 doi:10.20944/preprints201903.0261.v1

Article

ENTROPY OF REISSNER-NORDSTRÖM 3D BLACK HOLE IN ROEGENIAN ECONOMICS

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Abstract: The subject of this paper is to analyse the Math Principia of Economic 3D Black Holes in Roegenian economics. This idea is totally new in the related literature, excepting our papers. In details, we study two special problems: (i) math origin of economic 3D black holes, (ii) entropy and internal political stability depending on national income and the total investment, for economic RN 3D black hole. To solve these problems, it was necessary to jump from macroeconomic side to microeconomic side (a substantial approach so different), to complete the thermodynamics-economics dictionary with new entities, to introduce the flow between two macroeconomic systems, to study the Schwarzschild type metric properties on an economic 4D system, together with Rindler coordinates, Einstein 4D PDEs, and economic RN 3D black hole. In addition, we introduce some economic Ricci type flows or waves, for further research.

Keywords: Thermodynamics-economics dictionary; economic Einstein 4D PDEs; economic Schwarzschild type metric; economic 3D black holes; economic entropy.

MSC: 80A99, 91B74, 83C57

JEL Classification: B41

1. Thermodynamics-Economics Dictionary

There are many analogies between thermodynamics and economics. The first example is econophysics [3], [5], the second example is given by the papers [11]-[12], [14] the third example, is offered by the collection [20]-[34]. The analogies are created either by a dictionary for each problem or via a global dictionary. Therefore, each economic system would be similar to a thermodynamic system, in the sense that thermodynamics potentials and laws have correspondence in economics.

In the following, we reproduce the correspondence between the characteristic state variables and the laws of thermodynamics with the macro-economics as described in the papers [20]-[34], starting

 doi:10.20944/preprints201903.0261.v1

from the theory of Roegen [10]. They also allow us to include in economics the idea of an "economic black hole" [27], [29]-[33] with a similar meaning to the one in astrophysics [6], [9], [16], [17]. This idea is developed further as math origin of economic 3D black holes and entropy of Reissner-Nordström 3*D* Black Hole in Roegenian Economics.

THERMODYNAMICS		ECONOMICS
U=internal energy	...	G=growth potenţial
T=temperature	...	I=internal political stability
S=entropy	...	E=entropy
P=pressure	...	P=price level (inflation)
V=volume	...	Q=volume, structure, quality
M=total energy (mass)	...	Y=national income (income)
Q=electric charge	...	$\mathcal{I}$=total investment
J= angular momentum (spin)	...	J=economic angular momentum (economic spin)
M=M(S,Q,J)	...	Y=Y(E,$\mathcal{I}$,J)
μ_k=chemical potential	...	ν_k=economical potential
N_k=number of moles	...	$\mathcal{N}_k$=number of economic moles
W=mechanical work	...	W=wealth of the system
Q=heat	...	q=stock market
$T_H = \frac{\partial M}{\partial S}$=Hawking temperature	...	I_{BH}=BH-internal political stability
G = Newton constant	...	$\mathcal{G}$= universal economic constant
c = light velocity	...	c= maximum universal exchange speed
$\hbar$= normalized Planck constant	...	$\hbar$ = normalized economic quantum

The last three lines in previous dictionary were introduced here for the first time (required by economic Schwarzschild metric, and economic 3D black holes theory).

1.1. Thermodynamics differential laws

The process variables $W = mechanical\ work$ and $Q = heat$ are introduced into Carathéodory thermodynamics by $dW = PdV$ (the *first law*) and by elementary heat, respectively, $dQ = TdS$ (differential equality), for reversible processes, or $dQ < TdS$ (differential inequality), for irreversible processes (*second law*).

The Gibbs-Pfaff fundamental equation in thermodynamics is $dU - TdS + PdV + \sum_k \mu_k dN_k = 0$ (a combination of the *first law* and the *second law* in thermodynamics).

The *third law* of thermodynamics states that $\lim_{T \to 0} S = 0$.

1.2. Roegenian economics differential laws

The process variables in the economics W= *wealth of the system*, q = *stock market* are defined by $dW = Pdq$ (*first economic law, elementary wealth in the economy*) and $dq = IdE$ or $dq < IdE$ (*second economic law, elementary production of commodities*). A *commodity* is an economic good, a product of human labor, with a utility in the sense of life, for sale-purchase on the market in the economy.

Let us accept that the *Gibbs-Pfaff fundamental equation of economy* is $dG - IdE + PdQ + \sum_k \nu_k d\mathcal{N}_k = 0$ (combination of the *first law* and the *second law* in economics).

The *third law of economics* $\lim_{I \to 0} E = 0$ says that "if the internal political stability I tends to 0, the system is blocked, meaning entropy becomes $E = 0$, equivalent to maintaining the functionality of the economic system must cause disruption".

Econophysics applies methods of statistical physics and quantitative methods developed in physics to economic phenomena ([3], [5], [11]-[12], [14]). The name "econophysics" was coined by E. Stanley, a physicist from the Boston University, in 1996.

The long term association between Economics and Thermodynamics can be strengthened with new tools based on the foregoing dictionary. The previous thermodynamics-economics dictionary allows the transfer of information from thermodynamics to economics (see [20]-[34]), keeping the background of each discipline, that we think that was suggested by Roegen [10]. Of course, this new idea of thermodynamics-economics dictionary produces new concepts in econophysics.

A macro-economic system based on a Gibbs-Pfaff equation is controllable (see [31]).

Definition 1. *An economics structured similar to thermodynamics, via the previous dictionary, is called Roegenian economics.*

A partial history of attempts to connect physics and economics is given in [8], [14].

1.3. Gravity models in Physics and Economics

Newton's law of universal gravitation states that every particle attracts every other particle in the universe with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers. The equation for universal gravitation thus takes the form

$$F = G\,\frac{m_1 m_2}{r^2},$$

where F is the *gravitational force* acting between two objects, m_1 and m_2 are the masses of the objects, r is the distance between the centers of their masses, and G is the gravitational constant.

The *normalized Planck constant* $\hbar$ is a physical constant that is the quantum of action (central notion in quantum mechanics). The Planck constant represents the proportionality between the momentum and the quantum wavelength of not just the photon, but the quantum wavelength of any particle.

The previous dictionary adds to "gravity models of trade" [1], [4], [7], [19] another more significative *flow between two macroeconomic systems.*

Definition 2. *The economic center of gravity of a specific economic system is the region with the largest contribution to (national) income.*

In this sense, the distance r between two economic systems means the economic distance between two centers of economic gravity.

The flow between (national) incomes shows that every macro-economic system attracts every other macro-economic system by a force acting between the economic centers of gravity. The strength of this force is proportional to the product of the two national incomes, and inversely proportional to the square of the distance between them.

Definition 3. *Suppose $\mathcal{G}$ stands for universal economic constant (an economic constant of proportionality), Y stands for the (national) income, and r stands for the economic distance between the two macro-economic systems that are being studied. The formula*

$$\mathcal{F} = \mathcal{G}\,\frac{Y_1 Y_2}{r^2},$$

defines the flow between (national) incomes.

The *normalized economic quantum* $\hbar$ is the economic quantum of action.

2. Math origin of economic 3D black holes

We recall what economists commonly call **economic black holes** [2]: i) a **keynesian black hole** (in terms of liquidity trap) means a business activity or product on which large amounts of money are spent, but that does not produce any income or other useful result; ii) **liquidity black hole** means that

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trading financial assets becomes prohibitively expensive and asset prices collapse (1997 Asian crisis, 1998 Russian debt/LTCM crisis, 2007 subprime mortgage crisis).

In our papers [22]-[33] is described **a new concept of "economic 3D black hole" as a small part of a global economic system where the total income created is so strong that nothing can escape after falling beyond the horizon event (rising poverty)**. This kind of economic black hole is depicted with entropy E, national income (income) Y, total investment $\mathcal{I}$ and economic spin J. The national income is so large that it attracts all the economic resources of its neighbors. This is in fact the image through the previous dictionary of a Thermodynamics black hole.

Looking again on some papers regarding the Thermodynamics 3D Black Holes [6] and on our papers regarding the Economic 3D Black Holes [22]-[33], now we shall try to explain the infinitesimal roots of *Economic 3D Black Holes*, i.e., the *Math Principia of Economic 3D Black Holes*. To do that, we use an arithmetic economic time-space $\mathbb{R} \times \mathbb{R}^3$, with 4 dimensions. A point in this space has the coordinates (t, G, I, E), where t is the time, and the (G, I, E) are economic variables (G= potential growth, I= internal political stability, E= entropy), stripped of their true meaning and units of measure. Geometrically, the shorthand for all selected coordinates is x^μ, where the index μ take values $0; 1; 2; 3$. The national income Y, total investment $\mathcal{I}$, economic spin J are economic parameters.

To understand the relevant parameters and the geometry of economic black holes, we add some economic - geometric ingredients: the universal economic constant $\mathcal{G}$, an economic metric $g_{\mu\nu}$ of signature $(-;+;+;+)$ (which captures all the geometric and causal structure of economic time-space), the determinant of the metric $g = \det(g_{\mu\nu})$ (which is a negative number), the arc-length square $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, the Ricci tensor field $R_{\mu\nu}$, and the Ricci scalar $R = g^{\mu\nu}R_{\mu\nu}$. Also, we set $c = 1$.

The pseudo-Riemannian approach to economics is not only a mathematical curiosity but a useful technique to solve actual problems, since it might deliver local information of economic systems relying solely on global data.

Similar to the Einstein-Maxwell theory in Thermodynamics, we introduce the economic action

$$S = \frac{1}{16\pi\mathcal{G}} \int R\sqrt{-g}\; d^4x.$$

Theorem 1. *The Euler-Lagrange PDEs are the vacuum 4D Einstein PDEs*

$$R_{\mu\nu} = \frac{R}{2}\; g_{\mu\nu}.$$

Proof. The technique of obtaining the Euler-Lagrange PDEs is well-known. Equating to zero the variation, i.e.,

$$0 = \delta S = \frac{1}{16\pi\mathcal{G}} \int \left(\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R}{\sqrt{-g}} \frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} \right) \delta g^{\mu\nu}\; \sqrt{-g}\; d^4x,$$

taking $\delta g^{\mu\nu}$ arbitrary, we obtain the Euler-Lagrange PDEs (the equation of motion for the metric field),

$$\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R}{\sqrt{-g}} \frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} = 0$$

or the vacuum Einstein PDEs

$$R_{\mu\nu} = \frac{R}{2}\; g_{\mu\nu},$$

with the unknowns $g_{\mu\nu}$ (components of the metric). □

2.1. Schwarzschild type metric on 4D economic system

The Euler-Lagrange PDEs $\frac{\partial L}{\partial g^{\mu\nu}} = 0$, attached to the economic Lagrangian

$$L = \frac{1}{16\pi\mathcal{G}}\; R\sqrt{-g},$$

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are vacuum Einstein PDEs with the unknowns $g_{\mu\nu}$ (components of the pseudo-Riemannian metric). Consider the economic Schwarzschild metric $g_{\mu\nu}$ which is a spherically symmetric, static solution of previous Einstein PDEs. This metric is expected to describe the geometry of economic time-space outside or inside an economic black hole.

Let us use $\mathcal{G}$ as the universal economic constant, Y as the national income (parameter), t as the time, r as the radial coordinate, and Ω as the solid angle on a 2-sphere. Then economic Schwarzschild metric solution $g_{\mu\nu}$ is given by the arc-length square

$$ds^2 = -\left(1 - \frac{2\mathcal{G}Y}{r}\right) dt^2 + \left(1 - \frac{2\mathcal{G}Y}{r}\right)^{-1} dr^2 + r^2\, d\Omega^2.$$

If $r > 2\mathcal{G}Y$ (exterior), then the signature of the metric $g_{\mu\nu}$ is $(-,+,+,+)$. If $0 < r < 2\mathcal{G}Y$ (interior), then the signature of this metric $g_{\mu\nu}$ is $(+,-,+,+)$. The $r = 0$ singularity is known as a curvature singularity and is irremovable. The metric appears also to be singular at $r = 2\mathcal{G}Y$ because $g_{00} = 0$ (vanish) and $|g_{rr}| \to \infty$ (diverge). Let us show that the $r = 2\mathcal{G}Y$ singularity is a coordinate singularity and may be removed by an appropriate coordinate transformation. As example, the coordinate transformation

$$(t, r, \theta, \varphi) \to (u, r, \theta, \varphi),\ u = t - r - 2\mathcal{G}Y\ \ln(r - 2\mathcal{G}Y),\ du = -dt - \left(1 - \frac{2\mathcal{G}Y}{r}\right)^{-1} dr$$

produces

$$ds^2 = -\left(1 - \frac{2\mathcal{G}Y}{r}\right) du^2 - 2dudr + r^2(d\theta^2 + sin^2\theta d\varphi^2).$$

In the new coordinates (u, r, θ, φ), the components of the metric are non-singular at $r = 2\mathcal{G}Y$. Moreover, the previous exterior Schwarzschild solution may be analytically continued across the surface given by the equation $r = 2\mathcal{G}Y$ [13].

To better understand the nature of this apparent singularity, let us examine the geometry more closely near the *event horizon spherical surface* $r = 2\mathcal{G}Y$ of the exterior Schwarzschild solution. Much of the interesting economics (physics) having to do with the quantum properties of economic black holes comes from the region near the event horizon.

Theorem 2. *The pseudo-Riemannian metric*

$$ds^2 = -\left(1 - \frac{2\mathcal{G}Y}{r}\right) dt^2 + \left(1 - \frac{2\mathcal{G}Y}{r}\right)^{-1} dr^2 + r^2\, d\Omega^2.$$

is approximated by the pseudo-Riemannian metric

$$ds^2 = -\frac{\rho^2}{16\mathcal{G}^2Y^2}\, dt^2 + d\rho^2 + (2\mathcal{G}Y)^2 d\Omega^2.$$

Proof. To focus on the near horizon geometry in the region $r - 2\mathcal{G}Y \ll 2\mathcal{G}Y$, let us define $r - 2\mathcal{G}Y = \xi$, so that when $r \to 2\mathcal{G}Y$ we have $\xi \to 0$. The metric then takes the form

$$ds^2 = -\frac{\xi}{2\mathcal{G}Y + \xi}\, dt^2 + \frac{2\mathcal{G}Y + \xi}{\xi}\, d\xi^2 + (2\mathcal{G}Y + \xi)^2\, d\Omega^2.$$

Taking into account the inequality $\frac{\xi}{2\mathcal{G}Y} \ll 1$, we find the approximation

$$ds^2 = -\frac{\xi}{2\mathcal{G}Y}\, dt^2 + \frac{2\mathcal{G}Y}{\xi}\, d\xi^2 + (2\mathcal{G}Y)^2\, d\Omega^2,$$

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up to corrections that are of order $\frac{1}{2\mathcal{G}Y}$. Introducing a new coordinate ρ, by $\rho^2 = (8\mathcal{G}Y)\xi$, so that $\frac{2\mathcal{G}Y}{\xi}d\xi^2 = d\rho^2$, the metric takes a new form

$$ds^2 = -\frac{\rho^2}{16\mathcal{G}^2Y^2}dt^2 + d\rho^2 + (2\mathcal{G}Y)^2 d\Omega^2.$$

From this form of the metric it is clear that the coordinate ρ measures the geodesic radial distance. Note that the geometry factorizes. One factor is a 2-sphere of radius $2\mathcal{G}Y$ and the other is the (ρ, t) space

$$ds_2^2 = -\frac{\rho^2}{16\mathcal{G}^2Y^2}dt^2 + d\rho^2.$$

□

In the next subsection, we show that this $1+1$ dimensional time-space is just at Minkowski space written in funny coordinates called the *Rindler coordinates*.

2.2. Rindler coordinates

To understand Rindler coordinates and their relation to the near horizon geometry of the economic black hole, let us start with $1+1$ Minkowski space with the usual flat Minkowski metric $ds^2 = -dT^2 + dX^2$. Introducing the light-cone coordinate, $U = T + X$, $V = T - X$, this metric becomes $ds^2 = -dUdV$. Now we pass to Rindler coordinates (u, v), via an exponential change $U = \frac{1}{\kappa}e^{\kappa u}$, $V = -\frac{1}{\kappa}e^{-\kappa v}$, in which $ds^2 = -e^{\kappa(u-v)}\,dudv$.

We change again the coordinates via $u = t + x, v = t - x, \rho = \frac{1}{\kappa}e^{\kappa x}$. Then the metric becomes $ds^2 = -\rho^2\kappa^2\,dt^2 + d\rho^2$. Comparing with ds_2^2, we obtain the *surface economic gravity* $\kappa = \frac{1}{4\mathcal{G}Y}$ of the economic black hole.

For the economic Schwarzschild solution, one can think of it heuristically as the economic Newtonian acceleration $\frac{\mathcal{G}Y}{r_H^2}$ at the *horizon radius* $r_H = 2\mathcal{G}Y$. The surface economic gravity κ and the horizon radius r_H play an important role in describing the sense of an economic black hole. This analysis proves that the economic Schwarzschild time-space near the surface $r = 2\mathcal{G}Y$ is not singular at all. After all it looks exactly like a Cartesian product between a flat Minkowski space and a sphere of radius $2\mathcal{G}Y$. So the curvatures are inverse powers of the radius of curvature $2\mathcal{G}Y$ and hence are small for large $2\mathcal{G}Y$.

2.3. Economic Schwarzschild radius

An economic singularity or time-space singularity is a location in time-space where the economic metric field of an economic system becomes infinite in a way that does not depend on the coordinate system.

The quantities used to measure economic field strength are the scalar invariant curvatures of time-space, which includes a measure of the economic density $\frac{\partial Y}{\partial Q}$.

Let us give some economic information borrowed from Schwarzschild metric theory. The economic Schwarzschild radius is given now as $r_s = \dfrac{2\mathcal{G}Y}{c^2}$, where $\mathcal{G}$ is the universal economic constant, Y is the national income and c is the maximum universal exchange speed. The economic Schwarzschild radius is an economic parameter that appears in the economic Schwarzschild solution to Einstein's field equations, corresponding to the radius defining the economic event horizon of an economic Schwarzschild black hole. In fact, it is a characteristic radius associated with every (national) income.

Remark 1. *Let $R_{\alpha\beta\gamma\delta}$ be the curvature tensor field. An important quantity is the economic invariant given by*

$$R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} = \frac{12r_s^2}{r^6} = \frac{48\mathcal{G}^2Y^2}{c^4r^6},$$

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where r is the radial coordinate (measured as the circumference, divided by 2π, of a great circle of sphere centered around the economic system), and r_s is the economic Schwarzschild radius of the economic system, a scale factor which is related to its (national) income Y by $r_s = \frac{2\mathcal{G}Y}{c^2}$, where $\mathcal{G}$ is the universal economic constant.

3. Economic Reissner-Nordström (RN) 3D black hole

For this section we need an economic action containing an antisymmetric tensor field $F_{\mu\nu}$ (similar to electro-magnetic field strength). Explicitly, we set $c = 1$ and we use the following notations: $\mathcal{G}$ is universal economic constant, $R_{\mu\nu}$ is the Ricci tensor field, $R = g^{\mu\nu}R_{\mu\nu}$ is the Ricci scalar of the metric $g_{\mu\nu}$, the negative number $g = \det(g_{\mu\nu})$ is the determinant of the metric $g_{\mu\nu}$, and $F_{\mu\nu}$ is an economic field strength with $F^2 = g^{\mu\lambda}g^{\nu\sigma}F_{\mu\nu}F_{\lambda\sigma}$.

We introduce the economic action (multiple integral functional)

$$\frac{1}{16\pi\mathcal{G}} \int R\sqrt{-g}\, d^4x - \frac{1}{16\pi} \int F^2\sqrt{-g}\, d^4x$$

associated to the Lagrangian

$$L = \frac{1}{16\pi\mathcal{G}}\, R\sqrt{-g} - \frac{1}{16\pi}\, F^2\sqrt{-g}.$$

Let $\mathcal{I}$ be the total investment, Y be the national income, and $F_{tr} = \frac{\mathcal{I}^2}{r^2}$ be the non-zero economic field strength component. The pseudo-Riemannian metric of the economic system is now given via arc-length square

$$ds^2 = -\left(1 - \frac{2Y}{r} + \frac{\mathcal{I}^2}{r^2}\right) dt^2 + \left(1 - \frac{2Y}{r} + \frac{\mathcal{I}^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

We can recover the formulae for economic Schwarzschild metric taking the limit $\mathcal{I} \to 0$.

The most general static, spherically symmetric, charged solution of the Einstein - Maxwell theory gives the *economic Reissner-Nordström (RN) 3D black hole*.

From the previous metric we see that the event horizon for this solution is located there where $g_{tt} = 0$, or $1 - \frac{2Y}{r} + \frac{\mathcal{I}^2}{r^2} = 0$, or $r_\pm = Y \pm \sqrt{Y^2 - \mathcal{I}^2}$. Thus, r_+ defines the outer horizon of the economic black hole and r_- defines the inner horizon of the economic black hole. The area of the event horizon is given by $4\pi r_+^2$ (sphere). For an economic Schwarzschild black hole, the area is $A = 16\pi\mathcal{G}^2Y^2$, and the economic surface gravity is $\kappa = \frac{1}{4\mathcal{G}Y}$, where $\mathcal{G}$ is the universal economic constant.

Any economic system has an "internal political stability" and hence it has "economic entropy".

In 1974, the British physicist Stephen Hawking discovered that black holes have a characteristic temperature and are therefore capable of emitting radiation. Let's show that something similar happens to the economic black holes, i.e., there exists a characteristic "BH-internal political stability", namely, a marginal inclination to entropy $\frac{\partial Y}{\partial E}$.

Let $\hbar$ be the economic Planck constant. The "BH-internal political stability" and the "economic entropy" are given in terms of the surface economic gravity and horizon area by the formulae

$$I_{BH} = \frac{\kappa\hbar}{2\pi},\ E = \frac{Ac^3}{4\mathcal{G}\hbar},$$

where A means area of the economic black hole. Using geometrized units where $\mathcal{G} = 1, \hbar = 1, c = 1$, we can formulate

Theorem 3. *An economic black hole has two characteristics depending on (national) income and the total investment:*

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(i) The entropy

$$E = \pi r_+^2 = \pi \left(Y + \sqrt{Y^2 - \mathcal{I}^2} \right)^2.$$

(ii) The BH-internal political stability

$$I_{BH} = \frac{\kappa}{2\pi} = \frac{\sqrt{Y^2 - \mathcal{I}^2}}{2\pi \left[2Y \left(Y + \sqrt{Y^2 - \mathcal{I}^2} \right) - \mathcal{I}^2 \right]}.$$

(iii) The spaces $\{E, I\}$, $E \geq I \geq 0$, and $\{Y, \mathcal{I}\}$, $Y \geq \mathcal{I} \geq 0$, are diffeomorphic equivalent.

(iv) The total entropy (total BH-internal political stability) of the economic black hole is obtained by integration.

Hints: (i) Identify the horizon for the previous economic metric and examine the near horizon geometry to show that it has two-dimensional Rindler space-time as a factor.

(ii) Using the relation to the Rindler geometry, we determine the economic surface gravity κ as for the economic Schwarzschild black hole and thereby determine the internal political stability of the economic black hole.

(iii) In the extremal limit $Y \to \mathcal{I}$, the internal political stability vanishes but the entropy has a nonzero limit.

Corollary 1. *The entropy of the economic black hole is a convex function on the space $\{Y, \mathcal{I}\}$, $Y \geq \mathcal{I} \geq 0$.*

Corollary 2. *Let us consider either the function $(E, I) : D \subset \mathbb{R}^2 \to \mathbb{R}^2$ or the function $(I, E) : D \subset \mathbb{R}^2 \to \mathbb{R}^2$. The plot of the first component colored by the second component highlights entropic areas of economic interest, respectively areas of internal political stability influenced by entropy.*

Hint: We use the Maple subroutine "complexplot3d".

Finally, for the extremal Reissner - Nordström economic black hole, the near horizon geometry is of the form $AdS_2 \times S^2$, i.e., (r, t) is a two-dimensional Anti-de Sitter (AdS_2) and the second factor is the 2-sphere S^2.

3.1. Economic 4D Einstein PDEs and stress-energy tensor field

Accept $c = 1$ and denote $2\kappa = 16\pi\mathcal{G}$. Suppose that we use a full economic action

$$S = \int \left(\frac{1}{2\kappa} R + \mathcal{L} \right) \sqrt{-g}\, d^4x,$$

where $\mathcal{L}$ describes any economic fields appearing in the economics theory.

Theorem 4. *The Euler-Lagrange PDEs are the vacuum Einstein PDEs*

$$R_{\mu\nu} - \frac{R}{4}\, g_{\mu\nu} = kT_{\mu\nu}.$$

Proof. Again we shall use a well-known technique. The action principle then tells us that the variation of this action with respect to the inverse metric is zero, yielding

$$0 = \delta S = \int \left[\frac{1}{2\kappa} \left(\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R}{\sqrt{-g}} \frac{\delta \sqrt{-g}}{\delta g^{\mu\nu}} \right) + \frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}} \right] \delta g^{\mu\nu} \sqrt{-g}\, d^4x.$$

 doi:10.20944/preprints201903.0261.v1

Since this equality should hold for any variation $\delta g^{\mu\nu}$, we find

$$\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R}{\sqrt{-g}} \frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} = -2\kappa \frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}}.$$

Since $\frac{\delta R}{\delta g^{\mu\nu}} = R_{\mu\nu}$, these are the equations of motion for the metric tensor field $g_{\mu\nu}$, i.e., *Einstein PDEs*. The right hand side is proportional to the *economic stress-energy tensor field*

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}} = -2 \frac{\delta \mathcal{L}}{\delta g^{\mu\nu}} + g_{\mu\nu}\mathcal{L}.$$

□

4. Economic 4D Einstein PDEs and Ricci type flows or waves

Let $g_{\mu\nu}(x)$ be the components of a general economic metric and $R_{\mu\nu}$ the associated Ricci tensor on time-space $\mathbb{R} \times \mathbb{R}^3$. The first component of $x = (x^0, x^1, x^2, x^3)$ is the time $x^0 = t$. The solutions $g_{\mu\nu}(x)$ of the Einstein PDEs

$$R_{\mu\nu} = \frac{R}{2}\, g_{\mu\nu}$$

are "waves" with respect to $x^0 = t$.

Suppose that the components of the economic metric $g_{\mu\nu}(x)$ do not depend explicitly on t. Then we introduce a Ricci type flow $g(x, \tau)$ satisfying

$$\frac{\partial g_{\mu\nu}}{\partial \tau} = -2R_{\mu\nu}$$

and a Ricci type wave $g(x, \tau)$ by

$$\frac{\partial^2 g_{\mu\nu}}{\partial \tau^2} = -2R_{\mu\nu}.$$

An evolution metric $g(x, \tau)$, starting from $g_{\mu\nu}(x)$, determines an evolution of all geometric ingredients (connection, geodesics, curvature tensor field, Ricci tensor field, and scalar curvature).

On the other hand, we can introduce the economic flow

$$\frac{\partial g_{\mu\nu}}{\partial Y} = -2R_{\mu\nu}.$$

and the economic wave

$$\frac{\partial^2 g_{\mu\nu}}{\partial Y^2} = -2R_{\mu\nu},$$

where Y is the (national) income. The last two PDEs are ingredients in producing economic metrics with special properties.

The Ricci flow and the Ricci wave were introduced as tools to address a variety of non-linear problems in differential geometry and, in particular, the uniformization of compact Riemannian manifolds. The deformation variable t (or τ) that otherwise appears adhoc in mathematics, have meaning in physics, but produces difference of philosophy between economics, physics and mathematics regarding the applicability of the Ricci flow or the Ricci wave. In further papers, we shall be looking for economic interpretations of all previous flows and waves.

Open problem: Study the pseudo-Riemannian metric flows and pseudo-Riemannian metric dynamics (wave) generated by the covariant derivative of geometric vector fields (Killing vector field, conformal vector field, irrotational vector field, solenoidal vector field, and harmonic vector field) on pseudo-Riemannian manifolds. Such geometrical theories have anything economic meaning?

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5. Discussion

Our papers [20]-[34] are significant contributions to Roegenian economics because they set a vision and provides a framework for economics similar to thermodynamics based on a dictionary. In time we developed and study the following ideas: extrema with nonholonomic constraints [20], nonholonomic economic systems [21], economic geometric dynamics [22], black hole geometric thermodynamics [23], [24], Thermodynamics versus Economics [25], multitime optimal economic growth [26], [28], black hole models in economics [27], Geobiodynamics and Roegen type economy [29], nonholonomic geometry of economic systems [30], controllability of a nonholonomic macroeconomic system [31], optimal control on nonholonomic black holes [32], phase diagram for Roegenian economics and Geobiodynamics and Roegenian economic systems [33], economic cycles of Carnot type [34]. We interpret this collection of papers as a call to the economics-physics-mathematics community to respond to the current political forces that (inappropriately) shape our life. This article examines again the arguments and concludes that a proper dictionary may provide a productive way forward in econophysics. Of course, economics based on thermodynamics did not invalidate the traditional economy but confirmed new things that until now could not be explained directly.

Our theory addresses the role of mathematical context via proper dictionary, a topic absent from most mathematical papers. An implication of this argument is the need to strengthen the quality of the mathematics component in economics. The mathematical concepts turn up in entirely unexpected connections. Moreover, they often permit an unexpectedly close and accurate description of the economic phenomena in these connections. Also, because we do not understand always the reasons of Roegenian economics usefulness, we appreciate that a theory formulated in terms of mathematical concepts is uniquely appropriate.

Mathematics and physics play an important role in economics, but it is not so easy to recognize this. The uniqueness of the theories of mathematics and physics must impose the same think in economics. A proper answer to Roegenian economics would require elaborate theoretical work which has not been undertaken totally up to date. Our paper has a high degree of complexity because it uses techniques and ideas from differential geometry and thermodynamics to produce non-contradictory information in economics.

In this context, we show that an economic black hole has two characteristics depending on (national) *income* and the *total investment*: (i) the entropy, and (ii) the BH-internal political stability. The formulas found by us generate a diffeomorphism between the economic spaces $\{E, I\},\ E \geq I \geq 0$, and $\{Y, \mathcal{I}\},\ Y \geq \mathcal{I} \geq 0$, i.e., the pairs (entropy, internal political stability) and (national income, total investment) are deeply economically interconnected. The Maple subroutine "complexplot3d" applied to the pair (E, I) plots the first component E colored by the second component I, highlighting entropic areas of economic interest; applied to the pair (I, E) gives areas of internal political stability influenced by entropy. Future research will clarify the economic sense of these statements.

Author Contributions: All the authors contributed equally to the whole realization of the paper.

Funding: This research was funded by University Mediterranea of Reggio Calabria - Dept. of Law, Economics and Human Sciences, grant number "Decisions Lab 2019/1".

Acknowledgments: Many thanks go to University Mediterranea of Reggio Calabria - Dept. of Law, Economics and Human Sciences, grant number "Decisions Lab 2019/1", who funded this research. The subject of this paper was scientifically supported by the University Politehnica of Bucharest, University Mediterranea of Reggio Calabria, Institute of Geodynamics of Romanian Academy, and by the Academy of Romanian Scientists.

Conflicts of Interest: The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

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