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A NONLINEAR STUDY ON TIME EVOLUTION IN GHARANA TRADITION OF INDIAN CLASSICAL MUSIC

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ABSTRACT

Indian classical music is entirely based on the "Raga" structures. In Indian classical music, a "Gharana" or school refers to the adherence of a group of musicians to a particular musical style of performing a raga. The objective of this work was to find out if any characteristic acoustic cues exist which discriminates a particular gharana from the other. Another intriguing fact is if the artists of the same gharana keep their singing style unchanged over generations or evolution of music takes place like everything else in nature. In this work, we chose to study the similarities and differences in singing style of some artists from at least four consecutive generations representing four different gharanas using robust non-linear methods. For this, alap parts of a particular raga sung by all the artists were analyzed with the help of non linear multifractal analysis (MFDFA and MFDXA) technique. The spectral width obtained from the MFDFA method gives an estimate of the complexity of the signal whereas the cross correlation coefficient obtained from the MFDXA technique gives the degree of correlation between two nonlinear time series. The observations give a cue in the direction to the scientific recognition of "Guru-Shisya Parampara" (teacher-student tradition) – a hitherto much-heard philosophical term. Moreover the variation in the complexity patterns among various gharanas will give a hint of the characteristic feature of that particular gharana as well as the effect of globalization in the field of classical music happening through past few decades.

Keywords: *Guru-Shishya parampara, gharana*, Indian Classical Music, MFDFA, MFDXA

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"Evolution is nature's method of creation"... **Theodosius Dobzhansky**

1. INTRODUCTION

1.1. What is *"Gharana"* System of Indian Classical Music?

Indian classical music can be compared to the endless sky when its creative aspect is considered and *Raga* is the heart of it. Every performer of this genre is essentially a composer as well as an artist as while performing a *Raga* the way the notes are approached and rendered in musical phrases and the mood they convey are more important than the notes themselves. This leaves ample scope for improvisation within the structured framework of the *raga*. So, while rendering a particular *raga* different styles of presentation are observed across the country. The major differences in styles are named after different "*Gharanas"*. The coinage "Gharana" came from the Hindi word "Ghar" (House). It is commonly observed that the *gharanas* are named after different places, viz., Agra, Patiyala, Kirana, Gwalior, Indore etc. The naming of these *gharanas* mostly indicates towards the origination of these particular musical styles or ideologies. *Gharanas* have their basis in the traditional mode of musical training and education. Every *gharana* has its own distinct features; the main area of difference between *gharanas* being the manner in which the notes are sung. Though within a particular *gharana*, significant changes in singing styles are observed between generations of performers. These changes are ascribed to the individual creativity of that singer, which has led to the improvisation on the singing style of that particular *gharana*.

Now-a-days, the "*Gharana"* system in Hindustani classical music is affine to a number of ambiguous ideas among artists. According to a few, *gharana* system exists pretty distinctively, while some are against this thought. What defines the *gharana*? Is it really true that the artists of the same *gharana* keep their singing style unchanged over generations or evolution of music takes place like everything else in nature? These questions are still unanswered; at least from a scientific point of view. Also, the literature has very few scientific studies which attempted to analyze the significant musical styles that define a particular *gharana* (Silver & Burghardt, 1976; Wright & Wessel, 1998) or the difference in style observed between different *gharanas.* In this work, we tried to do an analytical comparison between the renditions of a *raga* sung by some artists who are said to belong to 4 different *gharanas* of Hindustani classical music.

1.2. A Brief Introduction to Different *Gharanas* of Hindustani Classical Music and *Guru-Shishya parampara*

A number of works in Western classical music tries to capture the evolution of style using different modes of music information retrieval (Weiß et.al, 2018; George, 2017; Park et.al, 2015), but this kind of evolutionary study is seldom seen in the Indian perspective. Under the Hindustani Classical Music, the tradition of "Gharana" (Brahaspati, 2002, Mahabharati, 2011) system holds special importance to many listeners. Perhaps, this feature is so unique that no where around the world can one find this sought of a tradition. The Gharana system is followed by both the North-Indian as well as the South-Indian forms of Indian classical music. In south India, the term Gharana is acknowledged by the word "Sampraya". In ancient times, there existed several Samprayas such as the "Shivmat", the "Bhramamat" and the "Bharatmat" (Pranjpay, 1992). It is believed that in ancient times, there existed a single form of the style of Indian Classical Music. However, the advent of the Muslims had a great impact on the Indian Classical Music and this created a division into this form of music. This lead to the regeneration of two forms of Indian Classical Music: the Carnatic Music (Usually this style is followed in the Southern parts of India and considered by some as the basic version of Indian Classical Music) and the Hindustani Music (The North Indian and the improvised version of the Indian Classical Music). One of the most unique and exclusive feature which is incorporated in the teaching of Indian Classical Music is the "Guru- Shishya" tradition (Hothi, 2013). Perhaps, in recent times, the education of Indian Classical Music is also imparted in several institutions, schools, colleges and universities. However, history and statistics reveal that even now the finest artists of the Indian Classical Music are produced through the "Guru-Shishya" tradition. In India, the Gharana system has contributed to all the three forms of music, that is vocal, instrumental and dance. The Gharana comes into existence through the confluence of the "Guru" and the "Shishya" (Chaubey, 1977). A wise "Guru" through his intelligence, aptitude and shear practice creates a sense of uniqueness and exclusivity and thereby inculcates a special eminence into his form of music. These attributes and traits are amicably transferred into the talented "Shishya" and the particular form of the performing arts thus becomes a tradition. These exceptional qualities are in fact so strong and prominent that the audiences can immediately recognize the Gharana i.e the similarity in singing style between two artists. It is believed that when so ever the form or style created by the founder "Guru" is carried forth till three generations; it turns in to the form of "Gharana". The name of the Gharana can be same as the name of the founder "Guru", or came be named after the place where the founder "Guru" resided. For example, in the field of Hindustani Vocal Music, there exists several Gharanas (Deshpandey 1973) such as the Gwalior Gharana, the Dilli Gharana, the Kirana Gharana, the Agra Gharana, the Patiyala Gharana, the Jaipur Gharana etc. Most of these *gharanas* are renowned for their significant style of *Khyal* while few *gharanas* like the Atrauli Gharana, the Vishnupur Gharana are more famous for their *Dhrupad* singing style whereas the Lucknow Gharana and the Benaras Gharana are world famous for their *Thumri* singing. Similarly, under the Instrumental Music the Senia Gharana, the Senia Maihar Gharana, the Etawah Gharana and the Imdadkhani Gharana hold special place

(Mankaran, 2000). Likewise, the Jaipur Gharana and the Lucknow Gharana are famous for dance (Shrivastav, 1985). Throughout the history of the Hindustani classical music tradition, students were often born into a musical family practicing the art of music in the guru-shishya tradition, which was passed down through hereditary means by the musically gifted members of the family (Qureshi, 2007, 1). Specific musical families, such as Dagar, the lineage of Ustad Allauddin Khan, the Gwalior gharana, or the Atrauli gharana, still exist and retain respect from the musical community. These musical lineages produce specific ways to render raga, and have perfected certain techniques used to perform ragas. Essentially, it was the traditions and specific treatment of ragas perfected in these *gharanas* that eventually made them recognizable to other students of raga, and these traditions created schools of music through their bloodlines: "it is thus a compound of social feature (the membership) and cultural feature (musical style)" (Newman, 1990, 146).

Thus, it becomes clear that Gharana denotes a particular musical style unbroken in a particular family for several (at least five) generations. It represents a family of musicians, a well-knit unit evolving, guarding and disseminating the distinctive style through its members, some of whom are well-known performers and some who are not. A rigorous scientific study to look for this features have been lacking in the literature as most of the previous studies looked into the aesthetic and prosodic features (Datta et.al, 2017) that distinguishes one *gharana* from another. Datta et.al (2017) makes use of projection pursuit techniques to analyze the similarity between different artists, means to classify the ragas objectively rather than perceptually. It takes the help of various linear features such as MFCC, RMS energy Spectral Irregularity/ Centroid to identify the features of a particular *gharana.* In this study, for the first time, we applied robust non-linear tools to extract multifractal features from the complete signal waveform which makes one particular *gharana* of music distinct from another, taking four generations of singers belonging to a particular *gharana.* The multifractal techniques used in our study have been discussed in depth in the next section.

1.3. Use of MFDFA/ MFDXA to Identify Different Singing Styles

Previous knowledge suggests that music signals have a complex behavior: at every instant components (in micro and macro scale: pitch, timbre, accent, duration, phrase, melody etc) are closely linked to each other (Bigerelle & Iost, 2000; Sengupta et.al, 2005). These properties are peculiar of systems with chaotic, self organized, and generally, non linear behavior. Therefore, the analysis of music using linear and deterministic frameworks seems unrealistic and a non-deterministic/chaotic approach is needed in understanding the speech/music signals. Fractal analysis of audio signals was first performed by Voss and Clarke (1975), who analyzed amplitude spectra of audio signals to find out a characteristic frequency f_c, which separates white noise (which is a measure of flatness of the power spectrum) at frequencies much lower than f_c from very correlated behavior ($\sim 1/f^2$) at frequencies much higher than f_c. Music data is a quantitative record of variations of a particular quality over a period of time. One way of analyzing it is to look for the geometric features to help towards categorizing the data in terms of

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concept (Devaney, 1989). Demos et.al, (2014) use the DFA technique to relate body movements of performers to the expression embedded in it. However, it is well-established experience that naturally evolving geometries and phenomena are rarely characterized by a single scaling ratio; different parts of a system may be scaling differently. That is, the clustering pattern is not uniform over the whole system. Such a system is better characterized as 'multifractal' (Lopes & Betrouni, 2009). A multifractal can be loosely thought of as an interwoven set constructed from sub-sets with different local fractal dimensions. Real world systems are mostly multifractal in nature. Music too, has non-uniform property in its movement (Su & Wu, 2006; Telesca & Lovallo, 2011). Jafari et.al, (2007) applied Multifractal Detrended Fluctuation Analysis (MFDFA) technique to assess the presence of multifractality in Bach's pitches and ascertained that the presence of multifractality is both due to long range correlation present in the pitches and broad probability distribution function. Su & Wu (2006) show that both melody and rhythm can be considered as multifractal objects by separating both of them as series of geometric points. Live performances encompass a variety of such musical features including tempo fluctuations (Holden et.al, 2009), notation and timbre variation to name a few. Several other researchers have used the fractal analysis technique to examine musical movements and musical structure (Das and Das, 2006; Patra & Chakraborty, 2013; Zlatintsi & Maragos, 2013; Rankin et.al, 2014). Thus, complexity variation of a certain music piece with time can be judged more rigorously using multifractal analysis which determines the multifractal scaling behavior of a time series featured by very irregular dynamics, with sudden and intense bursts of high-frequency fluctuations. To study such a signal with MFDFA would certainly be a better tool than Detrended Fluctuation Analysis (DFA) as DFA gives only a single scaling exponent. The MFDFA technique gives us a multifractal spectral width which is a measure of the inherent complexity of the music signal. The MFDFA was first conceived by Kantelhardt et.al (2002) as a generalization of the standard DFA (Peng et.al, 1994). Thus, the multifractal analysis of music signals can be efficiently used in analyzing the singing styles of different artists while performing a *raga*. We hypothesize that the change of multifractal spectral width of the signal will give us a cue about the variation of style among the artists belonging to different *gharanas* as well as artists from successive generations representing the same *gharana.* In this context, taking the entire signal as a time series for analysis can be interesting as we are considering all the properties as a whole to ratify the multifractal nature of music. Subsequently another non linear technique called Multifractal Detrended Cross correlation Analysis (MFDXA) (Zhou, 2008) was used to analyze the multifractal behaviors in the power-law cross-correlations between two time series data of music signals. With this technique, all segments of *raga* clips from the four generation of singers of a particular *gharana* are analyzed to find out a cross correlation coefficient (γ_x) which gives the degree of correlation between the different generations. For uncorrelated data, γ_x has a value 1 and the lower the value of γ_x more correlated is the data (Sanyal et.al, 2016). Thus a negative value of γ_x signifies that the two music signals have very high degree of correlation between them. We hypothesize that the singers for whom high degree of cross correlation (i. e lower value of γ_x) is obtained are the ones who have similarities in their singing styles, and the ones for whom lower degree of correlation is obtained are the ones who have improvised and deviated from the former's style of singing. Thus, we have been able to create a mathematical paradigm in which it is possible to quantify the evolutionary cues in a particular *gharana* of Hindustani music. In this work, we tried to investigate these cues based on which we would be able to identify the similarities and differences in singing styles between several artists.

2. EXPERIMENTAL DETAILS

2.1. Choice of Music Signals

The experimental part of this work was performed in two steps. In the first part, our objective was to study the similarity & changes in the singing pattern of a particular *raga* over generations of artists of the same *gharana*. In this study, the renditions of two *raga*s (*Bageshri* & *Jaijawanti*) containing *alap* & *vilambit bandish* (sung at a very low tempo) part sung by 4 artists of consecutive generations of a particular *gharana* (Patiyala) of Hindustani classical music were chosen for analysis. For each *raga* the chosen *bandish* was same for all the 4 artists. *Alap* is the opening section of a *raga* performance in typical Hindustani classical style. In the *alap* part the *raga* is introduced and the paths of its development are revealed using all the notes used in that particular *raga* and allowed transitions between them with proper distribution over time. *Alap* is usually accompanied by the tanpura drone only and sung at a slow tempo or sometimes without tempo. Then comes the *vilambit bandish* part where the lyrics and *tala* are introduced. A *Bandish* is a song composed keeping the structural framework of a *raga* intact, thus *bandish* provides the literature ingredient of the *raga* in each individual rendition for traditional structured singing. *Bandish* is usually performed with rhythmic accompaniment by a tabla or pakhawaj, a steady drone, and melodic accompaniment by a sarangi, harmonium etc. Vilambit is a type of *bandish* which is sung at a very slow tempo, or *laya,* of 10-40 beats per minute. The first paragraph of the song – *Sthayi* is followed by the second one – *Antara.*

In the second part of our work, we intended to study how the singing style of a particular *raga* varies from one *gharana* to another and how the style evolves for generations of artists of the same *gharana*. Renditions of *raga darbari* sung by artists of 4 different *gharana*s (viz. Agra, Gwalior, Kirana, and Patiala) of Hindustani classical music are chosen for this analysis. For each *gharana*, renditions by popular artists of at least 4 consecutive generations are selected.

2.2. Processing of Music Signals

All the signals are digitized at the rate of 44100 samples/sec 16 bit format. In the first experiment, *alap* part, *sthayi* & *antara* of the *bandish* part were cut separately from each rendition for detailed analysis. It is expected that in the *alap* part, note combinations or improvisations will differ for different vocalists while establishing the *raga* and hence there is significant variation in the length of the *alap* pieces chosen for our analysis. So, to minimize the variation due to improvisations, in case of each vocalist, about 20 seconds of the *alap* part were cut out which led only to identification of the *raga*. The said 20 seconds clips were selected by an eminent musician with more than 20 years of experience in performing Hindustani classical music. On the contrary, the *bandish* part has lesser chances of variation in note combinations as the same *bandish* is being sung by all the vocalists and keeping the melody structure almost same. Although for different vocalists significant variations in the scansion of the *bandish* i.e. the distribution of the lyrics over the whole cycle of the *tala* are expected. In the second part of our work, only *alap* part of the *raga* was cut separately from each rendition. In the *alap* part, variations in note combinations or improvisations are expected for different performers while establishing the *raga* and hence there is significant variation in the length of the *alap* parts. To keep parity, about 2 minutes of *alap* were cut out from each rendition which led primarily to identification of the *raga*. The said 2 minutes clips were selected by an eminent musician with more than 20 years of experience in performing Hindustani classical music. These clips were selected for analysis.

3. METHODOLOGY

In the first experiment, each of the chosen *alap* parts of 20 seconds duration was divided into 4 equal parts of 5 seconds and their multifractal spectral widths were calculated using the MFDFA technique. The variation in the spectral widths among the 4 vocalists was observed separately for the 2 chosen *raga*s. Similar observations were found in case of the *sthayi* and *antara* part of the *vilambit bandish*es. In the second part of our work, each *alap* part of 2 minutes duration was divided into 4 equal parts of 30 seconds and their multifractal spectral widths were calculated using the MFDFA technique. The variation in the spectral widths among the vocalists of successive generations was observed separately for all the chosen 4 *gharana*s. The detailed algorithm for MFDFA technique is given below.

Method of multifractal analysis of music signals:

The analysis of the music signals is done using MATLAB in this paper and for each step an equivalent mathematical representation is given which is taken from the prescription of Kantelhardt et.al (2002).

The complete procedure is divided into the following steps:

Step 1: converting the noise like structure of the signal into a random walk like signal. It can be represented as:

$$Y(i) = \sum(x_k - \bar{x}) \quad (1)$$

where $\bar{x}$ is the mean value of the signal

The integration reduced the level of noise present in experimental records and finite data.

Step 2: the whole length of the signal is divided into Ns no of segments consisting of certain no. of samples. For s as sample size and N the total length of the signal the segments are

$$Ns = \text{int}\left(\frac{N}{s}\right) \quad (2)$$

Step 3: The local RMS variation for any sample size s is the function F(s,v). This function can be written as follows:

$$F^2(s,v) = \frac{1}{s}\sum_{i=1}^{s}\{Y[(v-1)s+i] - y_v(i)\}^2$$

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For ν = N_s + 1........, 2 N_s, where $y_\nu(i)$ is the least square fitted value in the binν. In this work, a least square linear fit using first order polynomial (MF-DFA -1) is performed. The study can also be extended to higher orders by fitting quadratic, cubic, or higher order polynomials.

Step 4: The q-order overall RMS variation for various scale sizes can be obtained by the use of following equation

$$Fq(s) = \left\{\frac{1}{Ns}\sum_{v=1}^{Ns}[F^2(s,v)]^{\frac{q}{2}}\right\}^{\left(\frac{1}{q}\right)} \quad (3)$$

where q is an index that can take all possible values except zero, because in that case the factor 1/q is infinite.

Step 5: The scaling behaviour of the fluctuation function is obtained by drawing the log-log plot of Fq(s) vs. s for each value of q.

$$F_q(s) \sim s^{h(q)} \quad (4)$$

The h(q) is called the generalized Hurst exponent. The Hurst exponent is measure of self-similarity and correlation properties of time series produced by fractal. The presence or absence of long range correlation can be determined using Hurst exponent. A monofractal time series is characterized by unique h(q) for all values of q.

The generalized Hurst exponent h(q) of MF-DFA is related to the classical scaling exponent τ(q) by the relation

$$\tau(q) = qh(q) - 1 \quad (5)$$

A monofractal series with long range correlation is characterized by linearly dependent q order exponent τ(q) with a single Hurst exponent H. Multifractal signal on the other hand, possess multiple Hurst exponent and in this case, τ(q) depends non-linearly on q (Ashkenazy et.al, 2003).

The singularity spectrum f(α) is related to h(q) by

$$\alpha = h(q) + qh'(q)$$

$$f(\alpha) = q[\alpha - h(q)] + 1 \quad (6)$$

where α is the singularity strength and f(α) specifies the dimension of subset series that is characterized by α. The multifractal spectrum is capable of providing information about relative importance of various fractal exponents in the series e.g., the width of the spectrum denotes range of exponents. A quantitative characterization of the spectra may be obtained by least square fitting it to a quadratic function (Shimizu, Thurner & Ehrenberger, 2002) around the position of maximum α_0,

$$f(\alpha) = A(\alpha - \alpha_0)^2 + B(\alpha - \alpha_0) + C$$

where C is an additive constant C = f(α0) = 1. B indicates the asymmetry of the spectrum. It is zero for a symmetric spectrum. The width of the spectrum can be obtained by extrapolating the fitted curve to zero.

Width W is defined as,

$$W = \alpha_1 - \alpha_2,$$

$$\text{with } f(\alpha_1) = f(\alpha_2) = 0 \quad (7)$$

The width of the spectrum gives a measure of the multifractality of the spectrum. Greater is the value of the width W greater will be the multifractality of the spectrum. For a monofractal time series, the width will be zero as h(q) is independent of q.

The origin of multifractality in music signal time series can be verified by randomly shuffling the original time series data (Maity et.al, 2015). All long range correlations that existed in the original data are destroyed by this random shuffling and what remains is a totally uncorrelated sequence. Hence, if the multifractality of the original data was due to long range correlation, the shuffled data will show non-fractal scaling.

On the other hand, if the initial h(q) dependence does not change; i. e if $h(q)= h_{shuffled}(q)$, then the multifractality is not due to long range correlation but is a result of broad probability density function of the time series. If any series has multifractality both due to long range correlation as well as probability density function, then the shuffled series will have smaller width W than the original series.

Next, MFDXA technique was performed on *alap, sthayi bandish* and *antara bandish* part of *raga Bageshri* and *raga Jayjawanti* sung by four generations of singers of *Patiyala gharana.* Following section of this paper illustrates detailed methodology of the MFDXA technique.

Multifractal Detrended Cross Correlation Analysis (MF-DXA):

We have performed a cross-correlation analysis of correlation between different samples of the same raga following the prescription of Zhou (2008).

$x_{avg} = 1/N \sum_{i=1}^{N} x(i)$ and $y_{avg} = 1/N \sum_{i=1}^{N} y(i)$ (8)

Then we compute the profiles of the underlying data series x(i) and y(i) as

$X(i) \equiv [\sum_{k=1}^{i} x(k) - x_{avg}]$ for i = 1 . . . N (9)

$Y(i) \equiv [\sum_{k=1}^{i} x(k) - x_{avg}]$ for i = 1 . . . N (10)

The integration also reduces the level of measurement noise present in experimental records and finite data. Each of the integrated time series was divided to N_s non-overlapping bins where N_s = int (N/s) where s is the length of the bin. Now, since N is not a multiple of s, a short part of the series is left at the end. So in order to include this part of the series the entire process was repeated starting from the opposite end thus leaving a short part at the beginning thus obtaining $2N_s$ bins. For each bin, least square linear fit was performed and the fluctuation function is given by:

$F(s, \nu) = 1/s \sum_{i=1}^{s} \{Y[(\nu - 1)s + i] - y_\nu(i)\} \times \{X[(\nu - 1)s + i] - x_\nu(i)\}$

for each bin ν, ν = 1, . . . , N_s and

$F(s, \nu) = 1/s \sum_{i=1}^{s} \{Y[(\nu - Ns)s + i] - y_\nu(i)\} \times \{X[N - (\nu - Ns)s + i] - x_\nu(i)\}$

for ν = Ns + 1, . . . 2Ns where $x_\nu(i)$ and $y_\nu(i)$ is the least square fitted value in the bin ν.

The qth order detrended covariance Fq(s) is obtained after averaging over 2Ns bins.

$F_q(s) = \{1/2N_s \sum_{\nu=1}^{2Ns} [F(s,\nu)]^{q/2}\}^{1/q}$ (11)

where q is an index which can take all possible values except zero because in that case the factor 1/q blows up. The procedure can be repeated by varying the value of s. Fq(s) increases with increase in value of s. If the series is long range power correlated, then Fq(s) will show power law behavior

$$F_q(s) \sim s^{\lambda(q)}.$$

If such a scaling exists ln F_q will depend linearly on ln s, with λ(q) as the slope. Scaling exponent λ(q) represents the degree of the cross-correlation between the two time series. In general, λ(q) depends on q, indicating the presence of multifractality. In other words, we want to point out how two sound signals are cross-correlated in various time scales. We cannot obtain the value of λ(0) directly because F_q blows up at q = 0. Fq cannot be obtained by the normal averaging procedure; instead a logarithmic averaging procedure is applied

$$F_0(s) = \{1/4N_s \sum_{v=1}^{2Ns}[F(s,v)]\} \sim s^{\lambda(0)}. \quad (12)$$

For q = 2 the method reduces to standard DCCA. If scaling exponent λ(q) is independent of q, the cross-correlations between two time series are monofractal. If scaling exponent λ(q) is dependent on q, the cross-correlations between two time series are multifractal. Furthermore, for positive q, λ(q) describes the scaling behavior of the segments with large fluctuations and for negative q, λ(q) describes the scaling behavior of the segments with small fluctuations. Scaling exponent λ(q) represents the degree of the cross-correlation between the two time series x(i) and y(i). The value λ(q) = 0.5 denotes the absence of cross-correlation. λ(q) > 0.5 indicates persistent long range cross-correlations where a large value in one variable is likely to be followed by a large value in another variable, while the value λ(q) < 0.5 indicates anti-persistent cross-correlations where a large value in one variable is likely to be followed by a small value in another variable, and vice versa (Movahed & Hermanis, 2008).

Zhou found that for two time series constructed by binomial measure from p-model, there exists the following relationship (Zhou, 2008):

$$\lambda\,(q = 2) \approx [h_x\,(q = 2) + h_y\,(q = 2)]/2. \quad (13)$$

Podobnik and Stanley have studied this relation when q = 2 for monofractal Autoregressive Fractional Moving Average (ARFIMA) signals and EEG time series (Podobnik & Stanley,2008). In case of two time series generated by using two uncoupled ARFIMA processes, each of both is autocorrelated, but there is no power-law cross correlation with a specific exponent (Movahed & Hermanis, 2008). According to auto-correlation function given by:

$$C(\tau) = \langle[x(i+\tau) - \langle x\rangle][x(i) - \langle x\rangle]\rangle \sim \tau^{-\gamma}. \quad (14)$$

The cross-correlation function can be written as

$$C_x(\tau) = \langle[x(i+\tau) - \langle x\rangle][y(i) - \langle y\rangle]\rangle \sim \tau^{-\gamma_x} \quad (15)$$

where γ and γ_x are the auto-correlation and cross-correlation exponents, respectively. Due to the non-stationarities and trends superimposed on the collected data, direct calculation of these exponents are usually not recommended; rather the reliable method to calculate auto-correlation exponent is the DFA method, namely γ = 2 − 2h (q = 2) (Movahed & Hermanis, 2008). Recently, Podobnik et al., have demonstrated the relation between cross-correlation exponent, γ_x and scaling exponent λ(q) derived by Eq. (4) according to $\gamma_x = 2 - 2\lambda(q = 2)$ (Patra & Chakraborty, 2013). For uncorrelated data, γ_x has a value 1 and the lower the value of γ and γ_x more correlated is the data.

The cross-correlation coefficient, γ_x computed for the

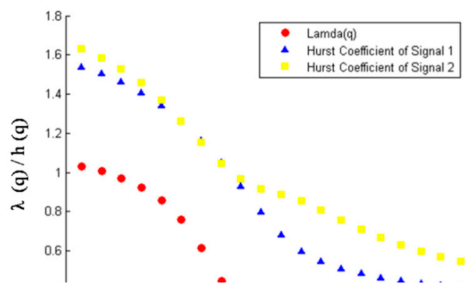


Fig. 1: Variation of λ (q) and h (q) for Artist 1 and Artist 2

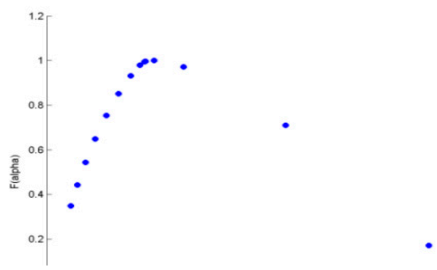


Fig. 2: Multifractal cross-correlated Spectrum of Artist 1 and 2 (*Sthayi Bandish_Raga Bageshri*)

four generation of singers gives the amount of similarities/differences amongst them while singing the *ragas*. A representative figure (**Fig. 1**) reports the variation of cross correlation exponent λ (q) with q for two particular samples (Artist 1 and Artist 2, *Sthayi Bandish_Raga Bageshri*), also the variation of h (q) with q for those two samples obtained from MFDFA technique are also shown in the same figure for comparison.

The variation of λ (q) with q for the two cross correlated signals (Artist 1 and 2) show that they are multifractal in nature. To illustrate further the presence of multifractality in the cross-correlated music signals, i.e. to have information about the distribution of degree of cross-correlation in various time scales, a representative multifractal spectrum was plotted for the two signals in **Fig. 2**. From the multifractal spectral width of the cross-correlated signals obtained from cross-correlations of Artist 1 and Artist 2, it becomes evident that the cross-correlated signal is again multifractal in nature.

4. RESULTS AND DISCUSSIONS

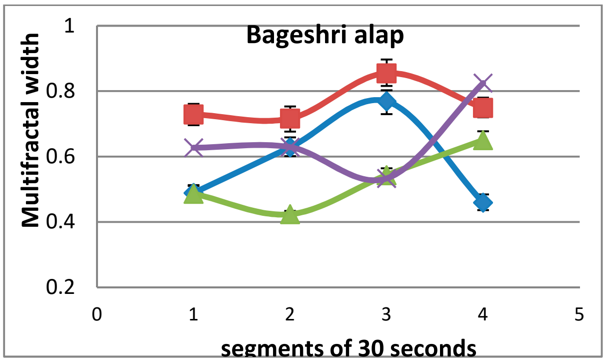
The multifractal spectral width (W) was computed for each part of the *raga* clips for all the four artists. Higher the value of W, higher is the degree of complexity present in the signal. Thus change in the spectral width due to change of note combinations, transitions between the notes; tempo and rhythm variations in a musical piece would be significantly important to characterize the singing style of different artists. **Table 1** show all the spectral width values where Artist 1 represents the oldest among the 4 artists of Patiyala *gharana* that we chose to study. Artists 2, 3 and 4 represent singers of consecutive generations from the same *gharana*.

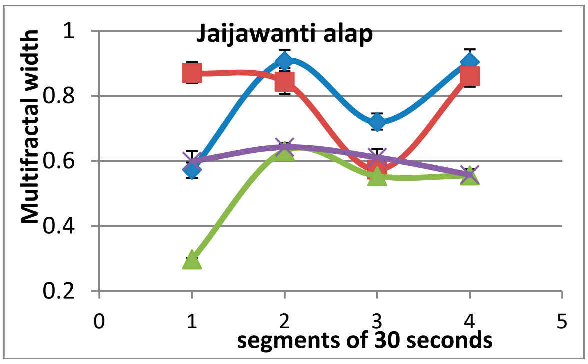
Table 1: Variation of the spectral width values for all 4 artists while rendering 2 chosen *raga*s

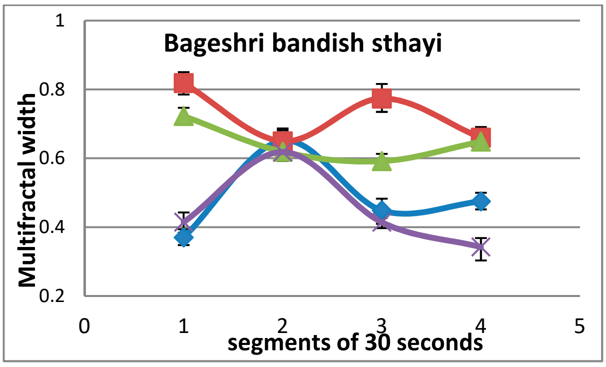
Raga used			Artist 1	Shuffled width	Artist 2	Shuffled width	Artist 3	Shuffled width	Artist 4	Shuffled width
		Part 1	0.49	0.11	0.73	0.02	0.49	0.07	0.63	0.01
		Part 2	0.63	0.09	0.72	0.03	0.42	0.03	0.63	0.03
		Part 3	0.77	0.07	0.86	0.01	0.54	0.09	0.53	0.03
	Alap	Part 4	0.46	0.11	0.75	0.02	0.65	0.07	0.83	0.06
		Part 1	0.37	0.09	0.82	0.02	0.72	0.06	0.41	0.03
		Part 2	0.65	0.08	0.65	0.05	0.62	0.05	0.62	0.01
		Part 3	0.45	0.08	0.77	0.07	0.59	0.11	0.41	0.03
	Bandish Sthayi	Part 4	0.48	0.10	0.66	0.06	0.65	0.09	0.34	0.03
		Part 1	0.28	0.13	0.49	0.03	0.47	0.09	0.48	0.02
		Part 2	0.55	0.09	0.67	0.04	0.54	0.05	0.55	0.02
		Part 3	0.59	0.09	0.66	0.04	0.56	0.10	0.46	0.03
Bageshri	*Bandish Antara*	Part 4	0.46	0.16	0.49	0.07	0.69	0.06	0.41	0.01
		Part 1	0.57	0.08	0.87	0.08	0.30	0.02	0.60	0.04
		Part 2	0.91	0.25	0.84	0.04	0.63	0.01	0.64	0.02
		Part 3	0.72	0.07	0.57	0.07	0.55	0.08	0.61	0.04
Jaijawanti	*Alap*	Part 4	0.90	0.09	0.86	0.04	0.56	0.04	0.56	0.02

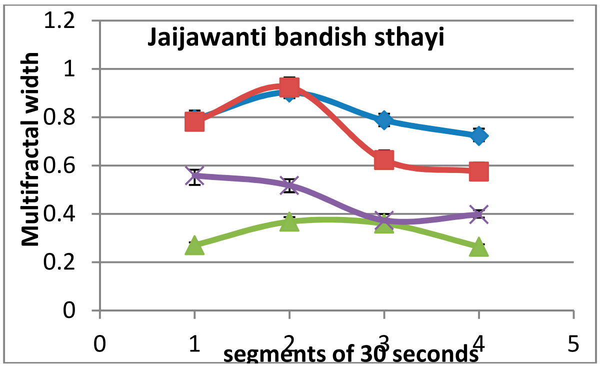
	Bandish Sthayi	Part 1	0.79	0.08	0.78	0.05	0.27	0.07	0.56	0.02
		Part 2	0.90	0.03	0.92	0.05	0.37	0.05	0.52	0.02
		Part 3	0.79	0.07	0.62	0.03	0.36	0.05	0.37	0.01
		Part 4	0.72	0.06	0.58	0.02	0.26	0.07	0.40	0.05
	Bandish Antara	Part 1	0.58	0.03	0.77	0.05	0.30	0.05	0.37	0.02
		Part 2	0.56	0.04	0.88	0.02	0.52	0.03	0.29	0.01
		Part 3	0.61	0.05	0.50	0.03	0.15	0.08	0.41	0.04
		Part 4	0.59	0.01	0.71	0.05	0.37	0.09	0.47	0.03

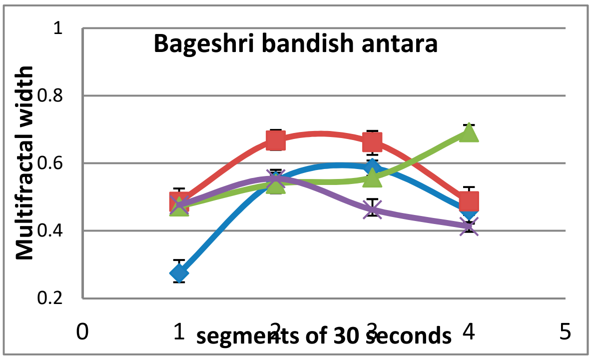
The following Fig. 3 and 4 represent the variation of W for the 4 artists while rendering the *alap* of *Raga Bageshri* and *Raga Jaijawanti* respectively. Fig. 5-6 represents the same for *Bandish Sthayi* part whereas Fig. 7-8 represents the same for *Bandish Antara* part. The error bars given in all the figures of Section 4 represent the computational errors introduced in the multifractal algorithm used in this work.











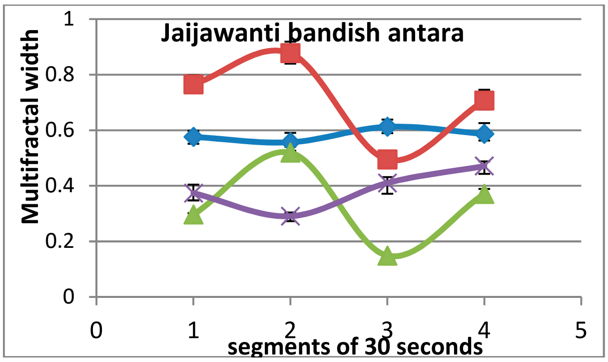


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Fig. 3-8: variation of spectral width in *alap* part, *bandish* shtayi and *antara* part of *raga Bageshri* and *raga jaijawanti* for all 4 artists.

The following observations can be drawn from a careful study of the figures:

1. From Table 1 it is evident that average spectral width is higher in case of *raga jaijawanti* than that of *raga Bageshri* for all 4 artists both in *alap* part and *bandish* part. Thus we can conclude overall complexity of *raga jaijawanti* is higher than *raga Bageshri*.
2. Comparing Fig. 3, 5, 7 with Fig. 4, 6, 8 we can easily say that spectral width variation among the 4 artists of same *gharana* is higher in case of *raga jaijawanti* than *raga Bageshri*. This complexity variation is more prominent in *bandish sthayi* and *antara* part than the *alap* part. This observation may be interpreted as following: In case of *raga Bageshri* the singing style of the oldest artist is maintained more by his successors than in case of *raga Jaijawanti* where their own styles were incorporated more.
3. The complexity variation among the 4 artists is least in case of *Bageshri antara* (Fig. 7) while largest in case of *jaijawanti antara* (Fig. 8).
4. In general for both *raga*s Artist 2 shows greater width than others in *alap* part as well as *bandish* part.
5. In case of *raga jaijawanti*, striking similarity is observed between Artist 1 and Artist 3 in the complexity variation pattern among the 4 parts of the *alap* as well as *sthayi* part whereas in *antara* part of the *bandish* Artist 3 resemble more with Artist 2 and Artist 4 resemble more with Artist 1, though the absolute values of the spectral width are much lesser for Artist 3 and Artist 4 than Artist 1 and Artist 2 in both *alap* and *bandish* parts. So, it is evident that while singing *raga jaijawanti* the artists of the younger generations are trying to incorporate the singing style of the artists from the older generations as per their choice.
6. In *alap* part of both *raga*s Artist 4 generally differs from others but in the *Bandish* part (both *sthayi* and *antara*) he shows resemblance with other artists.
7. In most of the time segments we get varying complexity which has a clear tendency to increase from Artist 1 to Artist 2 but mixed response in case of the contemporary artists.

These observations were further intensified by the findings of multifractal cross correlation analysis of the above mentioned sound clips. The multifractal cross correlation coefficient (γ_x) was computed for each pair of artists while singing the same part (viz. alap/ bandish sthayi/ bandish antara) of the same *raga*. Lower the value of γ_x, higher is the degree of correlation present between the two chosen signals. Thus MFDXA can compare the similarities and dissimilarities in the singing style of two artists quite efficiently. **Table 2** shows the variation of cross correlation coefficient among the artists of different generations while singing different parts of *Raga Bageshri* and *Raga Jaijawanti*. Here also Artist 1 represents the oldest generation among the chosen four and Artist 2, 3 and 4 represent the successive ones.

	Raga Bageshri			Raga Jaijawanti		
Artist	Alap	Bandish Sthayi	Bandish Antara	Alap	Bandish Sthayi	Bandish Antara

1vs2	-1.58	-0.02	0.08	-1.59	-0.26	-0.05
1vs3	-2.11	-2.12	-0.56	-2.21	-0.57	-0.47
1vs4	-0.57	-0.13	-0.44	-1.86	-0.91	-0.28
2vs3	1.88	0.58	0.34	1.33	0.44	0.04
2vs4	0.76	-0.03	-0.09	1.39	0.08	0.04
3vs4	-2.11	-0.09	-0.93	-1.75	-0.12	-0.02

Table 2: Variation of cross correlation coefficient among different generations of artists representing the same *gharana*

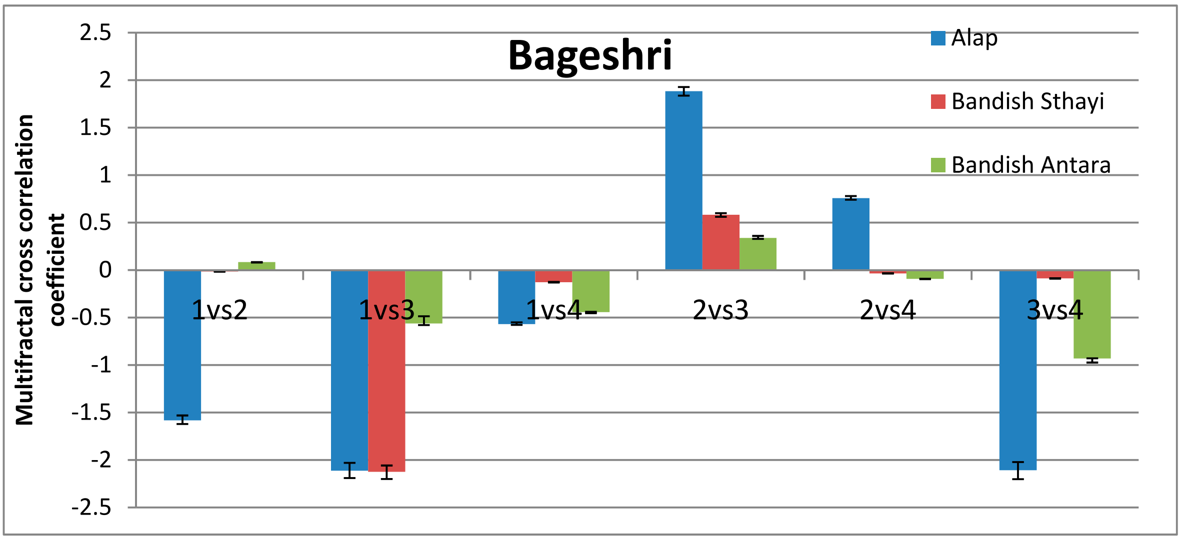

Fig. 9: Comparative analysis of style among 4 generations of artists while singing *raga Bageshri*

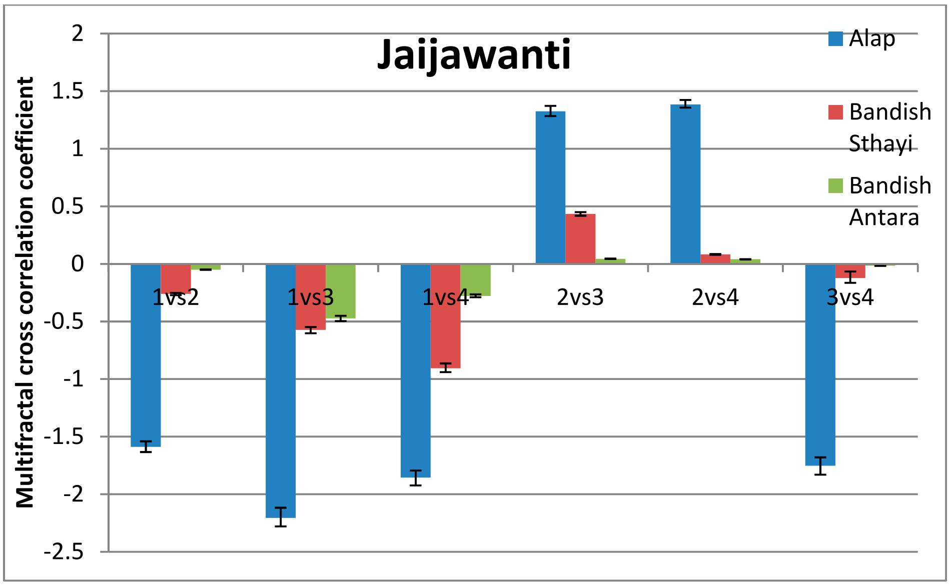

Fig. 10: Comparative analysis of style among 4 generations of artists while singing *raga Jaijawanti*

Fig. 9 and 10 clearly show that,

1) For both of the chosen renderings of *Raga Bageshri* and *Jaijawanti,* Artist 1 (first generation) and Artist 2 (second generation) show strong correlation in the *alap* part but in the *sthayi* and

antara of the *vilambit bandish* part the correlation is very poor. In case of *Jaijawanti sthayi* the correlation is little higher than the other *bandish* parts. This result indicates towards the probability that during singing a particular *raga,* Artist 2, being the direct disciple of Artist 1, followed the Guru's style in the *alap* portion but chose to deviate in most of the *bandish* part and incorporated his own style.

2) Artist 1 and Artist 3 (third generation) exhibit very strong correlation in the *alap* part as well as the whole *bandish* part of both *Raga Bageshri* and *Jaijawanti*. On average the correlation is higher in case of *alap* than in the *bandish* part. This finding gives a hint that may be the two artists sang very similar phrases in the *alap* part while establishing the *raga.* Also the chosen *bandish* being same for a particular *raga* for all the chosen four artists, similarity in the singing is anticipated. On the other hand, the findings of this analysis reveal strange information which features a striking dissimilarity in singing style between Artist 2 and Artist 3 during both the *ragas.* These two artists (Artist 2 & 3) exhibit extremely poor correlation in *bandish sthayi* and *antara* part of the two chosen *ragas,* whereas in the *alap* parts they show no correlation at all among themselves. This observation is striking as Artist 3 is direct disciple of Artist 2. Our results clearly indicate that Artist 3 preferred to follow Artist 1 from older generation rather than Artist 2, at least during singing the chosen renderings of *Raga Bageshri* and Raga *Jaijawanti.*
3) Artist 4 (fourth generation), being the direct disciple of Artist 3, show high similarity in style with his Guru, mainly in the *alap* portion of the two *ragas.* The correlation in the *bandish sthayi* and *antara* part is much lower than the *alap* part for these two artists. The *Bageshri antara* shows the highest correlation among all the *bandish* parts. The results yield that in the *bandish* part of the two *ragas,* Artist 4 follows Artist 1 more than Artist 3. Artist 4 and Artist 1 show sufficiently good correlation in all parts of the two *ragas,* though on average these correlations are lesser than that between Artist 1 and 3. This means Artist 4 resembles the style of Artist 1 while singing a particular *raga,* though clearly not as much as the resemblance between Artist 1 and 3. In case of Artist 4 also, very poor correlations with Artist 2 are observed during different parts of both the *ragas,* the *alap* portion of *Raga Bageshri* being completely uncorrelated for these two artists. This emphasizes on the uniqueness of singing style of Artist 2 (second generation) compared to the artists of other generations.

In the second part of our work, where we attempted a comparative study between four popular *gharanas* of Indian Classical Music, many more interesting results came out.

From the raw signals of the *alap* parts, few general observations are noted:

i. In Agra *gharana*, usually the *alap* part is sung through a long time (typically for 20-30 minutes, may be even more) where after introduction of the notes and movements of the *raga* special words like *"nom-tom"* are used to elaborate the *raga* in detail. Some artists of this *gharana* are known to sing very fast moving musical structures like tarana during the *alap*.

ii. For other 3 *gharana*s, a much shorter form of the *alap* i.e. an *Aochar* is usually sung at the beginning of a *raga* performance. Though it depends a lot on artist style, for Patiala, Gwalior and Kirana *gharana*, the *alap* part primarily features the main notes used in the *raga* with the essential phrases used to establish the *raga*. Detailed elaborations are usually sung after the introduction of the *bandish.*

Then the multifractal spectral width (W) was computed for each *alap* clips for all the chosen artists of successive generations for 4 different *gharana*s. Higher the value of W, higher is the degree of complexity present in the signal. Thus change in the spectral width due to change of note combinations, transitions between the notes and pause variations in a musical piece would be significantly important to characterize the singing style of different artists.

Agra gharana								
	Part 1	**Shuffled width**	**Part 2**	**Shuffled width**	**Part 3**	**Shuffled width**	**Part 4**	**Shuffled width**
Artist 1	0.38	0.02	0.44	0.04	0.33	0.01	0.38	0.03
Artist 2	0.61	0.08	0.88	0.11	0.69	0.09	0.76	0.10
Artist 3	0.72	0.07	0.90	0.14	0.67	0.11	0.92	0.17
Artist 4	0.5	0.03	0.77	0.06	0.67	0.05	0.91	0.09
Artist 5	0.67	0.06	0.53	0.08	0.58	0.04	0.61	0.06
Artist 6	0.42	0.02	0.30	0.01	0.39	0.02	0.34	0.03
Artist 7	0.53	0.04	0.55	0.07	0.48	0.06	0.41	0.05
Artist 8	0.53	0.03	0.45	0.09	0.45	0.08	0.39	0.06
Artist 9	0.60	0.03	0.69	0.12	0.68	0.11	0.63	0.08

Table 3: Variation of the spectral width values for all artists of Agra *gharana*

Total 9 artists representing Agra *gharana* were chosen for this analysis. Here Artist 1 represents the oldest (assigned as first generation) among all the artists of Agra *gharana* that we chose to study. Artists 2, 3, 4 and 5 are direct disciples of Artist 1 and they are almost contemporary. They are representing the second generation of this *gharana* in this study. Artist 6 and 7 are from third generation. Again within Agra *gharana*, other lineages are found, like artist 8 is direct disciple of artist 6 while artist 9 has learnt directly from Artist 7, hence Artist 8 and 9 represent the fourth generation. Thus all these 9 singers represent performers of consecutive generations from the same *gharana*.

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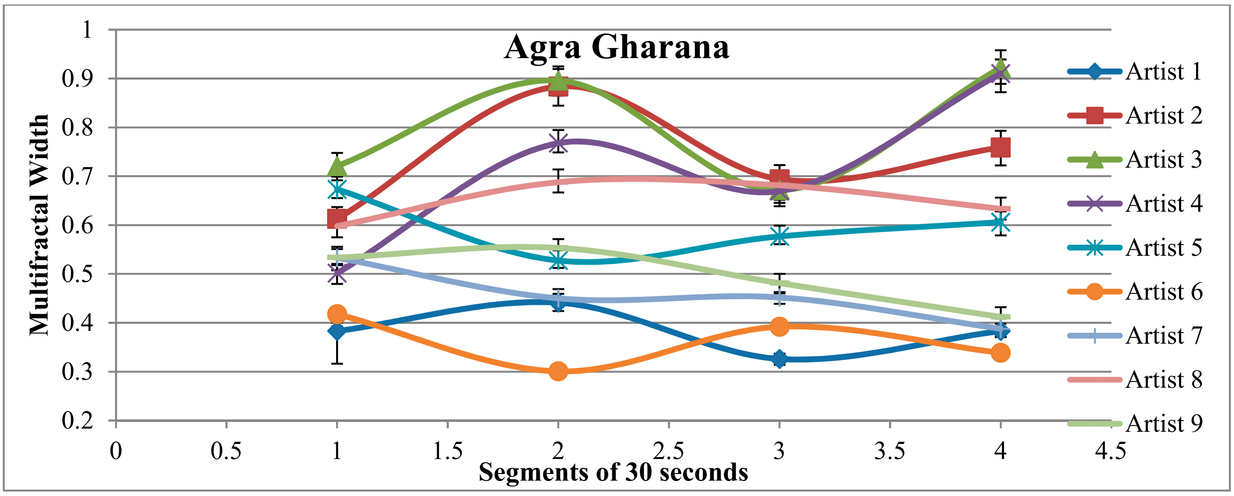

Fig. 11 represents the variation of multifractal spectral width for the artists of Agra *gharana* while rendering *Raga Darbari*

The following observations can be drawn from a careful study of the Fig. 11:

1. Striking similarities are found in the spectral width variation pattern between Artist 1 and Artist 2, 3 and 4. This probably indicates the inherence of style from tutor (*Guru*) to his direct disciples.
2. Despite of being another direct disciple of Artist 1, Artist 5 shows completely different pattern. This may have happened due to incorporation of own style by Artist 5.
3. If we consider Artist 1 as the first generation then Artist 2, 3, 4 & 5 will belong to second generation. Now it is quite evident from the above figure that in general artists of second generations are showing a much higher value of complexity than the first generation artist.
4. Artist 6 and 7 represent the third generation of artists belonging to Agra *gharana* whereas Artist 8 and 9 represent the fourth generation. Just like our previous observations, complexity change pattern for Artist 8 is similar to Artist 6 and for Artist 9 it resembles with that of Artist 7. In both cases the disciples show a higher value of complexity than their tutors.
5. Overall analysis indicates that in average artists of third and fourth generations show lesser value of complexity than second generation artists.

Kirana gharana								
	Part 1	**Shuffled width**	**Part 2**	**Shuffled width**	**Part 3**	**Shuffled width**	**Part 4**	**Shuffled width**
Artist 1	0.81	0.09	0.70	0.05	0.61	0.10	0.64	0.09
Artist 2	0.39	0.04	0.37	0.06	0.40	0.08	0.33	0.02
Artist 3	0.34	0.02	0.33	0.03	0.31	0.01	0.39	0.02
Artist 4	0.86	0.11	0.67	0.13	0.61	0.09	0.53	0.08
Artist 5	0.97	0.16	0.83	0.11	0.91	0.18	0.87	0.13

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Artist 6	0.49	0.04	0.44	0.07	0.44	0.06	0.38	0.03
Artist 7	0.57	0.07	0.53	0.09	0.48	0.04	0.50	0.06
Artist 8	0.47	0.03	0.45	0.05	0.44	0.02	0.42	0.09

Table 4: Variation of the spectral width values for all artists of Kirana *gharana*

In Kirana *gharana*, total 8 artists were chosen for this study. Similar to the previous notion, here also, Artist 1 (first generation) is the oldest one whose only direct disciple is Artist 4. All other artists of this *gharana* chosen for this analysis belong to a different lineage. Artist 2, 3, 4, 5 and 6 are contemporary and represent the second generation. Artist 7 is from third generation and Artist 8, representing fourth generation, is youngest of all.

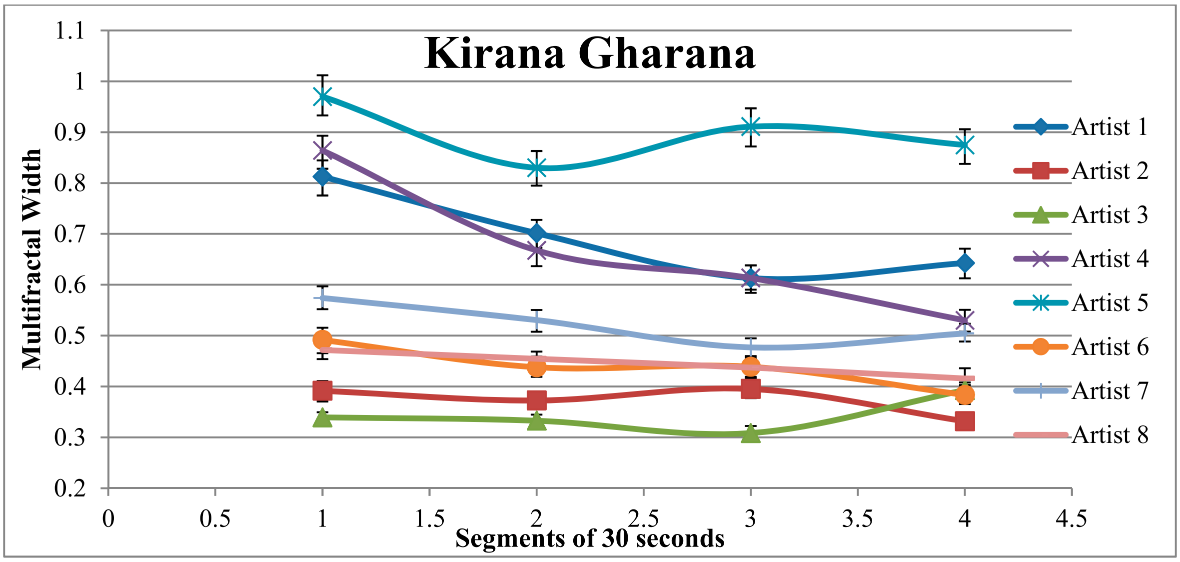

Fig. 12: Variation of multifractal spectral width for the artists of Kirana *gharana* while rendering *Raga Darbari*.

From the above figure we find that,

1) Artist 1 and Artist 4 show very similarity in complexity which indicates that Artist 4 follows the style of Artist 1.
2) Despite of being contemporary, Artist 2, 3 and 6 feature much lower complexity than artist 4. This indicates the presence of huge difference in style among different lineages belonging to same *gharana*.
3) Except Artist 5 who stands out with a much higher value of complexity than everyone else, all other artists of the second lineage show a much lower value of complexity than lineage 1.
4) Artist 7 and 8 follow the lineage of Artist 1. In this *gharana* also, Artist 7 and 8 from third and fourth generation respectively show lesser complexity values than artists of first and second generation.

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Gwalior gharana								
	Part 1	**Shuffled width**	**Part 2**	**Shuffled width**	**Part 3**	**Shuffled width**	**Part 4**	**Shuffled width**
Artist 1	0.57	0.05	0.48	0.05	0.43	0.03	0.48	0.02
Artist 2	0.55	0.08	0.53	0.06	0.58	0.05	0.43	0.06
Artist 3	0.56	0.08	0.50	0.09	0.58	0.12	0.65	0.16
Artist 4	0.40	0.01	0.48	0.01	0.46	0.02	0.41	0.03

Table 5: Variation of the spectral width values for all artists of Gwalior *gharana*

In case of Gwalior *gharana*, Artist 1, 2, 3 and 4 represent first, second, third and fourth generation respectively, though none is the absolute direct disciple of the artist representing the previous generation.

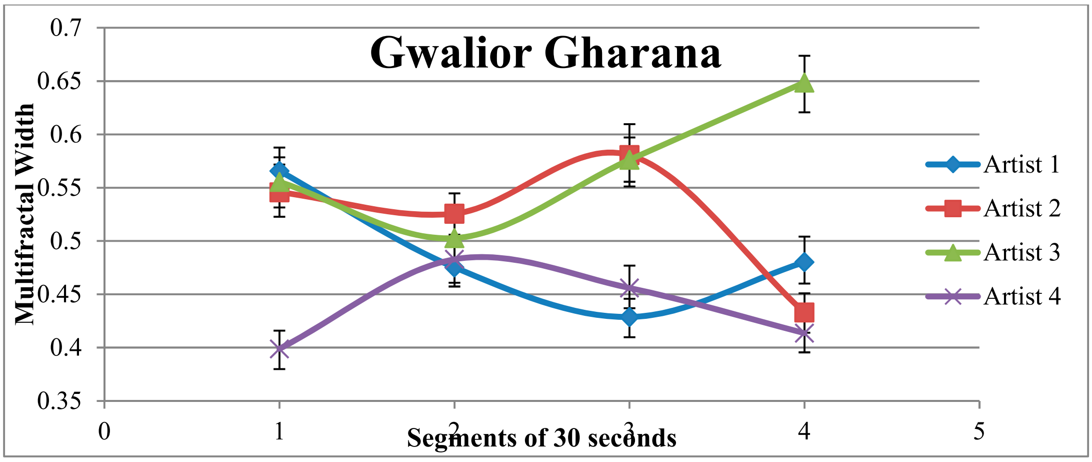

Fig. 13 represents the variation of multifractal spectral width for the artists of Gwalior *gharana* while rendering *Raga Darbari*.

Primary observation indicates, during performance of *raga Darbari* the complexity variation among all the artists of Gwalior *gharana* is much lesser compared to other three *gharana*s. Here Artist 2 and Artist 4 show some resemblance in the singing style. In this *gharana* too, the younger artists feature lesser value of complexity than the artist representing the second generation.

Patiala gharana								
	Part 1	**Shuffled width**	**Part 2**	**Shuffled width**	**Part 3**	**Shuffled width**	**Part 4**	**Shuffled width**
Artist 1	0.62	0.09	0.74	0.11	0.7	0.12	0.56	0.08
Artist 2	0.73	0.15	0.72	0.1	0.69	0.1	0.67	0.09
Artist 3	0.39	0.04	0.31	0.03	0.33	0.04	0.32	0.02
Artist 4	0.37	0.02	0.31	0.01	0.36	0.04	0.31	0.02

Table 6: Variation of the spectral width values for all artists of Patiala *gharana*

Artist 1, 2, 3 & 4 represent artists of consecutive generations of Patiala *gharana*, though in case of this *gharana* all the chosen artists are direct disciples of the artists chosen from their previous generations.

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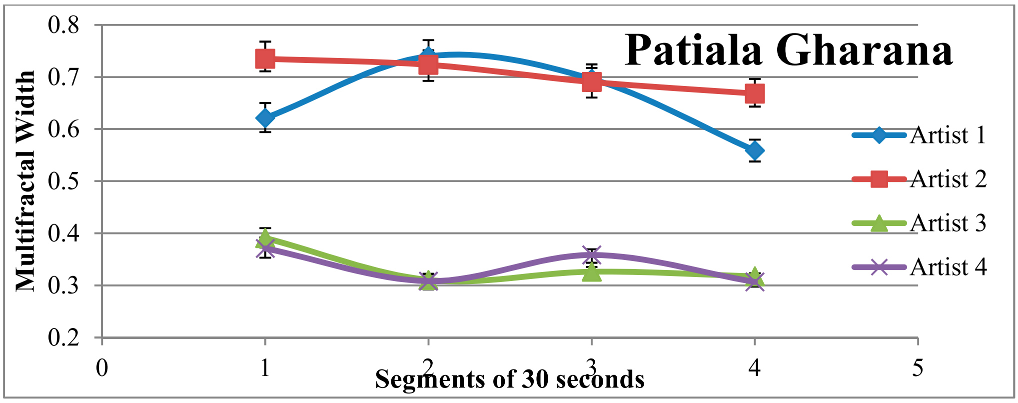

Fig. 14 represents the variation of multifractal spectral width for the artists of Patiala *gharana* while rendering *Raga Darbari*.

Here we observe Artist 1 and 2 show similar complexity values and the value is much lower in case of Artist 3 and 4. Artist 2 and 4 being direct disciples of Artist 1 and Artist 3 respectively, the results are expected. In Patiala *gharana* also, the artist from second generation features the highest average value of complexity. The prominent decrease in spectral width values for third and fourth generation artists may have caused due to a significant change in style.

5. CONCLUSION

The observations from our first part of analysis (using MFDFA technique) suggest that Artist 2 seems to be completely different in *bandish* as well as *alap* part in terms of complexity of the signal. The other artists, particularly Artist 3 and Artist 4 resemble Artist 1 in specific time segments. From the above information we may conclude that the contemporary artists Artist 3 and Artist 4 are trying to incorporate the style of Artist 1 in their singing sometimes. From this data we can see that in most of the time segments we get varying complexity which has a clear tendency to increase from Artist 1 to Artist 2 but mixed response in case of the contemporary artists. These results are completely supported by the MFDXA outcomes, as there too we see the uniqueness in the singing style of Artist 2 quite distinctly.

In the second part of our analysis we used one of the robust non linear techniques – MFDXA to assess the degree of correlation between singers of different generations of the same *gharana.* The concept of "*Guru-Shishya Parampara"* has been present in Hindustani Classical Music for a long time but a scientific foundation of the same have been lacking. For the first time, a rigorous scientific algorithm has been proposed here which tries to give a comprehensive idea on the concept of "*Guru-Shishya Parampara"* in Hindustani music. This technique goes into such microscopic depths of music signals recorded from different generations of artists of the same *gharana,* which is not possible by any other methods. The features extracted from this technique can give beautiful insights into how the singing style varies from one generation to other or even between singers of the same generation. A high degree of correlation indicates that the singing style of the two singers are similar even at the microscopic level, while a lower degree of

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correlation indicates that the singing styles are different for the two singers. The MFDXA results also provide lots of validations to strengthen the conclusion that artists of younger generations (Artist 3 & Artist 4) preferred to follow the style of Artist 1 from first generation rather than the style of Artist 2 in different parts of both the chosen *ragas*. This may have happened due to the aesthetical preferences of the younger generation artists. Analysis by these scientific techniques therefore yielded a clear cut comparison of the styles of four vocalists of a particular *gharana*, which is entirely new information even in the international scenario.

A similar study was performed for four different *gharanas* of Hindustani classical music using MFDFA technique to reach a more convincing conclusion. **Fig. 15** gives the averaged complexity for the singers of four successive generations belonging to four different *gharanas* of Indian Classical Music while singing *Raga Darbari.*

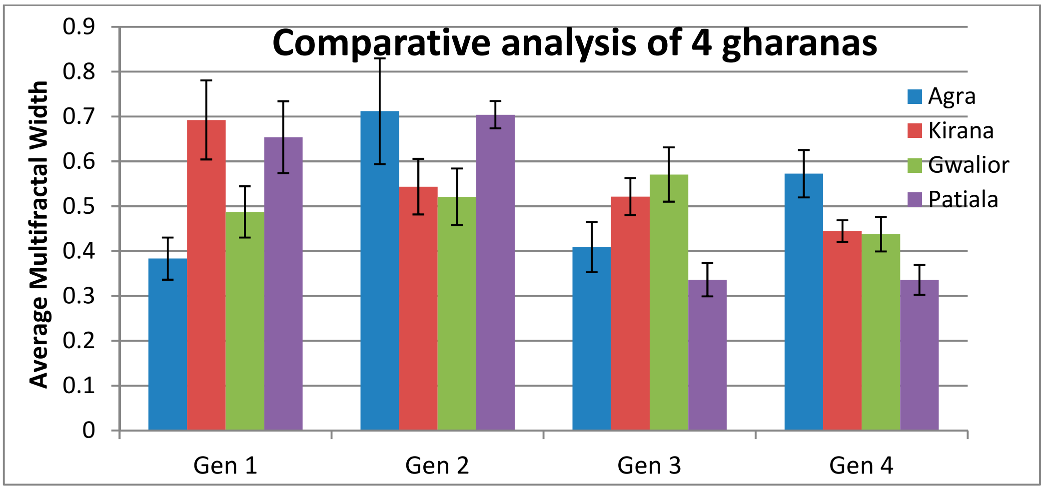

Fig. 15: A comparative analysis of 4 generations of artists from 4 different *gharanas*

In the above figure all the given error bars represent the standard deviation for different time segments of a particular *raga* (*Darbari Kanada* in this case) clip rendered by the artist(s) of a particular generation. Summarizing all the observations obtained from detailed analysis of the *alap* part of *raga Darbari* sung by various artists of different generations belonging to 4 different *gharana*s of Hindustani classical music, we can conclude that

1) Mostly complexity change with time shows strikingly similar pattern for the tutors and their direct disciples in all the 4 *gharana*s. This indicates that the style of the tutor is usually followed by his students. The outcomes of this study basically offers a scientific validation of "Guru – Shishya Parampara" tradition of Indian Classical Music using rigorous nonlinear techniques of Physics. Though in few cases, major deviations in the complexity change pattern are observed, indicating the possibility of incorporation of new style by the student. This again points out towards the flourishing of individual creativity which leads to infinite improvisations within the structured framework of a *raga.*

2) Another important observation is that significant difference in style is found among different lineages within same *gharana*. This indicates that though some major structural features remain

same for a particular *gharana* over generations, many finer structural differences are introduced by different artists through time.
3) In general for all *gharana*s, third and fourth generation artists show lesser average spectral width (W) values than first and second generation artists manifested in decreasing complexity with time. The main cause behind this probably indicates towards the miraculous advancement of technology in last few decades which has brought globalization in the field of classical music too. Now a day for a young artist the possibility of exposure to other *gharana*s of music is much higher than the earlier generations. This leads to mixture of certain styles among the young generation artists of all *gharana*s.

Thus, with the help of rigorous latest nonlinear analysis techniques like MFDFA and MFDXA, we have proposed an automated algorithm with which one can attempt to quantify the evolutionary effects in a musical performance (in this case, we have observed how the singing style of a *Raga* changes over generations of artists in different *gharanas*). The comparative analysis of singing styles of different *gharanas* as well as artists representing different generations using these nonlinear methods is a pilot study which can be further intensified by the extraction of notes, transitions between notes etc. The results can be verified by analysing the whole *alap* part as well as different other parts of the whole *raga* performance like *bandish, taan* etc. as well as more number of *ragas* should be taken into account. This work, as elaborated earlier, has a far reaching application in sustaining *guru-shishya parampara* in scientific perspective, even when training is arranged in distant mode.

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References:

Ashkenazy, Y., Baker, D. R., Gildor, H., & Havlin, S. (2003). Nonlinearity and multifractality of climate change in the past 420,000 years. Geophysical Research Letters, 30(22).
Bigerelle, M., & Iost, A. (2000). Fractal dimension and classification of music. *Chaos, Solitons & Fractals*, *11*(14), 2179-2192.
Brahaspati A. Sangeet Ratnakar, Sangeet Karyalay, Hathras, 2002
Chaubey S. K. Sangeet ke Gharano ki Charcha, Uttar Pradesh Hindi Granth Academy, Lucknow, 1977
Das, A., & Das, P. (2006). Fractal analysis of different eastern and western musical instruments. *Fractals*, *14*(03), 165-170.
Demos, A. P., Chaffin, R., & Kant, V. (2014). Toward a dynamical theory of body movement in musical performance. *Frontiers in psychology*, *5*, 477.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 April 2019 doi:10.20944/preprints201904.0157.v1

Deshpandey V. H. Gharanedar Gayaki, Oriental Longman Limited, New Delhi, 1973

Devaney, R. L. (1989). *An introduction to chaotic dynamical systems* (Vol. 13046). Reading: Addison-Wesley.

Georges, P. (2017). Western classical music development: a statistical analysis of composers similarity, differentiation and evolution. *Scientometrics*, *112*(1), 21-53.

Holden, J. G., Van Orden, G. C., & Turvey, M. T. (2009). Dispersion of response times reveals cognitive dynamics. *Psychological review*, *116*(2), 318.

Hothi, G. (2013). An Interrogative Analysis of the Imdadkhani Gharana of Hindustani Classical Music. *Indian Journal of Arts*, *1*(1), 10-12.

Jafari, G. R., Pedram, P., & Hedayatifar, L. (2007). Long-range correlation and multifractality in Bach's Inventions pitches. *Journal of Statistical Mechanics: Theory and Experiment*, *2007*(04), P04012.

Kantelhardt, J. W., Zschiegner, S. A., Koscielny-Bunde, E., Havlin, S., Bunde, A., & Stanley, H. E. (2002). Multifractal detrended fluctuation analysis of nonstationary time series. *Physica A: Statistical Mechanics and its Applications*, *316*(1-4), 87-114.

Lopes, R., & Betrouni, N. (2009). Fractal and multifractal analysis: a review. *Medical image analysis*, *13*(4), 634-649.

Maity, Akash Kumar, et al. "Multifractal Detrended Fluctuation Analysis of alpha and theta EEG rhythms with musical stimuli." Chaos, Solitons & Fractals 81 (2015): 52-67.

Mankaran V. Sangeet Saar, Raj Publishers, Jalandhar 2000

Movahed, M. S., & Hermanis, E. (2008). Fractal analysis of river flow fluctuations. Physica A: Statistical Mechanics and its Applications, 387(4), 915-932.

Park, D., Bae, A., Schich, M., & Park, J. (2015). Topology and evolution of the network of western classical music composers. *EPJ Data Science*, *4*(1), 2.

Patra, M., and Chakraborty, S. (2013). Analyzing the digital note progression of ragas within a thaat using fractal geometry. *International Journal of Advanced Computer and Mathematical Sciences* ISSN 2230-9624. 4, 148-153

Peng, C. K., Buldyrev, S. V., Havlin, S., Simons, M., Stanley, H. E., & Goldberger, A. L. (1994). Mosaic organization of DNA nucleotides. *Physical review e*, *49*(2), 1685.

Podobnik, B., & Stanley, H. E. (2008). Detrended cross-correlation analysis: a new method for analyzing two nonstationary time series. Physical review letters, 100(8), 084102.

Pranjpay S. S. Sangeet Bodh, Madya Pradesh Hindi Granth Academy, Bhopal, 1992

Qureshi, R. B. (1987). Musical sound and contextual input: a performance model for musical analysis. *Ethnomusicology*, *31*(1), 56-86.

Rankin, S. K., Fink, P.W., and Large, E. W. (2014). Fractal structure enables temporal prediction in music. *J. Acoust. Soc. Am.* 136, EL256

Sanyal, S., Banerjee, A., Patranabis, A., Banerjee, K., Sengupta, R., & Ghosh, D. (2016). A study on Improvisation in a Musical performance using Multifractal Detrended Cross Correlation Analysis. *Physica A: Statistical Mechanics and its Applications*, *462*, 67-83.

Sengupta, R., Dey, N., Datta, A. K., & Ghosh, D. (2005). Assessment of musical quality of tanpura by fractal-dimensional analysis. *Fractals*, *13*(03), 245-252.

Shimizu, Y. U., Thurner, S., & Ehrenberger, K. (2002). Multifractal spectra as a measure of complexity in human posture. Fractals, 10(01), 103-116.

Shrivastav H. Raga Parichay, Sangeet Sadan Prakashan, Allahabad, 1985

 doi:10.20944/preprints201904.0157.v1

Silver, B., & Burghardt, R. A. (1976). On becoming an Ustād: Six life sketches in the evolution of a Gharānā. *Asian Music*, 27-58.
Su, Z. Y., & Wu, T. (2006). Multifractal analyses of music sequences. *Physica D: Nonlinear Phenomena*, *221*(2), 188-194.
Telesca, L., & Lovallo, M. (2011, June). Revealing competitive behaviours in music by means of the multifractal detrended fluctuation analysis: application to Bach's Sinfonias. In *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences* (p. rspa20110118). The Royal Society.
Voss, R. F., & Clarke, J. (1975). 1/f noise in music and speech. *Nature*, *258*(5533), 317-318.
Weiß, C., Mauch, M., Dixon, S., & Müller, M. (2018). Investigating style evolution of Western classical music: A computational approach. *Musicae Scientiae*, 1029864918757595.
Wright, M., & Wessel, D. (1998). An Improvisation Environment for Generating Rhythmic Structures Based on North Indian" Tal" Patterns. In *ICMC*.
Zhou, W. X. (2008). Multifractal detrended cross-correlation analysis for two nonstationary signals. *Physical Review E*, *77*(6), 066211.
Zlatintsi, A., and Maragos, P. (2013). Multiscale fractal analysis of musical instrument signals with application to recognition. *IEEE Transactions on Audio, Speech, and Language Processing* 21, 737-748.