1}{\lambda x}\right)\;. \tag{10}$$

where m is the effective central mass of the configuration and where E is the generic energy constant, which is subject to quantization. Note that $q/2$ is the coupling constant from one charge to the other's field potential. This equation can be normalized as,

$$-\alpha_0 \frac{d^2\psi}{dx'^2} + V'(x')\psi = E'\psi\,; \tag{11}$$

$$\alpha_0 = \frac{\hbar^2\lambda^2}{mq\Phi_0}\,;\quad V'(x) = \frac{2V(x)}{q\Phi_0}\,;\ E'(x) = \frac{2E(x)}{q\Phi_0}\,;\ x' = \lambda x\,;\ d' = d\lambda\,.$$

The potential $V'(x)$ can be conveniently expanded as,

$$V'(x) = k_0 + k_2 x'^2 + \ldots\ldots\ , \tag{12}$$

where

$$k_0 = \frac{2}{d'^2} - \frac{4}{d'}\,;\ k_2 = \frac{6}{d'^4} - \frac{4}{d'^3}\,.$$

Hence, (11) can be written as,

$$-\alpha_0 \frac{d^2\psi}{dx'^2} + k_2 x'^2\psi = (E' - k_0)\psi\,. \tag{13}$$

Because all other quantities including α_0 are dimensionless as well. From (11), we have

$$\alpha_0 = \frac{\hbar^2\lambda^2}{mq\Phi_0}\,. \tag{14}$$

The quantity $q\Phi_0$ can be interpreted as the potential energy of the center of mass due to the field potential Φ. There are no other sources to produce the field's energy apart from the massive energy mc^2 in the center of mass. The identification of a relationship between the field's potential and this massive source energy requires a general relativistic analysis on the basis of Einstein's Field Equation. Inheriting the result for quarks, such as shown in [15, eq. (24)], under the electroweak relationship,

$$q^2 = 4\pi\varepsilon_0 g^2\hbar c\,, \tag{15}$$

we have,

$$\frac{q}{2}\Phi_0 = \frac{1}{mc^2}\frac{k_2\lambda^2(\hbar c)^2}{k_0^2}\,. \tag{16}$$

From (16) and (14)

$$\alpha_0 = \frac{k_0^2}{2k_2}\,. \tag{17}$$

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Note: Obviously, the simple relationship (17) heavily relies upon eq. (16) that has been invoked from previous work. I consider its re-derivation outside the scope of this text, because it can be readily reproduced from the published article [15].

Let us proceed by observing figure 1. It shows the potential (energy) $V'(x)$ of the muon's center of mass, defined by (12) and (10) as a function of its deviation from the spatial center. Each curve in the figure is characterized by a particular parameter value for the (normalized) spacing d' between the poles. There is a clear minimum for $d' = d'_{\min}$, an increase of the curvature for $d' < d'_{\min}$ and a decrease of the curvature for $d' > d'_{\min}$. If the two poles are spaced in the state of minimum potential ($d' = d'_{\min}$), the vibration energy of the muon is in the ground state. As long as the curves show a minimum with a negative value, the

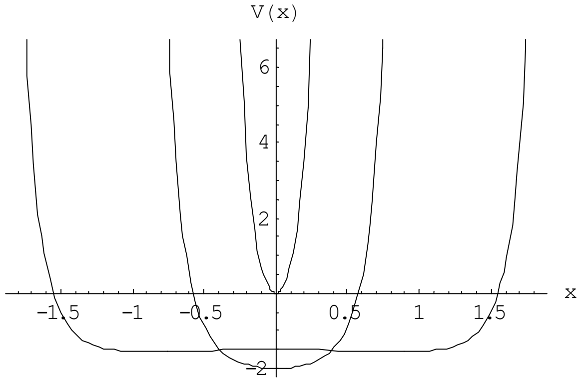

Figure 1: The potential energy of the muon's center of mass as a function of spacing between the poles. The stable ground state of the muon occurs at maximum binding energy (V(x) = -2; d' = 1). The binding energy is lost for spacing d' = 0.5.

configuration shows an amount of stability-preserving binding energy. It will be clear that the binding energy is lost for narrow spacing. In figure 2 it is illustrated that the energy constant level of first excitation in ground state may correspond with the ground state energy constant of the configuration at a smaller spacing.

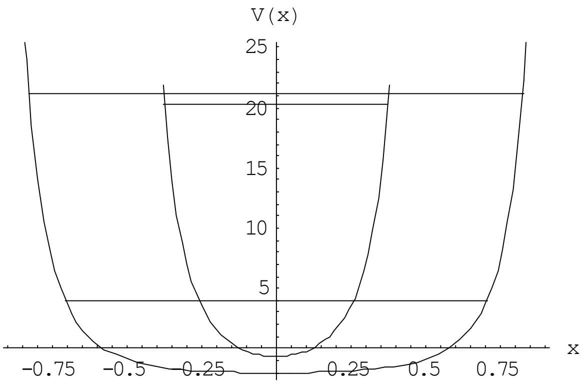

Figure 2: The jump from the muon state to the tauon state is a jump from the first excitation level of the muon state to the ground state of the heavier tauon. It happens at a spacing d' = 0.56, where the energy constants (not to be confused with the massive energies) match, under preservation of a slight amount of binding energy.

This is the reason why, under excitation conditions, the configuration may jump from the muon state to the tauon state. It will be clear that the jump to the level of second excitation cannot be made under preservation of negative binding energy. Hence, charged leptons beyond the tauon particle are non-existing.

Let us accept the muon's mass as a reference for the calculation of the tau particle's mass. A basic ingredient for the calculation is the basic quantum mechanical relationship between the mass m, the quantization step $\hbar\omega$ in its vibration energy and the curvature k_2 of the field potential that causes the vibration. Hence, from (11),

$$\frac{m\omega^2}{2} = q\Phi_0 k_2 \lambda^2 . \qquad (18)$$

The second ingredient is an expression that relates the strength quantity $q\Phi_0$ and the spatial parameter λ of an electron-type Dirac field. In a general relativistic analysis on the basis of Einstein's Field Equation [15, eq. (24)], it has been found that its ratio is frame independent and amounts to,

$$\frac{q\Phi_0}{\lambda} = \frac{\alpha\pi\hbar c}{4k_0 d'_{\min}}, \qquad (19)$$

where α is a dimensionless constant of order 1. Its meaning will soon be clear.

The third ingredient is the consideration that the field's energy is not an external field, but, instead, that the origin of the field's energy is due to the potential field of the two poles. Hence, the mass of the structure is nothing else but the created vibration energy. This means that the spacing $2d = 2d'_{\min}/\lambda$ equals half the wavelength of a standing wave, such that

$$\frac{2d'_{\min}}{\lambda} = \frac{\alpha c T}{2} = \frac{c}{2f} = \frac{1}{2}\frac{2\alpha\pi c}{\omega} \rightarrow \lambda = \frac{2\hbar\omega d'_{\min}}{\alpha\pi(\hbar c)}, \qquad (20)$$

where α is a dimensionless constant of order 1, as just mentioned.

The three ingredients hold for the basic ground state, i.e. the state where the effective mass of the anharmonic oscillator can be found by equating mc^2 with $\hbar\omega$. In the excitation mode these relationships have to be considered with care. This holds particularly for the relationships between Φ_0, λ and the pole spacing d. From (18) and (20), we have

$$mc^2 = \frac{8d'^2_{\min}}{(\alpha\pi)^2} k_2(d') q\Phi_0 . \qquad (21)$$

This expression allows relating the mass of the structure as a function of the spacing d'. Normalizing the massive energy on the muon's one, we have,

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$$\frac{mc^2}{mc^2\big|_{\text{muon}}} = \frac{k_2(d')}{k_2(d'_{\min})}. \tag{22}$$

It has to be emphasized that this mass is no longer made up by a standing like in the muon's case. The lower curve in figure 3 graphically shows the mass dependence as a function of the spacing between the poles.

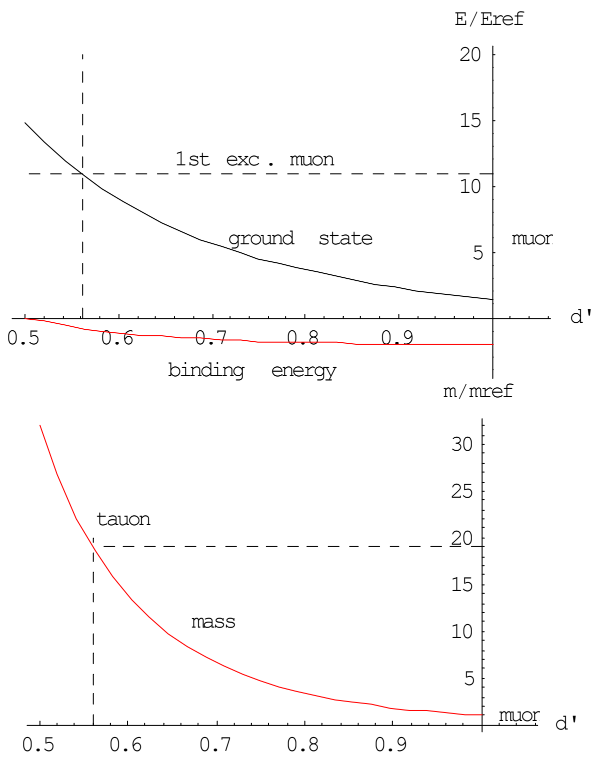

Figure 3: The lower curve shows the dependence of the lepton's physical mass on the pole spacing. The upper graph shows that the pole spacing of the tau particle is determined by the equilaty in vibration energy of the muon's first excitation level and the ground state vibration energy of the heavier tau particle. Note that the binding energy of the tau particle is just slightly negative.

In combination with the upper curve it illustrates how the mass of the tauon particle is related with the mass of the muon. At the spacing $d' = 0.56$ the vibration energy of the muon in first excitation equals the ground state energy of the heavier tau particle. Note that the binding energy, represented by the red curve, is still negative. The cross-over to a positive value occurs at $d' = 0.5$. The lower curve shows that, at spacing $d' = 0.56$, the relative mass value of the structure amounts to 18.9. This means that the tau particle's mass is expected being 1.89 GeV/c^2. This is rather close to the tau particle's PDG rest mass (1.78 GeV/c^2. The difference is due to a slight inaccuracy of the calculation. Where the upper curve of figure 3 is the result of a rather accurate numerical computation of the anharmonic oscillator, the lower curve is less accurate, because k_2 in (22) is determined as the result of the truncation of the series expansion (12).

Conclusion

The decay of a pion into a muon is a transformation of the bosonic quark-antiquark state into the fermionic state of two separate kernels with equal electric charge. Similarly as a quark and an electron, these kernels behave as Dirac particles, which have, next to the nuclear dipole moment from an elementary angular moment, a second dipole moment. This isospin is instrumental for the force equilibrium between the repulsive Coulomb-type interaction between the kernels and the attracting dipole moment field. The result is a quantum mechanical oscillator structure with a physical massive energy (vibration), solely composed by the standing wave energy between the kernels. A structure with a narrower spacing has, apart from vibration energy, massive kernels, but is not stable, unless its ground state energy equals the first-level excitation energy of the muon state. The analysis shows that only a single excitation can be maintained without violating the binding force condition of negative energy. This is the tauon particle state. Its calculated massive energy 1.89 GeV/c^2 corresponds fairly with the tau particle's PDG rest mass (1.78 GeV/c^2). It has been concluded further that the mass of the elementary charge, such as embodied by the electron, is the result of the spin-spin interaction between the second dipole moments of the pion's quark and antiquark.

Appendix A: The spin and the isospin of Dirac and Majorana particles

The quantum mechanical wave function of a Dirac particle and a Majorana particle

Let us first consider a generic free moving particle. Its Einsteinean energy is given as,

$$E_W = \sqrt{(m_0c^2)^2 + (c|\mathbf{p}|)^2}\,, \tag{A1}$$

where m_0 is the particle's rest mass and where $\mathbf{p}$ is the threevector momentum (ds/dt, not be confused with the fourvector momentum $\boldsymbol{p}$). Without loss of generality the particle's free motion can be aligned along the $x-$axis in a system of Carthesian coordinates for which we shall adopt the Hawking metric ($\mathrm{i}ct, x, y, z$), $\mathrm{i} = \sqrt{-1}$. Squaring (8) gives,

$$E_W^2 = -p_0^2c^2 = (m_0c^2)^2 + c^2p_1^2\,, \tag{A2}$$

which can be normalized as,

$$p_0'^2 + p_1'^2 + 1 = 0\,;\; p_\mu' = \frac{p_\mu}{m_0c}\,. \tag{A3}$$

Note: I prefer to use the Hawking metric ($\mathrm{i}ct, x, y, z$) or (+,+,+,+) to avoid the ugly minus sign in (-,+,+,+), which shows up as metric if the time dimension is defined as real instead of imaginary. As long as the temporal dimension is included, the bold italic notation for the vector $\boldsymbol{p}$ will be maintained.

Dirac wrote this equation as,

$$p_0'^2 + p_1'^2 + 1 = (\beta + \overline{\alpha}\cdot\boldsymbol{p}')(\beta + \overline{\alpha}\cdot\boldsymbol{p}') = 0\,, \text{ with } \overline{\alpha} = \overline{\alpha}(\alpha_0, \alpha_1) \text{ and } \boldsymbol{p}'(p_0', p_1')\,, \tag{A4a}$$

while Majorana wrote this equation as,

$$p_0'^2 + p_1'^2 + 1 = (\beta + \overline{\alpha} \cdot \boldsymbol{p}')(\beta - \overline{\alpha} \cdot \boldsymbol{p}') = 0, \text{ with } \overline{\alpha} = \overline{\alpha}(\alpha_0, \alpha_1) \text{ and } \boldsymbol{p}'(p_0', p_1'), \quad \text{(A4b)}$$

Thereby leaving freedom for the type of the number β and for the type of components of the two-dimensional vector $\overline{\alpha}$. The elaboration of the middle term is:

$$(\beta + \overline{\alpha} \cdot \boldsymbol{p}')(\beta \pm \overline{\alpha} \cdot \boldsymbol{p}') = (\beta + \sum_\mu \alpha_\mu p_\mu')(\beta \pm \sum_\nu \alpha_\nu p_{\nu_\nu}')$$
$$= \beta^2 + \sum_\nu \pm \beta\alpha_\nu p_\nu' + \sum_\mu \alpha_\mu \beta p_\mu' - \sum_\mu \sum_\nu \alpha_\mu \alpha_\nu p_\mu' p_\nu'$$
$$= \beta^2 + \sum_\nu (\pm\beta\alpha_\nu + \alpha_\nu\beta) p_\nu' \pm \sum_{\mu \neq \nu} (\alpha_\mu \alpha_\nu + \alpha_\nu \alpha_\mu) p_\mu' p_\nu' \pm \sum_\mu \alpha_\mu^2 p_\mu'^2 \quad \text{(A5)}$$

In this equation the upper sign in $\pm$ holds for the Dirac mode, while the lower sign holds for the Majorana mode. To satisfy Dirac's the following conditions should be true:

$$\alpha_\mu \alpha_\nu + \alpha_\nu \alpha_\mu = 0 \text{ if } \mu \neq \nu;\ \beta^2 = 1,\ \beta\alpha_\nu + \beta\alpha_\nu = 0 \text{ and } \alpha_\mu^2 = 1 \text{ for } \mu = 0,1, \quad \text{(A6a)}$$

while for the Majorana mode,

$$\alpha_\mu \alpha_\nu + \alpha_\nu \alpha_\mu = 0 \text{ if } \mu \neq \nu;\ \beta^2 = 1,\ -\beta\alpha_\nu + \beta\alpha_\nu = 0 \text{ and } \alpha_\mu^2 = -1 \text{ for } \mu = 0,1, \quad \text{(A6b)}$$

From these expressions it will be clear that the numbers α_μ and β have to be of special type. To this end we could use for the Dirac mode the following (Pauli) matrices,

$$\alpha_0 = \sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\quad \alpha_1 = \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \text{ and } \beta = \sigma_2 = \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}, \quad \text{(A7a)}$$

while for the Majorana mode we could use

$$\alpha_0 = \mathrm{i}\sigma_1 = \mathrm{i}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},\quad \alpha_1 = \mathrm{i}\sigma_3 = \mathrm{i}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \text{ and } \beta = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{(A7b)}$$

It can be verified that (A7a) meets the condition (A6a). Similarly, (A7b) meets the condition (A6b). It means that the Majorana condition can be derived from the Dirac condition, simply by replacing the real α- matrices by corresponding imaginary ones after swop and by replacing the imaginary β-matrix by the Identity matrix. The reason of the swop will be explained later.

Let us now invoke the basic theorem of quantum mechanics to associate a wave function with a particle. This is done by transforming momentum elements into wave function elements. Because in this simplified case the momentum relationship is two-dimensional, the wave function should be two-dimensional as well. Therefore, after transforming the momenta into operators on wave functions,

$$p_\mu' \to \hat{p}_\mu \psi \quad \text{with} \quad \hat{p}_\mu' = \frac{1}{m_0 c} \frac{\hbar}{\mathrm{i}} \frac{\partial}{\partial x_\mu}, \quad \text{(A8)}$$

(axiomatic quantum mechanical hypothesis), the momentum relationship (4) is transformed under consideration (A8) into the following two equivalent two-dimensional wave equations

$$[\alpha_0]\begin{bmatrix}\hat{p}_0'\psi_0\\ \hat{p}_0'\psi_1\end{bmatrix}+[\alpha_1]\begin{bmatrix}\hat{p}_1'\psi_0\\ \hat{p}_1'\psi_1\end{bmatrix}\pm[\beta]\begin{bmatrix}\psi_0\\ \psi_1\end{bmatrix}=0, \text{ and} \tag{A9}$$

Note that the Dirac condition is met for a single equation with a $+$ sign in front of β, while the Majorana condition falls apart into three possible modes , namely a single equation mode with either a $+$ sign, or a $-$sign, or a dual equation mode with two equations that should be simultaneously true, i.e., one with a $+$ sign, and a second one with a $-$sign. As will be shown, It is the latter one that makes the actual Majorana mode. The other two modes coincide with the Dirac mode.

With explicit expressions of the Pauli matrices for the Dirac case, we have

$$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}\hat{p}_0'\psi_0\\ \hat{p}_0'\psi_1\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}\hat{p}_1'\psi_0\\ \hat{p}_1'\psi_1\end{bmatrix}+\begin{bmatrix}0 & -\mathrm{i}\\ \mathrm{i} & 0\end{bmatrix}\begin{bmatrix}\psi_0\\ \psi_1\end{bmatrix}=0, \tag{A10}$$

In alternative terms, this reads as:

$$\hat{p}_0'\psi_1+\hat{p}_1'\psi_0+\mathrm{i}\psi_1=0 \quad \text{and} \quad \hat{p}_0'\psi_0-\hat{p}_1'\psi_1-\mathrm{i}\psi_0=0, \tag{A11}$$

Denormalizing (11)and writing it in matrix terms gives,

$$\begin{bmatrix}\hat{p}_1 & \hat{p}_0-\mathrm{i}m_0c\\ \hat{p}_0+\mathrm{i}m_0c & -\hat{p}_1\end{bmatrix}\begin{bmatrix}\psi_0\\ \psi_1\end{bmatrix}=0. \tag{A12}$$

Let this set be heuristically solved by

$$\psi_i(x,t)=u_i\exp\mathrm{i}(\frac{p_1}{\hbar}x-\frac{W}{\hbar}t), \tag{A13}$$

After substitution of (A-13a) into the upper signed part of (A-12) we find

$$\begin{bmatrix}p_1 & (\mathrm{i}W/c-\mathrm{i}m_0c)\\ (\mathrm{i}W/c+\mathrm{i}m_0c) & -p_1\end{bmatrix}\begin{bmatrix}u_0\\ u_1\end{bmatrix}=0. \tag{A14}$$

Non-trivial solutions for ψ_i are obtained if the determinant of the matrix is zero. In both cases this is true if:

$$(\frac{W}{c})^2=p_1^2+m_0^2c^2. \tag{A15}$$

This condition corresponds with the square of the Einsteinean relationship (A1). This marks the solution of Dirac's equation as a two-component solution A(13). The solution shows two different

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options for the amplitude ratio u_0 / u_1. These are not conflicting because of the quadratic nature of the determinant condition (A15).

$$\frac{u_0}{u_1} = -\mathrm{i}\frac{p_1}{(W/c + m_0 c)} = \mathrm{i}\frac{W/c - m_0 c}{p_1} \quad . \tag{A16}$$

Under non-relativistic condition ($|p_1| << m_0 c$) and positive energy $(W > 0)$, there is a large dominant component (u_0) as compared to a small minor component (u_1).

Let us now consider the less-known Majorana case. Denormalizing (A12a) and writing it in matrix terms gives,

$$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & -\mathrm{i} \end{bmatrix} \begin{bmatrix} \hat{p}'_0 \psi_0 \\ \hat{p}'_0 \psi_1 \end{bmatrix} + \begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix} \begin{bmatrix} \hat{p}'_1 \psi_0 \\ \hat{p}'_1 \psi_1 \end{bmatrix} \pm \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \psi_0 \\ \psi_1 \end{bmatrix} = 0 . \tag{A17}$$

This case represents three different possible modes, namely the single plus mode, the single minus mode and a dual mode where two different modes should be simultaneously be true. Let us first consider the single plus mode. It reads as the set,

$$\mathrm{i}\,\hat{p}'_0 \psi_0 + \mathrm{i}\,\hat{p}'_1 \psi_1 + \psi_0 = 0 \; . \qquad -\mathrm{i}\,\hat{p}'_0 \psi_1 + \mathrm{i}\,\hat{p}'_1 \psi_0 + \psi_1 = 0 \;\; ; \tag{A18}$$

Using the same procedure as before, (A18) is denormalized and written in matrix terms as,

$$\begin{bmatrix} \mathrm{i}\,\hat{p}_0 + m_0 c & \mathrm{i}\,\hat{p}_1 \\ \mathrm{i}\,\hat{p}_1 & -\mathrm{i}\hat{p}_0 + m_0 c \end{bmatrix} \begin{bmatrix} \psi_0 \\ \psi_1 \end{bmatrix} = 0 . \tag{A19}$$

Note the symmetry between (A19) and the corresponding one (A12) in the Dirac case. The symmetry is due to the swop between the Pauli matrices as mentioned while discussing (A7b).

Let this set be heuristically solved by

$$\psi_i(x,t) = u_i \exp \mathrm{i}(\frac{p_1}{\hbar} x - \frac{W}{\hbar} t) , \tag{A20}$$

After substitution of (A20) into (A19) we find

$$\begin{bmatrix} m_0 c + (W/c) & \mathrm{i}p_1 \\ \mathrm{i}p_1 & m_0 c - (W/c) \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \end{bmatrix} = 0 . \tag{A21}$$

Non-trivial solutions for ψ_i are obtained if the determinant of the matrix is zero. This is true if:

$$(\frac{W}{c})^2 = p_1^2 + m_0^2 c^2 . \tag{A22}$$

Similarly as in the Dirac case, this condition corresponds with the square of the Einsteinean relationship (A1). It makes (A20) a valid solution. Let us now consider the minus mode of the Majorana case. Analytically, it means simply changing the sign of $m_0 c$. Apart from the sign in the ratio u_0 / u_1, the solution is not different. In both cases and similarly in the Dirac case, it makes no relevant difference either if we would had taken the conjugate complex alternative for the tentative solutions (A13) and (1A9 by changing the sign of the argument of the exponential functions. Hence, both the plus mode and the minus mode of the Majorana case are not different from the Dirac case. The actual difference is made by accepting the dual equation mode that combines the plus mode and the minus mode. In that case two sets of solutions, $\psi_{01}(\psi_0, \psi_1)$ and $\psi_{23}(\psi_2, \psi_3)$ should be simultaneously true, where

$$\psi_0(x,t) = u_0 \exp\pm \mathrm{i}(\frac{p_1}{\hbar}x - \frac{W}{\hbar}t); \qquad \psi_2(x,t) = u_2 \exp\pm \mathrm{i}(\frac{p_1}{\hbar}x - \frac{W}{\hbar}t);$$

$$\psi_1(x,t) = u_1 \exp\pm \mathrm{i}(\frac{p_1}{\hbar}x - \frac{W}{\hbar}t); \quad \text{and} \quad \psi_3(x,t) = u_3 \exp\pm \mathrm{i}(\frac{p_1}{\hbar}x - \frac{W}{\hbar}t);$$

$$\frac{u_1}{u_0} = \mp \mathrm{i}\frac{p_1}{(W/c + m_0 c)}. \qquad \frac{u_2}{u_3} = \pm \mathrm{i}\frac{p_1}{(W/c + m_0 c)}. \tag{A23}$$

This allows equating $u_1 = u_3$ and $u_0 = u_2$ and subsequent combining to the wave function set $\psi(\psi_a, \psi_b)$, where

$$\psi_a = u_a \cos(\frac{p_1}{\hbar}x - \frac{W}{\hbar}t);$$

$$\psi_b = u_b \sin(\frac{p_1}{\hbar}x - \frac{W}{\hbar}t);$$

$$\frac{u_a}{u_b} = \frac{p_1}{(W/c + m_0 c)}. \tag{A24}$$

Summarizing: A Majorana particle, free moving in space in a Carthesian frame of coordinates along the motion direction, has a real wave function with two orthogonal components, whereas a free moving Dirac particle that is similarly aligned, has a complex wave function with two complex components. The Majorana mode is obtained from modifying Dirac's heuristic derivation, by a slight different composition of the Einsteinean energy and by modifying the real valued Pauli-matrices into imaginary ones.

The spin and the isospin of Dirac particles and Majorana particles

After having compared the wave functions of Dirac particles and Majorana particles, we wish to analyze their spins. If we write Dirac's decomposition of the Einsteinean energy as (4a), we may write Majorana's decomposition as,

$$E_W^2 = (\beta + \mathrm{i}\bar{\alpha} \cdot \boldsymbol{p}')(\beta - \mathrm{i}\bar{\alpha} \cdot \boldsymbol{p}') . \tag{A25}$$

Because of the relationships as developed by (A5) and (A6), we may capture both cases as,

$$E_W^2 = 1 + (\overline{\alpha} \cdot \boldsymbol{p}')(\overline{\alpha} \cdot \boldsymbol{p}') . \qquad \text{(A26)}$$

To proceed, we shall make use of a particular property of Pauli matrices, which states [16]

$$(\overline{\alpha} \cdot \mathbf{v})(\cdot\overline{\alpha} \cdot \mathbf{w}) = \mathbf{v} \cdot \mathbf{w} + |\mathbf{v} \times \mathbf{w}| . \qquad \text{(A27)}$$

Hence, from (A25)and (A26),

$$(\overline{\alpha} \cdot \boldsymbol{p}')(\overline{\alpha} \cdot \boldsymbol{p}') + 1 = \boldsymbol{p}' \cdot \boldsymbol{p}' + |\boldsymbol{p}' \times \boldsymbol{p}'| + 1 . \qquad \text{(A28)}$$

This might seem a trivial result, because the vector product of a vector with itself is zero. Hence, this is just a retrieval of the Einsteinean energy expression (A1). However, under influence of the presence of a conservative field forces, characterized by a (normalized) vector potential A', the expression changes under the change of momenta components,

$$\boldsymbol{p}' \to \boldsymbol{p}' + A' . \qquad \text{(A29)}$$

such that (A27) transforms to,

$$\boldsymbol{p}' \cdot \boldsymbol{p}' + |\boldsymbol{p}' \times \boldsymbol{p}'| + \to (\boldsymbol{p}' + A') \cdot (\boldsymbol{p}' + A') + |(\boldsymbol{p}' + A') \times (\boldsymbol{p}' + A')| \qquad \text{(A30)}$$

The vector product in this expression still seems being irrelevant, because of its zero value. This, however, changes after the quantum mechanical transform from momenta to operations on a wave function, defined by

$$p'_\mu \to \hat{p}_\mu \psi \quad \text{with} \quad \hat{p}'_\mu = \frac{1}{m_0 c} \frac{\hbar}{\mathrm{i}} \frac{\partial}{\partial x_\mu} . \qquad \text{(A31)}$$

Applying these transforms on the generic identity

$$(\mathbf{v} + \mathbf{w}) \times (\mathbf{v} + \mathbf{w}) = (\mathbf{v} \times \mathbf{x}) + (\mathbf{w} \times \mathbf{v})$$

we have

$$(\boldsymbol{p}' + \boldsymbol{A}') \times (\boldsymbol{p}' + \boldsymbol{A}') \to (\hat{\boldsymbol{p}}' \times \boldsymbol{A}')\psi + (\boldsymbol{A}' \times \hat{\boldsymbol{p}}')\psi . \qquad \text{(A32)}$$

Where the operator in the first term operates on ψ as well as on $\boldsymbol{A}'$, the operator in the second term only operates on ψ. As a consequence (A32) is evaluated as,

$$(\boldsymbol{p}' \times \boldsymbol{A}') + (\boldsymbol{A}' \times \boldsymbol{p}') = \frac{\hbar}{i m_0 c} \psi(\nabla \times A') . \qquad \text{(A33)}$$

Where the vector product of the momentum representation is zero, the equivalent wave function representation is not. Apparently, the expression (A1) of the Einsteinean energy under influence of spin changes via (A30) as,

$$(\boldsymbol{p}'+\boldsymbol{A}')(\boldsymbol{p}'+\boldsymbol{A}')+1\pm\left|\frac{\hbar}{\mathrm{i}m_0c}(\nabla\times\boldsymbol{A}')\right|=0\,. \tag{A34}$$

Generically, the fourvector potential $\boldsymbol{A}$ consists of a scalar component Φ next to a vector component $\mathbf{A}$. In spite of the particle's motion in one spatial direction, we shall suppose first that, next to a zero component A_x, the vectorial component has a zero valued transversal component A_y. Moreover, we suppose that the scalar component is orthogonal to the motion, i.e. independent of x. Hence

$$\boldsymbol{p}'=\boldsymbol{p}'(p_0',p_1')\,;\quad \boldsymbol{A}'=\boldsymbol{A}'(\mathrm{i}\frac{\Phi/c}{m_0c},0,0)\,. \tag{A35}$$

Note: The i factor in the scalar component is due to the (Hawking) metric choice (+,+,+,+) / (ict,x,y,z). It can be easily seen from the Lorenz gauge

$$\nabla\cdot\mathbf{A}+\frac{1}{c^2}\frac{\partial\Phi}{\partial t}=0\rightarrow\nabla\cdot\mathbf{A}+\mathrm{i}\frac{\partial\Phi/c}{\partial\mathrm{i}ct}=0\,. \tag{A36}$$

Note also that Φ/m_0c^2 is a dimensionless quantity. Under consideration of (A35), we have

$$\nabla\times\boldsymbol{A}'=\begin{bmatrix}\mathbf{e}_\mathrm{t} & \mathbf{e}_\mathrm{x} & \mathbf{e}_\mathrm{y}\\ \partial/\mathrm{i}ct & \partial/\partial x & \partial/\partial y\\ \mathrm{i}\Phi/m_0c^2 & 0 & 0\end{bmatrix}=-\frac{\partial}{\partial y}\mathrm{i}\frac{\Phi}{m_0c^2}\mathbf{e}_\mathrm{x} \tag{A37}$$

where $\mathbf{e}_\mathrm{x}$,$\mathbf{e}_\mathrm{y}$ and $\mathbf{e}_\mathrm{t}$, respectively , are unit vectors along the two spatial axes and the temporal axis. and the spatial axis. In this operation (A37) the virtue of the two-dimensional modeling becomes clear, because it allows including the temporal axis into the curl operation on the vector potential. Under consideration of (A37), the Einsteinean energy expression (A34) evolves as,

$$(p_0'+\frac{\mathrm{i}\Phi}{m_0c^2})^2+p_1'^2\pm\frac{\hbar}{m_0c}\frac{\partial}{\partial y}\frac{\Phi}{m_0c^2}+1=0\,. \tag{A38}$$

After denormalization,

$$(\frac{p_0}{m_0c}+\frac{\mathrm{i}\Phi}{m_0c^2})^2+(\frac{m_0v}{m_0c})^2\pm\frac{\hbar}{m_0c}\frac{\partial}{\partial y}\frac{\Phi}{m_0c^2}+1=0\,. \tag{A39}$$

Hence, under consideration of (2),

$$(\frac{E_W}{m_0c^2})^2 = -(\frac{p_0}{m_0c} + \frac{\mathrm{i}\Phi}{m_0c^2})^2 = (\frac{m_0v}{m_0c})^2 \pm \frac{\hbar}{m_0c}\frac{\partial}{\partial y}\frac{\Phi}{m_0c^2} + 1. \qquad \text{(A40)}$$

Note that p_0 is imaginary, like can be concluded from (A2). Hence, after squaring the left hand term of (A40) is a real quantity. The term in conjunction with p_0 represents the change of the temporal momentum that would show up under influence of a potential field energy Φ if present. Supposing that the first two terms in the most right-hand part are much smaller than 1,

$$E_W \approx (m_0c^2)(1 + \frac{1}{2}\frac{v^2}{c^2} \pm \frac{\hbar}{2c}\frac{1}{m_0}\frac{\partial}{\partial y}\frac{\Phi}{m_0c^2}) = (m_0c^2)(1 + \frac{1}{2}\frac{v^2}{c^2}) \pm \frac{\hbar}{2c}\frac{1}{m_0}\frac{\partial}{\partial y}\Phi. \qquad \text{(A41)}$$

The small value condition to move from (A40) to (A41) is not a prerequisite. It is adopted here to reveal the physical interpretation of $\hbar/2c$ as a virtual dipole moment $\hbar/2c$, because eventually, in the static condition, the torque disappears and the condition is true for tiny mass as well. The +/- term is the result of the quantum mechanical analysis enabled by the operator view. It is absent in the case that spin would have been ignored. It shows an additional amount of energy not present in classical mechanics. In the next paragraph we shall discuss it in more detail.

Physical interpretation of spin and isospin

Assigning a meaningful interpretation to the spin phenomenon requires an identification of the physical nature of the vector potential of the field of forces and on the coupling of the particles to such a field. In that respect, the behavior of a Dirac type charged particle (electron), is not much different from a quark. Let us first consider the electron.

Electrons

Let us consider E_S as the additional amount of energy due to spin as shown by (A41). It amounts to

$$E_S = \frac{\hbar}{2c}\frac{1}{m_0}\frac{\partial}{\partial y}\Phi. \qquad \text{(A42)}$$

We may interpret the potential Φ by the force equity F as

$$F = q\frac{\partial}{\partial y}\Phi_e = \frac{\partial}{\partial y}\Phi \to \Phi_e = \frac{\Phi}{q}, \qquad \text{(A43)}$$

where q is the electric charge of an electrical particle under consideration. Hence, from (A42) and A(43),

$$E_S = \frac{\hbar}{2c}\frac{q}{m_0}\frac{\partial}{\partial y}\Phi_e. \qquad \text{(A44)}$$

It is an additional amount of energy that would be executed by a torque force if a field would be present with a component perpendicular to the direction of the motion. It reveals a hidden electric moment μ_{el} to the amount of

$$\mu_{el} = \frac{\hbar}{2c}\frac{q}{m_0} \text{ } (\approx 3.09\ 10^{-32} \text{ C m for an electron}). \tag{A45}$$

To date no experiment has found a non-zero electric moment for the electric moment. The Particle Data Group states that, if it would exist, its value is $\mu_{el} < 0.87\ 10^{-30}$ C m. Curiously, in his classic paper on electrons, Dirac has identified an electric moment for electrons. However, he has waived it away, because in his calculation it showed up as an imaginary quantity. To support the conclusion that the electric moment of electrons is as real as the magnetic moment is, it is useful to include time-independent vector components in the vector potential. To do so, we expand (A37) to

$$\nabla \times \boldsymbol{A}' = \begin{bmatrix} \mathbf{e}_t & \mathbf{e}_x & \mathbf{e}_y \\ \partial / ict & \partial/\partial x & \partial/\partial y \\ \mathrm{i}\Phi / m_0 c^2 & A'_x & A'_y \end{bmatrix} = -\frac{\partial}{\partial y}\mathrm{i}\frac{\Phi}{m_0 c^2}\mathbf{e}_x + (\frac{\partial}{\partial x}A'_y - \frac{\partial}{\partial y}A'_x)\mathbf{e}_t \tag{A46}$$

Because of $\mathbf{B} = \nabla \times \mathbf{A}$, we have

$$\frac{\partial}{\partial x}A'_y - \frac{\partial}{\partial y}A'_x = B'_z . \tag{A47}$$

Because $\mathbf{e}_t$ is a unit vector along the imaginary axis, both contributions in (A46) are imaginary, thereby making a real contribution in (A34). Including this result into (A41) gives,

$$E_W \approx (m_0 c^2)(1 + \frac{1}{2}\frac{v^2}{c^2} \pm \frac{\hbar}{2c}\frac{1}{m_0}\frac{\partial}{\partial y}\frac{\Phi}{m_0 c^2} \mp \frac{\hbar}{2c}\frac{1}{m_0}\frac{B_z}{m_0 c}) \rightarrow$$
$$E_W \approx (m_0 c^2)(1 + \frac{1}{2}\frac{v^2}{c^2}) \pm \frac{\hbar}{2c}\frac{1}{m_0}\frac{\partial}{\partial y}\Phi \mp \frac{\hbar}{2}\frac{1}{m_0}B_z . \tag{A48}$$

Taking (43) into consideration it has to be noted that similarly as Φ is not the same as Φ_e, B is not the same as B_e. The difference is a factor q. Hence, apart from the electric moment μ_{el}, the electron contains a spin magnetic moment μ_m to the amount of

$$\mu_m = q\frac{\hbar}{2m_0} \approx (9.27\ 10^{-24} \text{ C m}^2 \text{ s}^{-1}), \tag{A49}$$

which is known as the Bohr magneton.

(Note: $\mu_m = g_e \frac{q}{2m_0} S$; $g_e = 2$; $S = \frac{\hbar}{2}$, where g_e is known as the gyromagnetic ratio).

The conclusion is that electrons show a magnetic dipole moment as well as an electric dipole moment. The latter one is too weak to be detected. Denoting the spin magnetic dipole moment for short as *spin*, it makes sense to denote the spin electric dipole moment as *isospin*. This view has been developed before by David Hestenes [..].

Let us proceed by considering the field potential. First, from the linear momentum. Second, from the angular momentum. As can be expected, the most simple expression for the field from the linear momentum will show up along the alignment axis of the dipole. Under consideration of a generic dipole moment $q_p d$, this potential field can be readily derived as,

$$\Phi_N(r) = \frac{q_p d}{4\pi\varepsilon_0 r^2} \rightarrow = \Phi_N(r) = \frac{q\hbar}{2mc}\frac{1}{4\pi\varepsilon_0 r^2}. \tag{A50}$$

Note that the charge q_p of the fictitious pole in the elementary dipole is different from the effective charge q of the electron. It is no more than just an auxiliary quantity. Something similar holds for the elementary angular moment $q\hbar/2m$. It can be viewed as a monopole q_p circulating with light speed at a distance d from a virtual center. Interpreting the angular momentum as a rotation of charge q_p with light speed at a fictitious radius $r_0 = 1/g_m\lambda$, we have

$$\frac{q}{m_0}\frac{\hbar}{2} = \frac{q_p c}{g_{m\lambda}\lambda} \rightarrow q_p = g_m \frac{q}{m_0}\frac{\hbar}{2}\frac{\lambda}{c}. \tag{A51}$$

The factor g_m can be interpreted as a gyromagnetic factor.

Hence,

$$\Phi_F(r) = \frac{q_p}{4\pi\varepsilon_0 r} = g_m \frac{q}{m_0}\frac{\hbar}{2}\frac{\lambda}{c}\frac{1}{4\pi\varepsilon_0 r} = \frac{g_m}{4\pi\varepsilon_0}\frac{q}{m_0}\frac{\hbar}{2}\frac{\lambda^2}{c}\frac{1}{\lambda r}, \tag{A52}$$

Along the alignment axis, the total potential field $\Phi_e(r)$ of the electron can be written as,

$$\Phi_e(r) = \Phi_F(r) + \Phi_N(r) = \frac{q\lambda^2}{4\pi\varepsilon_0}\frac{\hbar}{2cm_0}\left(\frac{1}{\lambda^2 r^2} - g_m \frac{1}{\lambda r}\right), \tag{A53}$$

where it is taken for granted that the electric forces from near field and far field have opposite signs.

Quarks

The difference between the quark case and the electron case is a consequence of a difference in the force equity expression (A43). Because the antiquark couples with the generic quantum mechanical coupling factor to the field of a quark, (43) changes simply into

$$F = g\frac{\partial}{\partial y}\Phi_{qu} = \frac{\partial}{\partial y}\Phi \to \Phi_{qu} = \frac{\Phi}{g}. \quad \text{(A54)}$$

After that, the analysis evolves along similar lines.

Appendix B: The interactions between the spins and the isospins in a meson

The spin spin interaction between the dipole moments of the quark and the antiquark can be analyzed similarly to the interaction between the spin of the orbital electron and the spin of the proton in the hydrogen atom. This interaction is known to be the cause of the hyperfine structrure in the spectral lines of the hydrogen gas. Similarly as the intrinsic angular momentum of an electron and a proton set up a magnetic dipole field, the intrinsic angular moment of a quark sets up a nuclear equivalent. Similarly as the magnetic moment of an electron experiences a potential energy from the field of the magnetic dipole from the proton spin, the nuclear equivalent of the magnetic moment of a pion quark experiences a potential energy from the nuclear equivalent of the magnetic dipole field from the spin of the antiquark. The difference between the hydrogen model and the pion is that we have to deal not only with the angular moments, but also with the non-angular dipole moments. Let us first consider the interaction between the angular moments.

It is well known that the magnetic field of a dipole that results from a current loop with an infinitesimal small dimension, as a consequence of a magnetic moment $\boldsymbol{\mu}_1$ is given by [9,p.157],

$$\mathbf{B}(r) = \frac{\mu_0}{4\pi}\{\frac{1}{r^3}[3(\boldsymbol{\mu}_1 \cdot \hat{\mathbf{r}}) - \boldsymbol{\mu}_1] + \frac{8\pi}{3}\boldsymbol{\mu}_1\delta^3(\mathrm{r})\}, \quad \text{(B1)}$$

where $\boldsymbol{\mu}_1$ is the magnetic moment and where $\hat{\mathbf{r}}$ is the unit vector in $r-$direction. The potential energy U_2 of a second particle with a magnetic moment $\boldsymbol{\mu}_2$, placed at a distance r apart, is

$$U_2(r) = \frac{\mu_0}{4\pi}\{\frac{1}{r^3}[3(\boldsymbol{\mu}_1 \cdot \hat{\mathbf{r}})(\boldsymbol{\mu}_2 \cdot \hat{\mathbf{r}}) - \boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2] + \frac{8\pi}{3}\boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2\delta^3(\mathbf{r})\}. \quad \text{(B2)}$$

In classical conditions, the second right-hand part in this expression does not play any role. However, if the two particles are of a quantum mechanical nature, therefore requiring a distributed wave function description, this is no longer true. As proven by Griffiths [9], it is in fact the only part that effectively contributes. The first right-hand part is angular dependent, implying a dependence of the mutual orientation of the two spins. Therefore, on the average, the contribution of this first part cancels out, while the magnitude of the second part is made up by the integral of all contributions within the sphere of overlap of the two wave functions. In such condition, the potential energy of any of the two particles resulting from interaction with the field of the other, as calculated from (B2), reduces to the constant contribution,

$$U_{1,2} = \frac{2\mu_0}{3}\frac{(\boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2)}{\pi d_0^3}. \quad \text{(B3)}$$

The formula applies to the positronium model, where a particle moves around an identical one at a distance 2 d_0, thereby making a circle with radius d_0 around the center of mass. In the hydrogen case, this result corresponds with the presence of the 21 cm line in its spectrum [9]. The magnetic moments amount to [9],

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$$\boldsymbol{\mu}_i = \frac{\gamma_s q_e}{2m_i}\boldsymbol{\sigma}_i, \tag{B4}$$

where γ_s is the gyromagnetic ratio $\gamma_s \approx 2$ and where $\boldsymbol{\sigma}_i$ are the spin vectors of the dipole. Hence, from (B3) and (B4),

$$U_{1,2} = \mu_0 \frac{2\gamma_s^2 q_e^2}{3m_1 m_2}\frac{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)}{\pi d_0^3}. \tag{B5}$$

The spins will align themselves in parallel or in anti-parallel, which gives, respectively,

$$\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 = \frac{\hbar^2}{4} \quad \text{and} \quad \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 = -\frac{3\hbar^2}{4} \tag{B6}$$

Hence, considering the case of parallel alignment, we have

$$U_{1,2} = \mu_0 \frac{\gamma_s^2 q_e^2}{6m_1 m_2}\frac{\hbar^2}{\pi d_0^3}. \tag{B7}$$

Let us now consider the nuclear case with the two dipole moments. The angular one is the classical one: a circular current creates the nuclear equivalent of a magnetic field, on which the other angular dipole moment interacts with the nuclear coupling factor. The non-angular one creates a different field, in fact an electric one, on which the second non-angular dipole moment interacts with the electric coupling factor. Hence, as a consequence from spin vectors $\boldsymbol{\sigma}_i$ there are two dipole-induced energetic contributions. How to derive the nuclear equivalent of (B7) ? We know that the electromagnetic force F_e and the (vectorial) far field force F_n between two charged quarks, spaced at distance $2d_0$, can be established from,

$$F(d_0 + r) + F(d_0 - r), \tag{B8}$$

where $F(r)$ is respectively given as $F(r) = F_e(r)$ and $F(r) = F_n(r)$, such that

$$F_e = -q_e \frac{\partial}{\partial r}\frac{q_e}{4\pi\varepsilon_0 r} \quad \text{and} \quad F_n = -g\frac{\partial}{\partial r}\Phi_0 \frac{\exp(-\lambda r)}{\lambda r}. \tag{B9}$$

There is no reason why these forces would be the same. What is clear, however, is, that $g\Phi_0/\lambda$ plays a similar role as $q_e^2/(4\pi\varepsilon_0)$, i.e.,

$$\frac{q_e^2}{4\pi\varepsilon_0} \leftrightarrow \frac{g\Phi_0}{\lambda}. \tag{B10}$$

Furthermore, as noted before, previous studies have revealed a nuclear equivalent for the electromagnetic fine structure relationship [15]. Where,

$$q_e^2 = 4\pi\varepsilon_0 g^2 \hbar c\,, \quad \text{we have} \quad \frac{\Phi_0}{\lambda} = \frac{\alpha\pi\hbar c}{2gd'_{\min}}\,. \tag{B11}$$

Expressions (B7) and (B11) enable establishing the potential energy under parallel spin as

$$U_a = \mu_0 \frac{\gamma_s^2 4\pi\varepsilon_0}{6m_1m_2}\frac{q_e^2}{4\pi\varepsilon_0}\frac{\hbar^2}{\pi d_0^3} \rightarrow \frac{4\pi\gamma_s^2}{6m_1m_2c^2}\frac{g\Phi_0}{\lambda}\frac{\hbar^2}{\pi d_0^3} \rightarrow \frac{1}{3}\frac{\alpha\pi\gamma_s^2(\hbar c)^3}{d'_{\min}m'_1m'_2d_0^3}\,. \tag{B12}$$

The second one is due to the nuclear equivalent of an electric dipole moment, to the amount of

$$\boldsymbol{\mu}_i = \frac{\gamma_s q_e}{2m_i c}\boldsymbol{\sigma}_i\,, \tag{B13}$$

Hence, from (B3) and (B13),

$$U_b = \frac{q_e^2}{\varepsilon_0 c^2}\frac{\gamma_s^2}{6m_1m_2}\frac{\hbar^2}{\pi d_0^3} = \frac{q_e^2}{4\pi\varepsilon_0 c^2}\frac{4\pi\gamma_s^2}{6m_1m_2}\frac{\hbar^2}{\pi d_0^3} = \frac{q_e^2}{4\pi\varepsilon_0\hbar c}\frac{\hbar}{c}\frac{4\pi\gamma_s^2}{6m_1m_2}\frac{\hbar^2}{\pi d_0^3}$$
$$= g^2\frac{2\gamma_s^2}{3m'_1m'_2}\frac{(\hbar c)^3}{d_0^3}\,. \tag{B14}$$

Hence, from (B12) and (B13), the ratio of the non-angular spin-spin interaction over the angular spin-spin interaction, is calculated as,

$$\frac{U_b}{U_a} = g^2\left(\frac{2d'_{\min}}{\alpha\pi}\right).. \tag{B15}$$

Taking into account that $d'_{\min} \approx 0.856$ and $\alpha \approx 0.69$, [15], this ratio is near to $g^2 \approx 1/137$ as predicted in the third paragraph of the main text, albeit that the fit Is not perfect.

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