

Performance Comparing and Analysis for Slot Allocation Model

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Abstract: The purpose of this study was to ascertain the effect of weight setting of objectives on displacement and implementation difficulty in slot allocation model. A linear integer programming model including three kinds of evaluation objectives with different weight is designed to compare and analysis displacement and implementation difficulty in slot allocation model. The average difficulty is very sensitive to the average displacement. The difficulty of implementation can be significantly reduced by weight setting with a little increase of displacement. Movements list descent according to priority or not have great impact on displacement and implementation difficulty in slot allocation model. Capacity is a key factor affecting displacement and implementation difficulties. The difficulty index proposed in this work is very useful in identifying which slot allocation scheme is better for decision maker. Our research can promote regulator to upgrade slot allocation policies.

Keywords Slot Allocation, Performance Comparing, Implementation Difficulty, Linear Integer Programming

1 Introduction

Most of the busiest airports worldwide experience serious congestion and delay problems, such as Beijing Capital International Airport (PEK), Shanghai Pudong International Airport (PVG), and Guangzhou Baiyun International Airport (CAN), tend to be severely congested ([Wu et al., 2018](#)). The experienced imbalance between supply and demand for air transport services forces all aviation stakeholders to drastically rethink airport capacity and its utilization and readdress the issue of experienced or anticipated capacity shortages ([Zografos et al., 2013](#)). Results of past research have proved that demand management could provide significant benefits at busy worldwide airports by permitting large delay reductions through limited interference with airline competitive scheduling ([Jacquillat and Odoni, 2015b](#)) or by evenly small substantial increase in declared capacity ([Zografos et al., 2012](#)). However, the latter aiming to build new capacity are capital intensive solutions, requiring significant implementation time, and are often subject to heated political debates. The need for an immediate relief to seriously congested airports calls for short to medium-term, demand-side solutions that are based on the optimum allocation and use of available airport capacity ([Zografos et al., 2017](#)). To control over-capacity scheduling, the most common demand management schemes fall into two categories: (i) approaches introducing market-driven or pure economic instruments (e.g., slot trading, auctions, congestion pricing) aiming to allocate capacity among competing users by considering real market (or approximations of) valuations of access to congested airport facilities ([Basso and Zhang, 2010](#), [Verhoef, 2010](#), [Zhang and Zhang, 2010](#), [Czerny and Zhang, 2011](#), [Le et al., 2008](#), [Vaze and Barnhart, 2012](#), [Grunewald, 2016](#)). (ii) efforts aiming to improve the efficiency by using administrative allocation mechanism ([Zografos et al., 2012](#), [Jacquillat and Odoni, 2013](#), [Pyrgiotis and Odoni, 2016](#), [Ribeiro et al., 2018](#), [Zografos et al., 2018](#), [Zografos et al., 2017](#), [Jacquillat and Odoni, 2015b](#), [Jacquillat and Odoni, 2015a](#), [Jacquillat and Odoni, 2018](#), [Benlic, 2018](#), [Zografos et al., 2013](#)).

In order to control the excessive demand of airports, Chinese Civil Aviation (CAA) has enacted slot regulation since 2010, but due to various reasons, satisfactory effect has not been achieved. Consequently, the

new slot regulation of CAA similar to that of International Air Transport Association (IATA) is introduced in April 2018. Although the Chinese airport flight delay has been improved after the implementation of these slot management methods, the unreasonable slot structure still exists (Li et al., 2016, Sun and Su, 2013, Hu et al., 2019). As the analysis of these articles indicated, the typical characteristics of slot structure at large Chinese airports are that the departure of flights is concentrated in the morning and the arrival of flights is concentrated in the evening. Because of this unreasonable schedule, the low punctuality rate of flights in the morning and evening rush hours is doomed. The inefficiency of slot allocation also exists in other countries (Picard et al., 2019). So, in order to further improve the arrival and departure punctuality rate, it is still necessary to further reasonably adjust the slot structure while accurately assesses and uses declared capacity.

A practical implication of the current slot allocation process is that it may schedule a slot request far away from its initially requested time interval. This, in turn, may decrease substantially the actual utility realized by the relevant airline in using the allocated slot, hence resulting in poor (if any at all) slot utilisation rates. Just as Zografos et al. (2018) pointed out that most existing models of flight scheduling typically do not consider acceptability of slot schedules. The more difficult the slot displacement is, the less likely the flight will accept the displacement. On the contrary, the less difficult the slot displacement is, the more likely the flight will accept it. Therefore, implementing difficulty index is proposed in this work instead of slot allocation acceptance index, which is constructed into the model and examined together with displacement and priority. Another key question is how to assure lower displacement for movements with higher priority? Is it possible that high priority own a low probability of displacement by our algorithm? Can priority be considered as the cost of adjusting the unit time to ensure that the high priority movement has a low probability of being adjusted (HPLA)? What are the differences in performance indicators when priority is batched into a computer program in priority order or in the order of morning to night (final timetable presentation) when priority is considered as the cost of adjusting the unit time? All these questions have not been investigated in the previous articles will be discussed or answered within this work.

The main contribution of this paper is three folds. The first is that the difficulty index of slot displacement for quantifying implementation difficulties of each movement is proposed. Secondly, a multi-objectives linear integer programming model and algorithm are developed to minimize the total compound cost of slot allocation. Thirdly, we investigated how to reduce the displacement amount and implementation difficulty while guarantee HPLA.

The remainder of this paper is organized as follows. Section 2 formulates the model, including the technical aspects of capturing the CAA regulation and difficulty in optimizing the allocation of slots. Section 3 is the method of solving problems. Section 4 presents the test and simulation results and their implications for schedule coordination practice. Section 5 summarizes our work and indicates directions for future research.

2 Proposed slot displacement models

Before describing the model, notations are stated as follows:

p_m : Priority of movement m .

f_m^D : Flight implementation difficulties

I_m^D : Flight difficulty index of one displacement unit

τ_m : Interval movement m required.

t_m : Interval movement m is scheduled.

n : Seats of flights corresponding movement m .

π_m : The average flight elapse time of movement m , to (arrival flight) or from (departure flight) this airport.

L_a : Coordination level parameters of this airport (main coordinator, auxiliary coordinated airport and uncoordinated airport; 7, 4, 1).

L_b : Coordination level parameters of associated airports.

x_m^t : { 0, 1 }, if movement m is scheduled at t interval.

C_{60} : Hourly capacity constraint;

C_{15} : 15-minute capacity constraint;

C_5 : 5-minute capacity constraint;

b_{me} : Corridor capacity constraints, $e = (1, 2 \dots E)$.

a_m^d : Movement m plan to operate on day d of a series day, usually series day expressed as [1, 2, 3, 4, 5, 6, 7]. a_m^d is set mandatorily as 1, this mean the same flight operating in one day have two movement number, such as m_1 and m_2 .

b_{mc} : The amount of a kind of capacity c consumed by movement m . In our model, c may be hourly capacity, 15-minute capacity, 5-minute capacity, or corridor capacity. In our model, the amount of each capacity consumed by a movement is 1, this mean all of $b_{mc_{60}}$, $b_{mc_{15}}$, b_{mc_5} and b_{me} are set as 1.

n : Number of seats.

The coordination time interval represents the unit of time (e.g., 5-min, 15-min, 60-min) used as the basis for capacity determination and slot allocation. Usually each time interval contains multiple slots. A movement corresponds to a takeoff or landing activity. A slot refers specifically to the interval occupied by one movement.

We are motivated by the idea that the less difficult it is to adjust slot, the easier it will be accepted; the more difficult it is to adjust slot, the harder it will be accepted. Therefore, the problem of slot displacement is transformed into the problem of minimizing the difficulty of slot displacement subjecting to the priority rule stipulated by CAA slot regulations and other operational constraints.

2.1 Difficulty index and difficulty of Displacement

Difficulty, f_m^D , is inversely proportional to flight time π_m , and is proportional to slot displacement. The shorter the flight distance, the easier the passenger chooses other modes of transport. The longer the flight distance, the less likely the passenger will switch to other modes of transport. In addition, the shorter the flight distance, the smaller the variation of elapse time, the longer the flight distance, the larger the variation of elapse time. The greater the flight distance, the more time the aircraft could gain or kill by adjusting its speed during flight ([Han et al., 2010](#), [Richard et al., 2011](#), [De Smedt and Berz, 2007](#), [Tang and Han, 2012](#), [Wang and Li, 2011](#), [Brooker, 2009](#), [Durand et al., 2010](#), [Han et al., 2017](#), [Rey et al., 2012](#), [Rosenow et al., 2019](#), [Haraldsdottir et al., 2007](#), [Cafieri and D'Ambrosio, 2017](#), [De Smedt and Putz, 2009](#), [Rezaee and Izadpanah, 2009](#)). Difficulty is subject to the coordination level of the connected airport. The larger the number of the coordination level of the connected airport, the more difficult the slot coordination is. On the contrary, the smaller the number, the easier the slot coordination is.

Flight implementation difficulties index I_m^D and difficulty f_m^D brought by difficulty expressed as follow:

$$I_m^D = \left[\left(\frac{n}{\pi_m} \right)^{\frac{1}{2}} \cdot (L_a L_b)^{\frac{3}{2}} \right] \quad (1)$$

$$f_m^D = |t_m - \tau_m| \cdot I_m^D = |t_m - \tau_m| \cdot \left[\left(\frac{n}{\pi_m} \right)^{\frac{1}{2}} \cdot (L_a L_b)^{\frac{3}{2}} \right] \quad (2)$$

2.2 Standardized priority

The CAA slot regulations enforced the requirement of the different priorities assigned to the various types of slot requests. This is achieved through a sequential approach that first allocates historic series of slots, followed by the "change to historic" series of slots, followed by new entrant slots, and, finally, by the remaining slots. The CAA slot regulations clearly stipulate that priority should be determined from high to low according to the product of allocation cardinality and time efficiency allocation coefficient of aviation enterprises. We standardize the

calculated priority values, so that the four types of priority are distributed within a clearly distinguishable range showed as in Table 1.

Table 1. Range of standardized priority

historic series of slots	[1501,2000]
“change to historic” series of slots	[1001,1500]
new entrant slots	[501,1000]
remaining slots	[1,500]

In the real world, priority determines the priority of slot selection. Similarly, in model computing, priority determines which movement can enter the model earlier and get the preferred slot. This is called High-Priority First (HPF).

2.3 Comprehensive displacement cost

As mentioned before, cost of one movement in most of traditional slot allocation models is as following:

$$f_m^O = |t_m - \tau_m| \cdot 1 \quad (3)$$

In expression (3), f_m^O is the interval displacement when required interval τ_m is replaced with scheduled interval t_m .

When priority is considered as the cost of adjusting the unit time, cost of one movement can be present as following.

$$f_m^P = |t_m - \tau_m| \cdot p_m \quad (4)$$

In our model, we integrate cost of displacement, difficulties and priorities as a whole in order to facilitate calculation and comparison. At the same time, three weight factors (w_1, w_2, w_3) are introduced artificially for the same reason. The comprehensive displacement cost of a movement, displaced from required interval τ_m to scheduled interval t_m , could be expressed as follows.

$$\delta_m^{ODP} = (w_1 \cdot 1 + w_2 \cdot I_m^D + w_3 \cdot p_m) = (w_1 \cdot 1 + w_2 \cdot \left[\left(\frac{n}{\pi_m} \right)^{\frac{1}{2}} (L_a L_b)^{\frac{3}{2}} \right] + w_3 \cdot p_m) \quad (5)$$

$$f_m^t = w_1 \cdot f_m^O + w_2 \cdot f_m^D + w_3 \cdot f_m^P = |t_m - \tau_m| \delta_m^{ODP} \quad (6)$$

We call δ_m^{ODP} as the comprehensive displacement cost factor considering implementation difficulties and priority. It is a pleasant found that δ_m^{ODP} is a constant, which makes it possible to solve it by Linear Integer Programming (LIP).

2.4 Displacement model for all flights

$$\text{minimize} \quad \sum_{m \in M} \sum_{t \in T} f_m^t x_m^t = \sum_{m \in M} \sum_{t \in T} |t_m - \tau_m| \delta_m^{ODP} x_m^t \quad (7)$$

$$\text{subject to} \quad \sum_{t \in S} x_m^t = 1, \quad m \in M \quad (8)$$

$$\sum_{m \in M} \sum_{t \in T_c^d} a_m^d b_{mc} x_m^t \leq u_c^{ds}, \quad c \in C, d \in D, s \in T_c \quad (9)$$

$$x_m^t \in \{0,1\}, \quad m \in M, \quad t \in S \quad (10)$$

The objective function (7) minimizes the overall displacement cost of all flights. Constraint (8) stipulates that every movement must be allocated to one interval. Constraint (9) specifies that total movement consumption cannot exceed capacity, for each constraint, day, and interval. Constraint (10) ensures that this model can be solved by integer programming method. There is no detailed description of the coefficient matrix in previous papers. In the next section, we will elaborate our proposed approach.

3 LIP for slot displacement models

In order to limit the flow of corridors, it is necessary to determine which corridor should be allocated to each flight. Firstly, the courses of all flights from the airport to the linked airport are calculated. Then, the courses for arrival or departure are sorted descending. If there are m corridors for arrival (or departure), then the arrival (or departure) courses are classified into m categories.

In general, the number of corridors for arrival is equal to the number of corridors for departure. Each corridor entrance must meet the capacity constraints. All movement was classified according to number of corridors and assigned evenly to corridor before executing LIP. By designing the variable $y_i^e = \{0,1\}, i = 1 \dots M, e = 1 \dots E$, the relationship matrix of movement and corridors is constructed as Table 2 show. The sum of each column must be 1, that is, each movement i must be assigned to a corridor e . The sum of each row is limited by the capacity of the corridor in every interval. This makes it easy to solve the capacity constraints of the corridor with linear programming, but with the increase of the number of flights and the number of corridors, the dimension of the constraints increases rapidly.

Table 2. relationship matrix of movement and corridors

e \ i	1	2	3	4	...	M	C_e
1	1	0	0	0	...	0	C_1
2	0	0	1	0	...	1	C_2
3	0	0	0	0	...	0	C_3
...
$E-1$	0	0	0	1	...	0	C_{E-1}
E	0	1	0	0	...	0	C_E

Considering that the flight execution cycle is usually at least once a week, and most flights operate every day, to reduce the computing time with the proposed approach, it is possible to determine the calendar days with the same set of requests, and then represent these as a single calendar day. In order to keep the accumulated rolling volume of flights from exceeding the hourly capacity, we consider three kinds of capacities in brackets (hourly capacity, 15-minute capacity and 5-minute capacity) to prevent this from happening. To prevent memory overflow, these three capacities are arranged separately, that is to say, the others are not active in the arrangement of one capacity, and in the order of hourly capacity, 15-minute capacity and 5-minute capacity sequentially. Because the limitation of 5-minute capacity has the greatest impact on the uniform distribution of time, the check of 5-minute capacity is at the end. When the proposed algorithm is used for 5-minute check, the dimension of the constraint is too large, and sometimes the memory is insufficient to execute. So we activate constrains of runway capacity and corridors capacity in batches according to the order of priority. The corresponding capacity is updated after each batch of arrangement. The following is the procedures of iterative linear integer programming algorithms based on data-splitting.

Iterative linear integer programming algorithms based on data-splitting

Priority calculation f_m^P

For $i=1:7$

Load movement in day 1

Set up hourly capacity c_{60} , 15-minute capacity c_{15} , 5-minute capacity c_5

Calculating the minimum total capacity, $Z = \text{Min}(24 \cdot c_{60}, 96 \cdot c_{15}, 288 \cdot c_5)$

Compared with M and Z , if M is larger than Z , the number of discarded requests is equal to $M-Z$

Use simplex method to arrange all remaining applications into each hourly period

Use simplex method to arrange the 15-minute period on the basis of the hour schedule

Use simplex method to arrange the 5-minute period on the basis of the 15-minute schedule

End

Table 3 is an example of describing the coefficient matrix in detail in the objective function and constrains within hours.

Table 3. Coefficient matrix and detailed expression when interval is based on hour

Execution batch equals $P=[M/Q]$	$\sum_{m \in M} \sum_{t \in T} f_m^t x_m^t$ $= \sum_{m \in M} \sum_{t \in T} t_m - \tau_m \delta_m^{DP} x_m^t$	f_m^t	$f_m^t x_m^t = f_1^1 x_{11} + f_1^2 x_{12} + f_1^3 x_{13} + \dots + f_1^{24} x_{124} +$ $f_2^1 x_{21} + f_2^2 x_{22} + f_2^3 x_{23} + \dots + f_2^{24} x_{224} +$ \dots $f_m^1 x_{m1} + f_m^2 x_{m2} + f_m^3 x_{m3} + \dots + f_m^{24} x_{m24}$	(a.0)
	$\sum_{m \in M} \sum_{t \in T_c^d} a_m^d b_{mc} x_m^t \leq u_c^{ds}$, $c \in \text{hour}, d \in [1, 2, \dots, 7],$ $S \in [1, 2, \dots, 24]$ $a_m^d = 1, b_{mc} = 1$	A_1	$x_{11} + x_{21} + x_{31} + \dots + x_{m1} \leq u_{hour}^1$	(a.1)
			$x_{12} + x_{22} + x_{32} + \dots + x_{m2} \leq u_{hour}^2$	(a.2)
			\vdots	\vdots
			$x_{124} + x_{224} + x_{324} + \dots + x_{m24} \leq u_{hour}^{24}$	(a.24)
	$\sum_{m \in M} \sum_{t \in T_c^d} a_m^d b_{mc} x_m^t \leq u_c^{ds}$, $c \in c1, \text{corridor } 1$ $d \in [1, 2, \dots, 7],$ $S \in [1, 2, \dots, 24]$ $a_m^d = 1, b_{mc} = 1$	Y	$y_1^1 x_{11} + y_2^1 x_{21} + y_3^1 x_{31} + \dots + y_m^1 x_{m1} \leq u_{c1}^1$	(b.1)
			$y_1^1 x_{12} + y_2^1 x_{22} + y_3^1 x_{32} + \dots + y_m^1 x_{m2} \leq u_{c1}^2$	(b.2)
			\vdots	\vdots
			$y_1^1 x_{124} + y_2^1 x_{224} + y_3^1 x_{324} + \dots + y_m^1 x_{m24} \leq u_{c1}^{24}$	(b.24)
	$\vdots \dots \vdots$		$\vdots \dots \vdots$	\vdots
$\sum_{m \in M} \sum_{t \in T_c^d} a_m^d b_{mc} x_m^t \leq u_c^{ds}$, $c \in c8, \text{corridor } 8$ $d \in [1, 2, \dots, 7],$ $S \in [1, 2, \dots, 24]$ $a_m^d = 1, b_{mc} = 1$	Y	$y_1^8 x_{11} + y_2^8 x_{21} + y_3^8 x_{31} + \dots + y_m^8 x_{m1} \leq u_{c8}^1$	(b.169)	
		$y_1^8 x_{12} + y_2^8 x_{22} + y_3^8 x_{32} + \dots + y_m^8 x_{m2} \leq u_{c8}^2$	(b.170)	
		\vdots	\vdots	
		$y_1^8 x_{124} + y_2^8 x_{224} + y_3^8 x_{324} + \dots + y_m^8 x_{m24} \leq u_{c8}^{24}$	(b.216)	
$\sum_{t \in T} x_m^t = 1, m \in Q$ $x_m^t \in \{0, 1\}, m \in Q$ $t \in [1, 2, \dots, 24]$	A_{eq}	$x_{11} + x_{12} + x_{13} + \dots + x_{124} = 1$	(d.1)	
		$x_{21} + x_{22} + x_{23} + \dots + x_{224} = 1$	(d.2)	
		\vdots	\vdots	
		$x_{m1} + x_{m2} + x_{m3} + \dots + x_{m24} = 1$	(d.m)	

4 Testing and results

The purpose of the test is to observe impact of changing weights of evaluation objectives on performance and answer previous proposed question. How to ensure HPLA in process of slot allocation? If the difficulty of slot displacement is regarded as an evaluation index or objective, how weights should be set to optimize slot allocation? Can priority be considered as the cost of adjusting the unit time to ensure HPLA? What are the differences when movements are batched into a computer program in priority order or in the order of morning to night (final timetable presentation)? All these questions are investigated in section 4.1.

4.1 Performance compare with different weight factors of evaluation objectives

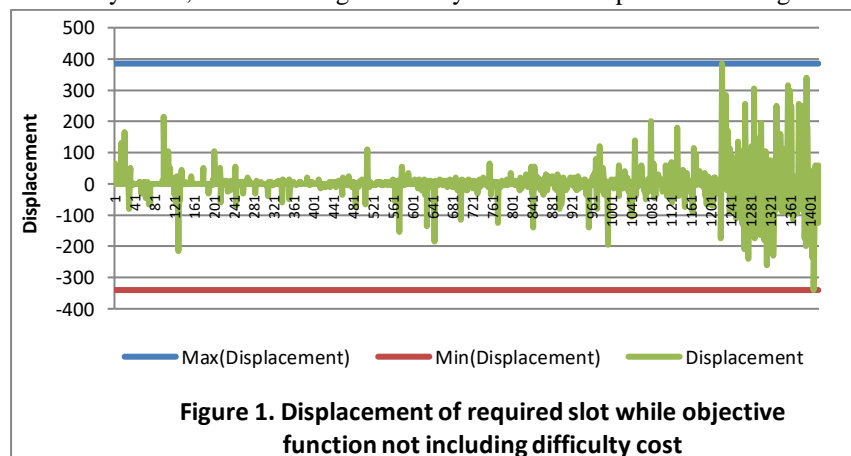
Weights setting and results with different weights of objective functions are showed in Table 4. In field of order in table 4, 'SD' means all movements in inputting table are sorted descent according to priority. Because of the length of the article, some of the corresponding graphs of the tests were ignored and labeled "N" in the table.

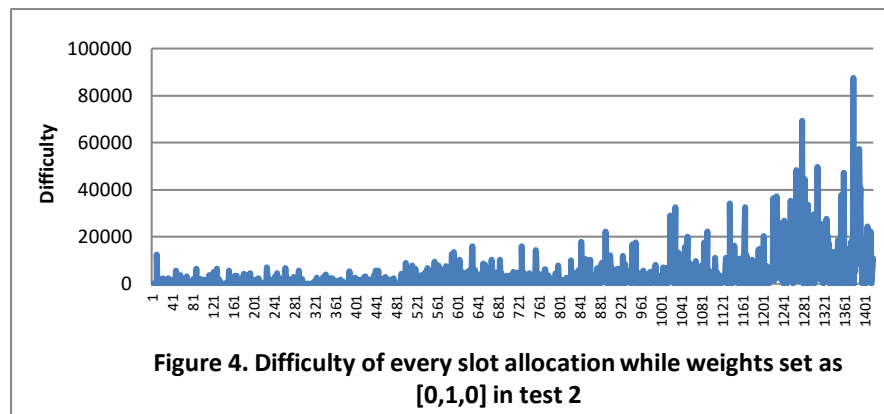
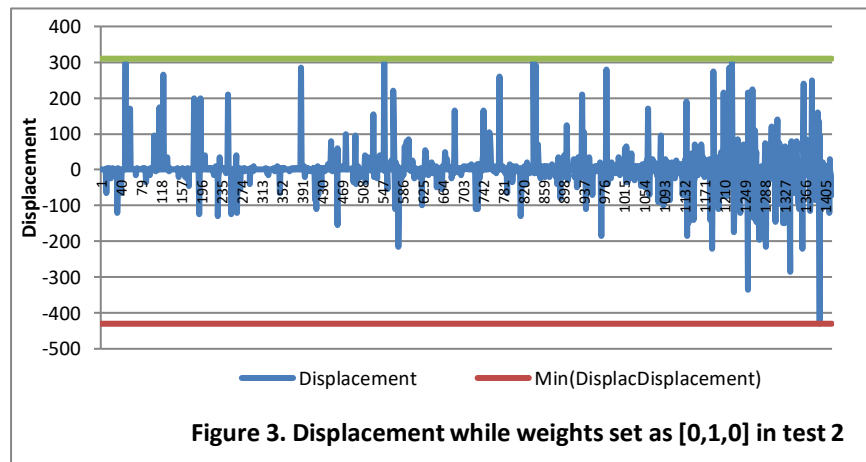
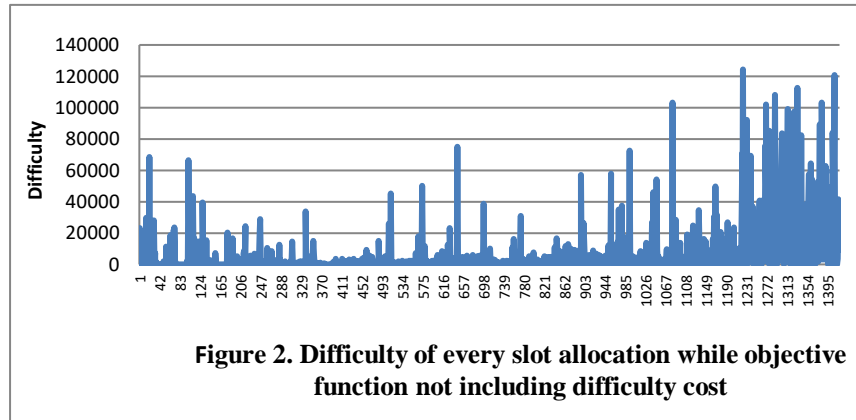
Table 4. Performance compare with different weight factors of evaluation objectives

Test s	Weights of disp,diff,prio	Total diff	Average diff	Total disp	Average disp	Capacity	Min (disp)	Max (disp)	Order	Figure
1	1,0,0	8.2E+06	5752.40	27200	19.18	88,23,7	-340	385	SD	1-2
2	0,1,0	4.5E+06	3177.89	29290	20.66	88,23,7	-430	310	SD	3-4
3	0,0,1	6.4E+06	4480.08	29220	20.61	88,23,7	-485	405	SD	5-6
4	0,0,9,0,1	5.1E+06	3582.03	30130	21.25	88,23,7	-565	360	SD	N
5	0,1,1	6.4E+06	4480.08	27015	19.05	88,23,7	-485	405	SD	N
6	0,0,5,0,5	6.6E+06	4650.95	29045	20.48	88,23,7	-485	340	SD	N
7	0,1,1	9.0E+06	6367.90	44330	31.26	78,20,7	-520	460	SD	N
8	0,0,5,0,5	7.2E+06	5064.14	28305	19.96	88,23,7	-585	375	not SD	7-9
9	1,0,1,0,9	5.2E+06	3670.50	29385	21.00	88,23,7	-545	350	SD	N
10	1,0,0	8.3E+06	5855.00	28170	20.00	88,23,8	-270	419	not SD	N
11	0.8, 0.01, 0.09	5.0E+06	3544.65	28440	20.06	88,23,8	-510	410	SD	N
12	100, 0.1, 0	5.3E+06	3707.60	28060	19.79	88,23,8	-545	365	SD	N

Note: disp=displacement, diff=difficulty, prio=priority, SD=sorting descent, N=not show, key information

Displacement and difficulty of slot allocation while weights is set as [1,0,0] are showed in figure 1-2. In this case, the slot with small priority (late entry procedure) has a greater probability of being adjusted, and the amount of slot displacement and probability of being adjusted will increase with priority decreasing. Displacement and difficulty of every slot allocation while weights is set as [0,1,0] are showed in figure 3-4. In this case, as the priority decreasing, the probability of displacement has the same trend as in the first case, but the difference is that even if the priority is high, there is more higher probability of being displacement than weights is set as [1,0,0]. Difficulty increases regularly in test 2, but the difficulty of test 1 fluctuates. In the first case, the average slot displacement is relatively small, but the average difficulty of the slot displacement is high in the second case.





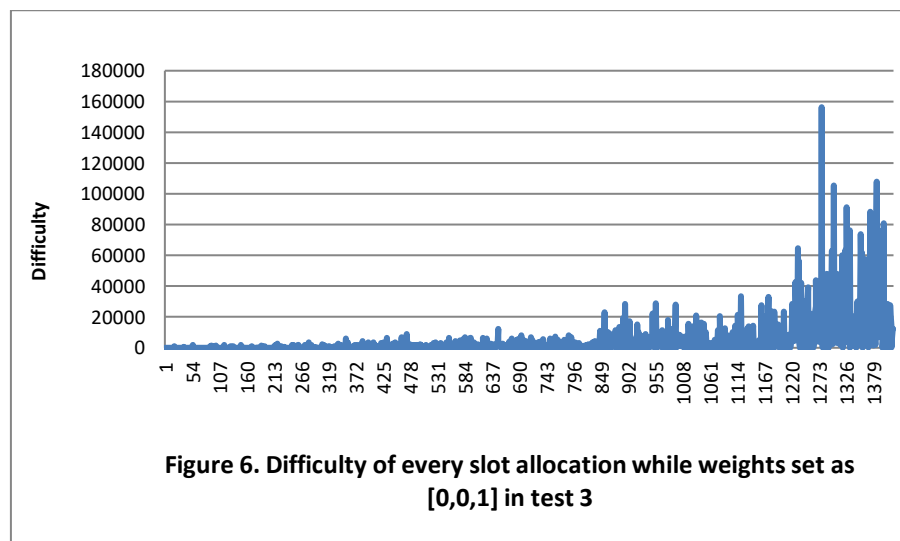
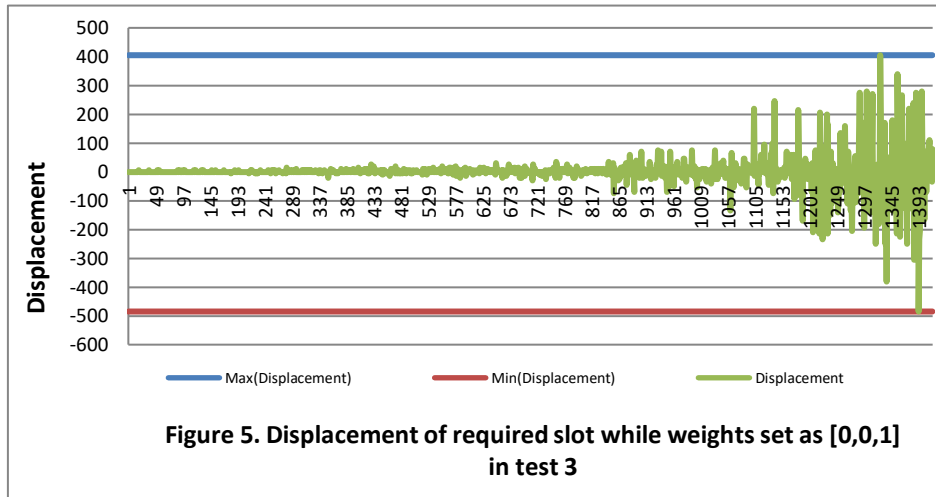
From table 4, we found that the following:

Average displacement and containment of minimum and maximum of displacement is the smallest when evaluation objective is just total displacement (test 1). Average difficulty is the smallest when evaluation objective is just total difficulty (test 2). When other conditions remain unchanged, if application is sent to the process without application sorting, then the average difficulty will increase, but the average displacement remain unchanged (test 6&8).

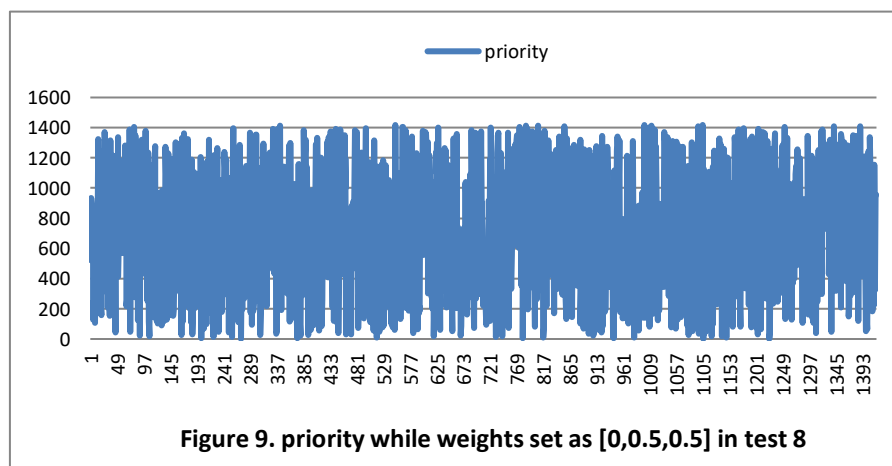
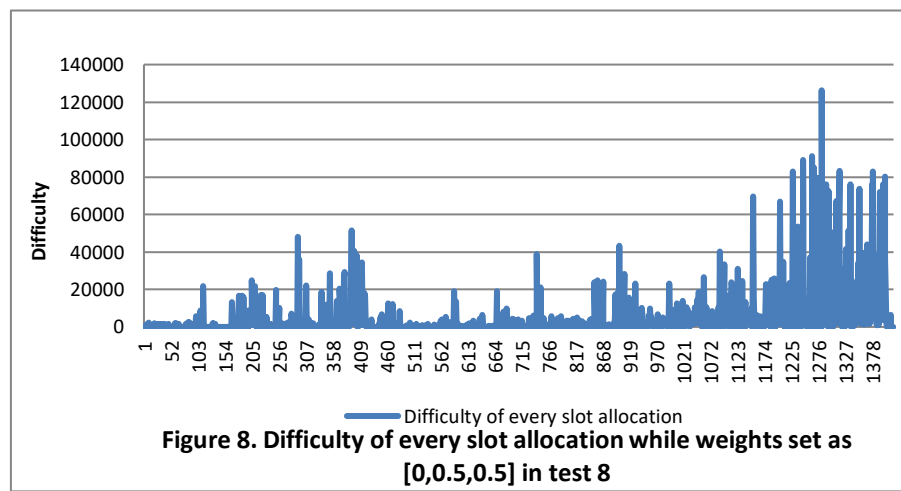
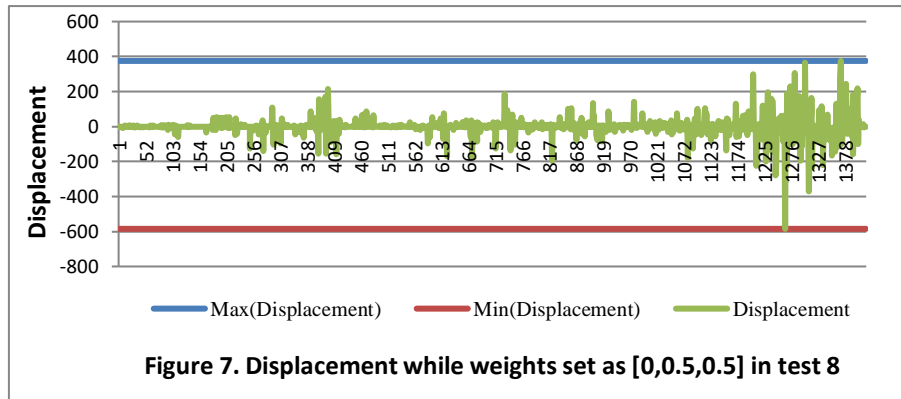
From test 4, 6&9, when difficulty and priority exist simultaneously in evaluation objectives, the changes of the weight of these two objectives mainly affect the change of difficulty, and the change of average displacement amount is almost unchanged. Furthermore, because the limited capacity (78) in test 7 is smaller than the

limited capacity ($7 \times 12 = 84$) in test 7, the average displacement amount and the displacement difficulty are significantly increased. This means that capacity is the key factor affecting the displacement.

Comparing test 3 and others, we found when priority is the solely evaluation objective, both of difficulty and amount of displacement will increase gradually with priority order decreasing (figure 5-6). This means movements with high priority have low probability of being adjusted and being adjusted greater.



From result of test 8 and figure 7-9, if movements are not list according to descent priority order, total of difficulty is the greatest in all tests with same capacity setting (88,23,7), but movements listed later in inputting data table have still more probability of being adjusted and being adjusted greater than a movement in the front of list. The reason is very obvious that when a movement listed at the end of roster, the remaining capacity becomes less and less, the probability of being adjusted becomes larger and larger.



4.2 Analysis of the correlation between average displacement and average difficulty

The Pearson correlation coefficient expression below is used to calculate the correlation coefficients of average displacement and average difficulty.

$$r = \frac{\sum XY - \sum X \sum Y}{\sqrt{(\sum X^2 - \frac{(\sum X)^2}{N})(\sum Y^2 - \frac{(\sum Y)^2}{N})}} \quad (11)$$

Mean displacement and average difficulty correlation coefficient $r = -0.54$, there is a negative correlation between them. That is to say, their relationship is mutually exclusive. If the average difficulty is sensitive to the

average displacement, there is an opportunity to increase a little of the average displacement to achieve a significant reduction in the average difficulty. Therefore, we need test the sensitivity of the average difficulty to the average displacement. Three column in table 4 (excluding test 7, because the capacity of this test is different from others) are extracted to table 5, and then all rows are ranked according to the average displacement in ascending order.

The first set of tests	average disp X	average diff Y
5	19.05	4480.08
1	19.18	5752.40
12	19.79	3707.60
8	19.96	5064.14
10	20.00	5855.00
11	20.06	3544.65
6	20.48	4650.95
3	20.61	4480.08
2	20.66	3177.89
9	21.00	3670.50
4	21.25	3582.03

The following formula is used to test the sensitivity of average diff to average displacement.

$$\delta_i^X = X_i - X_{i-1}, i=2 \dots N; \quad (12)$$

$$\delta_i^Y = Y_i - Y_{i-1}, i=2 \dots N; \quad (13)$$

$$\sigma = \frac{(\sum_{i=2}^N \delta_i^X)/N}{(\sum_{i=2}^N \delta_i^Y)/N} \frac{\sum_{i=2}^N \delta_i^X}{\sum_{i=2}^N \delta_i^Y}, i=2 \dots N; \quad (14)$$

Using the data in Table 5, we obtained the following sensitivity coefficients, $\sigma = -408.20$. This shows that the average difficulty is very sensitive to the average displacement. The average displacement only needs to increase a small amount, which can greatly reduce the implementation difficulty.

5 Discussion and conclusion

The test results show that even if we strictly implement the new slot regulation, different weights of performance objectives and even different processing technology, slot displacements and implementation difficulties are also different.

The average difficulty is very sensitive to the average displacement. This mean the average displacement only needs to increase a small amount, which can greatly reduce the implementation difficulty. It is worthwhile to increase some amount of slot displacement appropriately in return for a significant reduction in average execution difficulty. Especially when the number of seats on some regional flights is small, the air elapse time can be adjusted to a large margin, and the linked airports are uncoordinated airports, it is reasonable to make these flights be displaced more.

Priority cost is not suitable for giving too much weight, because too much weight will lead to increased difficulty and adjustment. Furthermore, HPLA is dominated guaranteed by sequencing in front of list. Therefore, airlines should pay more attention to “the position in the list” not priority itself. When applications are not list according to priority, great differences in priorities make it more difficult to implement them. For the management department of slot allocation, it is an effective way to ensure HPLA by inputting application to programs according to priority order.

The difficulty index proposed in this work is very useful in identifying which slot allocation scheme is better. These tests in Section 4.1 clearly imply this conclusion.

In summation, evaluation objectives including displacements, implementation difficulties and priority cost is suitable, and the weights of the latter two objectives should be set smaller, and the weights of difficulty should be greater than the weights of priority cost. However, the values of specific weights still need to be weighed by multi-stakeholders and specific market research.

The main limitation of the study is that priority is produced by simulation, because of limitation of data availability. This implies the relationship of implementation difficulty and priority need further research. We believe that with the improvement of technical means and awareness, it is necessary to discuss and study other goals or mixed objectives with greater vision. In further research, some new slot performance evaluation objectives and other constraints will be tried to make slot allocation more scientific and reasonable.

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