

1 Article

2 Estimating Wave Direction by Using Terrestrial 3 GNSS Reflectometry

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11 **Abstract:** The signal-to-noise ratio (SNR) data is part of the global navigation satellite systems
12 (GNSS) observables. In a marine environment, the oscillation of the SNR data can be used to derive
13 reflector heights. Since the attenuation of the SNR oscillation is related to the roughness of the sea
14 surface, the significant wave height (SWH) of the water surface can be calculated from the analysis
15 of the attenuation. The attenuation depends additionally on the relation between the coherent and
16 the incoherent part of the scattered power. The latter is a function of the correlation length of the
17 surface waves. Because the correlation length changes with respect to the direction of the line of
18 sight relative to the wave direction, the attenuation must show an anisotropic characteristic. In this
19 work, we present a method to derive the wave direction from the anisotropy of the attenuation of
20 the SNR data. The method is investigated based on simulated data as well by the analysis of
21 experimental data from a GNSS station in the North Sea.

22 **Keywords:** GNSS; Reflectometry; SNR; Wave Direction

23

24 1. Introduction

25 Since more than 20 years reflected GNSS signals are used as a tool to observe diverse
26 environmental conditions by estimating the properties of the reflecting surface. In 1993, Martin-Neira
27 [1] first proposed to use GNSS reflectometry (GNSS-R) in ocean altimetry. Since then, many
28 applications of GNSS-R were developed, reaching from satellite-based sea surface height (SSH)
29 measurements [2], to land-based observation of soil moisture [3] or snow depth [4] and SSH from
30 fixed stations [5] or moving ships [6]. In particular, the interference pattern technique (IPT) has
31 become popular since it uses of-the-shelf equipment, zenith-looking antennas and standard GNSS
32 observables. Therefore, this technique can be applied to data from many GNSS stations of already
33 existing networks.

34 Although all GNSS observables are influenced by multipath and can be used for the estimation
35 of the reflector height [7], IPT relates mostly on the analysis of the oscillating structure of the SNR
36 data because it is less influenced by cycle slips or atmospheric refraction as code or carrier phase data.
37 According to [8] and [9] the frequency of the oscillation is related to the height of the GNSS antenna
38 above a horizontal reflector. The full model [10] also takes into account the attenuation of the SNR
39 signal that depends on the roughness of the reflecting surface. If the reflector is a water surface, the
40 roughness is a measure of the significant wave height (SWH). The authors of [11] have reported that
41 the SWH can be derived from the elevation angle, at which the coherent part of the reflected signal
42 becomes smaller than the incoherent part. It was shown by [12] that the attenuation factor of the
43 oscillating SNR data can be used together with its amplitude and RMS to find the elevation angle ϵ_{coh}
44 at which the coherence is lost, while [13] have shown that the damping factor of the oscillating SNR
45 data can be used to find the significant wave height.

46 The authors of [11] compared their experimental results with data derived from the well-
 47 established Beckmann-Spizzichino model [14] for scattered reflection from a rough surface. This
 48 model describes the mean scattered field as a combination of a coherent and an incoherent part. While
 49 both parts depend on the roughness of the water surface, the incoherent part depends additionally
 50 on the correlation length of the surface. If the correlation length increases, the incoherent part
 51 increases, too. That can, as well as an increased roughness, lead to a domination of the incoherent
 52 part and yield a loss of coherence.

53 Up to now, in the SNR data analysis it was assumed that the correlation length of a sea surface
 54 is isotropic. However, a simple gedankenexperiment shows that the correlation length must depend
 55 on the wave direction and the direction from which the reflection comes: Consider a simple plane
 56 infinite wind-driven wave and define the direction from where the wave comes as the up-wind
 57 direction and its opposite as the down-wind direction. The direction perpendicular to the wave front
 58 is then referred to as the cross-wind direction. If we intersect the surface in the up-wind or down-
 59 wind direction and calculate the autocorrelation of the wave heights along the intersection, we will
 60 find a correlation length that is related to the wavelength. If we intersect the surface in any off-up-
 61 wind or off-down-wind direction, the wavelength along the intersection will become longer due to
 62 the geometrical stretching of the wave height distribution. If the direction of the intersection tends to
 63 the cross-wind direction, the correlation length tends to become infinitely long. Although this model
 64 is far too simple, it shows that it should be possible to derive the direction of waves from the
 65 anisotropy of ϵ_{coh} .

66 The aim of this work is to demonstrate the possibility to derive the direction of waves from the
 67 analysis of GNSS SNR data. In section 2, the basic theory of SNR data analysis and the scattered
 68 reflection model from Beckmann and Spizzichino is explained and discussed. Section 3 presents an
 69 investigation of the suggested method based on simulations of wave fields. In section 4, real data
 70 from a GNSS-equipped tide gauge in the North Sea is analysed and compared to data from a wave
 71 buoy and observations of wind directions. Section 5 concludes our findings.

72 2. Theoretical Background

73 2.1. Analysis of SNR data

74 The analysis in this work bases on the interference of a direct and a reflected signal that creates
 75 a characteristic oscillation in the signal-to-noise ratio (SNR). According to the full model from [10],
 76 the SNR is a combination of the direct and the reflected power related to the noise power. Under the
 77 assumption of a plane reflecting surface, the SNR can be decomposed into a trend and interference
 78 fringes that are attenuated with respect to the elevation angle:

$$\begin{aligned} \text{SNR} &= \text{trend} + \text{attenuation} \cdot \text{oscillation} \\ \text{trend} &= c_0 + c_1 t + c_2 t^2 + \dots \\ \text{attenuation} &= e^{-k^2 d^2 \sin^2 \epsilon}, \quad k = 2\pi/\lambda \\ \text{oscillation} &= \text{Amp} \cdot \cos(4\pi/\lambda h_{\text{ref},t} \sin \epsilon + \phi_0) \end{aligned} \quad (1)$$

79 Here, c_0 , c_1 and c_2 are the unknown parameters of a polynomial trend function of the time t , k is
 80 the wave number, λ is the wave length of the GNSS signal, d is the unknown damping coefficient, ϵ is
 81 the complementary angle of the incidence angle, Amp is the unknown amplitude of the SNR and ϕ_0
 82 is the unknown phase offset of the oscillation. The oscillation is governed by the reflector height $h_{\text{ref},t}$,
 83 which is the height of the antenna above the reflecting surface at the position of the specular point,
 84 which might be variable in time. With h_{APC} as the height of the antenna phase centre (APC) in a certain
 85 height datum and $h_{\text{tide},t}$ as time-variable water surface height in the same height datum, $h_{\text{ref},t}$ can be
 86 described as

$$h_{\text{ref},t} = h_{\text{APC}} - h_{\text{tide},t} + dh_{\text{sphere},t} \quad (2)$$

87 Since specular points can lie in a distance of several hundreds of meters away from the position of
 88 the antenna, the term $dh_{\text{sphere},t}$ corrects for the surface curvature in a spherical approximation. The
 89 complementary angle of the incidence angle ε defers from the elevation angle of the satellite due to the
 90 curvature of the reflecting surface and can be calculated according to [15] and [6]. The tropospheric
 91 refraction can be considered by a correction of ε derived from an astronomic refraction model [16].

92 The unknowns in eq. (1) can be derived from different methods. If h_{APC} and h_{tide} in eq. (2) are
 93 known, a non-linear least-squares adjustment for every satellite can be applied to estimate the
 94 individual unknown parameters. If h_{ref} is likewise unknown, it can be assumed that it is constant for all
 95 satellites observed at the same time. Due to the multimodality of h_{ref} , this parameter can be included in
 96 the non-linear adjustment only if good initial values are available. Otherwise, optimization techniques
 97 might be applied [17].

98 The amplitude at a specific angle ε can be calculated from the attenuation and the amplitude Amp
 99 in eq. (1) as

$$\text{Amp}_\varepsilon = \text{Amp} \cdot e^{-4k^2 d^2 \sin^2 \varepsilon} \quad (3)$$

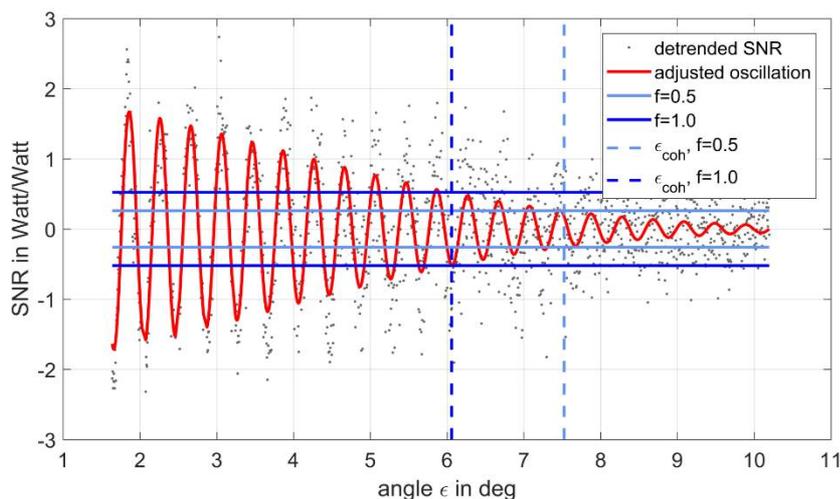
100 While the angle ε increases, Amp_ε becomes small in comparison to the noise of the SNR data and
 101 above a certain angle, it disappears in the noise. It is assumed that this is the cutoff angle ε_{coh} , at which
 102 the coherence is lost. The threshold at which the loss of coherence is assumed is a matter of definition.
 103 We suppose to use a threshold that is related to the standard deviation of the SNR data σ_{SNR} derived
 104 from the non-linear least-squares adjustment multiplied by a factor f . Under these assumptions the
 105 coherence is lost if

$$\text{Amp}_\varepsilon \leq f \cdot \sigma_{\text{SNR}} \quad (4)$$

106 The cutoff angle ε_{coh} is therefore deduced from eq. (3) and (4) as

$$\varepsilon_{\text{coh}} = a \sin \sqrt{\ln(f \cdot \sigma_{\text{SNR}} / \text{Amp}) / (-4k^2 d^2)}, \quad \text{for } f \cdot \sigma_{\text{SNR}} < \text{Amp} \quad (5)$$

107 Figure 1 shows a typical trend-reduced SNR data from the data set used in section 4. We plotted
 108 the threshold for a factor of 0.5 and 1.0 together with the resulting cutoff angle ε_{coh} . The actual value of
 109 the factor f is of minor importance for the investigation of the anisotropic behaviour of the cutoff angle
 110 ε_{coh} as long as it is constant for all satellites involved in the investigation of a particular sea state.



111
 112 **Figure 1.** Detrended SNR data from a GNSS receiver at a tide gauge station (grey dots) for GPS day
 113 190 of 2018, GPS PRN 8. The red line shows the adjusted oscillation, the horizontal continuous blue
 114 lines show the threshold according to eq. (4) for the factor f , vertical dashed blue lines present the
 115 corresponding cutoff angle ε_{coh} with 7.52° for $f=0.5$ and 6.06° for $f=1.0$.

116 2.1. Scattered Reflection

117 According to [11], the scattered reflection from a rough surface can be described by the Beckmann-
 118 Spizzichino model. It should be mentioned here that the description used in this work neglects
 119 shadowing or multiple scattering and can therefore only yield approximated results. Nevertheless, the
 120 model can be used to investigate the fundamental relations between surface roughness, correlation
 121 length and the loss of coherence.

122 We use here the notation from [18]. There, the mean scattered power $\langle E_2 E_2^* \rangle$ of the reflection
 123 from a rough surface is described based on the scattered power $\langle |E_{2ss}|^2 \rangle$ of the reflection from a
 124 smooth perfectly conducting surface as

$$\langle E_2 E_2^* \rangle = \langle |E_{2ss}|^2 \rangle \cdot e^{-g} \left(\rho_0^2 + \frac{\pi T^2 D^2}{A} \sum_{m=1}^{\infty} \frac{g^m}{m! m} e^{-v_{xy}^2 \frac{T^2}{4m}} \right) \quad (6)$$

125 where

$$g = \left(2\pi \frac{\sigma_h}{\lambda} (\cos \theta_i + \cos \theta_r) \right)^2$$

$$\rho_0 = \sin c(v_x X) \sin c(v_y Y)$$

$$D = \left(\frac{1 + \cos \theta_i \cos \theta_r - \sin \theta_i \sin \theta_r \cos \phi_r}{\cos \theta_i (\cos \theta_i + \cos \theta_r)} \right)$$

$$v_x = k(\sin \theta_i - \sin \theta_r \cos \phi_r) x$$

$$v_y = k(\sin \theta_r \sin \phi_r) y$$

$$v_{xy} = \sqrt{v_x^2 + v_y^2}$$

126 Here, σ_h is the standard deviation of the surface heights. For water surfaces, it is assumed to be
 127 approximately a quarter of the SWH [19]. Furthermore, λ is the wavelength of the GNSS signal, T is
 128 the correlation length of the reflecting surface, θ_i is the incidence angle with $\theta_i = 90^\circ - \varepsilon$, θ_r is the
 129 reflecting angle, ϕ_r is the azimuth of the reflection, X and Y are the dimensions of the reflecting area
 130 A into the coordinate direction x and y and k is again the wave number of the GNSS signal. We are
 131 interested in the scattered reflection into the direction of a specular reflection. Therefore, the incidence
 132 angle is equal to the reflecting angle and the azimuth of the reflection becomes zero. Since v_x and v_y
 133 become zero and D is 1 for that case, eq. (6) simplifies to

$$\langle E_2 E_2^* \rangle = \langle |E_{2ss}|^2 \rangle \cdot e^{-g} \left(1 + \frac{\pi T^2}{A} \sum_{m=1}^{\infty} \frac{g^m}{m! m} \right) \quad (7)$$

134 Similar to [11], the first term in the bracket in eq. (7) governs the coherent part while the second
 135 term governs the incoherent part. Hence, if the term

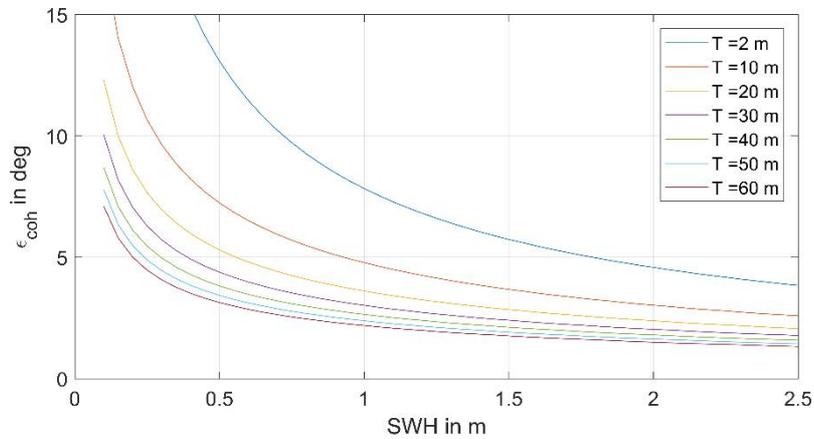
$$\text{incoh} = \frac{\pi T^2}{A} \sum_{m=1}^{\infty} \frac{g^m}{m! m} \quad (8)$$

136 becomes larger than the term $\text{coh} = 1$, the incoherent part dominates the mean scattered power. If the
 137 relation between incoh and coh reaches a particular value, the corresponding angle ε can be stated as
 138 the cutoff angle ε_{coh} . In accordance with [11] we use a value of 1 for the relation in this work.

139 The term incoh depends partly on the geometry of the scattered reflection since A is set as the
 140 area of the Fresnel zone. This depends on the reflector height and the angle ε , which also governs
 141 parameter g . Additionally, incoh depends on the sea state because eq. (8) contains the standard
 142 deviation of the water surface heights and the correlation length T of the surface. There is no firm
 143 definition for a value of the correlation length but it can be stated as the distance at which the
 144 correlation coefficient of the autocorrelation function of the surface height falls below a certain
 145 threshold. For real sea surfaces, the surface heights are mainly a result of the wind influence. Hence,
 146 the correlation length will be correlated with SWH. It is clear that the correlation length cannot
 147 become zero for such surfaces.

148 Figure 2 presents the resulting cutoff angle ε_{coh} for correlation lengths between 2 to 60 m and for
 149 different SWH at a reflector height of 12.3 m. It can be seen that for a particular SWH the cutoff angle
 150 ε_{coh} reaches values of remarkable differences in dependence of the correlation length T . Hence, it

151 should be possible to derive the wave direction if the cutoff angle can be derived from eq. (5)
 152 in several azimuthal directions and if the differences in the correlation length are large enough in these
 153 azimuthal directions. To clarify the range of the correlation length for sea surfaces, simulated wave
 154 fields will be used in section 3.



155
 156 **Figure 2.** Cutoff angle ϵ_{coh} as a function of different correlation lengths T and significant wave heights
 157 SWH. The cutoff angle was derived for $\text{incoh} = 1$.

158 3. Simulations

159 3.1. Waves

160 Wave fields of the sea surface from the real world are rarely available and do not allow to control
 161 the influencing parameters. Therefore, simulations of a wave field can be used for investigations, but
 162 the example of a plane wave field from the introduction is much too simple to allow for the
 163 exploration of the range of the correlation length. Hence, more realistic wave fields of the sea surface
 164 must be constructed.

165 Simulations of the three-dimensional height distribution of a sea surface are needed for scientific
 166 purposes, for example for the research of the interaction simulation of underwater gravity aided
 167 inertial navigation system [20] or computer graphics programming, for example to provide visual
 168 effects in print media or films [21]. Here, we used a model that take into account the stochastic but
 169 directional nature of the wind-driven waves by representing them as Gaussian stationary and ergodic
 170 processes. Consequently, they can be calculated as an infinite sum of simple cosine waves that
 171 propagate into azimuthal directions with variable amplitudes, frequencies and initial phases:

$$H(x, y) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} A_{i,j} \cos(k_i x \cos \theta_j + k_i y \sin \theta_j + \varphi_{ij}) \quad (9)$$

172 Here, k_i is the deep water wave number at angular frequency ω_i , while g is the gravitational
 173 acceleration. θ_j is the direction of the elementary wave and φ_{ij} is a random initial phase. The
 174 amplitude $A_{i,j}$ can be derived approximately from a directional wave spectrum as [22]

$$A_{i,j} \approx \sqrt{2S(\omega_i, \theta_j) \Delta\omega_i \Delta\theta_j} \quad (10)$$

175 where $\Delta\omega_i$ is an increment of ω_i and $\Delta\theta_j$ is that one of θ_j . $S(\omega_i, \theta_j)$ is the directional power
 176 spectrum that is composed by a power spectrum $S(\omega_i)$ and a directionality function $D(\theta_j)$ as

$$S(\omega_i, \theta_j) = S(\omega_i)D(\theta_j) \quad (11)$$

177 From the manifold of available power spectra, we used the JONSWAP spectrum [23] that is a
 178 modification of the Pierson-Moskowitz spectrum [24] for fetch-limited scenarios. This spectrum is

179 based on observation at the North Sea, the area of the experimental data that will be used in section
180 4. Here, we applied the formulation based on SWH and the wave peak period T_p [19].

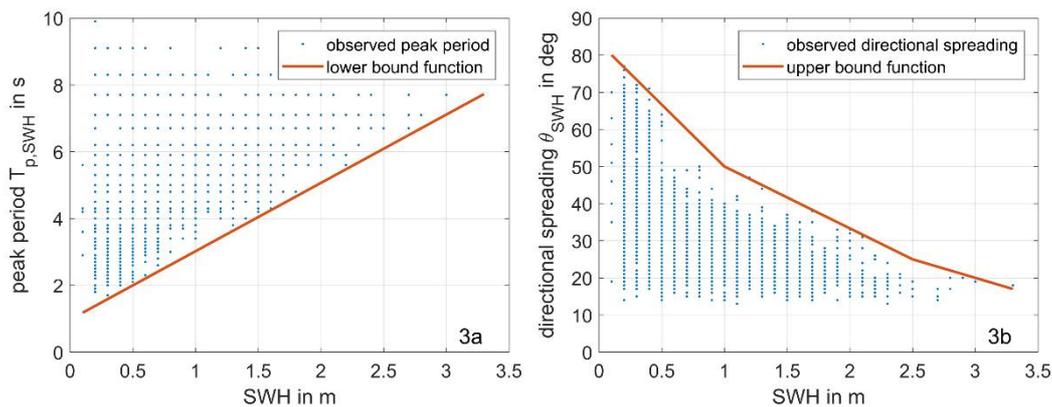
$$S(\omega) = A_\gamma \frac{5}{16} \text{SWH}^2 \omega_p^4 \omega^{-5} \exp\left(-\frac{5}{4} \left(\frac{\omega}{\omega_p}\right)^4\right) \gamma^{\exp\left(-0.5 \left(\frac{\omega - \omega_p}{\sigma \omega_p}\right)^2\right)} \quad (12)$$

181 where $\omega_p = 2\pi/T_p$, $A_\gamma = 1 - 0.287 \ln \gamma$ and $\gamma = 3.3$. The spectral width parameter σ must be
182 taken as 0.07 if $\omega \leq \omega_p$ or 0.09 if $\omega > \omega_p$. We used the directional function proposed by ITTC
183 (International Towing Tank Conference) [20]

$$D(\theta_j) = \frac{2}{\pi} \cos^2 \theta_j, \quad |\theta_j| \leq \frac{\pi}{2} \quad (13)$$

184 The simulations were carried out for $0.1 \leq \omega_i \leq 6.1$ Hz with an increment of $\Delta\omega_i = 0.2$ Hz,
185 whereby the main wave direction was set to zero, resulting in a west-to-east down-wind direction.
186 Hence, θ_j in eq. (9) and (13) are the directions of the spread of the wave. The increment $\Delta\theta_i$ was
187 set as one-tenth of the range from a minimal and a maximal wave direction θ_{\min} and θ_{\max} .

188 To provide realistic simulations, the parameters for wave peak period T_p as well as for θ_{\min}
189 and θ_{\max} were derived from real data observed over a period of two months at the wave buoy
190 ElbeWR in the North Sea at the Outer Elbe that was deployed by the German Federal Maritime and
191 Hydrographic Agency (Bundesamt für Seeschifffahrt und Hydrographie, BSH). Besides other data,
192 the buoy provides SWH, peak periods, wave principal directions and the wave directional spreading
193 with a temporal resolution of 30 minutes. Figure 3 shows observed data for peak periods (a) and
194 directional spreading (b). The peak period does not fall below a certain value for a specific SWH.
195 Hence, a linear function was fitted to derive the wave peak period $T_{p,\text{SWH}}$ as a function of SWH, which
196 will be used for the simulations. Likewise, the directional spreading does rarely exceed the plotted
197 upper bound from which a piecewise linear function is derived. The directional spreading θ_{SWH}
198 resulting from this function is used to define the minimal and a maximal wave direction as
199 $\theta_{\min} = -\theta_{\text{SWH}}/2$ and $\theta_{\max} = \theta_{\text{SWH}}/2$.



200

201 **Figure 3.** Observed peak period (a) and directional spreading (b) plotted over SWH for the wave buoy
202 ElbeWR. Red lines show the functions used to derive T_p and θ_{\min} and θ_{\max} for simulations.

203 The wave field was simulated for a 1x1 m grid with an extent of 1000 m for both x and y direction.
204 The range of SWH was set to be between 0.1 m and 2.5 m with an increment of 0.2 m. To avoid too
205 smooth surfaces for small SWH values, we added a normally distributed value with a mean of zero
206 and a standard deviation of $s = 5$ cm. Due to this and since the initial phase ϕ_{ij} from eq. (9) was
207 introduced as an evenly distributed random value within the range of 0 to 2π , the simulated wave

208 field is a random result. For every SWH, we carried out 100 simulations. Figure 4 presents an
 209 arbitrarily selected wave field derived for a SHW of 2.5 m.

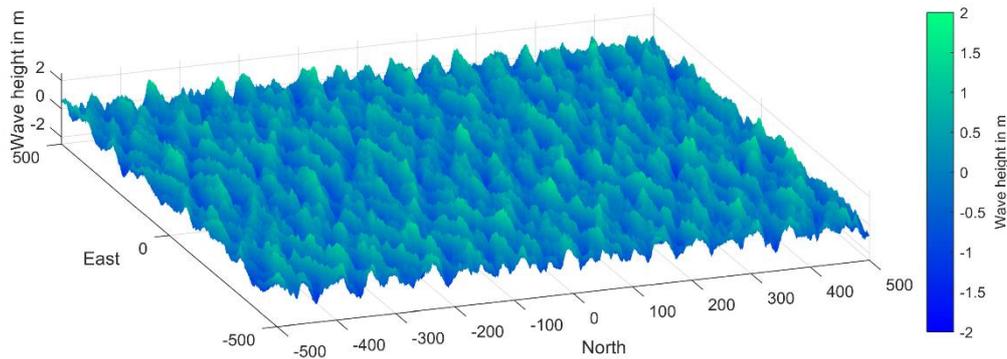
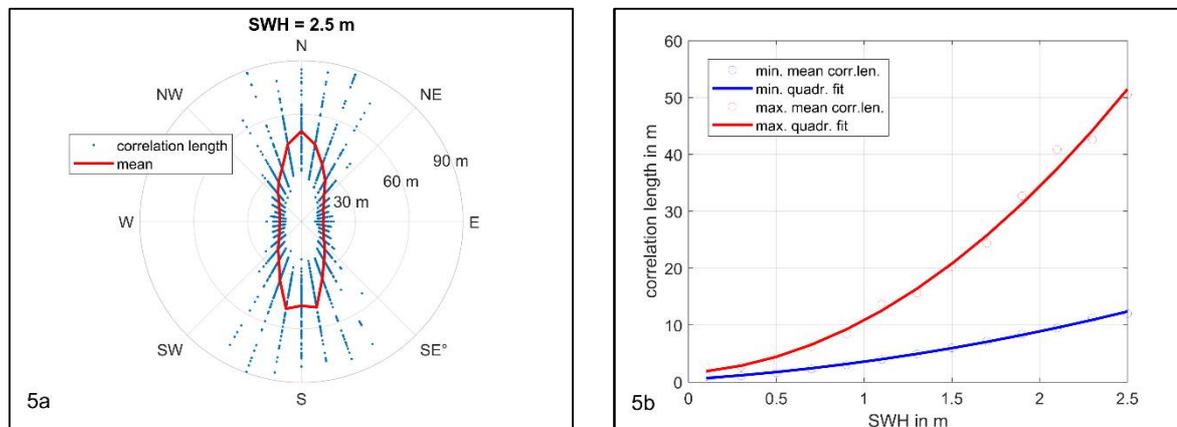


Figure 4. Simulated wave field for SWH = 2.5 m

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212 3.2. Calculation of Wave Direction

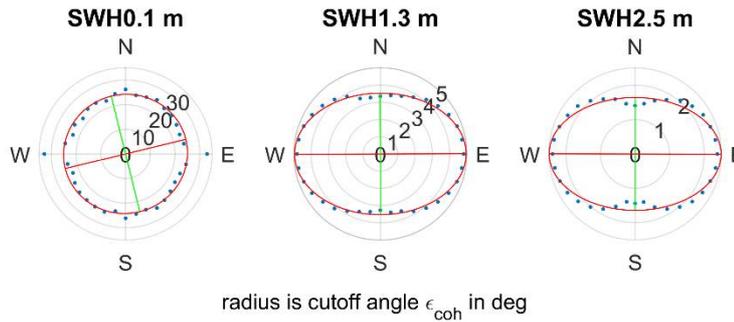
213 For every simulated wave field, we calculate the autocorrelation function for different azimuthal
 214 directions. To do so, we interpolated the wave heights along intersections in azimuth ranging from
 215 0° to 350° with an increment of 10° . Since the autocorrelation function show a behaviour of a damped
 216 cosine function [25], we defined the correlation length as the distance at which the autocorrelation
 217 becomes zero for the first time. The resulting 100 correlation lengths for a SWH of 2.5 m are plotted
 218 in Figure 5a together with the average value for the specific azimuth. The maximum and minimum
 219 average correlation length for all SWH are presented in Figure 5b. For both minimum and maximum
 220 values, a quadratic function was fitted, that allows to calculate the correlation length with respect to
 221 SWH.



222
 223
 224
 225

Figure 5. Correlation lengths for 100 simulated wave fields for a SWH of 2.5 (a). The red line shows the average correlation length for the corresponding azimuth. Minimum and maximum of the mean correlation length for the specific SWH and their quadratic fit (b).

226 The resulting average correlation length were used to calculate the cutoff angle ϵ_{coh} according to
 227 eq. (8) and the explanation from the previous section. Figure 6 shows the cutoff angles for the different
 228 azimuthal directions for SWH values of 0.1 m, 1.3 m and 2.5 m. The cutoff angles show a clear
 229 anisotropic behaviour. We estimated the semi-minor and semi-major axes and the azimuth of the
 230 semi-major axis of a fitting ellipse from least-squares adjustment. The difference of the axes' length
 231 for the first ellipse at a SWH of 0.1 m is not significant. Hence, the direction of the semi-major axis
 232 defers from the down-wind direction. For all $\text{SWH} \geq 0.3$ m, the difference of the axes' length is
 233 significant and the semi-major axes coincides with the down-wind direction.



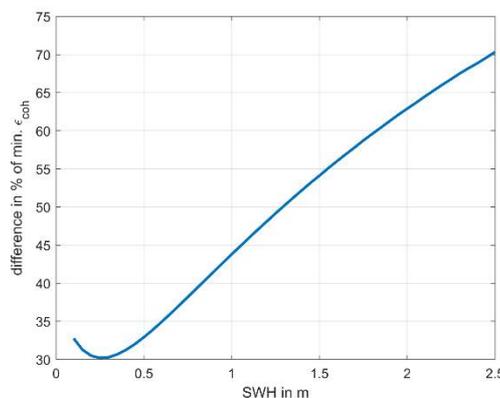
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235 **Figure 6.** Cutoff angles for three SWH values and different azimuth (blue dots). Ellipses in red present
 236 the least-squares fit with their semi-major (red line) and semi-minor (green line) axes. The axes at
 237 SHW of 0.1 m do not show a significant difference. All other semi-major axes coincide with the west-
 238 to-east down-wind direction.

239 The cutoff angle ϵ_{coh} can be calculated also for the minimal and maximal correlation length
 240 function presented in Figure 5b. As mentioned above, a random value with a standard deviation s of
 241 5 cm was added to the heights of the simulated wave field. This will have a larger influence on the
 242 wave fields with smaller SWH values. We took this into account by calculating σ_h in eq. (6) as of

$$\sigma_h \approx \sqrt{(\text{SWH} / 4)^2 + s^2} \quad (14)$$

243 The difference between the maximal and minimal cutoff angle can be related to the minimal
 244 cutoff angle. Figure 7 shows the according differences in percent. The differences are more
 245 pronounced for increased SWH. This can imply a more significant estimation of the wave direction
 246 for higher SWH and explains the discrepancy of the down-wind direction and the semi-major axis
 247 for a SHW of 0.1 m in Figure 6.



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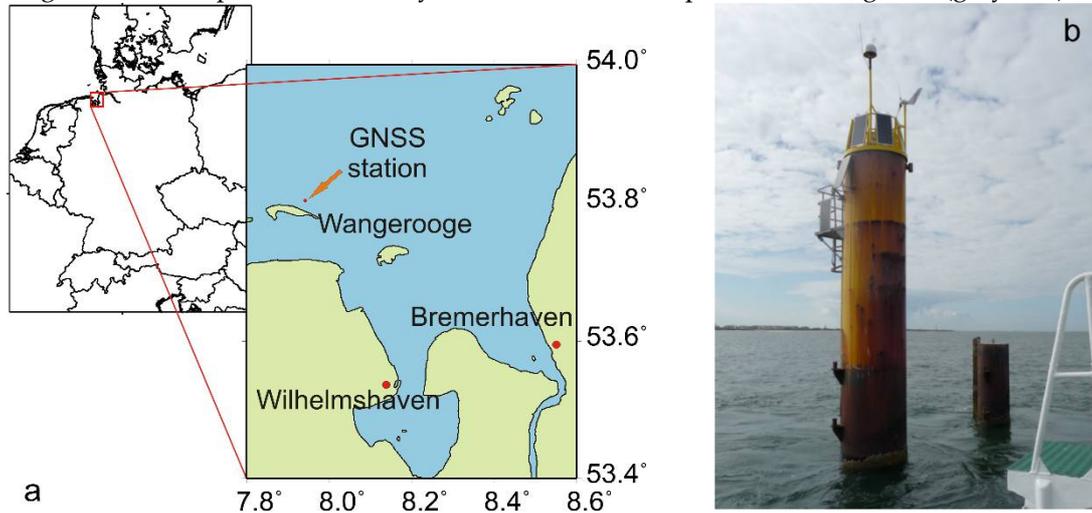
249 **Figure 7.** Differences of the maximum and minimum cutoff angles in relation to the minimum cutoff
 250 angle plotted over SWH. The difference is becoming larger and more pronounced with increasing
 251 SWH values.

252 Hence, it seems to be possible to derive wave directions from the analysis of the anisotropy of
 253 the cutoff angles ϵ_{coh} . At least for the simulated data, the semi-major axis coincides with down-wind
 254 direction. If this holds for real data too will be verified in section 4.

255 4. Validation With Experimental Data

256 A one-month GNSS data set for validation was collected from a Leica GR10 receiver and a Leica
 257 AR25.R3 antenna during July 2018 (GPS day of year 185 to 216). This equipment is operated by the
 258 German Federal Institute of Hydrology (Bundesanstalt für Gewässerkunde, BfG) and is installed atop
 259 of a pile at the tide gauge station TGW2 (Figure 8) approximately 1.7 km north of the coast of the

260 island of Wangerooge in the North Sea. The tide gauge station is located in a distance of about 24 km
 261 from the wave buoy ElbeWR in the North Sea that was mentioned in section 3. For purpose of
 262 comparison, the SWH observed at the buoy was corrected for the differences between the modelled
 263 SWH at the buoy and the tide gauge. The differences were derived from the numerical wave model
 264 CWAM of the German Weather Service (Deutscher Wetterdienst, DWD) (Kieser et al. 2013). The
 265 resulting SWH for the period covered by the GNSS data set is presented in Figure 9 (grey line).



266

Figure 8. Position of GNSS station north of the island of Wangerooge (a) and GNSS antenna installed atop of the tide gauge station TGW2 in the North (b) (photo BfG).

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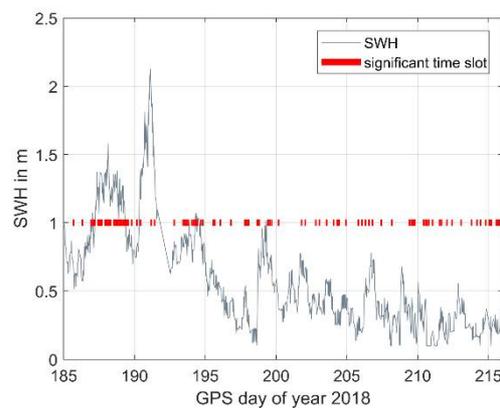
269 The distance from the APC to tide gauge zero was taken from information provided by BfG. The
 270 tide gauge readings with respect to the same tide gauge zero were achieved from freely available
 271 data of the German Federal Waterways and Shipping Administration (Wasserstraßen- und
 272 Schifffahrtsverwaltung des Bundes, WSV). Hence, the reflector height in eq. (1), ranging between
 273 about 10.1 m and 14.4 m, is known at all observation epochs and must not be deduced from GNSS
 274 SNR data. The GNSS data from GPS and GLONASS was collected in a sample rate of 1 second.
 275 Additional weather data was provided by the DWD from the close-by weather station Alte Weser
 276 Lighthouse.

277 The GNSS data for every satellite was split into ascending and descending tracks. To avoid
 278 influences from the shore of the island, only data with elevation angles over 1° were used. The
 279 attenuation of the SNR signal of this antenna type shows a strong degradation for elevation angles
 280 above about 10° (see Figure 1). Therefore, we restricted the data set for elevation angles below
 281 10° . All elevation angles were corrected for atmospheric refraction and curvature of the reflecting surface
 282 as mentioned in section 2. To allow for a good coverage of the horizon, we binned the data into 3h
 283 time slots. Since the evolution of the sea state is commonly a slow process, this bin size seems to be
 284 reasonable. Data sets were assigned to the time slots according to the time of their mid epochs to
 285 avoid cutting of data sets overlapping the bound of the time slots. Satellites tracks with elevation
 286 ranges of less than 3° were excluded from the analysis

287 In total, 252 time slots were analyzed with an average number of about 25 assigned satellite
 288 tracks per time slot. The unknown parameters of a polynomial trend function together with the
 289 amplitude Amp , the phase offset ϕ_0 and the damping coefficient d from eq. (1) were estimated from
 290 a least-squares adjustment individually for every satellite involved. We applied the Levenberg–
 291 Marquardt algorithm for the non-linear optimization problem to avoid divergence due to possible
 292 insufficient initial values for the damping coefficient.

293 The cutoff angles ε_{coh} were then calculated according to eq. (5), whereby a factor f of 1.0 was
 294 used. Likewise, the standard deviation of the cutoff angles was derived from a covariance
 295 propagation of the results from the least-squares adjustment. All resulting cutoff angles together with
 296 their corresponding azimuth assigned to the same time slot were then used to fit an ellipse by means
 297 of a weighted least-squares adjustment, while the weights were derived from the standard deviation

298 of the cutoff angles. For about 36 % of all time slots the adjustment yield significant differences of the
 299 semi-major and semi-minor axes of the ellipses. Figure 9 shows the SWH at tide gauge together with
 300 significant time slots in red (at SHW=1 m). About 73 % of the time slots with significant differences
 301 show an average SWH of more than 0.3 m, while about 61 % of the time slots with insignificant
 302 differences show an average SWH of less than 0.5 m. On GPS days 191 and 192 the SWH reached
 303 values of more than 2 m but only some slots of these days show significant results. This seems to
 304 contradict the finding from the simulation in section 3, that for higher SWH the results will be
 305 significant. However, in reality the oscillation of GNSS SNR observations at low elevation angles
 306 might become noisier due to shadowing effects or stronger tropospheric refraction. For the case of
 307 the tide gauge in the vicinity to the coast of the island, also breaking waves or converted shallow
 308 water waves might be included in the data set. An identification of such influence based on the
 309 existing data was not possible.

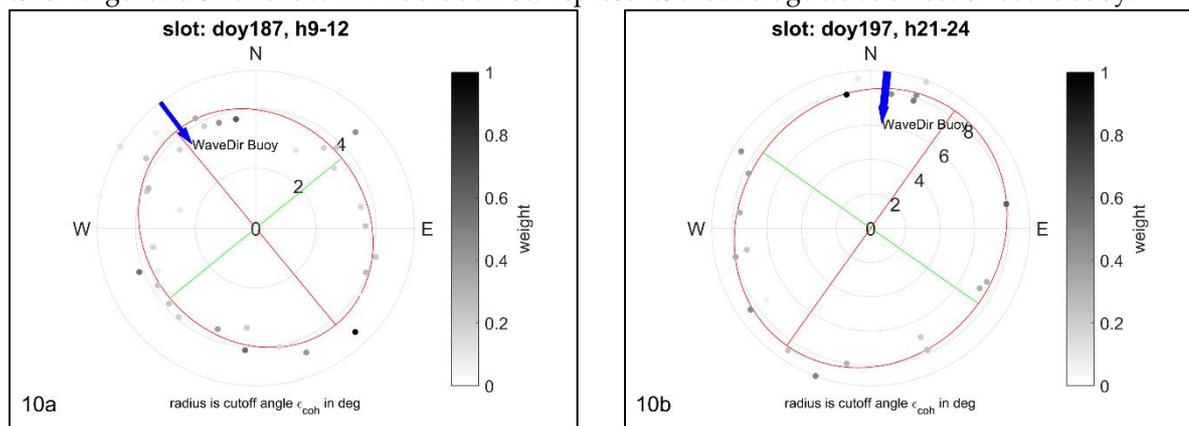


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Figure 9. SWH at tide gauge (grey) and significant time slots (red).

312 The resulting azimuthal directions of the major-semi axes can be compared to the average wave
 313 direction calculated for the time slots from the observations at the wave buoy. It must be mentioned
 314 that the directions of the major-semi axes do not allow distinguishing between down-wind or up-
 315 wind direction. Hence, the resulting azimuths are ambiguous by 180° . Figure 10 shows two typical
 316 results for larger and smaller SWH. The blue arrow represents the average wave direction at the buoy.



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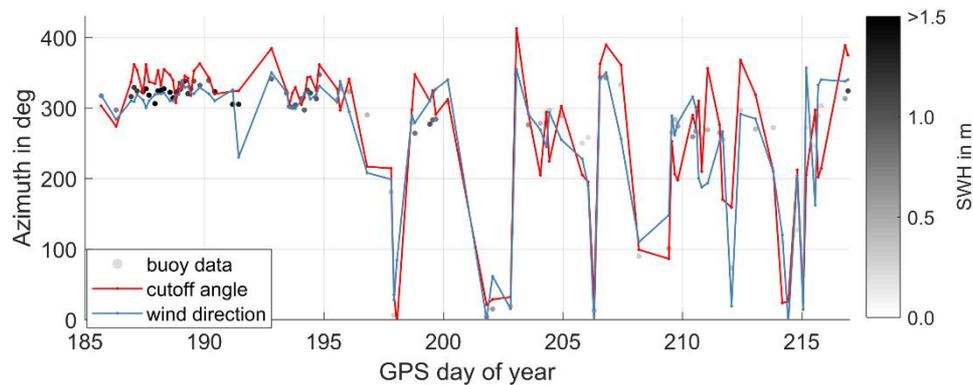
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Figure 10. Cutoff angles (grey dots, shading with respect to their weights), adjusted ellipse and average wave direction derived from buoy data, (a) SWH = 1.2 m, (b) SWH = 0.35 m.

320 All resulting azimuths were compared to the wave direction from the buoy. The down-wind or
 321 up-wind direction was calculated from the azimuths by adding or subtracting 180° so that the
 322 differences to the wave direction from the buoy become minimal. We carried out the same calculation
 323 for the wind direction that was taken from the mentioned weather data at the Alte Weser Lighthouse.
 324 Figure 11 shows a comparison of the data sets for all significant time slots. In particular, for larger
 325 SWH the similarity of the results from the analysis of the cutoff angle and the wave direction from

326 the buoy is obvious. It should be pointed out here, that the wave directions from the buoy show a
 327 spreading between 20° and 80° , depending on the SWH as mentioned in section 3.



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Figure 11. Wave direction from the buoy (grey dots, shading with respect to SWH), azimuth of major-semi axis derived from the analysis of the cutoff angles (red) and wind direction (blue).

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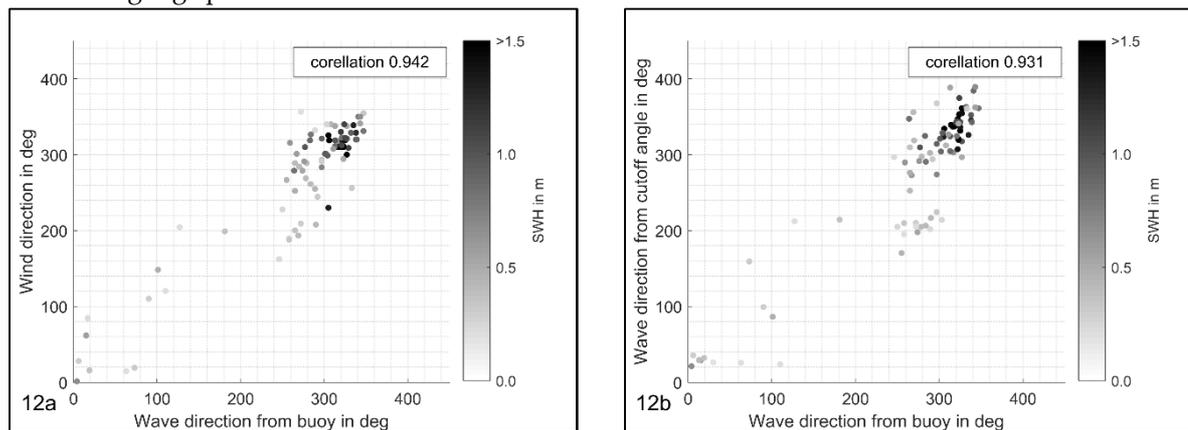
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The good agreement of the directions is emphasized by the high correlation between the data sets presented in Figure 12. The correlation coefficient between the wave direction from the buoy and the wind direction is about 0.94, while for the wave direction from the cutoff angles it reaches 0.93. It is remarkable that in both data plots a cluster of off-correlation values at a wave direction of about 290° occur. That indicates a possible discrepancy between the wave directions observed at the buoy and at the tide gauge position.



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Figure 12. Correlation between the wave direction from the buoy and the wind direction (a) and the wave direction from the buoy and the wave direction from the cutoff angles (b). The shading of the dots is with respect to SWH.

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5. Conclusions

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Besides its roughness, the correlation length of a sea surface is of major importance in the analysis of the interference pattern of GNSS SNR data. In general, higher SWH values yield a larger correlation length what results in an increase of the incoherent part of the mean scattered power from a reflecting water surface. Hence, the cutoff angles, the elevation angles at which the coherence is lost, become smaller with increased correlation length.

It is reasonable to assume that the correlation length of the waves of water surfaces depend on the direction of the line of sight. For directions that tend to the cross-wind direction the correlation length will be longer than for directions close to down-wind or up-wind direction. Therefore, it should be possible to notice the variation in the correlation length likewise in the variation of cutoff angles. In reverse, it should be possible to derive the wave directions from the variation of the cutoff angles if the variation of the correlation length is sufficiently large

353 It was shown in this work that for simulated but realistic directional wave fields the correlation
354 length vary strongly with the azimuth relative to the wave direction. Calculating the cutoff angles
355 under consideration of the anisotropy of the correlation length and estimating a fitted ellipse allows
356 to derive the wave direction from the azimuth of the semi-major axis. For smaller SWH the differences
357 of the semi-major and semi-minor axes might become insignificant, yielding incorrect wave
358 directions. For larger SWH these differences will at least theoretically be significant.

359 The findings from the simulations were verified and largely confirmed by the analysis of data
360 from a GNSS station in the North Sea. For significant results, the correlation to the wave direction
361 observed by a buoy is high and on the same level as the correlation between the wind direction and
362 the wave direction. The significance of the results might be improved if a reliable and automated
363 detection of corrupted data would be applicable.

364
365 **Data Availability Statement:** The GNSS data used in this study are available from German Federal Institute of
366 Hydrology (Bundesanstalt für Gewässerkunde, BfG) but restrictions apply to the availability of these data, which
367 were used under license for the current study, and so are not publicly available. Data are however available from
368 the corresponding author upon reasonable request and with permission of BfG.

369 The tide gauge readings used in this study are available from German Federal Waterways and Shipping
370 Administration (Wasserstraßen- und Schifffahrtsverwaltung des Bundes, WSV). The data are available from the
371 corresponding author upon reasonable request.

372 The weather data used in this study are available from German Weather Service (Deutscher Wetterdienst,
373 DWD). The data are available from the corresponding author upon reasonable request.

374 The data from the buoy and the SWH data from the model used in this study are available from German
375 Federal Maritime and Hydrographic Agency (Bundesamt für Seeschifffahrt und Hydrographie, BSH) and
376 German Weather Service (Deutscher Wetterdienst, DWD) but restrictions apply to the availability of these data,
377 which were used under license for the current study, and so are not publicly available. Data are however
378 available from the corresponding author upon reasonable request and with permission of BSH and DWD.

379 **Author Contributions:** J.R. had full access to all the data in the study and takes responsibility for the integrity
380 of the data and the accuracy of the data analysis. J.R. designed and performed research, simulated and analysed
381 data, J.R. and O.R. designed SNR analysis software and modelled SWH data, J.R. wrote the paper, O.R. and G.E.-
382 T. revised the article critically and approved the final version.

383 **Conflicts of Interest:** The authors declare no conflict of interest.

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