Statistical Mechanics-based Schrödinger treatment of Gravity

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Abstract

The entropic gravity conception proposes that what has been traditionally interpreted as unobserved dark matter might be merely the product of quantum effects. These effects would produce a novel sort of positive energy that translates into dark matter via $E = mc^2$. In the case of axions, this perspective as been shown to yield quite sensible, encouraging results [DOI:10.13140/RG.2.2.17894.88641] . There, a simple Schrödinger mechanism was utilized, in which his celebrated equation is solved with a potential function based on the microscopic Verlinde's entropic force advanced in [Physica A **511** (2018) 139]. In this effort we revisit such technique with regards to fermions' behavior (specifically, baryons).

KEYWORDS: Fermions, baryons, Emergent entropic force, Schrödringer equation, Gravitation.

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1 Introduction

1.1 Emergent entropy

In 2011 Verlinde [1] conceived the notion of of linking gravity to an entropic force. This idea was later proved valid [2], in a classical setting.

In [1], gravity crops up as a result of information concerning material bodies' positions, joining a thermal treatment of gravitation with 't Hooft's holographic principle. In such terms, gravitation ought to be regarded as an emergent phenomenon. These Verlinde's ideas were given great attention. For example, look at [3, 4]. An excellent review of the statistical mechanics of gravity was given by Padmanabhan [5].

Verlinde's conceptions motivated works on cosmology, the dark energy hypothesis, cosmological acceleration, cosmological inflation, and loop quantum gravity. The corresponding literature is very ample [4]. We like to cite Guseo's work [6], who showed that the local entropy function, linked to a logistic distribution, turns up to be catenary and vice versa. This is an invariance that can be connected to the Verlindes conjecture mentioned above. Guseo advances an original interpretation of the local entropy in a system [6].

1.2 Our goals in using Schrödinger's equation (SE)

Verlinde depicts gravity as an emergent phenomenon that originates in the quantum entanglement between small bits of space-time information [7]. Gravitation, viewed á la Verlinde as an emergent force, deviates at very short distances from Newton's form. The ensuing new gravitation-potential, if introduced now into Schrödinger equation (SE), should yield quantified states, and the associated energies would constitute a novel energy-source, not contemplated till now, save for our precedent axion treatment of Ref. [8]. Here we will proceed on the basis of a previous analysis [9] that uses the statistical treatment of quantum fermion gases. We effected in [9] the process described above and found a fermion-fermion gravitation force therefrom (here specifically, baryon-baryon). It turned out to be, as expected, proportional to $1/r^2$ for distances larger than one micron, but for smaller ones novel, more involved contributions arose. Accordingly, the ensuing potential $V(r)$ differed from the Newtonian one at short distances. We will now write down

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below the SE for *such* $V(r)$ and solve it, expecting to find new unknown till now quantum gravitational states for baryons.

1.3 Organizing our material

In Section 2 we review relevant details of [9]. We suitably approximate $V(r)$ so as to proceed in analytic fashion and set $V(r) = \sum_{i=1}^{3} V_i(r)$. Our central discussion is given in Section 3. There one solves our ensuing Schrdinger's equation for the baryon-baryon, Verlinde-like gravitation potential in separate fashion for each of its pieces. The piece V_1 becomes protagonist and yields our most important new findings. In order to illustrate on our problem's attack, we probe in Section 4 a perhaps daring conjecture concerning dark matter. Rough numerical estimates can be obtained. We end with some conclusions in Section 5.

2 Quantum gravitational potential $E_P(r)$ to be introduced in the SE

2.1 The gravitational potential function for N baryons of mass m

It was first derived in [9], where the following constants were introduced:

- a and b in the fashion
- $a = (3N)^{\frac{5}{2}} h^3$ and
- $b = 32\pi(\pi e m K)^{\frac{3}{2}}$, with a total baryons energy K
- $K = 10^{53} c^2$ Joules [10].

One ascertains in [9] that $\frac{\lambda 3 N k_B}{8\pi} = \frac{2}{3} GmM$, and the potential energy $E_P(r)$ acquires the form

$$E_P(r) = -GmM\frac{2b}{3a}\left\{\frac{r^2}{2}\ln\left(1-\frac{a}{br^3}\right) - \frac{a^{\frac{2}{3}}}{2b^{\frac{2}{3}}}\left\{\frac{1}{2}\ln\left[\frac{\left[r-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right]^2}{r^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}r+\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right] + \right.\right.$$

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$$\sqrt{3}\left[\arctan\left[\frac{2r+\left(\frac{a}{b}\right)^{\frac{1}{3}}}{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right]-\frac{\pi}{2}\right]\Bigg\}\Bigg\} \tag{2.1}$$

a critical result for us.

2.2 A Taylor approximation (TA) for $V(r)$

One can not analytically tackle the SE with such an awful E_P. Thus, we are forced to piece-wise approximate $V(r)$ in four different regions: $0 < r < r_0$, $r_0 < r < r_1$, $r_1 < r < r_2$ and $r > r_2$. r_1 will be made explicit below. r_0 is conjectured by us as being 10^{-10} centimeters, and $r_2 = (a/b)^{\frac{1}{3}}$

$$V(r) \approx V_1(r) + V_2(r) + V_3(r). + V_4(r) \tag{2.2}$$

For convenience we define

$$V_0 = -GmM\left(\frac{b}{a}\right)^{\frac{1}{3}}\frac{7\pi}{6\sqrt{3}}, \tag{2.3}$$

and call V_1 the TA, at zeroth order, for very small r. H stands for the Heaviside function.

$$V_1(r) = -GmM\left(\frac{b}{a}\right)^{\frac{1}{3}}\frac{7\pi}{6\sqrt{3}}H(r_0 - r) = V_0 H(r_0 - r) \tag{2.4}$$

For large r the pertinent approximation has been obtained in [9]:

$$V_3(r) = -\frac{GmM}{r}[H(r - r_1) - H(r - r_2)] \tag{2.5}$$

For intermediate $r-$values, $r_0 < r < r_1$ (There is experimental evidence to choose $r_1 = 25$ micrometers [11]). We call $W(r)$ the harmonic interpolating-form between the two fixed distance values $r_1 - r_0$. Thus,

$$V_2(r) = W(r) \tag{2.6}$$

For $V_4(r)$ we have

$$V_4(r) = \frac{2GmM}{3r}H(r - r_2) \tag{2.7}$$

We exactly solve below the SE for these four potentials.

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3 Exact solution of the SE

We deal with the (complete) SE and call m_r the reduced mass. One faces

$$U''(r) + \left[-\frac{l(l+1)}{r^2} - \frac{2m_r}{\hbar^2}V(r) + \frac{2m_r}{\hbar^2}E\right]U(r) = 0, \tag{3.1}$$

and analytically solve it piece-wise.

3.1 V_1's exact treatment

Let ϕ be the confluent hyper-geometric function [12]. For V_1 one has, for $E > V_0$ and (a definition to be used below) $s = \sqrt{\frac{8m_r(E-V_0)}{\hbar^2}}r$:

$$U_1''(r) + \left[-\frac{l(l+1)}{r^2} + \frac{2m_r}{\hbar^2}(E - V_0)\right]U_1(r) = 0, \tag{3.2}$$

whose solution is (A and B are two arbitrary constants)

$$U_{l1}(r) = A(-is)^{l+1}e^{-is}\phi\left(l+1, 2l+2; -is\right) - B(is)^{l+1}e^{is}\phi\left(l+1, 2l+2; is\right) \tag{3.3}$$

Thus, the radial solution $R_{l1}(r)$ adopts the appearance

$$R_{l1}(r) = A(-is)^{l+1}\frac{e^{-is}}{r}\phi\left(l+1, 2l+2; -is\right) - B(is)^{l+1}\frac{e^{is}}{r}\phi\left(l+1, 2l+2; is\right), \tag{3.4}$$

and, appealing to [12] and calling $\mathcal{J}$ to the Bessel function [12]

$$\phi(l+1, 2l+2; -is) = 2^{2l+1}e^{-i\pi(l+\frac{1}{2})}\Gamma\left(l+\frac{3}{2}\right)s^{-(l+\frac{1}{2})}e^{-i\frac{s}{2}}\mathcal{J}_{l+\frac{1}{2}}\left(\frac{s}{2}\right), \tag{3.5}$$

so that

$$R_{l1}(r) = 2^{2l+1}\Gamma\left(l+\frac{3}{2}\right)\frac{s^{-\frac{1}{2}]}}{r}\left(Be^{\frac{3\pi il}{2}}e^{3is2} - Ae^{-\frac{3\pi il}{2}}e^{-3is2}\right)\mathcal{J}_{l+\frac{1}{2}}\left(\frac{s}{2}\right). \tag{3.6}$$

Boundary conditions (BC). R_l must satisfy $R_l(r_0) = 0$ and $R_l'(r_0) = 0$. The two BE become now

$$\mathcal{J}_{l+\frac{1}{2}}\left(\frac{s_0}{2}\right) = 0, \tag{3.7}$$

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entailing that $s_0/2$ must be a zero of the Bessel function. This zero is denoted by $\chi_{l,n}$ [12].

$$s_0 = 2\chi_{l,n} \tag{3.8}$$

Energy is duly quantified and reads

$$E_{l,n} = \frac{\hbar^2}{2m_r}\frac{\chi_{l,n}^2}{r_0^2} + V_0 \tag{3.9}$$

This energy is something new in the baryonic stage, discovered right here. We will particularly interested below in the ground state energy $E_{l=0,\,n=1}$.

3.2 V_2's exact treatment

We have

$$U_2''(r) + \left[-\frac{l(l+1)}{r^2} + \frac{2m_r}{\hbar^2}[E - W(r)]\right] U_2(r) = 0, \tag{3.10}$$

Four operating BC are operative here: $R_{l2}(r_0) = 0$, $R_{l2}'(r_0) = 0$, $R_{l2}(r_1) = 0$, and $R_{l2}'(r_1) = 0$, and we can only satisfy three of them. Accordingly, the only solution is $R_{l2}(r) = 0$.

3.3 V_3's exact treatment

We face

$$U_3''(r) + \left[-\frac{l(l+1)}{r^2} + \frac{2m_r}{\hbar^2}\left(E + \frac{GmM}{r}\right)\right] U_3(r) = 0. \tag{3.11}$$

It is of help here to remember that Whitaker's function W solves the related differential equation

$$W'' + \left(-\frac{1}{4} + \frac{\lambda}{z} + \frac{\frac{1}{4} - \mu^2}{z^2}\right) W = 0. \tag{3.12}$$

Choose $E < 0$

Defining $\mu = l + \frac{1}{2}$, $\lambda = \frac{GmM}{\hbar}\sqrt{\frac{m_r}{2|E|}}$, it is clear that $s = \sqrt{\frac{8m_r|E|}{\hbar^2}}r$ for solving (3.11) one can write (A and B are arbitrary constants)

$$U_3(r) = AW_{\lambda,\mu}(s) - BW_{-\lambda,\mu}(-s), \tag{3.13}$$

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where $W_{\lambda,\mu}(z)$ is given by

$$W_{\lambda,\mu}(z) = \frac{(-1)^{2\mu} z^{\mu+\frac{1}{2}} e^{-\frac{z}{2}}}{\Gamma\left(\frac{1}{2}-\mu-\lambda\right)\Gamma\left(\frac{1}{2}+\mu-\lambda\right)} \left\{\sum_{k=0}^{\infty} \frac{\Gamma\left(k+\mu-\lambda+\frac{1}{2}\right)}{k!(2\mu+k)!} \otimes \right.$$

$$\left[\psi(k+1)+\psi(2\mu+k+1)-\psi\left(\mu+k-\lambda+\frac{1}{2}\right)-\ln z\right]+$$

$$(-z)^{-2\mu}\sum_{k=0}^{2\mu-1} \frac{\Gamma(2\mu-k)\,\Gamma\left(k-\mu-\lambda+\frac{1}{2}\right)}{k!}(-z)^k. \tag{3.14}$$

Here $2\mu+1$ is a natural number. The last sum vanishes for $\mu=0$. Accordingly,

$$R_{l3}(r) = r^{-1}[AW_{\lambda,\mu}(s) - BW_{-\lambda,\mu}(-s)]. \tag{3.15}$$

The operating BC here are $R_{l3}(r_1) = R'_{l3}(r_1) = 0$. They can be translated into

$$W'_{\lambda,\mu}(s_1) + \frac{W_{\lambda,\mu}(s_1)}{W_{\lambda,\mu}(-s_1)} W'_{-\lambda,\mu}(-s_1) = 0. \tag{3.16}$$

Let $\sigma_{l,n}$ be the zeroes of such an equation. Then,

$$s_1 = \sigma_{l,n}, \tag{3.17}$$

and the energy becomes quantized, as one should expect

$$E_{l,n} = -\frac{\hbar^2}{8m_r}\frac{\sigma_{l,n}^2}{r_1^2}. \tag{3.18}$$

Choose $E > 0$

We have $\mu = l+\frac{1}{2}$, $\lambda = -i\frac{GmM}{\hbar}\sqrt{\frac{m_r}{2E}}$ $s = \sqrt{\frac{8m_rE}{\hbar^2}}r$. Now, the solution becomes

$$U_3(r) = AW_{\lambda,\mu}(-is) - BW_{-\lambda,\mu}(is), \tag{3.19}$$

and then

$$R_{l3}(r) = r^{-1}[AW_{\lambda,\mu}(-is) - BW_{-\lambda,\mu}(is)]. \tag{3.20}$$

The operating BC are, once again, $R_{l3}(r_1) = R'_{l3}(r_1) = 0$, that translate into

$$W'_{\lambda,\mu}(-is_1) + \frac{W_{\lambda,\mu}(-is_1)}{W_{\lambda,\mu}(is_1)} W'_{-\lambda,\mu}(is_1) = 0. \tag{3.21}$$

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Denote by $\varsigma_{l,n}$ the zeroes of the above equation:

$$s_1 = \varsigma_{l,1}. \tag{3.22}$$

Energy becomes quantized again and the quantized eigenvalues become

$$E_{l,n} = \frac{\hbar^2}{8m_r}\frac{\varsigma_{l,n}^2}{r_1^2}. \tag{3.23}$$

In the two cases considered in the present Subsection, the separation between quantum energy levels is of the order of 10^{-17} Joules, entailing that one is facing a continuum-energy, as should be expected.

3.4 V_4's exact treatment

We face

$$U_4''(r) + \left[-\frac{l(l+1)}{r^2} + \frac{2m_r}{\hbar^2}\left(E + \frac{2GmM}{3r}\right)\right]U_4(r) = 0. \tag{3.24}$$

We remember again that Whitaker's function W solves the related differential equation

$$W'' + \left(-\frac{1}{4} + \frac{\lambda}{z} + \frac{\frac{1}{4} - \mu^2}{z^2}\right)W = 0. \tag{3.25}$$

Choose $E < 0$

Defining $\mu = l + \frac{1}{2}$, $\lambda = \frac{2GmM}{3\hbar}\sqrt{\frac{m_r}{2|E|}}$, it is clear that $s = \sqrt{\frac{8m_r|E|}{\hbar^2}}r$ for solving (3.11) one can write (A and B are arbitrary constants)

$$U_4(r) = AW_{\lambda,\mu}(s) - BW_{-\lambda,\mu}(-s), \tag{3.26}$$

And therefore

$$R_{l4}(r) = r^{-1}[AW_{\lambda,\mu}(s) - BW_{-\lambda,\mu}(-s)], \tag{3.27}$$

R_{l4} should verify $R_{l4}(r_2) = R_{l3}(r_2)$ and $R_{l4}'(r_2) = R_{l3}'(r_2)$
Observe that the energy is not quantized in this case

Choose $E > 0$

We have $\mu = l + \frac{1}{2}$, $\lambda = -i\frac{2GmM}{3\hbar}\sqrt{\frac{m_r}{2E}}$ $s = \sqrt{\frac{8m_rE}{\hbar^2}}r$. Now, the solution becomes

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$$U_4(r) = AW_{\lambda,\mu}(-is) - BW_{-\lambda,\mu}(is), \tag{3.28}$$

and then

$$R_{l4}(r) = r^{-1}[AW_{\lambda,\mu}(-is) - BW_{-\lambda,\mu}(is)]. \tag{3.29}$$

The operating BC are, once again, $R_{l4}(r_2) = R_{l3}(r_2)$ and $R'_{l4}(r_2) = R'_{l3}(r_2)$ The energy is not quantized again.

4 Interesting numerical baryonic assessment

Refer now to Eqs. (3.9) and (2.3), revisiting also the various definitions made at the start of Subsection (2.1). A nucleon's mass is $\sim 1.6 \times 10^{-27}$ Kg, from which we get m_r so as to obtain $E_{0,1} \sim 10^{-21}$ Joule and realize that $V_0 << E_{0,1}$. Since $mc^2 = 1.44 \times 10^{-10}$ Joule, we have $E_{0,0} << mc^2$. For axions the last inequality is just the opposite one (see [8]). For them $E_{0,1} >> mc^2$.

Now we can assess the total number N of baryons in the Universe as $N = K/mc^2$, with [10] $K = 10^{53} \times c^2$ Joule. The result is $N \sim 6.25 \times 10^{79}$.
$E_{0,1} \sim 10^{-21}$, and assuming that the major contribution of the baryonic-pairs of gravitationally interacting baryions comes from their ground state, we can estimate that their contribution to dark matter is $E_B \sim 2 \times 10^{77}$ eV, very small in comparison to the estimated value for dark matter of $K = 2.86 \sim 10^{84}$ eV [8]. In this last reference, though, it it see that the gravitationinteration between axions does significantly controbute to the extant amount of dark matter.

5 Discussion

We have here solved, for fermions, Schrödinger's equation (SE) for gravity. The logic on which this paper was written can be summarized as follows.

- We begun by adopting Verlinde's stance that gravity emerges from an entropy S (entropic force).
- In [9], for a gas of free fermions, we obtained 1) S, 2) Verlinde's entropic force F_e, and from it 3) gravity's potential $V(r)$. We also encountered

in [9] that $V(r)$ differs from the Newton's form at extremely short and extremely large distances..

The above potential $V(r)$ was approximated in suitable manner so as to be in a position to obtain analytical solutions to the pertinent SE for the potential $V(r)$.

The novel results of our treatment emerge at short distances (the V_1 component of $V(r)$). The ensuing low-lying SE-quantum states yield energy-eigenvalues (most importantly, the ground state $E_{0,1}$, not accounted for before. They produce, via Einstein's relation energy= mc^2, some quantity of matter, that we might identify as dark one. This Schrödinger-baryonic dark mass is insignificant, though. Baryons do not contribute to dark matter in this gravity fashion, which constitute an important result, we believe, since bosons do contribute [8]

6 Authors' contributions

Both authors worked on an equal footing in conceptualization and research.

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