THEORY OF $g$-$\mathfrak{T}_g$-COMPACTNESS

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ABSTRACT. Several specific types of generalized compactness of generalized topological spaces have been defined, investigated and related to compactness in ordinary topological spaces from time to time in the literature of topological spaces. Our recent research in the field of a new class of generalized compactness in a generalized topological space is reported herein as a starting point for more generalized classes.

KEY WORDS AND PHRASES. Generalized topological space, generalized compactness, generalized countable compactness, generalized sequential compactness, generalized local connectedness

1. INTRODUCTION

Among the most fundamental topological properties (briefly, $\mathcal{T}$-properties relative to ordinary topology, and $\mathcal{T}_g$-properties relative to generalized topology), the $\mathcal{T}$-properties, termed $\mathfrak{T}$-compactness and $g$-$\mathfrak{T}$-compactness in $\mathcal{T}$-spaces (ordinary and generalized compactness in ordinary topological spaces) and the $\mathcal{T}_g$-properties termed $\mathfrak{T}_g$-compactness and $g$-$\mathfrak{T}_g$-compactness in $\mathcal{T}_g$-spaces (ordinary and generalized compactness in generalized topological spaces) are verily the most important invariant properties [3, 4, 5, 7, 15, 16, 17, 20, 21, 22, 23, 25, 28, 29, 30, 31, 32, 33, 34, 35, 36]. In actual truth, $\mathfrak{T}$-compactness is an absolute property of a $\mathfrak{T}$-set [2, 13, 38, 33], and $g$-$\mathfrak{T}$-compactness, $\mathfrak{T}_g$-compactness and $g$-$\mathfrak{T}_g$-compactness, respectively, are absolute properties of a $g$-$\mathfrak{T}$-set, a $\mathfrak{T}_g$-set, and a $g$-$\mathfrak{T}_g$-set [18, 25, 29]. Typical examples of $g$-$\mathfrak{T}$-compactness in $\mathcal{T}$-spaces are $\alpha$, $\beta$, $\gamma$-compactness [10, 19, 26]; examples of $\mathfrak{T}_g$-compactness in $\mathcal{T}_g$-spaces are semi-$\alpha$, $s$, $gb$-compactness [7, 14, 29], whereas examples of $g$-$\mathfrak{T}_g$-compactness in $\mathcal{T}_g$-spaces are $bT^\alpha$, $\mu$-$rgb$, $\nu\pi$-compactness [5, 22, 37], among others.

In the literature of $\mathcal{T}_g$-spaces, the study of $g$-$\mathfrak{T}_g$-sets by various authors has produced some new classes of $g$-$\mathfrak{T}_g$-compactness in $\mathcal{T}_g$-spaces, similar in descriptions to $g$-$\mathfrak{T}$-compactness in $\mathcal{T}$-spaces [20, 21, 22, 25, 28, 30, 34, 35, 36]. In the paper

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Notes to the reader: The structures $\mathfrak{T} = (\Omega, \mathcal{T})$ and $\mathfrak{T}_g = (\Omega, \mathcal{T}_g)$, respectively, are topological spaces (briefly, $\mathcal{T}$-space and $\mathcal{T}_g$-space) with ordinary topology $\mathcal{T}$ and generalized topology $\mathcal{T}_g$ (briefly, topology and $g$-topology). Subsets of $\mathfrak{T}$, $\mathfrak{T}_g$, respectively, are called $\mathfrak{T}$, $\mathfrak{T}_g$-sets; subsets of $\mathcal{T}$, $\mathcal{T}_g$, respectively, are called $\mathcal{T}$, $\mathcal{T}_g$-open sets, and their complements are called $\mathcal{T}$, $\mathcal{T}_g$-closed sets. Generalizations of $\mathfrak{T}$-sets, $\mathcal{T}$-open and $\mathcal{T}$-closed sets in $\mathfrak{T}$, respectively, are called $\mathfrak{T}$-sets, $g$-$\mathcal{T}$-open and $g$-$\mathcal{T}$-closed sets; generalizations of $\mathfrak{T}_g$-sets, $\mathcal{T}_g$-open and $\mathcal{T}_g$-closed sets in $\mathfrak{T}_g$, respectively, are called $\mathfrak{T}_g$-sets, $g$-$\mathcal{T}_g$-open and $g$-$\mathcal{T}_g$-closed sets. Compactness in $\mathfrak{T}$ with $\mathfrak{T}$, $g$-$\mathcal{T}$-open coverings are called $\mathfrak{T}$, $g$-$\mathfrak{T}$-compactness, respectively; compactness in $\mathfrak{T}_g$ with $\mathfrak{T}_g$, $g$-$\mathfrak{T}_g$-open coverings are called $\mathfrak{T}_g$, $g$-$\mathfrak{T}_g$-compactness, respectively.
of [21], the authors introduced, gave characterizations and studied the properties of a new kind of \( g\)-\( T_\gamma \)-compactness, called soft \( \mu \)-compactness, in \( T_\gamma \)-spaces. [25] introduced two types of \( g\)-\( T_\nu \)-compactness, called \( \mu \)-semi-compactness and \( \mu \)-semi-Lindelöfness in \( T_\nu \)-spaces and, studied and gave characterizations of some of their properties. [34] introduced five types of \( g\)-\( T_\varphi \)-compactness in \( T_\varphi \)-spaces, called \( g \)-compactness, \( g \)-semi compactness, \( g \)-precompactness, \( g \)-\( \alpha \)-compactness, and \( g \)-\( \beta \)-compactness, and gave the characterizations of each of such \( g \)-compactness like properties and established the relationships among them. [36] gave the definition of the notion of \( D_\nu \)-compactness in \( T_\nu \)-spaces and studied its properties. The authors also investigated \( D_\mu \)-compactness in \( T_\mu \)-subspaces and products of \( T_\mu \)-spaces, just to name a few.

In regards to the above references, it results that, from every new type of \( g\)-\( T_\nu \)-set defined in a \( T_\nu \)-space, there can be defined a new type of \( g\)-\( T_\nu \)-connectedness in the \( T_\nu \)-space. Having defined a new class of \( g\)-\( T_\nu \)-sets and studied from it some \( T_\nu \)-properties in a \( T_\nu \)-space (see our works on theories of \( g\)-\( T_\nu \)-sets, \( g\)-\( T_\nu \)-maps, \( g\)-\( T_\nu \)-separation axioms, and \( g\)-\( T_\nu \)-connectedness), it seems, therefore, worth considering to introduce a new type of \( g\)-\( T_\nu \)-compactness in the \( T_\nu \)-space and discuss its \( T_\nu \)-properties accordingly. In this paper, we attempt to make a contribution to such a development by introducing a new theory, called Theory of \( g\)-\( T_\nu \)-Compactness, in which it is presented a new generalized version of \( T_\nu \)-connectedness in terms of the notions of \( g\)-\( T_\nu \)-sets, discussing the basic properties and giving its characterizations in this direction.

The paper is organised as follows: In Sect. 2, preliminary notions are described in Sect. 2.1 and the main results of \( g\)-\( T_\nu \)-compactness in a \( T_\nu \)-space are reported in Sect. 3. In Sect. 4, the establishment of the relationships among various types of \( g\)-\( T_\nu \)-compactness are discussed in Sect. 4.1. To support the work, a nice application of the concept of \( g\)-\( T_\nu \)-compactness in a \( T_\nu \)-space is presented in Sect. 4.2. Finally, Sect. 4.3 provides concluding remarks and future directions of the notion of \( g\)-\( T_\nu \)-compactness in a \( T_\nu \)-space.

2. Theory

2.1. Preliminaries. Though many of the notations and definitions had already been neatly discussed in complementary papers (see papers on theories of \( g\)-\( T_\nu \)-sets, \( g\)-\( T_\nu \)-maps, and \( g\)-\( T_\nu \)-separation axioms) however, we thought it necessary to recall some basic definitions and notations of most basic concepts presented in those papers.

The set \( \mathcal{U} \) denotes the universe of discourse, fixed within the framework of the theory of \( g\)-\( T_\nu \)-compactness and containing as elements all sets (\( \Lambda \)-sets: \( \Lambda \in \{ \Omega, \Sigma \} \); \( \mathcal{T}_\Lambda \); \( g\)-\( T_\Lambda \); \( g\)-\( \Sigma \)-\( \Lambda \)-sets; \( T_\varphi \); \( g\)-\( T_\varphi \); \( g\)-\( \Sigma \)-\( \varphi \)-\( \Lambda \)-sets, to name a few) considered in this theory, and \( I_n^\nu \) defined \( \{ \nu \in \mathbb{N}^0 : \nu \leq n \} \); index sets \( I_0^\nu \); \( I_*^\nu \); \( I_*^\infty \) are defined in an analogous way. Let \( \Lambda \in \{ \Omega, \Sigma \} \subset \mathcal{U} \) be a given set and let

\[
\mathcal{P}(\Lambda) \overset{\text{def}}{=} \{ O_{\varphi, \nu} \subseteq \Lambda : \nu \in I_*^\infty \}
\]

be the collection of all subsets \( O_{\varphi, 1}, O_{\varphi, 2}, \ldots \), of \( \Lambda \). Then every one-valued map of the type \( T_{\varphi, \Lambda} : \mathcal{P}(\Lambda) \rightarrow \mathcal{P}(\Lambda) \) satisfying \( T_{\varphi, \Lambda}(\emptyset) = \emptyset \); \( T_{\varphi, \Lambda}(O_{\varphi, \rho}) \subseteq O_{\varphi, \rho} \), and \( T_{\varphi, \Lambda}(\bigcup_{\nu \in I_*^\infty} O_{\varphi, \nu}) = \bigcup_{\nu \in I_*^\infty} T_{\varphi, \Lambda}(O_{\varphi, \nu}) \) is called a \( \varphi \)-topology on \( \Lambda \), and the structure \( \mathcal{T}_{\varphi, \Lambda} \overset{\text{def}}{=} (\Lambda, T_{\varphi, \Lambda}) \) is called a \( T_{\varphi, \Lambda} \)-space, on which no separation axioms are assumed unless otherwise mentioned [12, 11, 27].

On the other hand, if \( \mathcal{P}(\Gamma) \overset{\text{def}}{=} \{ O_{\varphi, \nu} \subset \Gamma : \nu \in I_*^\infty \} \) denotes the family of all
subsets $\mathcal{O}_{g,1}, \mathcal{O}_{g,2}, \ldots$, of any subset $\Gamma \subseteq \Lambda$ of $\Lambda$, then every one-valued restriction map of the type $\mathcal{T}_{g,\Gamma} : \mathcal{P}(\Gamma) \longrightarrow \{\mathcal{O}_g \cap \Gamma : \mathcal{O}_g \in \mathcal{T}_{g,\Lambda}\}$ defines a relative $g$-topology on $\Gamma$, and the structure $\mathcal{T}_{g,\Gamma} \overset{\text{def}}{=} (\Gamma, \mathcal{T}_{g,\Gamma})$ is called a $\mathcal{T}_{g,\Lambda}$-subspace. The operator $\text{cl}_{g,\Lambda} : \mathcal{P}(\Lambda) \rightarrow \mathcal{P}(\Lambda)$ carrying each $\mathcal{T}_{g,\Lambda}$-set $\mathcal{S}_g \subset \mathcal{T}_{g,\Lambda}$ into its closure $\text{cl}_{g,\Lambda}(\mathcal{S}_g) = \mathcal{T}_{g,\Lambda} - \text{int}_{g,\Lambda}(\mathcal{T}_{g,\Lambda} \setminus \mathcal{S}_g) \subset \mathcal{T}_{g,\Lambda}$ is called a $g$-closure operator and the operator $\text{int}_{g,\Lambda} : \mathcal{P}(\Lambda) \rightarrow \mathcal{P}(\Lambda)$ carrying each $\mathcal{T}_{g,\Lambda}$-set $\mathcal{S}_g \subset \mathcal{T}_{g,\Lambda}$ into its interior $\text{int}_{g,\Lambda}(\mathcal{S}_g) = \mathcal{T}_{g,\Lambda} - \text{cl}_{g,\Lambda}(\mathcal{T}_{g,\Lambda} \setminus \mathcal{S}_g) \subset \mathcal{T}_{g,\Lambda}$ is called a $g$-interior operator; for clarity, we will use $\text{cl}_{g}(-), \text{int}_{g}(-)$, respectively, instead of $\text{cl}_{g,\Lambda}(-), \text{int}_{g,\Lambda}(-)$.

Let $\mathcal{T}_{g,\Lambda}$ be a $\mathcal{T}_{g,\Lambda}$-space, let $\bar{\mathcal{U}}_{g} : \mathcal{P}(\Lambda) \rightarrow \mathcal{P}(\Lambda)$ denotes the absolute complement with respect to the underlying set $\Lambda \subset \mathcal{U}$, and let $\mathcal{S}_g \subset \mathcal{T}_{g,\Lambda}$ be any $\mathcal{T}_{g,\Lambda}$-set. The classes

\begin{equation}
\mathcal{T}_{g,\Lambda} \overset{\text{def}}{=} \{ \mathcal{O}_g \subset \mathcal{T}_{g,\Lambda} : \mathcal{O}_g \in \mathcal{T}_{g,\Lambda}\},
\end{equation}

respectively, denote the classes of all $\mathcal{T}_{g,\Lambda}$-open and $\mathcal{T}_{g,\Lambda}$-closed relative to the $g$-topology $\mathcal{T}_{g,\Lambda}$, and the classes

\begin{equation}
\mathcal{C}^{\text{sup}}_{\mathcal{T}_{g,\Lambda}}[\mathcal{S}_g] \overset{\text{def}}{=} \{ \mathcal{O}_g \in \mathcal{T}_{g,\Lambda} : \mathcal{O}_g \subseteq \mathcal{S}_g\},
\end{equation}

\begin{equation}
\mathcal{C}^{\text{inf}}_{\mathcal{T}_{g,\Lambda}}[\mathcal{S}_g] \overset{\text{def}}{=} \{ \mathcal{K}_g \in \mathcal{T}_{g,\Lambda} : \mathcal{K}_g \supseteq \mathcal{S}_g\},
\end{equation}

respectively, denote the classes of the $\mathcal{T}_{g,\Lambda}$-open subsets and $\mathcal{T}_{g,\Lambda}$-closed superset (complements of the $\mathcal{T}_{g,\Lambda}$-open subsets) of the $\mathcal{T}_{g,\Lambda}$-set $\mathcal{S}_g \subset \mathcal{T}_{g,\Lambda}$ relative to the $g$-topology $\mathcal{T}_{g,\Lambda}$. To this end, the $g$-closure and the $g$-interior of a $\mathcal{T}_{g,\Lambda}$-set $\mathcal{S}_g \subset \mathcal{T}_{g,\Lambda}$ in a $\mathcal{T}_{g,\Lambda}$-space $[6]$ define themselves as

\begin{equation}
\text{int}_{g,\Lambda}(\mathcal{S}_g) \overset{\text{def}}{=} \bigcup_{\mathcal{O}_g \in \mathcal{C}^{\text{sup}}_{\mathcal{T}_{g,\Lambda}}[\mathcal{S}_g]} \mathcal{O}_g, \quad \text{cl}_{g,\Lambda}(\mathcal{S}_g) \overset{\text{def}}{=} \bigcap_{\mathcal{K}_g \in \mathcal{C}^{\text{inf}}_{\mathcal{T}_{g,\Lambda}}[\mathcal{S}_g]} \mathcal{K}_g.
\end{equation}

In this work, by $\text{cl}_g \circ \text{int}_g(-), \text{int}_g \circ \text{cl}_g(-)$, and $\text{cl}_g \circ \text{int}_g \circ \text{cl}_g(-)$, respectively, are meant $\text{cl}_g(\text{int}_g(\cdot)), \text{int}_g(\text{cl}_g(\cdot)), \text{and} \text{cl}_g(\text{int}_g(\text{cl}_g(\cdot)))$; other composition operators are defined in a similar way. Furthermore, the backslash $\mathcal{T}_{g,\Lambda} \setminus \mathcal{S}_g$ refers to the set-theoretic difference $\mathcal{T}_{g,\Lambda} - \mathcal{S}_g$. The mapping $\text{op}_g : \mathcal{P}(\Lambda) \rightarrow \mathcal{P}(\Lambda)$ is called a $g$-operation on $\mathcal{P}(\Lambda)$ if the following statements hold:

\begin{equation}
\forall \mathcal{S}_g \in \mathcal{P}(\Lambda) \setminus \{\emptyset\} : \exists (\mathcal{O}_g, \mathcal{K}_g) \in \mathcal{T}_{g,\Lambda} \setminus \{\emptyset\} \times \mathcal{T}_{g,\Lambda} \setminus \{\emptyset\} : \text{op}_g(\emptyset) = \mathcal{O}_g \vee \text{op}_g(\emptyset) = \mathcal{K}_g, \quad (\mathcal{S}_g \subseteq \text{op}_g(\mathcal{O}_g)) \lor (\mathcal{S}_g \supseteq \text{op}_g(\mathcal{K}_g)),
\end{equation}

where $\text{op}_g : \mathcal{P}(\Lambda) \rightarrow \mathcal{P}(\Lambda)$ is called the ”complementary $g$-operation” on $\mathcal{P}(\Lambda)$ and, for all $\mathcal{T}_{g,\Lambda}$-sets $\mathcal{S}_g, \mathcal{S}_{g,\nu}, \mathcal{S}_{g,\mu} \in \mathcal{P}(\Lambda) \setminus \{\emptyset\}$, the following axioms are satisfied:

- **Ax. 1.** $(\mathcal{S}_g \subseteq \text{op}_g(\mathcal{O}_g)) \lor (\mathcal{S}_g \supseteq \text{op}_g(\mathcal{K}_g))$;
- **Ax. 11.** $(\text{op}_g(\mathcal{S}_g) \subseteq \text{op}_g \circ \text{op}_g(\mathcal{O}_g)) \lor (\text{op}_g(\mathcal{S}_g) \supseteq \text{op}_g \circ \text{op}_g(\mathcal{K}_g))$;
- **Ax. 111.** $(\mathcal{S}_{g,\nu} \subseteq \mathcal{S}_{g,\mu} \rightarrow \text{op}_g(\mathcal{O}_{g,\nu}) \subseteq \text{op}_g(\mathcal{O}_{g,\mu})) \lor (\mathcal{S}_{g,\mu} \subseteq \mathcal{S}_{g,\nu} \rightarrow \text{op}_g(\mathcal{K}_{g,\mu}) \supseteq \text{op}_g(\mathcal{K}_{g,\nu}))$;
- **Ax. 1111.** $(\text{op}_g(\bigcup_{\sigma=\nu,\mu} \mathcal{S}_{g,\sigma}) \subseteq \bigcup_{\sigma=\nu,\mu} \text{op}_g(\mathcal{O}_{g,\sigma})) \lor (\text{op}_g(\bigcup_{\sigma=\nu,\mu} \mathcal{S}_{g,\sigma}) \supseteq \bigcup_{\sigma=\nu,\mu} \text{op}_g(\mathcal{K}_{g,\sigma}))$. 


for some $\mathcal{T}_{g,\Lambda}$-open sets $\mathcal{O}_g$, $\mathcal{O}_g,\mu \in \mathcal{T}_{g,\Lambda} \setminus \{\emptyset\}$ and $\mathcal{T}_{g,\Lambda}$-closed sets $\mathcal{K}_g, \mathcal{K}_g,\nu, \mathcal{K}_g,\mu \in \sim \mathcal{T}_{g,\Lambda}$ [8, 24]. The class $L_g[\Omega] = L^g_\nu [\Omega] \times L^\mu_\nu [\Omega]$, where

\begin{equation}
L^g_\nu [\Lambda] \overset{\text{def}}{=} \{ op_{g,\nu} (\cdot) = (op_{g,\nu} (\cdot), \neg op_{g,\mu} (\cdot)) : (\nu, \mu) \in I^0_3 \times I^0_3 \}
\end{equation}

in the $\mathcal{T}_{g,\Lambda}$-space $\mathcal{T}_{g,\Lambda}$, stands for the class of all possible $g$-operators and their complementary $g$-operators in the $\mathcal{T}_{g,\Lambda}$-space $\mathcal{T}_{g,\Lambda}$. Its elements are defined as:

\begin{align*}
op_g (\cdot) & \in L^g_\nu [\Lambda] \overset{\text{def}}{=} \{ op_{g,0} (\cdot), op_{g,1} (\cdot), op_{g,2} (\cdot), op_{g,3} (\cdot) \} \\
= & \{ \text{int}_g (\cdot), \text{cl}_g \circ \text{int}_g (\cdot), \text{cl}_g \circ \text{cl}_g (\cdot) \}; \\
\neg op_g (\cdot) & \in L^g_\nu [\Lambda] \overset{\text{def}}{=} \{ \neg op_{g,0} (\cdot), \neg op_{g,1} (\cdot), \neg op_{g,2} (\cdot), \neg op_{g,3} (\cdot) \} \\
= & \{ \text{cl}_g (\cdot), \text{cl}_g \circ \text{int}_g (\cdot), \text{cl}_g \circ \text{cl}_g \circ \text{int}_g (\cdot) \}.
\end{align*}

A $\mathcal{T}_{g,\Lambda}$-set $\mathcal{S}_{g,\Lambda} \subset \mathcal{T}_{g,\Lambda}$ in a $\mathcal{T}_{g,\Lambda}$-space is called a $g$-$\mathcal{T}_{g,\Lambda}$-set if and only if there exist a pair $(\mathcal{O}_g, \mathcal{K}_g) \in \mathcal{T}_{g,\Lambda} \times \sim \mathcal{T}_{g,\Lambda}$ of $\mathcal{T}_{g,\Lambda}$-open and $\mathcal{T}_{g,\Lambda}$-closed sets, and a $g$-operator $op_g (\cdot) \in L_g[\Omega]$ such that the following statement holds:

\begin{equation}
(\exists \xi) \left[ (\xi \in \mathcal{S}_g) \wedge (G \mathcal{S}_g \subseteq op_g (\mathcal{O}_g)) \right] \wedge \left( \mathcal{S}_g \supseteq \neg op_g (\mathcal{K}_g) \right).
\end{equation}

The $g$-$\mathcal{T}_{g,\Lambda}$-set $\mathcal{S}_g \subset \mathcal{T}_{g,\Lambda}$ is said to be of category $\nu$ if and only if it belongs to the following class of $g$-$\nu$-$\mathcal{T}_{g,\Lambda}$-sets:

\begin{equation}
g-\nu-S[\mathcal{T}_{g,\Lambda}] \overset{\text{def}}{=} \{ \mathcal{S}_g \subset \mathcal{T}_{g,\Lambda} : (\exists \mathcal{O}_g, \mathcal{K}_g, op_{g,\nu} (\cdot)) \}
\end{equation}

It is called a $g$-$\nu$-$\mathcal{T}_{g,\Lambda}$-open set if it satisfies the first property in $g$-$\nu$-$S[\mathcal{T}_{g,\Lambda}]$ and a $g$-$\nu$-$\mathcal{T}_{g,\Lambda}$-closed set if it satisfies the second property in $g$-$\nu$-$S[\mathcal{T}_{g,\Lambda}]$. The classes of $g$-$\nu$-$\mathcal{T}_{g,\Lambda}$-open and $g$-$\nu$-$\mathcal{T}_{g,\Lambda}$-closed sets, respectively, are defined by

\begin{align*}
g-\nu-O[\mathcal{T}_{g,\Lambda}] & \overset{\text{def}}{=} \{ \mathcal{S}_g \subset \mathcal{T}_{g,\Lambda} : (\exists \mathcal{O}_g, op_{g,\nu} (\cdot)) [\mathcal{S}_g \subseteq op_{g,\nu} (\mathcal{O}_g)] \}, \\
g-\nu-K[\mathcal{T}_{g,\Lambda}] & \overset{\text{def}}{=} \{ \mathcal{S}_g \subset \mathcal{T}_{g,\Lambda} : (\exists \mathcal{K}_g, op_{g,\nu} (\cdot)) [\mathcal{S}_g \supseteq \neg op_{g,\nu} (\mathcal{K}_g)] \}.
\end{align*}

From these classes, the following relation holds:

\begin{align*}
g-S[\mathcal{T}_{g,\Lambda}] & = \bigcup_{\nu \in I^0_3} g-\nu-S[\mathcal{T}_{g,\Lambda}] \\
& = \bigcup_{\nu \in I^0_3} (g-\nu-O[\mathcal{T}_{g,\Lambda}] \cup g-\nu-K[\mathcal{T}_{g,\Lambda}]) \\
& = \bigcup_{\nu \in I^0_3} g-\nu-O[\mathcal{T}_{g,\Lambda}] \cup \bigcup_{\nu \in I^0_3} g-\nu-K[\mathcal{T}_{g,\Lambda}] \\
& = g-O[\mathcal{T}_{g,\Lambda}] \cup g-K[\mathcal{T}_{g,\Lambda}].
\end{align*}

By omitting the subscript $g$ in almost all symbols of the above definitions, we obtain very similar definitions but in a $\mathcal{T}_{\Lambda}$-space.

A $\mathcal{S}$-set $\mathcal{S} \subset \mathcal{T}_{\Lambda}$ in a $\mathcal{T}_{\Lambda}$-space is called a $g$-$\mathcal{T}_{\Lambda}$-set if and only if there exists a pair $(\mathcal{O}, \mathcal{K}) \in \mathcal{T}_{\Lambda} \times \sim \mathcal{T}_{\Lambda}$ of $\mathcal{T}_{\Lambda}$-open and $\mathcal{T}_{\Lambda}$-closed sets, and an operator $op (\cdot) \in L[\Lambda]$ such that the following statement holds:

\begin{equation}
(\exists \xi) \left[ (\xi \in \mathcal{S}) \wedge (\mathcal{S} \subseteq op (\mathcal{O})) \right] \wedge \left( \mathcal{S} \supseteq \neg op (\mathcal{K}) \right).
\end{equation}
The \( g\cdot g\)-set \( S \subset \mathfrak{T}_\lambda \) is said to be of category \( \nu \) if and only if it belongs to the following class of \( g\cdot g\)-\( \mathfrak{T}_\lambda \)-sets:

\[
g\cdot g\cdot S[\mathfrak{T}_\lambda] \overset{\text{def}}{=} \{ S \subset \mathfrak{T}_\lambda : (\exists \Omega, K, \text{op}_\nu (\cdot)) \ [S \subseteq \text{op}_\nu (\Omega)] \}
\]

(2.12)

It is called a \( g\cdot g\cdot \mathfrak{T}_\lambda \)-open set if it satisfies the first property in \( g\cdot g\cdot S[\mathfrak{T}_\lambda] \) and a \( g\cdot g\cdot \mathfrak{T}_\lambda \)-closed set if it satisfies the second property in \( g\cdot g\cdot S[\mathfrak{T}_\lambda] \). The classes of \( g\cdot g\cdot \mathfrak{T}_\lambda \)-open and \( g\cdot g\cdot \mathfrak{T}_\lambda \)-closed sets, respectively, are defined by

\[
g\cdot g\cdot O[\mathfrak{T}_\lambda] \overset{\text{def}}{=} \{ S \subset \mathfrak{T}_\lambda : (\exists \Omega, \text{op}_\nu (\cdot)) [S \subseteq \text{op}_\nu (\Omega)] \}
\]

(2.13)

\[
g\cdot g\cdot K[\mathfrak{T}_\lambda] \overset{\text{def}}{=} \{ S \subset \mathfrak{T}_\lambda : (\exists \Omega, \text{op}_\nu (\cdot)) [S \supseteq \neg \text{op}_\nu (\Omega)] \}. \]

As in the previous definitions, from these classes, the following relation holds:

\[
g\cdot g\cdot S[\mathfrak{T}_\lambda] = \bigcup_{\nu \in T_\lambda} g\cdot g\cdot S[\mathfrak{T}_\lambda] = \bigcup_{\nu \in T_\lambda} (g\cdot g\cdot O[\mathfrak{T}_\lambda] \cup g\cdot g\cdot K[\mathfrak{T}_\lambda]) = (\bigcup_{\nu \in T_\lambda} g\cdot g\cdot O[\mathfrak{T}_\lambda]) \cup (\bigcup_{\nu \in T_\lambda} g\cdot g\cdot K[\mathfrak{T}_\lambda]) = g\cdot O[\mathfrak{T}_\lambda] \cup g\cdot K[\mathfrak{T}_\lambda].
\]

(2.14)

The classes \( O[\mathfrak{T}_\lambda] \) and \( K[\mathfrak{T}_\lambda] \) denote the families of \( \mathfrak{T}_\lambda \)-open and \( \mathfrak{T}_\lambda \)-closed sets, respectively, in \( \mathfrak{T}_\lambda \), with \( S[\mathfrak{T}_\lambda] = O[\mathfrak{T}_\lambda] \cup K[\mathfrak{T}_\lambda] \); the classes \( O[\mathfrak{T}_\lambda] \) and \( K[\mathfrak{T}_\lambda] \) denote the families of \( \mathfrak{T}_\lambda \)-open and \( \mathfrak{T}_\lambda \)-closed sets, respectively, in \( \mathfrak{T}_\lambda \), with \( S[\mathfrak{T}_\lambda] = O[\mathfrak{T}_\lambda] \cup K[\mathfrak{T}_\lambda] \). Whenever we feel that the subscript \( \lambda \in \{ \Omega, \Sigma \} \) is understood from the context, it will be omitted for clarity. We are now in a position to present a carefully chosen set of terms used in the theory of \( g\cdot g\)-spaces between \( \mathfrak{T}_\lambda \)-spaces.

A \( (\mathfrak{T}_\lambda, \mathfrak{T}_\Sigma) \)-map and a \( (\mathfrak{T}_\Theta, \mathfrak{T}_\Sigma) \)-map, respectively, are mappings in the usual sense between \( \mathfrak{T}_\lambda \)-spaces and \( \mathfrak{T}_\lambda \)-spaces.

**Definition 2.1** \( (\mathfrak{T}_\lambda, \mathfrak{T}_\Sigma), (\mathfrak{T}_\Theta, \mathfrak{T}_\Sigma) \)-Maps. Let \( \mathfrak{T}_\lambda = (\Omega, \mathfrak{T}_\lambda) \) and \( \mathfrak{T}_\Sigma = (\Sigma, \mathfrak{T}_\Sigma) \) be \( \mathfrak{T} \)-spaces and, let \( \mathfrak{T}_\Theta = (\Omega, \mathfrak{T}_\Theta) \) and \( \mathfrak{T}_\Gamma = (\Sigma, \mathfrak{T}_\Gamma) \) be \( \mathfrak{T}_\Theta \)-spaces. Then, a map:

- \( \pi : \mathfrak{T}_\lambda \rightarrow \mathfrak{T}_\Sigma \) is called a \( (\mathfrak{T}_\lambda, \mathfrak{T}_\Sigma) \)-map from \( \mathfrak{T}_\lambda \) into \( \mathfrak{T}_\Sigma \).
- \( \pi : \mathfrak{T}_\Theta \rightarrow \mathfrak{T}_\Gamma \) is called a \( (\mathfrak{T}_\Theta, \mathfrak{T}_\Gamma) \)-map from \( \mathfrak{T}_\Theta \) into \( \mathfrak{T}_\Gamma \).

A \( g\cdot g\cdot (\mathfrak{T}_\lambda, \mathfrak{T}_\Sigma) \)-map is a generalization of a \( (\mathfrak{T}_\lambda, \mathfrak{T}_\Sigma) \)-map and, hence, is a distinguished mapping between \( \mathfrak{T} \)-spaces which does not exhibit mapping properties in the usual sense but does exhibit mapping properties in the generalized sense.

**Definition 2.2** \( g\cdot g\cdot (\mathfrak{T}_\Theta, \mathfrak{T}_\Sigma) \)-Map. Let \( \mathfrak{T}_\lambda = (\Omega, \mathfrak{T}_\lambda) \) and \( \mathfrak{T}_\Sigma = (\Sigma, \mathfrak{T}_\Sigma) \) be \( \mathfrak{T} \)-spaces, and let \( \text{op} (\cdot) \in \mathcal{L}[\Sigma] \). Then, a map \( \pi : \mathfrak{T}_\lambda \rightarrow \mathfrak{T}_\Sigma \) is called a \( g\cdot g\cdot (\mathfrak{T}_\lambda, \mathfrak{T}_\Sigma) \)-map if and only if, for every pair \( (\mathfrak{O}_\lambda, \mathfrak{K}_\lambda) \in \mathfrak{T}_\lambda \times \neg \mathfrak{T}_\lambda \) of \( \mathfrak{T}_\lambda \)-open and \( \mathfrak{T}_\lambda \)-closed sets in \( \mathfrak{T}_\lambda \) there corresponds a pair \( (\mathfrak{O}_\Sigma, \mathfrak{K}_\Sigma) \in \mathfrak{T}_\Sigma \times \neg \mathfrak{T}_\Sigma \) of \( \mathfrak{T}_\Sigma \)-open and \( \mathfrak{T}_\Sigma \)-closed sets in \( \mathfrak{T}_\Sigma \) such that the following statement holds:

\[
[\pi_\lambda (\mathfrak{O}_\lambda) \subseteq \text{op} (\mathfrak{O}_\lambda)] \lor [\pi_\lambda (\mathfrak{K}_\lambda) \supseteq \neg \text{op} (\mathfrak{K}_\lambda)].
\]

(2.15)
A \( g \) - \( (\Sigma, \Sigma) \)-map is said to be of category \( \nu \) if and only if it belongs to the following class of \( g \)-\( \nu \) - \( (\Sigma, \Sigma) \)-maps:

\[
g \cdot \nu \cdot M[\Sigma; \Sigma] \overset{\text{def}}{=} \left\{ \pi_\theta : (\forall \Omega_\omega, \Sigma_\sigma) (\exists \Omega_\sigma, \Sigma_\sigma, \text{op}_\nu(\cdot)) \left( (\pi_\theta(\Omega_\omega) \subseteq \text{op}_\nu(\Omega_\sigma) ) \lor (\pi_\theta(\Sigma_\omega) = \neg \text{op}_\nu(\Sigma_\sigma)) \right) \right\}.
\]

(2.16)

It is called a \( g \)-\( \nu \) - \( (\Sigma, \Sigma) \)-open map if it satisfies the first property in \( g \cdot \nu \cdot M[\Sigma; \Sigma] \) and a \( g \)-\( \nu \) - \( (\Sigma, \Sigma) \)-closed map if it satisfies the second property in \( g \cdot \nu \cdot M[\Sigma; \Sigma] \).

The classes of \( g \)-\( \nu \) - \( (\Sigma, \Sigma) \)-open and \( g \)-\( \nu \) - \( (\Sigma, \Sigma) \)-closed maps, respectively, are defined by

\[
g \cdot \nu \cdot M_\sigma[\Sigma; \Sigma] \overset{\text{def}}{=} \left\{ \pi_\theta : (\forall \Omega_\omega, \Sigma_\sigma) (\exists \Omega_\sigma, \Sigma_\sigma, \text{op}_\nu(\cdot)) \left( \pi_\theta(\Omega_\omega) \subseteq \text{op}_\nu(\Omega_\sigma) \right) \right\},
\]

\[
g \cdot \nu \cdot M_K[\Sigma; \Sigma] \overset{\text{def}}{=} \left\{ \pi_\theta : (\forall \Omega_\omega, \Sigma_\sigma) (\exists \Omega_\sigma, \Sigma_\sigma, \text{op}_\nu(\cdot)) \left( \pi_\theta(\Omega_\omega) \subseteq \text{op}_\nu(\Sigma_\sigma) \right) \right\}.
\]

(2.17)

From the class \( g \cdot \nu \cdot M[\Sigma; \Sigma] \), consisting of the classes \( g \cdot \nu \cdot M_\sigma[\Sigma; \Sigma] \) and \( g \cdot \nu \cdot M_K[\Sigma; \Sigma] \), respectively, of \( g \)-\( \nu \) - \( (\Sigma, \Sigma) \)-open and \( g \)-\( \nu \) - \( (\Sigma, \Sigma) \)-closed maps, where \( \nu \in T^3 \), there results in the following definition.

**Definition 2.3.** Let \( \Sigma = (\Omega, \Sigma) \) and \( \Sigma = (\Sigma, \Sigma) \) be \( T \)-spaces. If, for each \( \nu \in T^3 \), \( g \cdot \nu \cdot M_\sigma[\Sigma; \Sigma] \) and \( g \cdot \nu \cdot M_K[\Sigma; \Sigma] \), respectively, denote the classes of \( g \)-\( \nu \) - \( (\Sigma, \Sigma) \)-open and \( g \)-\( \nu \) - \( (\Sigma, \Sigma) \)-closed maps, then

\[
g \cdot M[\Sigma; \Sigma] = \bigcup_{\nu \in T^3} g \cdot \nu \cdot M[\Sigma; \Sigma] \bigcup_{\nu \in T^3} g \cdot \nu \cdot M_K[\Sigma; \Sigma].
\]

(2.18)

As above, the \( g \cdot (\Sigma, \Sigma) \)-map is a generalization of the \( (\Sigma, \Sigma) \)-map and, thus, is a distinguished mapping between \( T \)-spaces which does not exhibit mapping properties in the usual sense but does exhibit mapping properties in the generalized sense.

**Definition 2.4.** \( g \)-\( \nu \) - \( (\Sigma, \Sigma) \)-Map. Let \( \Sigma = (\Omega, \Sigma) \) and \( \Sigma = (\Sigma, \Sigma) \) be \( T \)-spaces, and let \( \text{op}_\nu(\cdot) \in L^3[\Sigma] \). Then, a map \( \pi_\theta : \Sigma \to \Sigma \) is called a \( g \)-\( (\Sigma, \Sigma) \)-map if and only if, for every pair \( (\Omega_\omega, \Sigma_\sigma) \in T^3 \times \neg T^3 \) of \( T \)-open and \( \neg T^3 \)-closed sets in \( \Sigma \), there corresponds a pair \( (\Omega_\sigma, \Sigma_\sigma) \in T^3 \times \neg T^3 \) of \( T \)-open and \( \neg T^3 \)-closed sets in \( \Sigma \) such that the following statement holds:

\[
[\pi_\theta(\Omega_\omega, \Sigma_\sigma) \subseteq \text{op}_\nu(\Omega_\sigma, \Sigma_\sigma)] \lor [\pi_\theta(\Sigma_\sigma, \Omega_\sigma) \subseteq \neg \text{op}_\nu(\Sigma_\omega, \Omega_\sigma)].
\]

(2.19)

A \( g \cdot (\Sigma, \Sigma) \)-map is said to be of category \( \nu \) if and only if it belongs to the following class of \( g \)-\( \nu \) - \( (\Sigma, \Sigma) \)-maps:

\[
g \cdot \nu \cdot M[\Sigma, \Sigma] \overset{\text{def}}{=} \left\{ \pi_\theta : (\forall \Omega_\omega, \Sigma_\sigma) (\exists \Omega_\sigma, \Sigma_\sigma, \text{op}_\nu(\cdot)) \left( (\pi_\theta(\Omega_\omega) \subseteq \text{op}_\nu(\Omega_\sigma) ) \lor (\pi_\theta(\Sigma_\sigma) = \neg \text{op}_\nu(\Sigma_\sigma)) \right) \right\}.
\]

(2.20)

In the above description, it is called a \( g \)-\( \nu \) - \( (\Sigma, \Sigma) \)-open map if it satisfies the first property in \( g \)-\( \nu \cdot M[\Sigma, \Sigma] \) and a \( g \)-\( \nu \) - \( (\Sigma, \Sigma) \)-closed map if it satisfies
the second property in $g\nu-M\{\Sigma_{\nu},\nu\}$. The classes of $g\nu$-$(\Sigma_{\nu},\nu)$-open maps and $g\nu$-$(\Sigma_{\nu},\nu)$-closed maps, respectively, are defined by

$$
g\nu-M_{\emptyset}\{\Sigma_{\nu},\nu\} \overset{\text{def}}{=} \{\pi_{g}: (\forall O_{g}\omega) \left( \exists O_{g}\omega, \text{op}_{g\nu}(\cdot) \right) \}
$$

$$
g\nu-M_{K}\{\Sigma_{\nu},\nu\} \overset{\text{def}}{=} \{\pi_{g}: (\forall K_{\omega}) \left( \exists K_{\omega}, \text{op}_{g\nu}(\cdot) \right) \}
$$

(2.21)

From the class $g\nu-M\{\Sigma_{\nu},\nu\}$, consisting of the classes $g\nu-M_{\emptyset}\{\Sigma_{\nu},\nu\}$ and $g\nu-M_{K}\{\Sigma_{\nu},\nu\}$ of $g\nu$-$(\Sigma_{\nu},\nu)$-open and $g\nu$-$(\Sigma_{\nu},\nu)$-closed maps, where $\nu \in I_{0}$, respectively, there results in the following definition.

**Definition 2.5.** Let $\xi_{\nu} = (\Omega, T_{\nu})$ and $\xi_{\nu} = (\Sigma, T_{\nu})$ be $T_{\nu}$-spaces. If, for each $\nu \in I_{0}$, $g\nu-M_{\emptyset}\{\Sigma_{\nu},\nu\}$ and $g\nu-M_{K}\{\Sigma_{\nu},\nu\}$ respectively denote the classes of $g\nu$-$(\Sigma_{\nu},\nu)$-open and $g\nu$-$(\Sigma_{\nu},\nu)$-closed maps, then

$$
g\nu-M\{\Sigma_{\nu},\nu\} = \bigcup_{\nu \in I_{0}} g\nu-M\{\Sigma_{\nu},\nu\} = \bigcup_{\nu \in I_{0}} (g\nu-M_{\emptyset}\{\Sigma_{\nu},\nu\} \cup g\nu-M_{K}\{\Sigma_{\nu},\nu\})
$$

(2.22)

**Definition 2.6.** ($g\nu$-$(\Sigma_{\nu},\nu)$)-Continuous. Let $\xi_{\nu} = (\Omega, T_{\nu})$ and $\xi_{\nu} = (\Sigma, T_{\nu})$ be $T_{\nu}$-spaces, and let $\text{op}_{g\nu}(\cdot) \in L_{g}[\Omega]$. Then, a map $\pi_{g} : \xi_{\nu} \rightarrow \xi_{\nu}$ is said to be $g\nu$-$(\Sigma_{\nu},\nu)$-continuous if and only if, for every pair $(\text{O}_{g\nu}\omega, K_{g\nu}) \in T_{\nu}$, $K_{g\nu}$-closed sets in $\xi_{\nu}$ there corresponds a pair $(\text{O}_{g\nu}\omega, K_{g\nu}) \in T_{\nu} \times \text{op}_{g\nu}(\cdot)$ and $T_{\nu}\text{op}_{g\nu}(\cdot)$-closed sets in $\xi_{\nu}$ such that the following statement holds:

(2.23) \[ \pi_{g}^{-1}(\exists \text{O}_{g\nu}\omega, K_{g\nu}) \supseteq \pi_{g}\text{op}_{g\nu}(\cdot) \]

A $g\nu$-$(\Sigma_{\nu},\nu)$-continuous map is said to be of category $\nu$ if and only if it belongs to the following class of $g\nu$-$(\Sigma_{\nu},\nu)$-continuous maps:

$$
g\nu-C\{\Sigma_{\nu},\nu\} = \bigcup_{\nu \in I_{0}} g\nu-C\{\Sigma_{\nu},\nu\} = \bigcup_{\nu \in I_{0}} \{\pi_{g}: (\exists \text{O}_{g\nu}\omega, K_{g\nu}) \supseteq \pi_{g}\text{op}_{g\nu}(\cdot) \}
$$

(2.24)

**Definition 2.7.** Let $\xi_{\nu} = (\Omega, T_{\nu})$ and $\xi_{\nu} = (\Sigma, T_{\nu})$ be $T_{\nu}$-spaces. If, for each $\nu \in I_{0}$, $g\nu-C\{\Sigma_{\nu},\nu\}$ denotes the class of $g\nu$-$(\Sigma_{\nu},\nu)$-continuous maps, then

(2.25) \[ g\nu-C\{\Sigma_{\nu},\nu\} = \bigcup_{\nu \in I_{0}} g\nu-C\{\Sigma_{\nu},\nu\}. \]

**Definition 2.8.** ($g\nu$-$(\Sigma_{\nu},\nu)$)-Irresolute. Let $\xi_{\nu} = (\Omega, T_{\nu})$ and $\xi_{\nu} = (\Sigma, T_{\nu})$ be $T_{\nu}$-spaces, and let $\text{op}_{g\nu}(\cdot) \in L_{g}[\Omega]$. Then, a map $\pi_{g} : \xi_{\nu} \rightarrow \xi_{\nu}$ is said to be $g\nu$-$(\Sigma_{\nu},\nu)$-irresolute if and only if, for every pair $(\text{O}_{g\nu}\omega, K_{g\nu}) \in T_{\nu} \times \text{op}_{g\nu}(\cdot)$ and $T_{\nu}\text{op}_{g\nu}(\cdot)$-closed sets in $\xi_{\nu}$ there corresponds a pair $(\text{O}_{g\nu}\omega, K_{g\nu}) \in T_{\nu} \times \text{op}_{g\nu}(\cdot)$ and $T_{\nu}\text{op}_{g\nu}(\cdot)$-closed sets in $\xi_{\nu}$ such that the
following statement holds:
\[ \{\pi^{-1}_g (\partial \mathcal{O}_{g,\sigma}) \subseteq \partial \mathcal{O}_{g,\omega}: \vee [\pi^{-1}_g (\neg \partial \mathcal{O}_{g,\sigma})] \supseteq \neg \partial \mathcal{O}_{g,\omega}] \}. \]  
\(2.26\)

A \(g\) \((\mathcal{X}_{g,\Omega}, \mathcal{X}_{g,\Sigma})\)- irresolute map is said to be of category \(\nu\) if and only if it belongs to the following class of \(g\)-\(\nu\)-\((\mathcal{X}_{g,\Omega}, \mathcal{X}_{g,\Sigma})\)- irresolute maps:
\[ g\-\nu\-I [\mathcal{X}_{g,\Omega}; \mathcal{X}_{g,\Sigma}] \text{ def } \{ \pi_g : (\forall \mathcal{O}_{g,\sigma}, \mathcal{K}_{g,\sigma} \exists \mathcal{O}_{g,\omega}, \mathcal{K}_{g,\omega}, \partial \mathcal{O}_{g,\sigma}, \partial \mathcal{K}_{g,\sigma}, \partial \mathcal{O}_{g,\omega}, \partial \mathcal{K}_{g,\omega}) \} \]
\[ [\pi^{-1}_g (\partial \mathcal{O}_{g,\sigma}) \subseteq \partial \mathcal{O}_{g,\omega}: \vee [\pi^{-1}_g (\neg \partial \mathcal{O}_{g,\sigma})] \supseteq \neg \partial \mathcal{O}_{g,\omega}] \} \]
\(2.27\)

**Definition 2.9.** Let \(\mathcal{X}_{g,\Omega} = (\Omega, T_{g,\Omega})\) and \(\mathcal{X}_{g,\Sigma} = (\Sigma, T_{g,\Sigma})\) be \(T_g\)-spaces. If, for each \(\nu \in I^0_2\), \(g\-\nu\-I [\mathcal{X}_{g,\Omega}; \mathcal{X}_{g,\Sigma}]\) denotes the class of \(g\-\nu\-I\) \((\mathcal{X}_{g,\Omega}, \mathcal{X}_{g,\Sigma})\)- irresolute maps, then
\[ g\-I [\mathcal{X}_{g,\Omega}; \mathcal{X}_{g,\Sigma}] = \bigcup_{\nu \in I^0_2} g\-\nu\-I [\mathcal{X}_{g,\Omega}; \mathcal{X}_{g,\Sigma}]. \]  
\(2.28\)

In regards to the above descriptions, by a \(g\)-\(T_g\)-open set and a \(g\)-\(T_g\)-closed set are meant a \(T_g\)-open set \(\mathcal{O}_g \in T_g\) and a \(T_g\)-closed set \(\mathcal{K}_g \in \neg T_g\) satisfying \(\mathcal{O}_g \subseteq \partial \mathcal{O}_g\) (and \(\neg \mathcal{K}_g \subseteq \neg \partial \mathcal{K}_g\), respectively. Likewise, by a \(g\)-\(T_g\)-open set of category \(\nu\) and \(g\)-\(T_g\)-closed set of category \(\nu\) are meant a \(T_g\)-open set \(\mathcal{O}_g \in T_g\) and a \(T_g\)-closed set \(\mathcal{K}_g \in \neg T_g\) satisfying \(\mathcal{O}_g \subseteq \partial \mathcal{O}_g\) (and \(\neg \mathcal{K}_g \subseteq \neg \partial \mathcal{K}_g\), respectively; \(g\)-\(T_g\)-sets of category \(\nu\) will be called \(g\-\nu\)-\(T_g\)-sets.

Given the \(\mathcal{X}_g\)-sets \(\mathcal{R}_g, \mathcal{S}_g \subseteq \mathcal{X}_g\), \(\mathcal{R}_g\) is said to be equivalent to \(\mathcal{S}_g\), written \(\mathcal{R}_g \sim \mathcal{S}_g\) if and only if, there exists a \(\mathcal{X}_g\)-map \(\pi_g : \mathcal{R}_g \rightarrow \mathcal{S}_g\) which is bijective. A \(\mathcal{X}_g\)-set \(\mathcal{S}_g \subseteq \mathcal{X}_g\) is finite if and only if \(\mathcal{S}_g = \emptyset\) or \(\mathcal{S}_g \sim I^*_\mu\) for some \(\mu \in I^*\); otherwise, the \(\mathcal{X}_g\)-set \(\mathcal{S}_g\) is said to be infinite. A \(\mathcal{X}_g\)-set \(\mathcal{R}_g \subseteq \mathcal{X}_g\) is countable if and only if it is finite or denumerable.

By adding a \(g\)-\(T_g\)-separation axiom of type H, called \(g\-T_{g,H}\)-axiom, to the axioms for a \(T_g\)-space \(\mathcal{T}_g = (\Omega, T_g)\) to obtain a \(g\)-\(T_{g,H}\)-space \(g\-T_{g,H} \text{ def } (\Omega, g\-T_{g,H})\) is meant that, for every disjoint pair \((\xi, \zeta) \in \mathcal{T}_g \times \mathcal{T}_g\) of points in \(\mathcal{T}_g\), there exists a disjoint pair \((\mathcal{O}_g, \mathcal{O}_g) \in T_g \times T_g\) of \(T_g\)-open sets such that \(\xi \in \partial \mathcal{O}_g\) \((\mathcal{O}_g, \mathcal{O}_g)\) and \(\xi \in \partial \mathcal{O}_g\) \((\mathcal{O}_g, \mathcal{O}_g)\). The definition follows:

**Definition 2.10.** \((g\-T_{g,H})\)-Space. A \(T_g\)-space \(\mathcal{T}_g = (\Omega, T_g)\) endowed with a \(g\-T_{g,H}\)-axiom is called a \(g\-T_{g,H}\)-space \(g\-T_{g,H} \text{ def } (\Omega, g\-T_{g,H})\).

**Definition 2.11.** \((g\-\mathcal{X}_g)\)-Sets Sequence. Let \(g\-\nu\-S [\mathcal{X}_g] \subseteq \mathcal{X}_g\) be the class of \(g\-\mathcal{X}_g\)-sets of category \(\nu\) in a \(T_g\)-space \(\mathcal{T}_g = (\Omega, T_g)\). The symbol \(\langle \mathcal{S}_g, \alpha : \alpha \in I^*_\nu \rangle \) denotes a sequence of \(g\-\mathcal{X}_g\)-sets of category \(\nu\) in \(\mathcal{X}_g\) that has been indexed by \(I^*_\nu \subseteq I^*_\nu\), inheriting its order from \(I^*_\nu\), and the corresponding index mapping \(\phi : \alpha \mapsto \mathcal{S}_g, \alpha\) denotes the \(\alpha\)-th term of the sequence.

Throughout, the relation \(\langle \mathcal{R}_g, \alpha \rangle_{\alpha \in I^*_\nu} < \langle \mathcal{S}_g, \alpha \rangle_{\alpha \in I^*_\nu}\) means that the one preceding "<" is a subsequence of the other following "<". Suppose a \(\mathcal{X}_g\)-set \(\mathcal{R}_g \subseteq \mathcal{X}_g\) is related to a sequence \(\langle \mathcal{S}_g, \alpha \in g\-S [\mathcal{X}_g] \rangle_{\alpha \in I^*_\nu}\) by the relation \(\mathcal{R}_g \subseteq \bigcup_{\alpha \in I^*_\nu} \mathcal{S}_g, \alpha\), then \(\mathcal{R}_g\) is said to be covered by a sequence \(\langle \mathcal{S}_g, \alpha \in g\-S [\mathcal{X}_g] \rangle_{\alpha \in I^*_\nu}\), whose cardinality is at most \(\sigma \in I^*_\nu\). The definition follows:
DEFINITION 2.12 ($g$-$\mathcal{F}_g$-Covering). Let $S_0 \subset \mathcal{F}_g$ be a $\mathcal{F}_g$-set in a $\mathcal{T}_g$-space $\mathcal{F}_g$. Then, for every $\nu \in I_3^0$:

1. $S_0$ is said to be "covered" by a sequence $\langle U_{\nu, \alpha} \in g$-$\mathcal{O}[\mathcal{F}_g] \rangle_{\alpha \in I_3^g}$ of $g$-$\mathcal{O}[\mathcal{F}_g]$-open sets whose cardinality is at most $\sigma \in I_3^g$ if and only if $S_0 \subseteq \bigcup_{\alpha \in I_3^g} U_{\nu, \alpha}$.

2. $S_0$ is said to be "covered" by a sequence $\langle V_{\nu, \alpha} \in g$-$\mathcal{K}[\mathcal{F}_g] \rangle_{\alpha \in I_3^g}$ of $g$-$\mathcal{K}[\mathcal{F}_g]$-open sets whose cardinality is at most $\sigma \in I_3^g$ if and only if $S_0 \subseteq \bigcup_{\alpha \in I_3^g} V_{\nu, \alpha}$.

Since $g$-$\mathcal{O}[\mathcal{F}_g] = \bigcup_{\nu \in I_3^g} g$-$\mathcal{O}[\mathcal{F}_g]$, $g$-$\mathcal{K}[\mathcal{F}_g] = \bigcup_{\nu \in I_3^g} g$-$\mathcal{K}[\mathcal{F}_g]$, and $g$-$\mathcal{S}[\mathcal{F}_g] = g$-$\mathcal{O}[\mathcal{F}_g] \cup g$-$\mathcal{K}[\mathcal{F}_g]$, the sequences $\langle S_{\nu, \alpha} \in g$-$\mathcal{S}[\mathcal{F}_g] \rangle_{\alpha \in I_3^g}$, $\langle U_{\nu, \alpha} \in g$-$\mathcal{O}[\mathcal{F}_g] \rangle_{\alpha \in I_3^g}$, and $\langle V_{\nu, \alpha} \in g$-$\mathcal{K}[\mathcal{F}_g] \rangle_{\alpha \in I_3^g}$, respectively, are simply said to be a $g$-$\mathcal{F}_g$-covering, a $g$-$\mathcal{F}_g$-open covering, and a $g$-$\mathcal{F}_g$-closed covering of $S_0$ whose cardinality is at most $\sigma \in I_3^g$.

DEFINITION 2.13 ($g$-$\mathcal{F}_g$-Subcovering). Let $\langle S_{\nu, \alpha} \in g$-$\mathcal{S}[\mathcal{F}_g] \rangle_{\alpha \in I_3^g}$ be a $g$-$\mathcal{F}_g$-covering of a $\mathcal{F}_g$-set $S_0 \subset \mathcal{F}_g$ in a $\mathcal{T}_g$-space $\mathcal{F}_g$, and let $\theta : I_3^g \rightarrow I_3^g$ be an index mapping. Then the map

$$
\theta : \langle S_{\nu, \alpha} \in g$-$\mathcal{S}[\mathcal{F}_g] \rangle_{\alpha \in I_3^g} \longrightarrow \langle S_{\nu, \theta(\alpha)} \in g$-$\mathcal{S}[\mathcal{F}_g] \rangle_{(\alpha, \theta(\alpha)) \in I_3^g \times I_3^g}
$$

is said to realise a " $g$-$\mathcal{F}_g$-subcovering" $\langle S_{\nu, \theta(\alpha)} \rangle_{(\alpha, \theta(\alpha)) \in I_3^g \times I_3^g}$ of $S_0$ from the $g$-$\mathcal{F}_g$-covering $\langle S_{\nu, \alpha} \rangle_{\alpha \in I_3^g}$ if and only if $S_0 \subseteq \bigcup_{(\alpha, \theta(\alpha)) \in I_3^g \times I_3^g} S_{\nu, \theta(\alpha)}$.

Thus, $\langle S_{\nu, \theta(\alpha)} \rangle_{(\alpha, \theta(\alpha)) \in I_3^g \times I_3^g} \preceq \langle S_{\nu, \alpha} \rangle_{\alpha \in I_3^g}$ is equivalent to this definition, meaning that, for every $\theta(\alpha) \in I_3^g \subseteq I_3^g$, there exists $\alpha \in I_3^g \subseteq I_3^g$ such that $S_{\nu, \theta(\alpha)} = S_{\nu, \alpha}$. It is plain that, for every $\sigma \in I_3^g$, $\theta(\sigma) = card(I_3^g) \leq card(I_3^g) = \sigma$.

DEFINITION 2.14 ($g$-$\mathcal{F}_g$-Compact Set). A $\mathcal{F}_g$-set $S_0 \subset \mathcal{F}_g$ of a $\mathcal{T}_g$-space $\mathcal{F}_g$ is said to be $g$-$\mathcal{F}_g$-compact if and only if, for every $g$-$\mathcal{F}_g$-open covering $\langle U_{\nu, \alpha} \in g$-$\mathcal{O}[\mathcal{F}_g] \rangle_{\alpha \in I_3^g}$,

$$
\exists \langle U_{\nu, \theta(\alpha)} \rangle_{(\alpha, \theta(\alpha)) \in I_3^g \times I_3^g} : S_0 \subseteq \bigcup_{(\alpha, \theta(\alpha)) \in I_3^g \times I_3^g} U_{\nu, \theta(\alpha)}
$$

where $\theta(\sigma) = card(I_3^g) \leq card(I_3^g) = \sigma$. The class of all $g$-$\mathcal{F}_g$-compact sets of category $\nu \in I_3^g$ is:

$$
g$-$\mathcal{F}_g$-A[\mathcal{F}_g] \overset{\text{def}}{=} \left\{ S_0 : \exists \langle U_{\nu, \alpha} \in g$-$\mathcal{O}[\mathcal{F}_g] \rangle_{\alpha \in I_3^g} \right\}
$$

Thus, by a $g$-$\mathcal{F}_g$-compact set is meant a type of set $\mathcal{F}_g$-set every $g$-$\mathcal{F}_g$-open covering of which has a finite $g$-$\mathcal{F}_g$-open subcovering [25, 34, 35]. Further, it is clear from the context that, $g$-$\mathcal{A}[\mathcal{F}_g] = \bigcup_{\nu \in I_3^g} g$-$\mathcal{F}_g$-A[\mathcal{F}_g]$; its elements, then, are simply called $g$-$\mathcal{F}_g$-compact sets. Stated differently, the above definition says that,
given any sequence \( \langle U_{\theta, \alpha} \in g-O \left[ \mathcal{T}_\theta \right] \rangle_{\alpha \in I^*_\theta} \) of \( g-\mathcal{T}_\theta \)-open sets of \( S_\theta \subset \mathcal{T}_\theta \) such that every point \( \xi \in S_\theta \) belongs to at least one \( U_{\theta, \alpha} \), \( \alpha \in I^*_\theta \), it is possible to select from \( \langle U_{\theta, \alpha} \rangle_{\alpha \in I^*_\theta} \) a finite number of \( g-\mathcal{T}_\theta \)-open sets \( U_{\theta, \theta(1)}, U_{\theta, \theta(2)}, \ldots, U_{\theta, \theta(\gamma)} \) whose union covers all of \( S_\theta \).

**Remark 2.15.** Since \( \langle U_{\theta, \theta(\alpha)} \rangle_{(\alpha, \theta(\alpha)) \in I^*_\theta \times I^*_\theta} \prec \langle U_{\theta, \alpha} \in g-O \left[ \mathcal{T}_\theta \right] \rangle_{\alpha \in I^*_\theta} \), \( g-\mathcal{T}_\theta \)-compactness of a \( \mathcal{T}_\theta \)-set is defined in terms of relatively \( g-\mathcal{T}_\theta \)-open sets.

The concept of \( g-\mathcal{T}_\theta \)-refinement is now given in the following definition.

**Definition 2.16 (\( g-\mathcal{T}_\theta \)-Refinement).** A \( g-\mathcal{T}_\theta \)-covering \( \langle S_{\theta, \alpha} \in g-S \left[ \mathcal{T}_\theta \right] \rangle_{\alpha \in I^*_\theta} \) of a \( \mathcal{T}_\theta \)-set \( S_\theta \subset \mathcal{T}_\theta \) of a \( \mathcal{T}_\theta \)-space \( \mathcal{T}_\theta = (\Omega, \mathcal{T}_\theta) \) is a “\( g-\mathcal{T}_\theta \)-refinement” of another \( g-\mathcal{T}_\theta \)-covering \( \langle \mathcal{R}_{\theta, \beta} \in g-S \left[ \mathcal{T}_\theta \right] \rangle_{\beta \in I^*_\theta} \) of the same \( \mathcal{T}_\theta \)-set \( S_\theta \) if and only if:

\[
(2.32) \quad \left( \forall \alpha \in I^*_\theta \right) \left( \exists \beta \in I^*_\theta \right) [S_{\theta, \alpha} \subseteq \mathcal{R}_{\theta, \beta}].
\]

In the event that \( S_\theta = \Omega \), \( \langle S_{\theta, \alpha} \in g\nu-S \left[ \mathcal{T}_\theta \right] \rangle_{\alpha \in I^*_\theta} \) is a \( g-\nu-\mathcal{T}_\theta \)-covering of \( \mathcal{T}_\theta \) if \( \Omega = \bigcup_{\alpha \in I^*_\theta} S_{\theta, \alpha} \). Accordingly, \( \langle S_{\theta, \theta(\alpha)} \in g\nu-S \left[ \mathcal{T}_\theta \right] \rangle_{(\alpha, \theta(\alpha)) \in I^*_\theta \times I^*_\theta} \) is a \( g-\nu-\mathcal{T}_\theta \)-subcovering of \( \mathcal{T}_\theta \) if the relation \( \Omega = \bigcup_{(\alpha, \theta(\alpha)) \in I^*_\theta \times I^*_\theta} S_{\theta, \theta(\alpha)} \) holds, where \( \theta(\sigma) = \text{card}(I_{\theta(\sigma)}) < \text{card}(I^*_\theta) < \infty \). The definition follows.

**Definition 2.17 (\( g-\nu-\mathcal{T}_\theta^{[A]} \)-Space).** A \( \mathcal{T}_\theta \)-space \( \mathcal{T}_\theta = (\Omega, \mathcal{T}_\theta) \) is called a \( g-\nu-\mathcal{T}_\theta^{[A]} \)-space \( g-\nu-\mathcal{T}_\theta^{[A]} \) if and only if each \( g-\mathcal{T}_\theta \)-open covering \( \langle U_{\theta, \alpha} \in g-O \left[ \mathcal{T}_\theta \right] \rangle_{\alpha \in I^*_\theta} \) of \( \mathcal{T}_\theta \) has a finite \( g-\nu-\mathcal{T}_\theta \)-open subcovering.

In the sequel, by \( g-\nu-\mathcal{T}_\theta^{[CA]} \) def \( (\Omega, g-\nu-\mathcal{T}_\theta^{[CA]}) \), \( g-\nu-\mathcal{T}_\theta^{[SA]} \) def \( (\Omega, g-\nu-\mathcal{T}_\theta^{[SA]}) \), and \( g-\nu-\mathcal{T}_\theta^{[LA]} \) def \( (\Omega, g-\nu-\mathcal{T}_\theta^{[LA]}) \), respectively, are meant countably, sequentially, and locally \( g-\nu-\mathcal{T}_\theta^{[A]} \)-spaces, as are easily understood. Finally, by a \( g-\nu-\mathcal{T}_\theta^{[E]} \)-space \( g-\mathcal{T}_\theta^{[E]} \) is meant \( g-\mathcal{T}_\theta^{[E]} = \bigvee_{\nu \in I^*_\theta} g-\nu-\mathcal{T}_\theta^{[E]} = (\Omega, \bigvee_{\nu \in I^*_\theta} g-\nu-\mathcal{T}_\theta^{[E]}) = (\Omega, g-\mathcal{T}_\theta^{[E]}) \), where \( E \in \{ A, CA, SA, LA \} \).

The main results of the theory of \( g-\mathcal{T}_\theta \)-compactness are presented in the following sections.

## 3. Main Results

In a \( \mathcal{T}_\theta \)-space, any \( g-\mathcal{T}_\theta \)-subcovering is a \( g-\mathcal{T}_\theta \)-refinement as proved in the following theorem.

**Theorem 3.1 (\( g-\mathcal{T}_\theta \)-Refinement).** In a \( \mathcal{T}_\theta \)-space \( \mathcal{T}_\theta = (\Omega, \mathcal{T}_\theta) \), any \( g-\mathcal{T}_\theta \)-subcovering \( \langle S_{\theta, \theta(\alpha)} \rangle_{(\alpha, \theta(\alpha)) \in I^*_\theta \times I^*_\theta} \) derived from a \( g-\mathcal{T}_\theta \)-covering \( \langle S_{\theta, \alpha} \in g-S \left[ \mathcal{T}_\theta \right] \rangle_{\alpha \in I^*_\theta} \) is a \( g-\mathcal{T}_\theta \)-refinement.

**Proof.** Let \( \langle S_{\theta, \theta(\alpha)} \rangle_{(\alpha, \theta(\alpha)) \in I^*_\theta \times I^*_\theta} \) be any \( g-\mathcal{T}_\theta \)-subcovering derived from a \( g-\mathcal{T}_\theta \)-covering \( \langle S_{\theta, \alpha} \in g-S \left[ \mathcal{T}_\theta \right] \rangle_{\alpha \in I^*_\theta} \) in a \( \mathcal{T}_\theta \)-space \( \mathcal{T}_\theta = (\Omega, \mathcal{T}_\theta) \). Then, it results that \( \langle S_{\theta, \theta(\alpha)} \rangle_{(\alpha, \theta(\alpha)) \in I^*_\theta \times I^*_\theta} \prec \langle S_{\theta, \alpha} \rangle_{\alpha \in I^*_\theta} \). Thus,

\[
(\forall \theta(\alpha) \in I^*_\theta) \left( \exists \alpha \in I^*_\theta \right) [S_{\theta, \theta(\alpha)} \subseteq S_{\theta, \alpha}].
\]
Therefore, the $g$-$T_0$-subcovering $\langle S_{g,\beta}^{\theta} \rangle_{(\alpha,\beta)\in I_1^g \times I_2^g}$ derived from the $g$-$T_0$-covering $\langle S_{g,\alpha}^{\beta} \rangle_{\alpha\in I_1^g}$ is therefore a $g$-$T_0$-refinement. This completes the proof of the theorem.

A necessary and sufficient condition for a $T_0$-set of a $T_0$-space to be $g$-$T_0$-compact may well be stated as thus.

Theorem 3.2. Let $S_0 \subset T_0$ be a $T_0$-set of a $T_0$-space $T_0 = (\Omega, T_0)$. Then, $S_0 \in g\cdot\mathcal{L}(T_0)$ if and only if, for each $g$-$T_0$-open covering $\langle U_{g,\alpha} \in g\cdot\mathcal{O}(T_0) \rangle_{\alpha \in I_1^g}$ of $S_0$, there is a finite $g$-$T_0$-open subcovering $\langle U_{g,\beta}^{\alpha} \rangle_{(\alpha,\beta)\in I_1^g \times I_2^g}$ of $S_0$:

$S_0 \in g\cdot\mathcal{L}(T_0) \iff (\forall \langle U_{g,\alpha} \in g\cdot\mathcal{O}(T_0) \rangle_{\alpha \in I_1^g}) \left( \exists \left( \bigcup_{(\alpha,\beta)\in I_1^g \times I_2^g} U_{g,\beta}^{\alpha} \right) \subseteq \bigcup_{(\alpha,\beta)\in I_1^g \times I_2^g} U_{g,\beta}^{\alpha} U_{g,\alpha} \right).$

(3.1)

Proof. Necessity. Let $S_0 \in g\cdot\mathcal{L}(T_0)$ in $T_0$, and let $\langle U_{g,\alpha} \in g\cdot\mathcal{O}(T_0) \rangle_{\alpha \in I_1^g}$ be a $g$-$T_0$-open covering of $S_0$. Then, $S_0 \subseteq \bigcup_{\alpha \in I_1^g} U_{g,\alpha}$ and, consequently, $S_0 = \bigcup_{\alpha \in I_1^g} U_{g,\alpha} \cap S_0$. Therefore, $\langle U_{g,\alpha} \cap S_0 \rangle_{\alpha \in I_1^g}$ is a $g$-$T_0$-open covering of $S_0$ by relatively $g$-$T_0$-open sets $U_{g,1} \cap S_0, U_{g,2} \cap S_0, \ldots, U_{g,\alpha} \cap S_0 \in g\cdot\mathcal{O}(T_0)$. Since $S_0 \in g\cdot\mathcal{L}(T_0)$, there is a finite $g$-$T_0$-open subcovering $\langle U_{g,\beta}^{\alpha} \rangle_{(\alpha,\beta)\in I_1^g \times I_2^g}$ of $S_0$ such that $S_0 = \bigcup_{(\alpha,\beta)\in I_1^g \times I_2^g} U_{g,\beta}^{\alpha}$. Thus, it follows that $S_0 \subseteq \bigcup_{(\alpha,\beta)\in I_1^g \times I_2^g} \bigcup_{(\alpha,\beta)\in I_1^g \times I_2^g} U_{g,\beta}^{\alpha} \cap S_0$.

Sufficiency. Conversely, suppose that, for every $g$-$T_0$-open covering $\langle U_{g,\alpha} \in g\cdot\mathcal{O}(T_0) \rangle_{\alpha \in I_1^g}$ of $S_0$, $\langle U_{g,\alpha} \rangle_{\alpha \in I_1^g}$ has a finite $g$-$T_0$-open subcovering of the type $\langle U_{g,\beta}^{\alpha} \rangle_{(\alpha,\beta)\in I_1^g \times I_2^g}$ of $S_0$. It must be shown that, given a $g$-$T_0$-open covering $\langle U_{g,\beta} \rangle_{\beta \in I_2^g}$ of $S_0$, there is a finite $g$-$T_0$-open subcovering $\langle U_{g,\beta}^{\alpha} \rangle_{(\beta,\alpha)\in I_2^g \times I_1^g}$ of $S_0$ such that $S_0 = \bigcup_{(\beta,\alpha)\in I_2^g \times I_1^g} U_{g,\beta}^{\alpha}$. For every $\beta \in I_2^g$, since $U_{g,\beta} \in g\cdot\mathcal{O}(T_0)$ is a relatively $g$-$T_0$-open set in $S_0$, there exists a $g$-$T_0$-open set $U_{g,\beta} \in g\cdot\mathcal{O}(T_0)$ such that $U_{g,\beta} \cap S_0 = U_{g,\beta} \cap S_0$. But $S_0 = \bigcup_{\beta \in I_2^g} U_{g,\beta} \subseteq \bigcup_{\beta \in I_2^g} U_{g,\beta} \cap S_0,\ldots, U_{g,\mu} \in g\cdot\mathcal{O}(T_0)$, there is a finite $g$-$T_0$-open subcovering $\langle U_{g,\beta}^{\alpha} \rangle_{(\beta,\alpha)\in I_2^g \times I_1^g}$ of $S_0$ such that $S_0 \subseteq \bigcup_{(\beta,\alpha)\in I_2^g \times I_1^g} \bigcup_{(\beta,\alpha)\in I_2^g \times I_1^g} U_{g,\beta}^{\alpha}$. Thus, $S_0 = \left( \bigcup_{(\beta,\alpha)\in I_2^g \times I_1^g} U_{g,\beta}^{\alpha} \right) \cap S_0 = \bigcup_{(\beta,\alpha)\in I_2^g \times I_1^g} \left( U_{g,\beta}^{\alpha} \cap S_0 \right) = \bigcup_{(\beta,\alpha)\in I_2^g \times I_1^g} U_{g,\beta}^{\alpha}.\beta$
Hence, it results that the $g$-$\mathbb{T}_g$-open covering $\langle \hat{U}_{g,\beta} \in g-O[\mathbb{T}_g] \rangle_{\beta \in I_{g}^*}$ of $S_g$ by relatively $g$-$\mathbb{T}_g$-open sets $\hat{U}_{g,1}, \hat{U}_{g,2}, \ldots, \hat{U}_{g,\sigma} \in g-O[\mathbb{T}_g]$ has a finite $g$-$\mathbb{T}_g$-open subcovering $\langle U_{g,\phi(\beta)} \rangle_{(\beta, \phi(\beta)) \in I_{g}^* \times I_{g}(\alpha)}$ of $S_g$.

Q.E.D.

The following theorem states that, any finite union of $g$-$\mathbb{T}_g$-compact sets in a $\mathbb{T}_g$-space $\mathbb{T}_g = (\Omega, \mathcal{T}_g)$ is $g$-$\mathbb{T}_g$-compact in $\mathbb{T}_g$.

**Theorem 3.3.** If $S_{g,1}, S_{g,2}, \ldots, S_{g,\mu} \in g-A[\mathbb{T}_g]$ be $\mu \geq 1$ $g$-$\mathbb{T}_g$-compact sets in a $\mathbb{T}_g$-space $\mathbb{T}_g = (\Omega, \mathcal{T}_g)$, then $\bigcup_{\alpha \in I_{g}^*} S_{g,\alpha} \in g-A[\mathbb{T}_g]$.

**Proof.** Let $S_{g,1}, S_{g,2}, \ldots, S_{g,\mu} \in g-A[\mathbb{T}_g]$ be $\mu \geq 1$ $g$-$\mathbb{T}_g$-compact sets in $\mathbb{T}_g$. Then, for every $\alpha \in I_{g}^*$, there exists $\langle U_{g,\phi(\alpha)} \rangle_{(\alpha, \phi(\alpha)) \in I_{g}^* \times I_{g}(\alpha)} \prec \langle U_{g,\phi} \rangle \in g-O[\mathbb{T}_g]$ where $I_{g}(\sigma) \subseteq I_{g}^*$, such that $S_{g,\alpha} \subseteq \bigcup_{(\phi, (\alpha, \phi)) \in I_{g}^* \times I_{g}(\alpha)} U_{g,\phi(\alpha)}$ holds. Consequently,

$$\bigcup_{\alpha \in I_{g}^*} S_{g,\alpha} \subseteq \bigcup_{\alpha \in I_{g}^*} \left( \bigcup_{(\alpha, \phi(\alpha)) \in I_{g}^* \times I_{g}(\alpha)} U_{g,\phi(\alpha)} \right) \subseteq \bigcup_{(\alpha, \phi(\alpha)) \in I_{g}^* \times I_{g}(\alpha)} U_{g,\phi(\alpha)}.$$

Hence, it follows that, $\bigcup_{\alpha \in I_{g}^*} S_{g,\alpha} \in g-A[\mathbb{T}_g]$ in $\mathbb{T}_g$. The proof of the theorem is complete.

Q.E.D.

An arbitrary $\mathbb{T}_g$-set of a $\mathbb{T}_g$-space which is equivalent to the index set $I_{g}^*$ for some $\mu < \infty$ is necessarily $g$-$\mathbb{T}_g$-compact and thus, the theorem follows.

**Theorem 3.4.** If $S_g \subseteq \mathbb{T}_g$ be any finite $\mathbb{T}_g$-set of a $\mathbb{T}_g$-space $\mathbb{T}_g = (\Omega, \mathcal{T}_g)$, then $S_g \in g-A[\mathbb{T}_g]$.

**Proof.** Let $S_g \subseteq \mathbb{T}_g$ be any finite $\mathbb{T}_g$-set of a $\mathbb{T}_g$-space $\mathbb{T}_g = (\Omega, \mathcal{T}_g)$. Then, there exist $\langle O_{g,\phi(\alpha)} \rangle_{(\alpha, \phi(\alpha)) \in I_{g}^* \times I_{g}(\alpha)} \prec \langle O_{g,\phi} \rangle \in g-O[\mathbb{T}_g]$ such that the relation $\bigcup_{\xi \in S_g} \{ \xi \} \subseteq \bigcup_{(\alpha, \phi(\alpha)) \in I_{g}^* \times I_{g}(\alpha)} O_{g,\phi(\alpha)}$ holds. Since $O_{g,\alpha} \subseteq \mathcal{O}_g(O_{g,\alpha})$ for every $\alpha \in I_{g}^*$ and, $\bigcup_{\xi \in S_g} \{ \xi \} = S_g$, it results that,

$$S_g \subseteq \bigcup_{(\alpha, \phi(\alpha)) \in I_{g}^* \times I_{g}(\alpha)} O_{g,\phi(\alpha)} \subseteq \bigcup_{(\alpha, \phi(\alpha)) \in I_{g}^* \times I_{g}(\alpha)} O_{g,\phi(\alpha)} \subseteq \bigcup_{\alpha \in I_{g}^*} \mathcal{O}_g(O_{g,\phi(\alpha)}).$$

Therefore, $S_g \subseteq \bigcup_{(\alpha, \phi(\alpha)) \in I_{g}^* \times I_{g}(\alpha)} \mathcal{O}_g(O_{g,\phi(\alpha)})$. But, for every $(\alpha, \phi(\alpha)) \in I_{g}^* \times I_{g}(\alpha)$, $\mathcal{O}_g(O_{g,\phi(\alpha)}) \in g-O[\mathbb{T}_g]$. Consequently, for every $(\alpha, \phi(\alpha)) \in I_{g}^* \times I_{g}(\alpha)$,
there exists \( U_{g,\vartheta(a)} \in g\cdot\mathcal{O}[\mathcal{T}_g] \) such that \( U_{g,\vartheta(a)} = \text{op}_g\left(\mathcal{O}_{g,\vartheta(a)}\right) \). Thus, \( S_g \subseteq \bigcup_{\alpha,\vartheta(a) \in I_g \times I_{g}^{\alpha}} U_{g,\vartheta(a)} \) and hence, \( S_g \in g\cdot\mathcal{A}[\mathcal{T}_g] \). This completes the proof of the theorem.

Q.E.D.

Since finite \( \mathcal{T}_g \)-sets of a \( \mathcal{T}_g \)-space are always \( g\cdot\mathcal{T}_g \)-compact, an immediate consequence of the above theorem is the following corollary.

**Corollary 3.5.** Let \( S_g \subseteq \mathcal{T}_g \) be a \( \mathcal{T}_g \)-set of a discrete \( \mathcal{T}_g \)-space \( \mathcal{T}_g = (\Omega, \mathcal{T}_g) \). Then, \( S_g \in g\cdot\mathcal{A}[\mathcal{T}_g] \) if and only if it is a finite \( \mathcal{T}_g \)-set.

An immediate consequence of the above theorem is the following proposition.

**Proposition 3.6.** If \( \mathcal{T}_g = (\Omega, \mathcal{T}_g) \) is a finite strong \( \mathcal{T}_g \)-space, then it is a \( g\cdot\mathcal{T}_g^{[\mathcal{A}]} \)-space \( g\cdot\mathcal{T}_g^{[\mathcal{A}]} = (\Omega, g\cdot\mathcal{T}_g^{[\mathcal{A}]}); \)

\[
(3.4) \quad (\mathcal{T}_g = (\Omega, \mathcal{T}_g)) \cap (\text{card} (\Omega) < \infty) \Rightarrow g\cdot\mathcal{T}_g^{[\mathcal{A}]} = (\Omega, g\cdot\mathcal{T}_g^{[\mathcal{A}]}); 
\]

Proof. Let \( \mathcal{T}_g = (\Omega, \mathcal{T}_g) \) be a finite strong \( \mathcal{T}_g \)-space with \( \Omega = \{\xi_{\alpha} : \alpha \in I_{g}^{\mu}\} \) and \( \mu < \infty \). Since \( \mathcal{T}_g \) is a finite strong \( \mathcal{T}_g \)-space, \( (\mathcal{O}_{g,\alpha})_{\alpha \in I_{g}^{\mu}} \) is a \( \mathcal{T}_g \)-open covering of \( \Omega \), then, for every \( \alpha \in I_{g}^{\mu} \), there exists a \( \vartheta(a) \in I_{g}^{\alpha} \) such that \( \xi_{\alpha} \in \mathcal{O}_{g,\vartheta(a)} \). Thus, \( \Omega = \bigcup_{\alpha \in I_{g}^{\mu}} \{\xi_{\alpha} \subseteq \bigcup_{(\alpha,\vartheta(a)) \in I_{g}^{\mu} \times I_{g}^{\alpha}} \mathcal{O}_{g,\vartheta(a)} \} \) and consequently, \( (\mathcal{O}_{g,\vartheta(a)})_{(\alpha,\vartheta(a)) \in I_{g}^{\mu} \times I_{g}^{\alpha}} \) is a \( \mathcal{T}_g \)-open subcovering of \( \Omega \). But, for every \( (\alpha,\vartheta(a)) \in I_{g}^{\mu} \times I_{g}^{\alpha} \), \( \mathcal{O}_{g,\vartheta(a)} \) is \( \text{op}_g\left(\mathcal{O}_{g,\vartheta(a)}\right) \in g\cdot\mathcal{O}[\mathcal{T}_g] \). Consequently, for each \( (\alpha,\vartheta(a)) \in I_{g}^{\mu} \times I_{g}^{\alpha} \), there corresponds a \( \mathcal{U}_{g,\vartheta(a)} \in g\cdot\mathcal{O}[\mathcal{T}_g] \) such that \( \mathcal{U}_{g,\vartheta(a)} = \text{op}_g\left(\mathcal{O}_{g,\vartheta(a)}\right) \). Thus, \( \Omega \subseteq \bigcup_{(\alpha,\vartheta(a)) \in I_{g}^{\mu} \times I_{g}^{\alpha}} \mathcal{U}_{g,\vartheta(a)} \). Hence, \( \mathcal{T}_g = (\Omega, \mathcal{T}_g) \) is a \( g\cdot\mathcal{T}_g^{[\mathcal{A}]} \)-space \( g\cdot\mathcal{T}_g^{[\mathcal{A}]} = (\Omega, g\cdot\mathcal{T}_g^{[\mathcal{A}]}); \)

The proof of the proposition is complete.

To prove that a \( \mathcal{T}_g \)-set is not \( g\cdot\mathcal{T}_g \)-compact, one only has to exhibit one \( \mathcal{T}_g \)-open covering of the \( \mathcal{T}_g \)-set with no finite \( g\cdot\mathcal{T}_g \)-open subcovering. The proposition follows.

**Proposition 3.7.** If \( \mathcal{T}_g = (\Omega, \mathcal{T}_g) \) be a \( \mathcal{T}_g \)-space generated by unit \( \mathcal{T}_g \)-sets of \( \Omega \), then any infinite \( \mathcal{T}_g \)-set \( S_g \subseteq \mathcal{T}_g \) is not \( g\cdot\mathcal{T}_g \)-compact.

Proof. Let \( S_g \subseteq \mathcal{T}_g \) be any infinite \( \mathcal{T}_g \)-set of a \( \mathcal{T}_g \)-space \( \mathcal{T}_g = (\Omega, \mathcal{T}_g) \) generated by unit \( \mathcal{T}_g \)-sets of \( \Omega \). Then, since \( \{x\} \in \mathcal{T}_g \) and \( \{x\} \subseteq \text{op}_g\left(\{x\}\right) \) hold for every \( \{x\} \subseteq S_g \), it follows that, for every \( \xi \in S_g \), \( \{\xi\} \subseteq \text{op}_g\left(\{\xi\}\right) \). Consequently, \( S_g = \bigcup_{\xi \in S_g} \{\xi\} \subseteq \bigcup_{\xi \in S_g} \text{op}_g\left(\{\xi\}\right) \). Clearly, \( \text{op}_g\left(\{\xi\}\right) \in g\cdot\mathcal{O}[\mathcal{T}_g] \) for every \( \xi \in S_g \) and therefore, there exists, for each \( \xi \in S_g \), a \( U_{g,\xi} \in g\cdot\mathcal{O}[\mathcal{T}_g] \) such that \( U_{g,\xi} = \text{op}_g\left(\{\xi\}\right) \). Hence, \( S_g \subseteq \bigcup_{\xi \in S_g} U_{g,\xi} \), implying that \( \{U_{g,\xi}\}_{\xi \in S_g} \) is an infinite \( g\cdot\mathcal{T}_g \)-open covering of \( S_g \). Consequently, there exists no finite \( g\cdot\mathcal{T}_g \)-open subcovering \( \{U_{g,\xi}\}_{\xi \in S_g} \times U_{g,\xi} \subseteq \mathcal{S}_g \) such that \( S_g \subseteq \bigcup_{\xi \in S_g} U_{g,\xi} \). This completes the proof of the theorem.

Q.E.D.

From the above two propositions, the corollary follows.

**Corollary 3.8.** If \( \mathcal{T}_g = (\Omega, \mathcal{T}_g) \) be a \( \mathcal{T}_g \)-space generated by unit \( \mathcal{T}_g \)-sets of \( \Omega \) and \( S_g \subseteq \mathcal{T}_g \), then \( S_g \in g\cdot\mathcal{A}[\mathcal{T}_g] \) if and only if it is a finite \( \mathcal{T}_g \)-set in \( \mathcal{T}_g \).

A \( \mathcal{T}_g \)-set \( S_g \subseteq \mathcal{T}_g \) is called a \( \mathcal{T}_g \)-open set if \( S_g \subseteq \bigcup_{\mathcal{O}_{g} \in \mathcal{C}_{\mathcal{T}_g}[\mathcal{T}_g]} \mathcal{O}_{g} \). But, for every \( \mathcal{O}_{g} \in \mathcal{T}_g \), \( \mathcal{O}_{g} \subseteq \text{op}_g\left(\mathcal{O}_{g}\right) \) and, consequently, \( S_g \subseteq \bigcup_{\mathcal{O}_{g} \in \mathcal{C}_{\mathcal{T}_g}[\mathcal{T}_g]} \mathcal{O}_{g} \subseteq \bigcup_{\mathcal{O}_{g} \in \mathcal{C}_{\mathcal{T}_g}[\mathcal{T}_g]} \mathcal{O}_{g} \).
\( \bigcup_{g \in \mathcal{C}(T_\theta)} \text{op}_g (O_\theta) \), meaning that, \( g \)-\( T_\theta \)-openness is implied by \( T_\theta \)-openness. Accordingly, \( g \)-\( T_\theta \)-compactness implies \( T_\theta \)-compactness. The theorem follows.

**Theorem 3.9.** Let \( S_\theta \subseteq T_\theta \) be any \( T_\theta \)-set of a \( T_\theta \)-space \( T_\theta = (\Omega, T_\theta) \). If \( S_\theta \) be \( g \)-\( T_\theta \)-compact, then it is also \( T_\theta \)-compact:

\[
S_\theta \subseteq g-A [T_\theta] \Rightarrow S_\theta \in A [T_\theta].
\]

**Proof.** Let \( S_\theta \subseteq T_\theta \) be any \( T_\theta \)-set of a \( T_\theta \)-space \( T_\theta = (\Omega, T_\theta) \) and suppose \( S_\theta \in g-A [T_\theta] \). Since \( S_\theta \) is \( g \)-\( T_\theta \)-compact, there exists a \( g \)-\( T_\theta \)-open covering \( \langle U_\theta, \alpha \in g \cdot O[T_\theta] \rangle \) of \( S_\theta \) which has a \( g \)-\( T_\theta \)-open subcovering \( \langle U_\theta, \theta(\alpha) \rangle \) of \( S_\theta \) such that \( S_\theta \subseteq \bigcup_{(\alpha, \theta(\alpha)) \in I_\theta} U_\theta, \theta(\alpha) \). The assertion that, \( U_\theta, \theta(\xi) \in g \cdot A [T_\theta] \) for every \( (\alpha, \theta(\alpha)) \in I_\theta \times I_\theta(\alpha) \) implies the existence of \( O_\theta, \theta(\xi) \in T_\theta \) such that, \( U_\theta, \theta(\xi) \subseteq \text{op}_g (O_\theta, \theta(\xi)) \) for every \( (\alpha, \theta(\alpha)) \in I_\theta \times I_\theta(\alpha) \). Consequently,

\[
S_\theta = \bigcup_{(\alpha, \theta(\alpha)) \in I_\theta \times I_\theta(\alpha)} (O_\theta, \theta(\alpha) \cap S_\theta)
\]

\[
\subseteq \bigcup_{(\alpha, \theta(\alpha)) \in I_\theta \times I_\theta(\alpha)} (O_\theta, \theta(\alpha) \cap \text{op}_g (O_\theta, \theta(\xi)))
\]

\[
\subseteq \bigcup_{(\alpha, \theta(\alpha)) \in I_\theta \times I_\theta(\alpha)} O_\theta, \theta(\xi),
\]

thereby implying, \( S_\theta \subseteq \bigcup_{(\alpha, \theta(\alpha)) \in I_\theta \times I_\theta(\alpha)} O_\theta, \theta(\xi) \). Hence, \( S_\theta \in g \cdot A [T_\theta] \) implies \( S_\theta \in A [T_\theta] \). The proof of the theorem is complete.

Q.E.D.

A situation in which a \( T_\theta \)-set fails to be \( g \)-\( T_\theta \)-compact is contained in the following proposition.

**Proposition 3.10.** If \( S_\theta \subseteq T_\theta \) be any infinite \( T_\theta \)-set of a discrete \( T_\theta \)-space \( T_\theta = (\Omega, T_\theta) \), then \( S_\theta \notin g \cdot A [T_\theta] \).

**Proof.** Let \( S_\theta \subseteq T_\theta \) be a \( T_\theta \)-set of a discrete \( T_\theta \)-space \( T_\theta = (\Omega, T_\theta) \). Then, \( S_\theta \in g \cdot A [T_\theta] \) if and only if it is a finite \( T_\theta \)-set. Since \( T_\theta \) is a discrete \( T_\theta \)-space, consider the class \( \{ \xi : \xi \in S_\theta \} \) of unit \( T_\theta \)-sets of \( S_\theta \). Clearly, the relation \( S_\theta \subseteq \bigcup_{\xi \in S_\theta} \{ \xi \} \subseteq \bigcup_{\xi \in S_\theta} \text{op}_g (\{ \xi \}) \) holds and, for every \( \xi \in S_\theta \), \( \text{op}_g (\{ \xi \}) \in g \cdot O [T_\theta] \). Accordingly, for every \( \xi \in S_\theta \), set \( \text{op}_g (\{ \xi \}) = U_\theta, \xi \). Then, \( \langle U_\theta, \xi \rangle \in g \cdot O [T_\theta] \) is an infinite \( g \)-\( T_\theta \)-open covering of \( S_\theta \). Consequently, \( \langle U_\theta, \xi \rangle \subseteq S_\theta \) contains a finite \( g \)-\( T_\theta \)-open covering \( \langle U_\theta, \xi(\xi) \rangle \subseteq S_\theta \times I_\theta \times I_\theta(\alpha) \subseteq (U_\theta, \xi) \subseteq S_\theta \) such that \( S_\theta \subseteq \bigcup_{(\xi, \theta(\xi)) \in S_\theta \times I_\theta} U_\theta, \theta(\xi) \). Hence, \( S_\theta \notin g \cdot A [T_\theta] \). The proof of the theorem is complete.

Q.E.D.

The above proposition motivates us to postulate the following corollary.

**Corollary 3.11.** Let \( T_\theta = (\Omega, T_\theta) \) to be a \( T_\theta \)-space. If \( T_\theta \) is a \( g \cdot T_\theta \)-space \( g \cdot T_\theta = (\Omega, g \cdot T_\theta) \), then it is also a \( T_\theta \)-space \( T_\theta = (\Omega, T_\theta) \).

In terms of \( g \)-\( T_\theta \)-closed sets, the notion of \( g \)-\( T_\theta \)-compactness may be characterized in the following way.
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THEOREM 3.12. A necessary and sufficient conditions for a $T_\theta$-space $\mathcal{F}_g = (\Omega, \mathcal{T}_\theta)$ to be a $g$-$T^a_\omega$-space $g$-$T^a_\omega = (\Omega, g$-$T^a_\omega)$ is that, whenever a sequence $\langle V_\delta, \alpha \rangle \in g$-$K[\mathcal{T}_g]_{\alpha \in I^*_\omega}$ of $g$-$T_\omega$-closed sets is such that $\cap_{\alpha \in I^*_\omega} V_\delta, \alpha = \emptyset$, then there exists $\langle V_\beta, \beta(\alpha) \rangle_{\alpha, \beta(\alpha) \in I^*_\omega \times I^*_\omega} \prec \langle V_\delta, \alpha \rangle_{\alpha \in I^*_\omega}$ such that the relation $\cap_{\alpha, \beta(\alpha) \in I^*_\omega \times I^*_\omega} V_\delta, \alpha = \emptyset$ holds.

PROOF. Suppose $\mathcal{F}_g$ is a $g$-$T^a_\omega$-space $g$-$T^a_\omega = (\Omega, g$-$T^a_\omega)$ and a sequence $\langle V_\delta, \alpha \rangle \in g$-$K[\mathcal{T}_g]_{\alpha \in I^*_\omega}$ of $g$-$T_\omega$-closed sets is given such that $\cap_{\alpha \in I^*_\omega} V_\delta, \alpha = \emptyset$. Then, $\bigcup_{\alpha \in I^*_\omega} U_\delta, \alpha = \bigcup_{\alpha \in I^*_\omega} C(V_\delta, \alpha) = C\big(\cap_{\alpha \in I^*_\omega} V_\delta, \alpha\big) = \Omega$, so that $\langle U_\delta, \alpha \rangle \in g$-$O[\mathcal{T}_g]_{\alpha \in I^*_\omega}$ is a $g$-$T_\omega$-open covering of $\mathcal{F}_g$. Thus, there exists a $g$-$T_\omega$-open subcovering $\langle U_\delta, \beta(\alpha) \rangle_{\alpha, \beta(\alpha) \in I^*_\omega \times I^*_\omega} \prec \langle U_\delta, \alpha \rangle_{\alpha \in I^*_\omega}$ and, thus, $\cap_{\alpha, \beta(\alpha) \in I^*_\omega \times I^*_\omega} V_\delta, \beta(\alpha) = C\big(\cup_{\alpha, \beta(\alpha) \in I^*_\omega \times I^*_\omega} U_\delta, \beta(\alpha)\big) = \emptyset$.

Sufficiency. Conversely, suppose that, for every $\langle V_\delta, \alpha \rangle \in g$-$K[\mathcal{T}_g]_{\alpha \in I^*_\omega}$ of $g$-$T_\omega$-closed sets such that $\cap_{\alpha \in I^*_\omega} V_\delta, \alpha = \emptyset$, there exists a $g$-$T_\omega$-open subcovering given by $\langle V_\beta, \beta(\alpha) \rangle_{\alpha, \beta(\alpha) \in I^*_\omega \times I^*_\omega} \prec \langle V_\delta, \alpha \rangle_{\alpha \in I^*_\omega}$ such that $\cap_{\alpha, \beta(\alpha) \in I^*_\omega \times I^*_\omega} V_\beta, \beta(\alpha) = \emptyset$. Further, let $\langle U_\delta, \alpha \rangle \in g$-$O[\mathcal{T}_g]_{\alpha \in I^*_\omega}$ stand for a $g$-$T_\omega$-open covering of $\mathcal{F}_g$. Then $\cup_{\alpha \in I^*_\omega} C(U_\delta, \alpha) = \emptyset$. Thus $\cap_{\alpha, \beta(\alpha) \in I^*_\omega \times I^*_\omega} \big(\cup_{\alpha, \beta(\alpha) \in I^*_\omega \times I^*_\omega} U_\delta, \beta(\alpha)\big) = \emptyset$ and $\langle U_\delta, \alpha \rangle_{\alpha \in I^*_\omega}$ is a $g$-$T_\omega$-open subcovering of $\mathcal{F}_g$.

If $\mathcal{T}_g, \Gamma = (\Gamma, \mathcal{T}_\Gamma)$ be a $T_\omega$-space such that $(\Gamma, \mathcal{T}_\Gamma) \subseteq (\Omega, \mathcal{T}_\omega, \Omega)$ and $(\Gamma, \mathcal{T}_\Gamma) \subseteq (\Sigma, \mathcal{T}_\Sigma, \Sigma)$, where $\mathcal{T}_\omega, \Omega = (\Omega, \mathcal{T}_\omega, \Omega)$ and $\mathcal{T}_\Sigma, \Sigma = (\Sigma, \mathcal{T}_\Sigma, \Sigma)$ are two $T_\omega$-spaces satisfying $(\Omega, \mathcal{T}_\omega, \Omega) \neq (\Sigma, \mathcal{T}_\Sigma, \Sigma)$, then $\mathcal{T}_\Gamma : \mathcal{P}(\Gamma) \to \mathcal{P}(\Gamma)$ is the same whether $\mathcal{F}_g, \Gamma = (\Gamma, \mathcal{T}_\Gamma)$ is a $g$-$T^a_\omega$-space $g$-$T^a_\omega = (\Omega, g$-$T^a_\omega)$ depends only on the elements forming the structure $(\Gamma, \mathcal{T}_\Gamma)$. Therefore, the $g$-$T^a_\omega$-compactness of a $T_\omega$-subspace $\mathcal{F}_g, \Gamma = (\Gamma, \mathcal{T}_\Gamma)$ of a $T_\omega$-space $\mathcal{T}_\omega, \Omega = (\Omega, \mathcal{T}_\omega, \Omega)$ may be related to $\mathcal{T}_\omega, \Omega : \mathcal{P}(\Omega) \to \mathcal{P}(\Omega)$ by virtue of the following theorem.

THEOREM 3.13. Let $\Gamma \subseteq \Omega$ be a $g$-$A$-set of a $T_\omega$-space $\mathcal{F}_g = (\Omega, \mathcal{T}_\omega)$. Then, the following statements are equivalent:

1. $\Gamma \in g$-$A[\mathcal{T}_g]$ with respect to the absolute $g$-topology $T_\theta : \mathcal{P}_g(\Omega) \to \mathcal{P}_g(\Omega)$.
2. $\Gamma \in g$-$A[\mathcal{T}_g]$ with respect to the relative $g$-topology $T_\theta, \Gamma : \mathcal{P}_g(\Gamma) \to \mathcal{P}_g(\Gamma) \equiv T_\theta, \Gamma \equiv \{O_\theta \cap \Gamma : \emptyset \subseteq O_\theta \in \mathcal{T}_g\} = \{O_\theta \cap \Gamma : O_\theta \in \mathcal{T}_g\}$. 

PROOF. 1. $\implies$ 2. Let $\langle U_\delta, \alpha \rangle_{\alpha \in I^*_\omega}$ be a $g$-$T_\omega$-covering of $\Gamma$ with respect to the relative $g$-topology $T_\theta, \Gamma : \mathcal{P}_g(\Gamma) \to \mathcal{P}_g(\Gamma) \equiv T_\theta, \Gamma \equiv \{O_\theta \cap \Gamma : O_\theta \in \mathcal{T}_g\}$. The relative $g$-topology being $T_\theta, \Gamma : \mathcal{P}_g(\Gamma) \to T_\theta, \Gamma \equiv \{O_\theta \cap \Gamma : O_\theta \in \mathcal{T}_g\}$; it consequently follows that, for every $\alpha \in I^*_\omega$, there exists $U_\delta, \alpha \in \mathcal{T}_g$ such that $\cup_{\alpha \in I^*_\omega} U_\delta, \alpha = \emptyset$. Thus, $\cup_{\alpha \in I^*_\omega} U_\delta, \alpha$ and therefore, $\langle U_\delta, \alpha \rangle_{\alpha \in I^*_\omega}$ is a $g$-$T_\omega$-open covering of $\Gamma$ with respect to the absolute $g$-topology $T_\theta : \mathcal{P}_g(\Omega) \to \mathcal{P}_g(\Omega)$. By virtue of 1., $\Gamma \in g$-$A[\mathcal{T}_g]$ with respect to $\mathcal{T}_\omega$ and consequently, a finite $g$-$T_\omega$-open subcovering $\langle U_\delta, \beta(\alpha) \rangle_{\alpha, \beta(\alpha) \in I^*_\omega \times I^*_\omega} \prec \langle U_\delta, \alpha \rangle_{\alpha \in I^*_\omega}$
exists where, for every \((\alpha, \vartheta(\alpha)) \in I^*_g \times I^*_g(\vartheta))\), \(\mathcal{U}_{g, \vartheta(\alpha)} = \text{op}_g (\widehat{\mathcal{O}}_{g, \vartheta(\alpha)} \cap \Gamma)\). But then

\[
\Gamma \subseteq \Gamma \cap \left( \bigcup_{(\alpha, \vartheta(\alpha)) \in I^*_g \times I^*_g(\vartheta)} \widehat{\mathcal{O}}_{g, \vartheta(\alpha)} \right) = \bigcup_{(\alpha, \vartheta(\alpha)) \in I^*_g \times I^*_g(\vartheta)} (\widehat{\mathcal{O}}_{g, \vartheta(\alpha)} \cap \Gamma) = \bigcup_{(\alpha, \vartheta(\alpha)) \in I^*_g \times I^*_g(\vartheta)} \mathcal{U}_{g, \vartheta(\alpha)}.
\]

Thus, it follows that the \(g\)-\(\mathcal{T}_g\)-open covering \(\langle \mathcal{U}_{g, \alpha} \rangle_{\alpha \in I^*_g}\) contains a finite \(g\)-\(\mathcal{T}_g\)-open subcovering \(\langle \mathcal{U}_{g, \alpha} \rangle_{(\alpha, \vartheta(\alpha)) \in I^*_g \times I^*_g(\vartheta)}\) of \(\Gamma\) with respect to the relative \(g\)-topology \(\mathcal{T}_{g, \Gamma} : \mathcal{P}_g(\Gamma) \rightarrow \mathcal{T}_{g, \Gamma}\). Hence, \((\Gamma, \mathcal{T}_{g, \Gamma})\) is a \(g\)-\(\mathcal{T}_g^{[\Lambda]}\)-space. This proves that \(\text{ii} \Rightarrow \text{I}\).

1. \(\Leftarrow \text{ii}\). Let \(\langle \mathcal{U}_{g, \alpha} \rangle_{\alpha \in I^*_g}\) be a \(g\)-\(\mathcal{T}_g\)-open covering of \(\Gamma\) with respect to the absolute \(g\)-topology \(\mathcal{T}_g : \mathcal{P}_g(\Omega) \rightarrow \mathcal{P}_g(\Omega)\). For every \(\alpha \in I^*_g\), there exists, then, \(\widehat{\mathcal{O}}_{g, \alpha} \in \mathcal{O}_g\) such that \(\mathcal{U}_{g, \alpha} = \text{op}_g (\widehat{\mathcal{O}}_{g, \alpha})\). For every \(\alpha \in I^*_g\), set \(\mathcal{O}_{g, \alpha} = \widehat{\mathcal{O}}_{g, \alpha} \cap \Gamma\). Consequently, \(\Gamma \subseteq \bigcup_{\alpha \in I^*_g} \mathcal{U}_{g, \alpha}\) implies

\[
\Gamma \subseteq \Gamma \cap \left( \bigcup_{\alpha \in I^*_g} \mathcal{U}_{g, \alpha} \right) = \bigcup_{\alpha \in I^*_g} (\Gamma \cap \mathcal{U}_{g, \alpha}) = \bigcup_{\alpha \in I^*_g} \text{op}_g (\widehat{\mathcal{O}}_{g, \alpha}) = \bigcup_{\alpha \in I^*_g} \text{op}_g (\mathcal{O}_{g, \alpha}) = \bigcup_{\alpha \in I^*_g} \text{op}_g (\mathcal{O}_{g, \alpha}).
\]

and from which it results that, \(\Gamma \subseteq \bigcup_{\alpha \in I^*_g} \text{op}_g (\mathcal{O}_{g, \alpha})\). Since \(\mathcal{O}_{g, \alpha} \in \mathcal{T}_{g, \Gamma}\) and \(\text{op}_g (\mathcal{O}_{g, \alpha}) \in g\)-\(\mathcal{O}_g(\mathcal{T}_g)\) for every \(\alpha \in I^*_g\), set \(\mathcal{U}_{g, \alpha} = \text{op}_g (\mathcal{O}_{g, \alpha})\). Then, \(\langle \mathcal{U}_{g, \alpha} \rangle_{\alpha \in I^*_g}\) is a \(g\)-\(\mathcal{T}_g\)-open covering of \(\Gamma\) with respect to the relative \(g\)-topology \(\mathcal{T}_{g, \Gamma} : \mathcal{P}_g(\Gamma) \rightarrow \mathcal{T}_{g, \Gamma}\). But, by hypothesis, \(\Gamma \in g\)-\(\mathcal{T}_g^{[\Lambda]}\) with respect to the relative \(g\)-topology \(\mathcal{T}_{g, \Gamma} : \mathcal{P}_g(\Gamma) \rightarrow \mathcal{T}_{g, \Gamma}\), and, therefore, a finite \(g\)-\(\mathcal{T}_g\)-open subcovering \(\langle \mathcal{U}_{g, \alpha} \rangle_{(\alpha, \vartheta(\alpha)) \in I^*_g \times I^*_g(\vartheta)} \prec \langle \mathcal{U}_{g, \alpha} \rangle_{\alpha \in I^*_g}\) exists. Accordingly,

\[
\Gamma \subseteq \bigcup_{(\alpha, \vartheta(\alpha)) \in I^*_g \times I^*_g(\vartheta)} \mathcal{U}_{g, \vartheta(\alpha)} = \bigcup_{(\alpha, \vartheta(\alpha)) \in I^*_g \times I^*_g(\vartheta)} \text{op}_g (\widehat{\mathcal{O}}_{g, \vartheta(\alpha)}) = \bigcup_{(\alpha, \vartheta(\alpha)) \in I^*_g \times I^*_g(\vartheta)} \text{op}_g (\mathcal{O}_{g, \vartheta(\alpha)}) = \bigcup_{(\alpha, \vartheta(\alpha)) \in I^*_g \times I^*_g(\vartheta)} \text{op}_g (\mathcal{O}_{g, \vartheta(\alpha)}).
\]

Thus, it results that \(\mathcal{U}_{g, \alpha} \in I^*_g\) is reducible to a finite \(g\)-\(\mathcal{T}_g\)-open subcovering \(\langle \mathcal{U}_{g, \alpha} \rangle_{(\alpha, \vartheta(\alpha)) \in I^*_g \times I^*_g(\vartheta)}\) with respect to the absolute \(g\)-topology \(\mathcal{T}_g : \mathcal{P}_g(\Omega) \rightarrow \mathcal{P}_g(\Omega)\).
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$\mathcal{P}_g(\Omega)$. Hence, $\Gamma \in \mathfrak{g}^{-A}[\mathcal{T}_g]$ with respect to the absolute $\mathfrak{g}$-topology $\mathcal{T}_g : \mathcal{P}_g(\Omega) \rightarrow \mathcal{P}_g(\Omega)$. Thus proves that $i.$ is implied by $ii.$

Of the concept of $\mathfrak{g}$-$\mathcal{T}_g$-compactness, an equivalent statement is contained in the following theorem.

**Theorem 3.14.** Let $\mathcal{T}_g = (\Omega, \mathcal{T}_g)$ be a $\mathcal{T}_g$-space. Then, the following statements are equivalent:

1. $\mathcal{T}_g$ is a $\mathfrak{g}$-$\mathcal{T}_g^{[A]}$-space $\mathfrak{g}$-$\mathcal{T}_g^{[A]} = (\Omega, \mathfrak{g}$-$\mathcal{T}_g^{[A]}$).
2. For every sequence $\langle \mathcal{V}_{\varphi, \alpha} \in \mathfrak{g}$-$K[\mathcal{T}_g] \rangle_{\alpha \in I^*_g}$ of $\mathfrak{g}$-$\mathcal{T}_g$-closed sets of $\mathcal{T}_g$, $\bigcap_{\alpha \in I^*_g} \mathcal{V}_{\varphi, \alpha} = \emptyset$ implies $\langle \mathcal{V}_{\varphi, \alpha} \in \mathfrak{g}$-$K[\mathcal{T}_g] \rangle_{\alpha \in I^*_g}$ contains a finite subsequence $\langle \mathcal{V}_{\varphi, \beta(\alpha)} \rangle_{(\alpha, \beta(\alpha)) \in I^*_g \times I^*_g}$ of $\mathfrak{g}$-$\mathcal{T}_g$-closed sets with $\bigcap_{(\alpha, \beta(\alpha)) \in I^*_g \times I^*_g} \mathcal{V}_{\varphi, \beta(\alpha)} = \emptyset$.

**Proof.** $i. \rightarrow ii.$ Suppose $\bigcap_{\alpha \in I^*_g} \mathcal{V}_{\varphi, \alpha} = \emptyset$. Then, by virtue of De Morgan’s Law, it follows that $\Omega = \emptyset = \emptyset \bigcup (\bigcap_{\alpha \in I^*_g} \mathcal{V}_{\varphi, \alpha}) = \bigcup_{\alpha \in I^*_g} \mathcal{C}(\mathcal{V}_{\varphi, \alpha}) = \bigcup_{\alpha \in I^*_g} \mathcal{U}_{\varphi, \alpha}$. Therefore, $\langle \mathcal{U}_{\varphi, \alpha} \in \mathfrak{g}$-$O[\mathcal{T}_g] \rangle_{\alpha \in I^*_g}$ is a $\mathfrak{g}$-$\mathcal{T}_g$-open covering of $\mathcal{T}_g$. But since $\mathcal{T}_g$ is, by hypothesis, a $\mathfrak{g}$-$\mathcal{T}_g^{[A]}$-space $\mathfrak{g}$-$\mathcal{T}_g^{[A]} = (\Omega, \mathfrak{g}$-$\mathcal{T}_g^{[A]}$), there exists a finite subsequence $\langle \mathcal{V}_{\varphi, \beta(\alpha)} \rangle_{(\alpha, \beta(\alpha)) \in I^*_g \times I^*_g}$ of $\mathfrak{g}$-$\mathcal{T}_g$-open sets such that $\Omega = \bigcup_{(\alpha, \beta(\alpha)) \in I^*_g \times I^*_g} \mathcal{U}_{\varphi, \beta(\alpha)}$. Thus, by De Morgan’s Law, it follows that $\emptyset = \emptyset = \emptyset \bigcup (\bigcup_{(\alpha, \beta(\alpha)) \in I^*_g \times I^*_g} \mathcal{U}_{\varphi, \beta(\alpha)}) = \bigcap_{(\alpha, \beta(\alpha)) \in I^*_g \times I^*_g} \mathcal{V}_{\varphi, \beta(\alpha)}$. This proves that $i.$ implies $ii.$

$i. \leftarrow ii.$ Let $\langle \mathcal{U}_{\varphi, \alpha} \in \mathfrak{g}$-$O[\mathcal{T}_g] \rangle_{\alpha \in I^*_g}$ is a $\mathfrak{g}$-$\mathcal{T}_g$-open covering of $\mathcal{T}_g$. Then, $\Omega = \bigcup_{\alpha \in I^*_g} \mathcal{U}_{\varphi, \alpha}$. Moreover, by De Morgan’s Law, $\emptyset = \emptyset = \emptyset \bigcup (\bigcap_{\alpha \in I^*_g} \mathcal{U}_{\varphi, \alpha}) = \bigcap_{\alpha \in I^*_g} \mathcal{C}(\mathcal{U}_{\varphi, \alpha}) = \bigcap_{\alpha \in I^*_g} \mathcal{V}_{\varphi, \alpha}$. Thus, $\langle \mathcal{V}_{\varphi, \alpha} \in \mathfrak{g}$-$K[\mathcal{T}_g] \rangle_{\alpha \in I^*_g}$ is a sequence of $\mathfrak{g}$-$\mathcal{T}_g$-closed sets and, by above, has an empty intersection. By hypothesis, it follows, then, that there exists a finite subsequence $\langle \mathcal{V}_{\varphi, \beta(\alpha)} \rangle_{(\alpha, \beta(\alpha)) \in I^*_g \times I^*_g}$ of $\mathfrak{g}$-$\mathcal{T}_g$-closed sets such that $\bigcap_{(\alpha, \beta(\alpha)) \in I^*_g \times I^*_g} \mathcal{V}_{\varphi, \beta(\alpha)} = \emptyset$. Thus, by virtue of De Morgan’s Law, it results that $\emptyset = \emptyset = \emptyset = \emptyset \bigcup (\bigcup_{(\alpha, \beta(\alpha)) \in I^*_g \times I^*_g} \mathcal{U}_{\varphi, \beta(\alpha)}) = \bigcap_{(\alpha, \beta(\alpha)) \in I^*_g \times I^*_g} \mathcal{V}_{\varphi, \beta(\alpha)}$. Accordingly, $\mathcal{T}_g$ is a $\mathfrak{g}$-$\mathcal{T}_g^{[A]}$-space $\mathfrak{g}$-$\mathcal{T}_g^{[A]} = (\Omega, \mathfrak{g}$-$\mathcal{T}_g^{[A]}$) and, hence, $i.$ is implied by $ii.$

It might be conjectured that a $\mathfrak{g}$-$\mathcal{T}_g$-closed set of a $\mathfrak{g}$-$\mathcal{T}_g^{[A]}$-space is $\mathfrak{g}$-$\mathcal{T}_g$-compact. In actual fact this conjecture holds, as proved in the following proposition.

**Proposition 3.15.** If $\mathcal{S}_g \in \mathfrak{g}$-$K[\mathcal{T}_g]$ be a $\mathfrak{g}$-$\mathcal{T}_g$-closed set of a $\mathfrak{g}$-$\mathcal{T}_g^{[A]}$-space $\mathfrak{g}$-$\mathcal{T}_g^{[A]} = (\Omega, \mathfrak{g}$-$\mathcal{T}_g^{[A]}$), then $\mathcal{S}_g \in \mathfrak{g}$-$A[\mathcal{T}_g]$:

\[\text{(3.6)}\]

**Proof.** Let it be assumed that $\mathcal{S}_g \in \mathfrak{g}$-$K[\mathcal{T}_g]$ is a $\mathfrak{g}$-$\mathcal{T}_g$-closed set of a $\mathfrak{g}$-$\mathcal{T}_g^{[A]}$-space $\mathfrak{g}$-$\mathcal{T}_g^{[A]} = (\Omega, \mathfrak{g}$-$\mathcal{T}_g^{[A]}$). Then, $\emptyset = \emptyset \bigcup (\bigcap_{\alpha \in I^*_g} \mathcal{U}_{\varphi, \alpha})$: that is, $\Omega \setminus \mathcal{S}_g$ is a $\mathfrak{g}$-$\mathcal{T}_g$-open set in $\mathfrak{g}$-$\mathcal{T}_g^{[A]}$. Let $\langle \mathcal{U}_{\varphi, \alpha} \in \mathfrak{g}$-$O[\mathcal{T}_g] \rangle_{\alpha \in I^*_g}$ be a $\mathfrak{g}$-$\mathcal{T}_g$-open covering of $\mathcal{S}_g$ in $\mathfrak{g}$-$\mathcal{T}_g^{[A]}$ and, for every $\alpha \in I^*_g$, set $\mathcal{U}_{\varphi, \alpha} = \mathcal{U}_{\varphi, \alpha} \cup \mathcal{C}(\mathcal{S}_g)$. Then, $\langle \mathcal{U}_{\varphi, \alpha} \rangle_{\alpha \in I^*_g}$ is a $\mathfrak{g}$-$\mathcal{T}_g$-open covering of $\Omega$. But since $\mathfrak{g}$-$\mathcal{T}_g^{[A]} = (\Omega, \mathfrak{g}$-$\mathcal{T}_g^{[A]}$) is a $\mathfrak{g}$-$\mathcal{T}_g^{[A]}$-space, there exists a finite $\mathfrak{g}$-$\mathcal{T}_g$-open subcovering $\langle \mathcal{U}_{\varphi, \theta(\alpha)} \rangle_{(\alpha, \theta(\alpha)) \in I^*_g \times \theta(I^*_g)} \subset \langle \mathcal{U}_{\varphi, \alpha} \rangle_{\alpha \in I^*_g}$ such that $\Omega \subseteq \bigcup_{(\alpha, \theta(\alpha)) \in I^*_g \times \theta(I^*_g)} \mathcal{U}_{\varphi, \theta(\alpha)}$, where $\mathcal{U}_{\varphi, \theta(\alpha)} = \mathcal{U}_{\varphi, \theta(\alpha)} \cup \mathcal{C}(\mathcal{S}_g)$ for every $(\alpha, \theta(\alpha)) \in$
Therefore, \( \langle U_\delta, \delta(\alpha) \rangle \) is a finite \( g \)-\( \mathcal{I} \)-open subcovering of \( S_\delta \). Hence, \( S_\delta \subseteq g \cdot A \{ \mathcal{I} \} \). The proof of the proposition is complete.

Another way of stating the notion of \( g \)-\( \mathcal{T} \)- compactness is by the aid of the notion of finite intersection property.

**Definition 3.16.** A sequence \( \langle S_{\beta, \alpha} \in g \cdot S \{ \mathcal{I} \} \rangle \) of \( g \)-\( \mathcal{I} \)- sets is said to have the "finite intersection property" if and only if every finite subsequence of the type \( \langle S_{\beta, \alpha} \rangle \subseteq I_\alpha \times I_\alpha \) has a non-empty intersection:

\[
\forall (\alpha, \beta(\alpha)) \in I_\alpha \times I_\alpha : \bigcap_{\alpha \in I_\alpha} S_{\beta, \alpha} \neq \emptyset.
\]

Granted the above definition, we can now state the notion of \( g \)-\( \mathcal{T} \)- compactness in terms of the families of \( g \)-\( \mathcal{T} \)-closed sets of a \( g \)-\( \mathcal{T} \)-space.

**Theorem 3.17.** A \( g \)-\( \mathcal{T} \)-space \( g \cdot \mathcal{I} \) is a \( g \)-\( \mathcal{T} \)-space \( g \cdot \mathcal{I} \) if and only if every sequence \( \langle V_{\beta, \alpha} \in g \cdot K \{ \mathcal{I} \} \rangle \subseteq I_\alpha \times I_\alpha \) of \( g \)-\( \mathcal{I} \)-closed sets which has the finite intersection property has a non-empty intersection.

**Proof.** Necessity. Let the \( g \)-\( \mathcal{T} \)-space \( g \cdot \mathcal{I} \) be a \( g \)-\( \mathcal{T} \)-space \( g \cdot \mathcal{I} \), and let \( \langle V_{\beta, \alpha} \in g \cdot K \{ \mathcal{I} \} \rangle \subseteq I_\alpha \times I_\alpha \) be a sequence of \( g \)-\( \mathcal{I} \)-closed sets such that \( \bigcup_{\alpha \in I_\alpha} V_{\beta, \alpha} = \emptyset \). For every \( \alpha \in I_\alpha \), set \( U_{\delta, \alpha} = \mathcal{C} (V_{\beta, \alpha}) \), and consider the sequence \( \langle U_{\delta, \alpha} \in g \cdot O \{ \mathcal{I} \} \rangle \subseteq I_\alpha \times I_\alpha \) of \( g \)-\( \mathcal{I} \)-open sets. Since \( \bigcup_{\alpha \in I_\alpha} U_{\delta, \alpha} = \bigcup_{\alpha \in I_\alpha} \mathcal{C} (V_{\beta, \alpha}) = \mathcal{C} \left( \bigcap_{\alpha \in I_\alpha} V_{\beta, \alpha} \right) = \Omega \), it follows that \( \langle U_{\delta, \alpha} \in g \cdot O \{ \mathcal{I} \} \rangle \subseteq I_\alpha \times I_\alpha \) is a \( g \)-\( \mathcal{I} \)-open covering of \( g \cdot \mathcal{I} \). But \( g \cdot \mathcal{I} \) is a \( g \)-\( \mathcal{T} \)-space \( g \cdot \mathcal{I} \) and, therefore, there exists a \( g \)-\( \mathcal{I} \)-closed subcovering \( \langle U_{\delta, \beta(\alpha)} \rangle \subseteq I_\alpha \times I_\alpha \) such that

\[
\Omega = \bigcup_{(\alpha, \beta(\alpha)) \in I_\alpha \times I_\alpha} U_{\delta, \beta(\alpha)} = \bigcup_{(\alpha, \beta(\alpha)) \in I_\alpha \times I_\alpha} \mathcal{C} (V_{\beta, \alpha}) = \mathcal{C} \left( \bigcap_{(\alpha, \beta(\alpha)) \in I_\alpha \times I_\alpha} V_{\beta, \alpha} \right).
\]

This implies that \( \bigcap_{(\alpha, \beta(\alpha)) \in I_\alpha \times I_\alpha} V_{\beta, \alpha} \neq \emptyset \). Hence, it follows that, if a sequence \( \langle V_{\beta, \alpha} \in g \cdot K \{ \mathcal{I} \} \rangle \subseteq I_\alpha \times I_\alpha \) of \( g \)-\( \mathcal{I} \)-closed sets of \( g \)-\( \mathcal{T} \)-space \( g \cdot \mathcal{I} \) has the finite intersection property, then \( \bigcap_{(\alpha, \beta(\alpha)) \in I_\alpha \times I_\alpha} V_{\beta, \alpha} \neq \emptyset \).

**Sufficiency.** Conversely, suppose that \( g \cdot \mathcal{I} \) is a \( g \)-\( \mathcal{T} \)-space in which every sequence \( \langle V_{\beta, \alpha} \in g \cdot K \{ \mathcal{I} \} \rangle \subseteq I_\alpha \times I_\alpha \) of \( g \)-\( \mathcal{I} \)-closed sets which has the finite intersection property has a non-empty intersection. Then, for every subsequence \( \langle V_{\beta, \alpha} \rangle \subseteq I_\alpha \times I_\alpha \), the relation \( \bigcap_{(\alpha, \beta(\alpha)) \in I_\alpha \times I_\alpha} V_{\beta, \alpha} \neq \emptyset \) holds. Consequently, \( \bigcap_{(\alpha, \beta(\alpha)) \in I_\alpha \times I_\alpha} V_{\beta, \alpha} \neq \emptyset \). Otherwise, in other words, \( \bigcap_{(\alpha, \beta(\alpha)) \in I_\alpha \times I_\alpha} V_{\beta, \alpha} \neq \emptyset \) for every \( I_\alpha \subseteq I_\alpha \) implies \( \bigcap_{(\alpha, \beta(\alpha)) \in I_\alpha \times I_\alpha} V_{\beta, \alpha} \neq \emptyset \). But this is the contrapositive statement of \( \bigcap_{(\alpha, \beta(\alpha)) \in I_\alpha \times I_\alpha} V_{\beta, \alpha} = \emptyset \) implies that there exists \( I_\alpha \subseteq I_\alpha \) such that \( \bigcap_{(\alpha, \beta(\alpha)) \in I_\alpha \times I_\alpha} V_{\beta, \alpha} = \emptyset \). It results that, every sequence \( \langle V_{\beta, \alpha} \in g \cdot K \{ \mathcal{I} \} \rangle \subseteq I_\alpha \times I_\alpha \) of \( g \)-\( \mathcal{I} \)-closed sets of \( g \)-\( \mathcal{T} \)-space, \( \bigcap_{(\alpha, \beta(\alpha)) \in I_\alpha \times I_\alpha} V_{\beta, \alpha} = \emptyset \) implies...
\[ \langle V_{\xi,a} \in g-K[\mathfrak{T}_g] \rangle_{\alpha \in I_2} \] contains a finite subsequence \( \langle V_{\xi,\beta(a)} \rangle_{(\alpha,\beta(a)) \in I_2 \times I_2} \) of \( g-\mathfrak{T}_g \)-closed sets with \( \bigcap_{(\alpha,\beta(a)) \in I_2 \times I_2} V_{\xi,\beta(a)} = \emptyset \). Hence, \( g-\mathfrak{T}_g^{[A]} \) is a \( g-\mathfrak{T}_g^{[A]} \)-space \( g-\mathfrak{T}_g^{[A]} = (\Omega, g-\mathfrak{T}_g^{[A]}) \).

An interesting remark may well be given at this stage.

**Remark 3.18.** In particular, if the \( g-\mathfrak{T}_g^{(H)} \)-space \( g-\mathfrak{T}_g^{(H)} = (\Omega, g-\mathfrak{T}_g^{(H)}) \) is a \( g-\mathfrak{T}_g^{[A]} \)-space \( g-\mathfrak{T}_g^{[A]} = (\Omega, g-\mathfrak{T}_g^{[A]}) \) and the elements of \( \langle V_{\xi,a} \in g-K[\mathfrak{T}_g] \rangle_{\alpha \in I_2} \) forms a descending sequence \( V_{\xi,1} \supset V_{\xi,2} \supset \cdots \supset V_{\xi,a} \supset \cdots \) of non-empty \( g-\mathfrak{T}_g \)-closed sets, then \( \bigcap_{\alpha \in I_2} V_{\xi,a} \neq \emptyset \). Such property in its own right is weaker than \( g-\mathfrak{T}_g \)-compactness. In fact, it indicates the sense in which \( g-\mathfrak{T}_g \)-compactness asserts that the \( g-\mathfrak{T}_g^{(H)} \)-space \( g-\mathfrak{T}_g^{(H)} \) has enough points, namely, at least enough points to yield one point in each such intersection of a descending sequence \( V_{\xi,1} \supset V_{\xi,2} \supset \cdots \supset V_{\xi,a} \supset \cdots \) of non-empty \( g-\mathfrak{T}_g \)-closed sets.

The notion of \( g-\mathfrak{T}_g \)-compactness may be characterized in terms of \( g-\mathfrak{T}_g \)-open neighborhood in the following manner.

**Theorem 3.19.** A necessary and sufficient conditions for a \( \mathfrak{T}_g \)-space \( \mathfrak{T}_g = (\Omega, \mathfrak{T}_g) \) to be a \( g-\mathfrak{T}_g^{[A]} \)-space \( g-\mathfrak{T}_g^{[A]} = (\Omega, g-\mathfrak{T}_g^{[A]}) \) is that, whenever for each \( \xi \in \mathfrak{T}_g \) a \( g-\mathfrak{T}_g \)-open neighborhood of \( \xi \) is given, there is a finite collection \( C_\xi = \{ \xi_\eta : \eta \in I_\alpha \} \) of points \( \xi_1, \xi_2, \ldots, \xi_n \in \mathfrak{T}_g \) such that \( \Omega = \bigcup_{\xi \in C_\xi} \text{op}_g(\mathfrak{T}_g) \).

**Proof. Necessity.** Suppose \( \mathfrak{T}_g \) is a \( g-\mathfrak{T}_g^{[A]} \)-space \( g-\mathfrak{T}_g^{[A]} = (\Omega, g-\mathfrak{T}_g^{[A]}) \). Let there be given for each \( \xi \in \mathfrak{T}_g \) a \( g-\mathfrak{T}_g \)-open neighborhood of \( \xi \). For each \( \xi \in \mathfrak{T}_g \), there is a \( \mathfrak{T}_g \)-open set \( U_{\xi,\xi} \subset \mathfrak{T}_g \) satisfying \( \xi \in U_{\xi,\xi} \subset \text{op}_g(\mathfrak{T}_g) \). Thus, for every \( \xi \in \mathfrak{T}_g \), \( U_{\xi,\xi} \subset \text{op}_g(\mathfrak{T}_g) \), and, consequently, \( \langle U_{\xi,\xi} \subset \text{op}_g(\mathfrak{T}_g) \rangle_{\xi \in \mathfrak{T}_g} \) is a \( g-\mathfrak{T}_g \)-open covering of \( \mathfrak{T}_g \). Since \( \mathfrak{T}_g \) is a \( g-\mathfrak{T}_g^{[A]} \)-space \( g-\mathfrak{T}_g^{[A]} = (\Omega, g-\mathfrak{T}_g^{[A]}) \), there is a finite \( g-\mathfrak{T}_g \)-open subcovering \( \langle U_{\xi,\xi} \subset \text{op}_g(\mathfrak{T}_g) \rangle_{\mu \in I_2} \). But, for every \( \mu \in I_2 \), \( \xi_\mu \in U_{\xi,\xi} \subset \text{op}_g(\mathfrak{T}_g) \), whence \( \Omega = \bigcup_{\xi \in I_2} \text{op}_g(\mathfrak{T}_g) = \bigcup_{\xi \in \mathfrak{T}_g} \text{op}_g(\mathfrak{T}_g) \) is a \( g-\mathfrak{T}_g^{[A]} \)-space \( g-\mathfrak{T}_g^{[A]} = (\Omega, g-\mathfrak{T}_g^{[A]}) \).

**Sufficiency.** Conversely, suppose that whenever, for each \( \xi \in \mathfrak{T}_g \) a \( g-\mathfrak{T}_g \)-open neighborhood of \( \xi \) is given, there is a finite collection \( C_\xi = \{ \xi_\eta : \eta \in I_\alpha \} \) of points \( \xi_1, \xi_2, \ldots, \xi_n \in \mathfrak{T}_g \) such that \( \Omega = \bigcup_{\xi \in C_\xi} \text{op}_g(\mathfrak{T}_g) \). Let \( \langle U_{\xi,\alpha} \subset \text{op}_g(\mathfrak{T}_g) \rangle_{\alpha \in I_2} \) be a \( g-\mathfrak{T}_g \)-open covering of \( \mathfrak{T}_g \). Then for each \( \xi \in \mathfrak{T}_g \), there exists an \( \alpha = \alpha(\xi) \) such that \( \xi \in U_{\xi,\alpha(\xi)} \), and hence, \( U_{\xi,\alpha(\xi)} \subset \text{op}_g(\mathfrak{T}_g) \) for every \( \xi, \alpha(\xi) \in \mathfrak{T}_g \times I_\alpha \). By hypothesis, there is, a finite collection \( C_\xi = \{ \xi_\eta : \eta \in I_\alpha \} \) of points \( \xi_1, \xi_2, \ldots, \xi_n \in \mathfrak{T}_g \) such that \( \Omega = \bigcup_{\xi \in C_\xi} U_{\xi,\alpha(\xi)} \) and thus, \( \mathfrak{T}_g \) is a \( g-\mathfrak{T}_g^{[A]} \)-space \( g-\mathfrak{T}_g^{[A]} = (\Omega, g-\mathfrak{T}_g^{[A]}) \).

In the following statements, the notion of \( g-\mathfrak{T}_g \)-compactness is related to the \( g-\mathfrak{T}_g^{(H)} \)-axioms of their \( g-\mathfrak{T}_g^{(H)} \)-spaces.

**Lemma 3.20.** If \( S_\xi \subset g-\mathfrak{T}_g \) be a \( g-\mathfrak{T}_g \)-compact set of a \( g-\mathfrak{T}_g^{(H)} \)-space \( g-\mathfrak{T}_g^{(H)} = (\Omega, g-\mathfrak{T}_g^{(H)}) \) and suppose \( \xi \notin S_\xi \), then there exists \( \langle U_{\xi,\alpha}, U_{\xi,\beta} \rangle \in \text{op}_g(\mathfrak{T}_g) \times \text{op}_g(\mathfrak{T}_g) \) such that \( (\{ \xi \}, S_\xi) \subseteq \langle U_{\xi,\alpha}, U_{\xi,\beta} \rangle \) and \( \bigcap_{\mu = \alpha, \beta} U_{\xi,\mu} = \emptyset \).

**Proof.** Let \( S_\xi \subset g-\mathfrak{T}_g \) be a \( g-\mathfrak{T}_g \)-compact set of a \( g-\mathfrak{T}_g^{(H)} \)-space \( g-\mathfrak{T}_g^{(H)} = (\Omega, g-\mathfrak{T}_g^{(H)}) \) and suppose \( \xi \notin S_\xi \). Since \( \xi \notin S_\xi \), it results that \( \xi \in S_\xi \) implies...
\(\xi \notin \{\zeta\}\). But by hypothesis, \(\mathfrak{T}_0\) is a \(g^{\mathfrak{T}_0}(H)\)-space \(g^{\mathfrak{T}_0}(H) = (\Omega, g^{\mathfrak{T}_0}(H))\) and therefore, there exists \((U_{\xi,\zeta}, \hat{U}_{\xi,\zeta}) \in g^{\mathfrak{T}_0}[\mathfrak{T}_0] \times g^{\mathfrak{T}_0}[\mathfrak{T}_0]\) such that \((\xi, \zeta) \in U_{\xi,\zeta} \times \hat{U}_{\xi,\zeta}\) and \(U_{\xi,\zeta} \cap \hat{U}_{\xi,\zeta} = \emptyset\). Hence, it follows that \(S_0 \subseteq \bigcup_{\xi \in S_0} \hat{U}_{\xi,\zeta}\), meaning that \(\langle \hat{U}_{\xi,\zeta} \rangle_{\xi \in S_0}\) is a \(g^{\mathfrak{T}_0}\)-open covering of \(S_0\). But \(S_0 \in g^{A}[\mathfrak{T}_0]\). Consequently, there exists \(\langle U_{\xi,\zeta}(\mu) \rangle_{(\mu, \zeta) \in I_0^2 \times S_0} \prec \langle \hat{U}_{\xi,\zeta} \rangle_{\xi \in S_0}\) such that \(S_0 \subseteq \bigcup_{(\mu, \zeta) \in I_0^2 \times S_0} U_{\xi,\zeta}(\mu)\). Now let
\[
U_{\xi,\alpha} = \bigcup_{(\mu, \zeta) \in I_0^2 \times S_0} U_{\xi,\zeta}(\mu), \quad U_{\xi,\beta} = \bigcup_{(\mu, \zeta) \in I_0^2 \times S_0} \hat{U}_{\xi,\zeta}(\mu).
\]
It is evidently that, \((U_{\xi,\alpha}, U_{\xi,\beta}) \in g^{\mathfrak{T}_0}[\mathfrak{T}_0] \times g^{\mathfrak{T}_0}[\mathfrak{T}_0]\), since \((U_{\xi,\zeta}(\mu), \hat{U}_{\xi,\zeta}(\mu)) \in g^{\mathfrak{T}_0}[\mathfrak{T}_0] \times g^{\mathfrak{T}_0}[\mathfrak{T}_0]\) for every \((\mu, \zeta) \in I_0^2 \times S_0\). Furthermore, \((\{\xi\}, S_0) \subseteq (U_{\xi,\alpha}, U_{\xi,\beta})\), since \(\xi \in U_{\xi,\zeta}(\mu)\) for every \((\mu, \zeta) \in I_0^2 \times S_0\). Lastly, let it be claimed that \(\bigcap_{\alpha=0,\beta} U_{\xi,\mu} = \emptyset\). Then, \(U_{\xi,\zeta}(\mu) \cap U_{\xi,\zeta}(\mu) = \emptyset\) for every \((\mu, \zeta) \in I_0^2 \times S_0\) which, in turn, implies that \(U_{\xi,\alpha} \cap U_{\xi,\beta} = \emptyset\) for every \((\mu, \zeta) \in I_0^2 \times S_0\). Hence,
\[
\bigcap_{\mu=\alpha,\beta} U_{\xi,\mu} = U_{\xi,\alpha} \cap \left( \bigcup_{(\mu, \zeta) \in I_0^2 \times S_0} U_{\xi,\zeta}(\mu) \right) = \bigcup_{(\mu, \zeta) \in I_0^2 \times S_0} (U_{\xi,\alpha} \cap U_{\xi,\zeta}(\mu)) = \bigcup_{(\mu, \zeta) \in I_0^2 \times S_0} \emptyset = \emptyset.
\]
This completes the proof of the lemma. Q.E.D.

**Theorem 3.21.** Let \(S_0 \in \mathfrak{g}^{\mathfrak{T}_0}[\mathfrak{T}_0]\) be a \(g^{\mathfrak{T}_0}(H)\)-compact set of a \(g^{\mathfrak{T}_0}(H)\)-space \(\mathfrak{T}_0^{\mathfrak{T}_0}(H) = (\Omega, g^{\mathfrak{T}_0}(H))\). If \(\xi \notin S_0\), then there exists a \(g^{\mathfrak{T}_0}\)-open set \(U_0 \in g^{\mathfrak{T}_0}[\mathfrak{T}_0]\) such that \(\xi \in U_0 \subseteq \mathfrak{C}(S_0)\).

**Proof.** Let \(S_0 \in \mathfrak{g}^{\mathfrak{T}_0}[\mathfrak{T}_0]\) be a \(g^{\mathfrak{T}_0}(H)\)-compact set of a \(g^{\mathfrak{T}_0}(H)\)-space \(\mathfrak{T}_0^{\mathfrak{T}_0}(H) = (\Omega, g^{\mathfrak{T}_0}(H))\) and suppose \(\xi \notin S_0\). Since \(\mathfrak{T}_0\) is a \(g^{\mathfrak{T}_0}(H)\)-space \(\mathfrak{T}_0^{\mathfrak{T}_0}(H) = (\Omega, g^{\mathfrak{T}_0}(H))\), there exists then \((U_0, \hat{U}_0) \in g^{\mathfrak{T}_0}[\mathfrak{T}_0] \times g^{\mathfrak{T}_0}[\mathfrak{T}_0]\) such that \((\{\xi\}, S_0) \subseteq (U_0, \hat{U}_0)\) and \(\hat{U}_0 \cap U_0 = \emptyset\). Hence, \(U_0 \cap S_0 = \emptyset\) and consequently, \(\xi \notin S_0 \subseteq \mathfrak{C}(S_0)\). This proves the theorem. Q.E.D.

**Proposition 3.22.** If \(S_0 \in \mathfrak{g}^{\mathfrak{T}_0}[\mathfrak{T}_0]\) be a \(g^{\mathfrak{T}_0}(H)\)-compact set of a \(g^{\mathfrak{T}_0}(H)\)-space \(\mathfrak{T}_0^{\mathfrak{T}_0}(H) = (\Omega, g^{\mathfrak{T}_0}(H))\), then \(S_0 \in \mathfrak{g}^{\mathfrak{K}}[\mathfrak{T}_0]\) in \(g^{\mathfrak{T}_0}(H)\).

**Proof.** Let \(S_0 \in \mathfrak{g}^{\mathfrak{T}_0}[\mathfrak{T}_0]\) be a \(g^{\mathfrak{T}_0}(H)\)-compact set of a \(g^{\mathfrak{T}_0}(H)\)-space \(\mathfrak{T}_0^{\mathfrak{T}_0}(H) = (\Omega, g^{\mathfrak{T}_0}(H))\). It must be proved that \(S_0 \in \mathfrak{g}^{\mathfrak{K}}[\mathfrak{T}_0]\) which is equivalent to prove that \(\mathfrak{C}(S_0) \in g^{\mathfrak{T}_0}[\mathfrak{T}_0]\) in \(g^{\mathfrak{T}_0}(H)\). Let \(\xi \in \mathfrak{C}(S_0)\); that is, \(\xi \notin S_0\). Since \(\xi \notin S_0\) there exists a \(g^{\mathfrak{T}_0}(H)\)-open set \(U_0, \xi \in g^{\mathfrak{T}_0}[\mathfrak{T}_0]\) such that \(\xi \in U_0, \xi \subseteq \mathfrak{C}(S_0)\). Consequently, \(\mathfrak{C}(S_0) = \bigcup_{\xi \in \mathfrak{C}(S_0)} U_0, \xi\). Therefore, \(\mathfrak{C}(S_0) \in g^{\mathfrak{T}_0}[\mathfrak{T}_0]\), since \(U_0, \xi \in g^{\mathfrak{T}_0}[\mathfrak{T}_0]\) for every \(\xi \in \mathfrak{C}(S_0)\). Hence, \(S_0 \in \mathfrak{g}^{\mathfrak{K}}[\mathfrak{T}_0]\) in \(g^{\mathfrak{T}_0}(H)\). This proves the proposition. Q.E.D.

**Lemma 3.23.** If \(\mathfrak{T}_0 = (\Omega, \mathfrak{T}_0)\) be a \(\mathfrak{T}_0\)-space whose \(g^{\mathfrak{T}_0}\)-topology \(\mathfrak{T}_0 : \mathcal{P}(\Omega) \to \mathcal{P}(\Omega)\) is cofinite on \(\Omega\), then \(\mathfrak{T}_0\) is a \(g^{\mathfrak{T}_0}(H)\)-space \(g^{\mathfrak{T}_0}(H) = (\Omega, g^{\mathfrak{T}_0}(H))\).

**Proof.** Let \(\mathfrak{T}_0 = (\Omega, \mathfrak{T}_0)\) be a \(\mathfrak{T}_0\)-space whose \(g^{\mathfrak{T}_0}\)-topology \(\mathfrak{T}_0 : \mathcal{P}(\Omega) \to \mathcal{P}(\Omega)\) is cofinite on \(\Omega\) and suppose \((U_{\xi,\alpha} \in g^{\mathfrak{T}_0}[\mathfrak{T}_0])_{\alpha \in I_0^2}\) be a \(g^{\mathfrak{T}_0}\)-open covering of...
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**Lemma.**

Then, \( \Omega \). Then, \( g \) is a finite

**Theorem 3.24.** If \((R_x, S_y) \in g-A [T^g_x] \times g-A [T^g_y] \) be a pair of disjoint \( g-T^g_x \)-compact sets of a \( g-T^g_x \)-space \( g-T^g_x \) (\( \Omega, g-T^g_x \)), then there exists a pair \( (U_{g,a}, U_{g,b}) \in g-O [T^g_x] \times g-O [T^g_y] \) of disjoint \( g-T^g_x \)-open sets such that \((R_x, S_y) \subseteq (U_{g,a}, U_{g,b}) \). 

**Proof.** Let \((R_x, S_y) \in g-A [T^g_x] \times g-A [T^g_y] \) be a pair of disjoint \( g-T^g_x \)-compact sets of a \( g-T^g_x \)-space \( g-T^g_x \) (\( \Omega, g-T^g_x \)) and suppose \( \xi \in R_x \). Then, since \( R_x \cap S_y = \emptyset \), it results that \( \xi \notin S_y \). But by hypothesis, \( S_y \in g-A [T^g_x] \) and consequently, there exists \( (U_{g,a}, U_{g,b}) \in g-O [T^g_x] \times g-O [T^g_y] \) such that \( \{ \xi \}, S_y \subseteq \{ U_{g,a}, U_{g,b} \} \) and \( U_{g,a} \cap U_{g,b} = \emptyset \). Since \( \xi \in U_{g,a} \), it follows that \( \{ U_{g,a}, U_{g,b} \} \) is a \( g-T^g_x \)-open covering of \( R_x \). Since \( R_x \in g-A [T^g_x] \), a \( g-T^g_x \)-open subcovering

\[
\bigcup_{\xi \in R_x \times R_y} \bigcup_{(\xi, \xi) \in S_y \times S_y} U_{g,a} \o_{(\xi, \xi)} \quad \bigcup_{\xi \in R_x \times R_y} U_{g,b} \o_{(\xi, \xi)}
\]

where \( R_x \subseteq R_x \) is finite, can be selected so that \( R_x \subseteq \bigcup_{(\xi, \xi) \in S_y \times S_y} U_{g,a} \o_{(\xi, \xi)} \). Furthermore, \( S_y \subseteq \bigcup_{\xi \in R_x \times R_y} U_{g,a} \o_{(\xi, \xi)} \), where \( S_y \subseteq S_y \) is finite, since \( S_y \subseteq S_y \). Now let

\[
U_{g,a} = \bigcup_{(\xi, \xi) \in S_y \times S_y} U_{g,a} \o_{(\xi, \xi)}, \quad U_{g,b} = \bigcup_{(\xi, \xi) \in S_y \times S_y} U_{g,b} \o_{(\xi, \xi)}.
\]

Observe that \((R_x, S_y) \subseteq (U_{g,a}, U_{g,b}) \). Moreover, \( (U_{g,a}, U_{g,b}) \in g-O [T^g_x] \times g-O [T^g_y] \), since \( U_{g,a} \o_{(\xi, \xi)} \in g-O [T^g_x] \) for every \( \xi \in R_x \) and \( U_{g,b} \o_{(\xi, \xi)} \in g-O [T^g_y] \) for every \( \xi \in R_y \). The proof of the theorem is complete when the statement \( U_{g,a} \cap U_{g,b} = \emptyset \) is proved. First observe that, for every \( \xi \in R_x \) and \( \xi \in R_y \), the relation \( U_{g,b} \cap U_{g,b} = \emptyset \) implies \( U_{g,a} \cap U_{g,b} = \emptyset \). Consequently,

\[
\bigcap_{\mu=\alpha, \beta} U_{g,\mu} = \bigcup_{(\xi, \xi) \in R_x \times R_y} U_{g,\xi} \cap U_{g,\beta} = \bigcup_{(\xi, \xi) \in R_x \times R_y} (U_{g,\xi} \cap U_{g,\beta}) = \bigcup_{(\xi, \xi) \in R_x \times R_y} \emptyset = \emptyset.
\]

This proves the theorem. Q.E.D.
We next show that $g$-$\mathcal{T}_\theta$-compactness is an absolute property that is preserved under $g$-$\mathcal{T}_\theta$-continuous maps.

**Theorem 3.25.** Let $\mathcal{T}_\theta \Omega = (\Omega, \mathcal{T}_\theta \Omega)$ and $\mathcal{T}_\theta \Sigma = (\Sigma, \mathcal{T}_\theta \Sigma)$ be $\mathcal{T}_\theta$-spaces. If $\pi_\theta : \mathcal{T}_\theta \Omega \to \mathcal{T}_\theta \Sigma$ is a $g$-$(\mathcal{T}_\theta \Omega, \mathcal{T}_\theta \Sigma)$-continuous map and, $S_\theta, \omega \in g\mathcal{A}[\mathcal{T}_\theta \Sigma]$ in $\mathcal{T}_\theta \Omega$, then $\text{im}(\pi_\theta|_{S_\theta, \omega}) \in g\mathcal{A}[\mathcal{T}_\theta \Sigma]$ in $\mathcal{T}_\theta \Sigma$.

**Proof.** Let $\mathcal{T}_\theta \Omega = (\Omega, \mathcal{T}_\theta \Omega)$ and $\mathcal{T}_\theta \Sigma = (\Sigma, \mathcal{T}_\theta \Sigma)$ be given $\mathcal{T}_\theta$-spaces, $\pi_\theta \in g\mathcal{C}[\mathcal{T}_\theta \Omega; \mathcal{T}_\theta \Sigma]$, $S_\theta, \omega \in g\mathcal{A}[\mathcal{T}_\theta \Omega]$ in $\mathcal{T}_\theta \Omega$ and, suppose $\langle U_\theta, \alpha \rangle_{\alpha \in I_\theta}$ be a $g$-$\mathcal{T}_\theta$-open covering of $\text{im}(\pi_\theta|_{S_\theta, \omega})$ in $\mathcal{T}_\theta \Sigma$. Then,

$$S_\theta, \omega \subseteq \pi_\theta^{-1} \circ \pi_\theta (S_\theta, \omega) \subseteq \bigcup_{\alpha \in I_\theta} U_\theta, \alpha \subseteq \bigcup_{\alpha \in I_\theta} \pi_\theta^{-1} (U_\theta, \alpha).$$

Thus, $\langle \pi_\theta^{-1} (U_\theta, \alpha) \rangle_{\alpha \in I_\theta}$ is a $g$-$\mathcal{T}_\theta$-open covering of $S_\theta, \omega$ in $\mathcal{T}_\theta \Sigma$, because $\pi_\theta \in g\mathcal{C}[\mathcal{T}_\theta \Omega; \mathcal{T}_\theta \Sigma]$ and, for every $\alpha \in I_\theta$, $U_\theta, \alpha \in g\mathcal{O}[\mathcal{T}_\theta \Omega]$ implies $\pi_\theta^{-1} (U_\theta, \alpha) \in g\mathcal{O}[\mathcal{T}_\theta \Sigma]$. But, the relation $S_\theta, \omega \in g\mathcal{A}[\mathcal{T}_\theta \Sigma]$ holds and, consequently, there exists $\langle \pi_\theta^{-1} (U_\theta, \alpha) \rangle_{\alpha \in I_\theta} \prec \langle \pi_\theta^{-1} (U_\theta, \alpha) \rangle_{\alpha \in I_\theta}$ such that the relation $S_\theta, \omega \subseteq \bigcup_{\alpha \in I_\theta} U_\theta, \alpha \subseteq \bigcup_{\alpha \in I_\theta} \pi_\theta^{-1} (U_\theta, \alpha)$ holds. Accordingly,

$$\pi_\theta (S_\theta, \omega) \subseteq \pi_\theta \circ \pi_\theta^{-1} \left( \bigcup_{\alpha \in I_\theta} U_\theta, \alpha \right) = \bigcup_{\alpha \in I_\theta} U_\theta, \alpha.$$ 

Thus, $\langle U_\theta, \alpha \rangle_{\alpha \in I_\theta}$ is a $g$-$\mathcal{T}_\theta$-open subcovering of $\text{im}(\pi_\theta|_{S_\theta, \omega})$ and hence, $\text{im}(\pi_\theta|_{S_\theta, \omega}) \in g\mathcal{A}[\mathcal{T}_\theta \Sigma]$ in $\mathcal{T}_\theta \Sigma$. The proof of the theorem is complete. Q.E.D.

We now show that $g$-$\mathcal{T}_\theta$-compactness is an absolute property that is also preserved under $g$-$\mathcal{T}_\theta$-irresolute maps.

**Theorem 3.26.** Let $S_\theta, \omega \subseteq \mathcal{T}_\theta \Omega$ be a $\mathcal{T}_\theta$-set and let $\pi_\theta \in g\mathcal{I}[\mathcal{T}_\theta \Omega; \mathcal{T}_\theta \Sigma]$ be a $g$-$(\mathcal{T}_\theta \Omega, \mathcal{T}_\theta \Sigma)$-irresolute map, where $\mathcal{T}_\theta \Omega = (\Omega, \mathcal{T}_\theta \Omega)$ and $\mathcal{T}_\theta \Sigma = (\Sigma, \mathcal{T}_\theta \Sigma)$ are $\mathcal{T}_\theta$-spaces. If $S_\theta, \omega \in g\mathcal{A}[\mathcal{T}_\theta \Omega]$, then $\text{im}(\pi_\theta|_{S_\theta, \omega}) \in g\mathcal{A}[\mathcal{T}_\theta \Sigma]$.

**Proof.** Let $S_\theta, \omega \subseteq \mathcal{T}_\theta \Omega$ be a $\mathcal{T}_\theta$-set and let $\pi_\theta \in g\mathcal{I}[\mathcal{T}_\theta \Omega; \mathcal{T}_\theta \Sigma]$ be a $g$-$(\mathcal{T}_\theta \Omega, \mathcal{T}_\theta \Sigma)$-irresolute map, where $\mathcal{T}_\theta \Omega = (\Omega, \mathcal{T}_\theta \Omega)$ and $\mathcal{T}_\theta \Sigma = (\Sigma, \mathcal{T}_\theta \Sigma)$ are $\mathcal{T}_\theta$-spaces. Suppose $S_\theta, \omega \in g\mathcal{A}[\mathcal{T}_\theta \Omega]$, let $\langle U_\theta, \alpha \in g\mathcal{O}[\mathcal{T}_\theta \Sigma] \rangle_{\alpha \in I_\theta}$ be any $g$-$\mathcal{T}_\theta$-open covering of $\pi_\theta (S_\theta, \omega)$ in $\mathcal{T}_\theta \Sigma$. Then, since $\pi_\theta \in g\mathcal{I}[\mathcal{T}_\theta \Omega; \mathcal{T}_\theta \Sigma]$, it follows, evidently, that the relation $S_\theta, \omega \subseteq \bigcup_{\alpha \in I_\theta} \pi_\theta^{-1} (U_\theta, \alpha)$ holds. On the other hand, since $S_\theta, \omega \in g\mathcal{A}[\mathcal{T}_\theta \Omega]$, it results that, a $g$-$\mathcal{T}_\theta$-open subcovering $\langle U_\theta, \alpha \rangle_{\alpha \in I_\theta} \prec \langle U_\theta, \alpha \rangle_{\alpha \in I_\theta}$ exists such that the relation $S_\theta, \omega \subseteq \bigcup_{\alpha \in I_\theta} U_\theta, \alpha \subseteq \bigcup_{\alpha \in I_\theta} \pi_\theta^{-1} (U_\theta, \alpha)$ holds. Consequently, it follows, then, that $\pi_\theta (S_\theta, \omega) \subseteq \bigcup_{\alpha \in I_\theta} U_\theta, \alpha$ and, hence, $\text{im}(\pi_\theta|_{S_\theta, \omega}) \in g\mathcal{A}[\mathcal{T}_\theta \Sigma]$. The proof of the theorem is complete. Q.E.D.

**Lemma 3.27.** Let $g\mathcal{T}_\theta^2 \Omega = (\Omega, g\mathcal{T}_\theta^2 \Omega)$ be a $g$-$\mathcal{T}_\theta^2 \Omega$-space. If $S_\theta \in g\mathcal{K}[g\mathcal{T}_\theta^2 \Omega]$, then $S_\theta \in g\mathcal{A}[g\mathcal{T}_\theta^2 \Omega]$ in $g\mathcal{T}_\theta^2 \Omega$.

**Proof.** Let $g\mathcal{T}_\theta^2 \Omega = (\Omega, g\mathcal{T}_\theta^2 \Omega)$ be a $g$-$\mathcal{T}_\theta^2 \Omega$-space and suppose $S_\theta \in g\mathcal{K}[g\mathcal{T}_\theta^2 \Omega]$. Suppose $\langle U_\theta, \alpha \in g\mathcal{O}[g\mathcal{T}_\theta^2 \Omega] \rangle_{\alpha \in I_\theta}$ is a $g$-$\mathcal{T}_\theta^2 \Omega$-open covering of $S_\theta$, then $\Omega = \ldots$
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$(\bigcup_{\alpha \in I_g} U_{\theta, \alpha}) \cup \mathcal{C}(S_g) = \bigcup_{\alpha \in I_g} (U_{\theta, \alpha} \cup \mathcal{C}(S_g))$, meaning that $(U_{\theta, \alpha} \cup \mathcal{C}(S_g))_{\alpha \in I_g}$ is a $g\cdot \Sigma_g$-open covering of $S_g$ because, $S_g \in g\cdot K[g\cdot \Sigma_g]$ implies $\mathcal{C}(S_g) \in g\cdot O[g\cdot \Sigma_g]$.

On the other hand, $g\cdot \Sigma_g$ is, by hypothesis, a $g\cdot \Sigma_g$-space. Thus, there exists $(U_{\theta, \theta}(\alpha))_{(\alpha, \theta(\alpha)) \in I_g \times I_g} \times (U_{\theta, \theta}(\alpha))_{\alpha \in I_g}$ such that $\Omega = (\bigcup_{(\alpha, \theta(\alpha)) \in I_g \times I_g} U_{\theta, \theta}(\alpha)) \cup \mathcal{C}(S_g)$. But $S_g \cap \mathcal{C}(S_g) = 0$ and, hence, $S_g \subseteq \bigcup_{(\alpha, \theta(\alpha)) \in I_g \times I_g} U_{\theta, \theta}(\alpha)$. This shows that any $g\cdot \Sigma_g$-open covering $(U_{\theta, \theta}(\alpha))_{\alpha \in I_g}$ of $S_g$ contains a finite $g\cdot \Sigma_g$-open subcovering $(U_{\theta, \theta}(\alpha))_{(\alpha, \theta(\alpha)) \in I_g \times I_g}$ and hence, $S_g \in g\cdot A[g\cdot \Sigma_g]$ in $g\cdot \Sigma_g$. The proof of the lemma is complete.

Q.E.D.

THEOREM 3.28. Let $g\cdot \Sigma_g = (\Omega, g\cdot \Sigma_g) = \bigcup_{\alpha \in I_g} U_{\theta, \alpha}$ be a $g\cdot \Sigma_g$-space and let $g\cdot \Sigma_g = (\Sigma, g\cdot \Sigma_g)$ be a $g\cdot \Sigma_g$-space. If the $g\cdot \Sigma_g$-map $\pi_\theta : g\cdot \Sigma_g \rightarrow g\cdot \Sigma_g$ is one-one and $g\cdot \Sigma_g$-continuous, then $g\cdot \Sigma_g \simeq \pi_\theta(g\cdot \Sigma_g)$.

PROOF. Let $g\cdot \Sigma_g = (\Omega, g\cdot \Sigma_g)$ be a $g\cdot \Sigma_g$-space and let $g\cdot \Sigma_g = (\Sigma, g\cdot \Sigma_g)$ be a $g\cdot \Sigma_g$-space, and suppose $\pi_\theta : g\cdot \Sigma_g \rightarrow g\cdot \Sigma_g$ is a one-one $g\cdot (g\cdot \Sigma_g; g\cdot \Sigma_g)$-continuous map. Clearly, $\pi_\theta : g\cdot \Sigma_g \rightarrow g\cdot \Sigma_g$ is onto, and since it is, by hypothesis, a one-one $g\cdot (g\cdot \Sigma_g; g\cdot \Sigma_g)$-continuous map, it follows that $\pi_\theta^{-1} : g\cdot \Sigma_g \rightarrow g\cdot \Sigma_g$ exists. It must be shown that $\pi_\theta^{-1} \in g\cdot C[g\cdot \Sigma_g; g\cdot \Sigma_g]$. It must be shown that $\pi_\theta^{-1} \in g\cdot C[g\cdot \Sigma_g; g\cdot \Sigma_g]$. Recall that $\pi_\theta^{-1} : g\cdot \Sigma_g \rightarrow g\cdot \Sigma_g$ is a one-one $g\cdot (g\cdot \Sigma_g; g\cdot \Sigma_g)$-continuous map and only if, for every $K_{\theta, \omega} \subseteq g\cdot \Sigma_g$, $\pi_\theta^{-1}(K_{\theta, \omega}) \subseteq \im(\pi_\theta)$. Clearly, $K_{\theta, \omega} \subseteq \im(\pi_\theta)$. But, $\pi_\theta^{-1}(K_{\theta, \omega}) \subseteq \im(\pi_\theta)$.

Accordingly, $\pi_\theta^{-1} \in g\cdot C[g\cdot \Sigma_g; g\cdot \Sigma_g]$ and hence, $g\cdot \Sigma_g \simeq \pi_\theta(g\cdot \Sigma_g)$. The proof of the theorem is complete.

Q.E.D.

A $g\cdot \Sigma_g$-space coincides with a $g\cdot \Sigma_g$-space upon satisfaction of some condition as given in the following proposition.

PROPOSITION 3.29. Let $g\cdot \Sigma_g = (\Omega, g\cdot \Sigma_g) = \bigcup_{\alpha \in I_g} U_{\theta, \alpha}$ be a $g\cdot \Sigma_g$-space and let $g\cdot \Sigma_g = (\Omega, g\cdot \Sigma_g)$ be a $g\cdot \Sigma_g$-space. If $g\cdot \Sigma_g \supseteq g\cdot \Sigma_g$, then $g\cdot \Sigma_g = g\cdot \Sigma_g$.

PROOF. Let $g\cdot \Sigma_g = (\Omega, g\cdot \Sigma_g)$ be a $g\cdot \Sigma_g$-space and $g\cdot \Sigma_g = (\Omega, g\cdot \Sigma_g)$, a $g\cdot \Sigma_g$-space, and suppose $g\cdot \Sigma_g \supseteq g\cdot \Sigma_g$. Further, consider the $g\cdot \Sigma_g$-map $\pi_\theta : g\cdot \Sigma_g \rightarrow g\cdot \Sigma_g$ defined by $\pi_\theta(\xi) = \xi$. Since $g\cdot \Sigma_g \supseteq g\cdot \Sigma_g$, for every $O_{\theta, \alpha} \subseteq g\cdot \Sigma_g$, there exist $O_{\theta, \theta}(\alpha) \subseteq g\cdot \Sigma_g$ such that $\pi_\theta^{-1}(O_{\theta, \theta}(\alpha)) = O_{\theta, \theta}(\alpha)$. Consequently, $\pi_\theta : g\cdot \Sigma_g \rightarrow g\cdot \Sigma_g$ is a one-one and onto $g\cdot (g\cdot \Sigma_g; g\cdot \Sigma_g)$-continuous map from a $g\cdot \Sigma_g$-space $g\cdot \Sigma_g$ to a $g\cdot \Sigma_g$-space $g\cdot \Sigma_g$ and therefore, $g\cdot \Sigma_g \simeq \pi_\theta(g\cdot \Sigma_g)$. Hence, $g\cdot \Sigma_g = g\cdot \Sigma_g$. The proof of the proposition is complete.

Q.E.D.
Below is defined a new concept called $g$-$\Sigma_g$-accumulation point, by means of which further characterisation on $g$-$\Sigma_g$-compactness can be established.

**Definition 3.30 ($g$-$\Sigma_g$-Accumulation Point).** A point $\xi \in \Sigma_g$ of a $\Sigma_g$-space $\Sigma_g = (\Omega, \mathcal{T}_g)$ is called a "$g$-$\Sigma_g$-accumulation point" (or, "$g$-$\Sigma_g$-limit point," "$g$-$\Sigma_g$-cluster point," "$g$-$\Sigma_g$-derived point") of a $\Sigma_g$-set $S_0 \subset \Sigma_g$ if and only if every $g$-$\Sigma_g$-open set $U_{g, \xi} \in g-O[\Sigma_g]$ containing $\xi$ (whether $\xi \in S_0$ or $\xi \notin S_0$) contains at least a point $\zeta \in \Sigma_g \setminus \{\xi\}$:

\[(3.8) \quad \xi \in U_{g, \xi} \in g-O[\Sigma_g] \Rightarrow S_0 \cap (U_{g, \xi} \setminus \{\xi\}) \neq \emptyset.\]

The set of all $g$-$\Sigma_g$-accumulation points, denoted by $\text{der}_g(S_0) \subset \Sigma_g$, is called the "$g$-$\Sigma_g$-derived set of $S_0".

Making use of the notion of $g$-$\Sigma_g$-accumulation point, we further introduce another concept called *countable $g$-$\Sigma_g$-compactness*, possessed by all $g$-$\Sigma_g$-compact sets.

**Definition 3.31.** A $\Sigma_g$-set $S_0 \subset \Sigma_g$ of a $\Sigma_g$-space $\Sigma_g = (\Omega, \mathcal{T}_g)$ is said to be "countably $g$-$\Sigma_g$-compact" if and only if every infinite $\Sigma_g$-subset $\mathcal{R}_0 \subset S_0$ of $S_0$ has at least one $g$-$\Sigma_g$-accumulation point $\xi \in S_0$.

The statement relating the notions of $g$-$\Sigma_g$-compactness and countable $g$-$\Sigma_g$-compactness is contained in the following theorem.

**Theorem 3.32.** If $S_0 \in g-A[\Sigma_g]$ be a $g$-$\Sigma_g$-compact set of a $\Sigma_g$-space $\Sigma_g = (\Omega, \mathcal{T}_g)$, then it is also countably $g$-$\Sigma_g$-compact in $\Sigma_g$.

**Proof.** Let $S_0 \in g-A[\Sigma_g]$ be a $g$-$\Sigma_g$-compact set of a $\Sigma_g$-space $\Sigma_g = (\Omega, \mathcal{T}_g)$ and, suppose $\mathcal{R}_0 \subset S_0$ be an infinite $\Sigma_g$-subset of $S_0$. Equivalently proved, it must be shown that, the assumption that $\mathcal{R}_0$ has no $g$-$\Sigma_g$-accumulation point $\xi \in S_0$ leads to a contradiction. Since $\mathcal{R}_0 \subset S_0$ is, by assumption, an infinite $\Sigma_g$-subset of $S_0$ with no $g$-$\Sigma_g$-accumulation point $\xi \in S_0$, it follows, then, that, for every $\xi \in S_0$, there exists a $g$-$\Sigma_g$-open set $U_{g, \xi} \in g-O[\Sigma_g]$ which contains at most one point $\zeta \in \mathcal{R}_0$.

It may be remarked, in passing, that $(U_{g, \xi})_{\xi \in S_0}$ is a $g$-$\Sigma_g$-open covering of the $g$-$\Sigma_g$-compact set $S_0 \in g-A[\Sigma_g]$ for, $S_0 \subseteq \bigcup_{\xi \in S_0} U_{g, \xi}$. Consequently, there exists a $g$-$\Sigma_g$-open subcovering $(\langle U_{g, \vartheta(\xi)} \rangle_{\xi \in S_0 \times S_0} \times S_0) \prec (\langle U_{g, \vartheta(o) \in I^*_g} \rangle_{o \in I^*_g})$, where $\hat{S}_0 \subset S_0$, such $\mathcal{R}_0 \subset S_0$, $\mathcal{R}_0 \subseteq \bigcup_{(\xi, \vartheta(\xi)) \in S_0 \times S_0} U_{g, \vartheta(\xi)}$. But, for every $(\xi, \vartheta(\xi)) \in S_0 \times \hat{S}_0$, $U_{g, \vartheta(\xi)}$ contains at most one point $\zeta \in \mathcal{R}_0$. Therefore, the infinite $\Sigma_g$-subset $\mathcal{R}_0$ of $S_0$, satisfying $\mathcal{R}_0 \subseteq \bigcup_{(\xi, \vartheta(\xi)) \in S_0 \times \hat{S}_0} U_{g, \vartheta(\xi)}$, can contain at most $\eta = \text{card}(\hat{S}_0) < \infty$ points. Accordingly, it follows that every infinite $\Sigma_g$-subset $\mathcal{R}_0 \subset S_0$ of $S_0$ contains a $g$-$\Sigma_g$-accumulation point $\xi \in S_0$. Hence, $S_0 \in g-A[\Sigma_g]$ is also countably $g$-$\Sigma_g$-compact in $\Sigma_g$. This completes the proof of the theorem. \(Q.E.D.\)

An immediate consequence of the above theorem is the following corollary.

**Corollary 3.33.** Every $\Sigma_g$-space $\Sigma_g = (\Omega, \mathcal{T}_g)$ having the property that every countable $g$-$\Sigma_g$-open covering $(\langle U_{g, \alpha} \in g-O[\Sigma_g] \rangle_{o \in I^*_g})$ of $\Sigma_g$ contains a finite $g$-$\Sigma_g$-open subcovering $(\langle U_{g, \alpha, \alpha} \rangle_{(\xi, \alpha) \in I^*_g \times I^*_g}) \prec (\langle U_{g, \alpha} \rangle_{o \in I^*_g})$ of $\Sigma_g$ is a countably $g$-$\Sigma_g[A]$-space $g$-$\Sigma_g[A] = (\Omega, g$-$\Sigma_g[A]).$
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DEFINITION 3.34. A $\Sigma_\alpha$-set $S_\alpha$ of a $T_\beta$-space $\mathcal{T}_\alpha = (\Omega, T_\alpha)$ is "sequentially $g$-$\Sigma_\alpha$-compact" if and only if every sequence $\{\xi_\alpha \in S_\alpha\}_{\alpha \in I_{\infty}}$ in $S_\alpha$ contains a subsequence $\{\xi_{\alpha}(\omega)\}_{(\alpha, \omega) \in I_{\infty} \times I_{\infty}}$ which converges to a point $\xi \in S_\alpha$.

THEOREM 3.35. Let $\pi_\alpha : \mathcal{T}_\alpha \to \mathcal{T}_\beta$ be a $g$-$(\mathcal{T}_\alpha, \mathcal{T}_\beta)$-continuous map, where $\mathcal{T}_\alpha = (\Omega, T_\alpha)$ and $\mathcal{T}_\beta = (\Omega, T_\beta)$ are $T_\beta$-spaces. If $S_{\beta \omega} \subset \mathcal{T}_\beta$ is a sequentially $g$-$\Sigma_\alpha$-compact set in $\mathcal{T}_\beta$, then $\text{im}(\pi_\alpha|_{S_{\beta \omega}}) \subset \mathcal{T}_\alpha$ is also a sequentially $g$-$\Sigma_\alpha$-compact set in $\mathcal{T}_\alpha$.

**Proof.** Let $\pi_\alpha \in g-C[\mathcal{T}_\alpha; \mathcal{T}_\beta]$, where $\mathcal{T}_\alpha = (\Omega, T_\alpha)$ and $\mathcal{T}_\beta = (\Omega, T_\beta)$ are $T_\beta$-spaces, and suppose $S_{\beta \omega} \subset \mathcal{T}_\beta$ is sequentially $g$-$\Sigma_\alpha$-compact in $\mathcal{T}_\beta$. If $\{\xi_{\alpha} \in S_{\beta \omega}\}_{\alpha \in I_{\infty}}$ is a sequence in $\text{im}(\pi_\alpha|_{S_{\beta \omega}}) \subset \mathcal{T}_\alpha$, then there exists a subsequence $\{\xi_{\alpha} \in S_{\beta \omega}\}_{(\alpha, \omega) \in I_{\infty} \times I_{\infty}}$ in $S_{\beta \omega}$ such that $\xi_{\alpha(\omega)} = \xi_\alpha$ for every $\alpha \in I_{\infty}$. But, by hypothesis, $S_{\beta \omega} \subset \mathcal{T}_\beta$ is sequentially $g$-$\Sigma_\alpha$-compact in $\mathcal{T}_\beta$. Therefore, there exists a subsequence $\{\xi_{\alpha} \in S_{\beta \omega}\}_{(\alpha, \omega) \in I_{\infty} \times I_{\infty}}$ in $S_{\beta \omega}$ such that $\xi_{\alpha(\omega)} = \xi_\alpha$ converges to a point $\xi \in S_{\beta \omega}$. On the other hand, $\pi_\alpha \in g-C[\mathcal{T}_\alpha; \mathcal{T}_\beta]$ and, therefore, $\pi_\alpha : \mathcal{T}_\alpha \to \mathcal{T}_\beta$ is sequentially $g$-$\Sigma_\alpha$-continuous. Consequently, $\{\pi_\alpha(\xi_{\alpha})\}_{(\alpha, \omega) \in I_{\infty} \times I_{\infty}}$ converges to $\pi_\alpha(\xi) \in \text{im}(\pi_\alpha|_{S_{\beta \omega}})$. Hence, it follows that $\text{im}(\pi_\alpha|_{S_{\beta \omega}}) \subset \mathcal{T}_\alpha$ is sequentially $g$-$\Sigma_\alpha$-compact in $\mathcal{T}_\alpha$. Q.E.D.

PROPOSITION 3.36. Let $\pi_\alpha : \mathcal{T}_\alpha \to \mathcal{T}_\beta$ be a $g$-$(\mathcal{T}_\alpha, \mathcal{T}_\beta)$-continuous map, where $\mathcal{T}_\alpha = (\Omega, T_\alpha)$ and $\mathcal{T}_\beta = (\Omega, T_\beta)$ are $T_\beta$-spaces. If $S_{\beta \omega} \subset \mathcal{T}_\beta$ is a $g$-$\Sigma_\alpha$-compact set in $\mathcal{T}_\beta$, then $\text{im}(\pi_\alpha|_{S_{\beta \omega}}) \subset \mathcal{T}_\alpha$ is also a $g$-$\Sigma_\alpha$-compact set in $\mathcal{T}_\alpha$.

**Proof.** Let $\pi_\alpha \in g-C[\mathcal{T}_\alpha; \mathcal{T}_\beta]$, where $\mathcal{T}_\alpha = (\Omega, T_\alpha)$ and $\mathcal{T}_\beta = (\Omega, T_\beta)$ are $T_\beta$-spaces, and suppose $\{U_{\alpha, \omega} \in O[\mathcal{T}_\alpha]\}_{\alpha \in I_{\infty}}$ is a $\Sigma_\alpha$-open covering of $S_{\beta \omega} = \pi_\alpha^{-1}(S_{\beta \omega})$, because $O[\mathcal{T}_\alpha] \subset g-O[\mathcal{T}_\beta]$. Since $S_{\beta \omega} \subset \mathcal{T}_\beta$ is a finite $g$-$\Sigma_\alpha$-open subcovering $\{U_{\alpha, \omega} \in O[\mathcal{T}_\alpha]\}_{\alpha \in I_{\infty}}$ exists, that $S_{\beta \omega} \subset \bigcup_{\alpha \in I_{\infty}} U_{\alpha, \omega}$. Since $\pi_\alpha \in g-C[\mathcal{T}_\alpha; \mathcal{T}_\beta]$, it follows, consequently, that $\pi_\alpha(S_{\beta \omega}) \subset \bigcup_{\alpha \in I_{\infty}} U_{\alpha, \omega}$. Therefore, $\{\pi_\alpha(U_{\alpha, \omega})\}_{\alpha \in I_{\infty}} \subset O[\mathcal{T}_\beta]$ is a finite $\Sigma_\alpha$-open subcovering of $S_{\beta \omega} \subset \mathcal{T}_\beta$. Hence, $\text{im}(\pi_\alpha|_{S_{\beta \omega}}) \subset \mathcal{T}_\alpha$ is also $\Sigma_\alpha$-compact in $\mathcal{T}_\alpha$. The proof of the proposition is complete. Q.E.D.

THEOREM 3.37. Let $\pi_\alpha : \mathcal{T}_\alpha \to \mathcal{T}_\beta$ be a $g$-$(\mathcal{T}_\alpha, \mathcal{T}_\beta)$-continuous map, where $\mathcal{T}_\alpha = (\Omega, T_\alpha)$ and $\mathcal{T}_\beta = (\Omega, T_\beta)$ are $T_\beta$-spaces. If $S_{\beta \omega} \subset \mathcal{T}_\beta$ is a countably $g$-$\Sigma_\alpha$-compact set in $\mathcal{T}_\beta$, then $\text{im}(\pi_\alpha|_{S_{\beta \omega}}) \subset \mathcal{T}_\alpha$ is also a countably $g$-$\Sigma_\alpha$-compact set in $\mathcal{T}_\alpha$.

**Proof.** Let $\pi_\alpha \in g-C[\mathcal{T}_\alpha; \mathcal{T}_\beta]$, where $\mathcal{T}_\alpha = (\Omega, T_\alpha)$ and $\mathcal{T}_\beta = (\Omega, T_\beta)$ are $T_\beta$-spaces, and suppose $S_{\beta \omega} \subset \mathcal{T}_\beta$ is a countably $g$-$\Sigma_\alpha$-compact set in $\mathcal{T}_\beta$. To prove that $\text{im}(\pi_\alpha|_{S_{\beta \omega}}) \subset \mathcal{T}_\alpha$ is countably $g$-$\Sigma_\alpha$-compact, let $S_{\beta, \sigma} \subset \pi_\alpha(S_{\beta \omega})$ be an infinite $\Sigma_\alpha$-subset of $\text{im}(\pi_\alpha|_{S_{\beta \omega}})$. Then, a denumerable $\Sigma_\alpha$-subset $R_{\beta, \sigma}$ of $S_{\beta, \sigma}$ exists. Since $R_{\beta, \sigma} \subset S_{\beta, \sigma} \subset \text{im}(\pi_\alpha|_{S_{\beta \omega}}) = \pi_\alpha(S_{\beta \omega})$, the proof of the theorem is complete. Q.E.D.
there exists a denumerable \(\mathcal{T}_g\)-subset \(\mathcal{R}_{g,\omega} = \{\xi_\alpha : \alpha \in I^*_g\} \subset S_{g,\omega}\), with 
\[ \pi_\alpha(\xi_\alpha) = \xi_\alpha \text{ for every } \alpha \in I^*_g. \]
But, by hypothesis, \(S_{g,\omega} \subset \mathcal{T}_g\), so \(\mathcal{R}_{g,\omega}\) contains a \(g\)-\(\mathcal{T}_g\)-accumulation point \(\xi \in S_{g,\omega}\). Thus, \(\xi \in \mathcal{R}_{g,\omega} \cup \text{der}(\mathcal{R}_{g,\omega}) \subseteq \mathcal{R}_{g,\omega}\) and \(\pi_\xi(\xi) \in \text{im}(\pi_\xi|_{\mathcal{R}_{g,\omega}}) = \pi_\xi(\mathcal{S}_{g,\omega})\); evidently, 
\[ \text{der}(\mathcal{R}_{g,\omega}) \in g-K[\mathcal{T}_g,\Omega] \text{ and, therefore, a } g\)-\(\mathcal{T}_g\)-closed set \(\mathcal{V}_g \subset g-K[\mathcal{T}_g,\Omega] \exists \text{ such that, } \text{der}(\mathcal{R}_{g,\omega}) = \mathcal{V}_g. \]
But, by hypothesis, \(\pi_\xi \in g-C[\mathcal{T}_g,\Omega,\xi])\). Consequently, \(\pi_\xi(\mathcal{R}_{g,\omega} \cup \text{der}(\mathcal{R}_{g,\omega})) \subseteq \pi_\xi(\mathcal{R}_{g,\omega}) \cup \text{der}(\pi_\xi(\mathcal{R}_{g,\omega}) = \mathcal{R}_{g,\sigma} \cup \text{der}(\mathcal{R}_{g,\sigma}). \)
Thus, \(\xi \in \mathcal{R}_{g,\omega} \cup \text{der}(\mathcal{R}_{g,\omega}) \) and, therefore, \(\pi_\xi(\xi) \in \mathcal{R}_{g,\sigma} \cup \text{der}(\mathcal{R}_{g,\sigma}). \)
Now, \(\pi_\xi(\xi) \in \mathcal{R}_{g,\sigma} \cup \text{der}(\mathcal{R}_{g,\sigma}), \) so let it be claimed that \(\pi_\xi(\xi) \in \mathcal{R}_{g,\sigma}\).

1. Case \(\xi \notin \mathcal{R}_{g,\omega}\). If \(\xi \notin \mathcal{R}_{g,\omega}\), then \(\pi_\xi(\xi) \notin (\mathcal{R}_{g,\omega}) = \mathcal{R}_{g,\sigma}\). But, \(\pi_\xi(\xi) \in \mathcal{R}_{g,\omega} \cup \text{der}(\mathcal{R}_{g,\sigma})\) and, consequently, \(\pi_\xi(\xi) \in \mathcal{R}_{g,\omega} \cup \text{der}(\mathcal{R}_{g,\omega})\) of \(\mathcal{R}_{g,\sigma}\).

2. Case \(\xi \in \mathcal{R}_{g,\omega}\). If \(\xi \in \mathcal{R}_{g,\omega}\), choose a \(\mu \in I^*_g\) such that \(\xi = \xi_\mu\). Then, \(\xi \notin \mathcal{R}_{g,\omega} = \{\xi_\alpha : \alpha \in I^*_g \setminus \{\mu\}\} \) and, every \(g\)-\(\mathcal{T}_g\)-open set \(U_\xi,\xi \in g-O[\mathcal{T}_g]\) containing \(\xi\) contains at least a point \(\xi \in \mathcal{R}_{g,\omega} = \{\xi_\alpha : \alpha \in I^*_g \setminus \{\mu\}\} \) and, therefore, \(\xi \in \mathcal{R}_{g,\omega}\) of \(\mathcal{R}_{g,\sigma}\).

**PROPOSITION 3.38.** If \(S_{\theta} \subset \mathcal{T}_g\) be a sequentially \(g\)-\(\mathcal{T}_g\)-compact set of a \(\mathcal{T}_g\)-space \(\mathcal{T}_g = (\Omega, \mathcal{T}_g)\), then every countable \(g\)-\(\mathcal{T}_g\)-open covering \(\{U_{\theta,\alpha} \in g-O[\mathcal{T}_g] : \alpha \in I^*_{\mathcal{T}_g}\}\) of the \(g\)-\(\mathcal{T}_g\)-compact set \(S_{\theta}\) is reducible to a finite \(g\)-\(\mathcal{T}_g\)-open subcovering of the type \(\{U_{\theta,\alpha} : (\alpha, \theta(\alpha)) \in I^*_g \times I^*_\mathcal{T}_g\} \subset S_{\theta}\).

**PROOF.** Let it be assumed that \(S_{\theta} \subset \mathcal{T}_g\) is a sequentially \(g\)-\(\mathcal{T}_g\)-compact infinite set of a \(\mathcal{T}_g\)-space \(\mathcal{T}_g = (\Omega, \mathcal{T}_g)\). Furthermore, assume that there exists a countable \(g\)-\(\mathcal{T}_g\)-open covering \(\{U_{\theta,\alpha} \in g-O[\mathcal{T}_g] : \alpha \in I^*_{\mathcal{T}_g}\}\) of \(S_{\theta}\) with no finite \(g\)-\(\mathcal{T}_g\)-open subcovering \(\{U_{\theta,\alpha} : (\alpha, \theta(\alpha)) \in I^*_g \times I^*_\mathcal{T}_g\} \subset S_{\theta}\). Finally, introduce the sequence \(\{\xi_\alpha \in S_{\theta}, \alpha \in I^*_{\mathcal{T}_g}\}\) and define its elements in the following manner. Let \(\theta(1) \in I^*_\mathcal{T}_g\) be the smallest integer in \(I^*_\mathcal{T}_g\) such that \(S_{\theta} \cap \bigcup_{\alpha \in I^*_\mathcal{T}_g} U_{\theta,\alpha} \neq \emptyset\); choose \(\xi_1 \in S_{\theta} \cap \bigcup_{\alpha \in I^*_\mathcal{T}_g} U_{\theta,\alpha}\). Let \(\theta(2) \in I^*_\mathcal{T}_g\) be the least integer larger than \(\theta(1)\) in \(I^*_\mathcal{T}_g\) such that \(S_{\theta} \cap \bigcup_{\alpha \in I^*_\mathcal{T}_g} U_{\theta,\alpha} \neq \emptyset\); choose \(\xi_2 \in \bigcup_{\alpha \in I^*_\mathcal{T}_g} (S_{\theta} \cap \bigcup_{\alpha \in I^*_\mathcal{T}_g} U_{\theta,\alpha})\). Note that, such a point \(\xi_2\) always exists, for otherwise \(U_{\theta,\alpha(1)}\) covers \(S_{\theta}\). Continuing in this way, the properties of \(\{\xi_\alpha \in S_{\theta}, \alpha \in I^*_{\mathcal{T}_g}\}\) for every \(\alpha \in I^*_\mathcal{T}_g\) \(\setminus \{1\}\), are

\[ \xi_\alpha \in S_{\theta} \cap \bigcup_{\alpha \in I^*_\mathcal{T}_g} U_{\theta,\alpha}, \quad \xi_\alpha \notin \bigcup_{\alpha \in I^*_\mathcal{T}_g \setminus \{\alpha\}} \bigcup_{\alpha \in I^*_\mathcal{T}_g \setminus \{\alpha\}} U_{\theta,\alpha(\alpha)}, \quad \theta(\alpha) > \theta(\alpha - 1). \]

Let it be claimed that the sequence \(\{\xi_\alpha \in I^*_\mathcal{T}_g\}\) has no convergent subsequence of the type \(\{\xi_\alpha(\alpha) : (\alpha, \theta(\alpha)) \in I^*_g \times I^*_\mathcal{T}_g\} \subset S_{\theta}\). Suppose \(\xi \in S_{\theta}\), then there exists a \(\mu \in I^*_\mathcal{T}_g\) such that \(\xi \in U_{\theta,\mu}\). Now, \(S_{\theta} \cap U_{\theta,\mu} \neq \emptyset\) since, \(\xi \in S_{\theta} \cap U_{\theta,\mu}\).
Thus, there exists $\nu \in I^*_{\partial(\sigma)}$ such that, $U_{\cdot, \partial(\nu)} = U_{\cdot, \partial(\mu)}$. But, by the properties of the sequence $\langle \xi_\alpha \rangle_{\alpha \in I^*_\sigma}$, $\alpha > \partial(\nu)$ implies $\xi_\alpha \notin U_{\cdot, \partial(\mu)}$. Accordingly, since $\xi \in U_{\cdot, \partial(\alpha)} \in g-O[I_{\sigma}]$ no subsequence $\langle (\xi_\alpha, \partial(\alpha)) \rangle_{\alpha \in I^*_\sigma \times I^*_\sigma} \subseteq \langle (\xi_\alpha)_{\alpha \in I^*_\sigma} \rangle$ of $\langle (\xi_\alpha)_{\alpha \in I^*_\sigma} \rangle$ converges to $\xi \in S_{\cdot, \partial}$. But, $\xi$ was arbitrary and, hence, $S_{\cdot, \partial} \subseteq \mathcal{T}_{\cdot, \partial}$ is not sequentially $g-\mathcal{T}_{\cdot, \partial}$-compact in $\mathcal{T}_{\cdot, \partial}$. The proof of the proposition is complete. Q.E.D.

**Definition 3.39** ($g-\mathcal{T}_{\cdot, \partial}$-Neighborhood). Let $\xi \in \mathcal{T}_{\cdot, \partial}$ be a point in a $T_{\cdot, \partial}$-space $\mathcal{T}_{\cdot, \partial} = (\Omega, \mathcal{T}_{\cdot, \partial})$. A $\mathcal{T}_{\cdot, \partial}$-subset $\mathcal{N}_{\cdot, \partial} \subseteq \mathcal{T}_{\cdot, \partial}$ of $\mathcal{T}_{\cdot, \partial}$ is a “$g-\mathcal{T}_{\cdot, \partial}$-neighborhood of $\xi$” if and only if $\mathcal{N}_{\cdot, \partial}$ is a $g-\mathcal{T}_{\cdot, \partial}$-superset of a $g-\mathcal{T}_{\cdot, \partial}$-open set $U_{\cdot, \partial, \xi} \in g-O[I_{\sigma}] \subseteq \mathcal{T}_{\cdot, \partial}$ containing $\xi$:

\[
(\xi, \mathcal{N}_{\cdot, \partial}, U_{\cdot, \partial, \xi}) \in \mathcal{T}_{\cdot, \partial} \times \mathcal{T}_{\cdot, \partial} \times g-O[I_{\sigma}] : \xi \in U_{\cdot, \partial, \xi} \subseteq \mathcal{N}_{\cdot, \partial},
\]

The class of all $g-\mathcal{T}_{\cdot, \partial}$-neighborhoods of $\xi \in \mathcal{T}_{\cdot, \partial}$, defined as

\[
(3.10) \quad g-N[\xi] \overset{\text{def}}{=} \{ \mathcal{N}_{\cdot, \partial} \subseteq \mathcal{T}_{\cdot, \partial} : (\exists U_{\cdot, \partial, \xi} \in g-O[I_{\sigma}]) (\xi \in U_{\cdot, \partial, \xi} \subseteq \mathcal{N}_{\cdot, \partial}) \}
\]

is called the ”$g-\mathcal{T}_{\cdot, \partial}$-neighborhood system of $\xi$.”

There exist $T_{\cdot, \partial}$-spaces which are not $g-\mathcal{T}_{\cdot, \partial}$-compact, but have instead a local version of $g-\mathcal{T}_{\cdot, \partial}$-compactness, and such local $T_{\cdot, \partial}$-property, called local $g-\mathcal{T}_{\cdot, \partial}$-compactness, is formalized in the following definition.

**Definition 3.40** (Locally $g-\mathcal{T}_{\cdot, \partial}$-Compact). A $\mathcal{T}_{\cdot, \partial}$-set $S_{\cdot, \partial} \subseteq \mathcal{T}_{\cdot, \partial}$ of a $T_{\cdot, \partial}$-space $\mathcal{T}_{\cdot, \partial} = (\Omega, \mathcal{T}_{\cdot, \partial})$ is said to be ”locally $g-\mathcal{T}_{\cdot, \partial}$-compact” if and only if, given any $(\xi, \mathcal{N}_{\cdot, \partial}, \xi) \in S_{\cdot, \partial} \times g-N[\xi]$, there is a $g-\mathcal{T}_{\cdot, \partial}$-neighborhood $\mathcal{N}_{\cdot, \partial, \xi} \in g-N[\xi]$ of $\xi$ such that $\mathcal{N}_{\cdot, \partial, \xi} \subseteq \mathcal{N}_{\cdot, \partial}$ and $\mathcal{N}_{\cdot, \partial, \xi} \cup \mathcal{d} \mathcal{e} \mathcal{r}(\mathcal{N}_{\cdot, \partial, \xi}) \subseteq g-A[I_{\sigma}]$.

The localisation of $g-\mathcal{T}_{\cdot, \partial}$-compactness is the requirement that small $g-\mathcal{T}_{\cdot, \partial}$-open sets have the desired $g-\mathcal{T}_{\cdot, \partial}$-compactness property even though the $T_{\cdot, \partial}$-space as a whole may not. The following theorem shows that we are dealing with a valid generalization of $g-\mathcal{T}_{\cdot, \partial}$-compactness.

**Theorem 3.41.** If $S_{\cdot, \partial} \in g-A[I_{\sigma}]$ be a $g-\mathcal{T}_{\cdot, \partial}$-compact set of a $T_{\cdot, \partial}$-space $\mathcal{T}_{\cdot, \partial} = (\Omega, \mathcal{T}_{\cdot, \partial})$, then it is also locally $g-\mathcal{T}_{\cdot, \partial}$-compact in $\mathcal{T}_{\cdot, \partial}$.
By virtue of the above theorem, it follows that a \( g\)-\( \mathcal{F}_0 \)-compact set of a \( T_0 \)-space has a \( g\)-\( \mathcal{F}_0 \)-open covering necessarily. This is embodied in the following corollary.

**Corollary 3.42.** Every \( T_0 \)-space \( \mathcal{F}_0 = (\Omega, T_0) \) having the property that every local \( g\)-\( \mathcal{F}_0 \)-open covering \( \{ \mathcal{U}_{\alpha} : \alpha \in g\mathcal{O}[\mathcal{F}_0] \} \) of \( \mathcal{F}_0 \) contains a finite \( g\)-\( \mathcal{F}_0 \)-open subcovering \( \{ \mathcal{U}_{\alpha} : \alpha \in g\mathcal{O}[\mathcal{F}_0] \} \) of \( \mathcal{F}_0 \) is a \( g\)-\( T_0[A] \)-space \( g\mathcal{F}_0[A] = (\Omega, g\mathcal{T}_0[A]) \).

For \( g\)-(\( \mathcal{F}_0, \Omega \), \( \mathcal{F}_0, \Sigma \))-continuous maps, the following theorem, which shows that local \( g\)-\( \mathcal{F}_0 \)-compactness is a \( T_0 \)-invariant, presents itself.

**Theorem 3.43.** Let \( \pi_0 : \mathcal{F}_0, \Omega \rightarrow \mathcal{F}_0, \Sigma \) be a \( g\)-(\( \mathcal{F}_0, \Omega \), \( \mathcal{F}_0, \Sigma \))-continuous map, where \( \mathcal{F}_0, \Omega = (\Omega, T_0, \mathcal{O}) \) and \( \mathcal{F}_0, \Sigma = (\Omega, T_0, \mathcal{O}) \) are \( T_0 \)-spaces. If \( \mathcal{S}_{\mathcal{F}_0, \Omega} \subset \mathcal{F}_0 \) is a locally \( g\)-\( \mathcal{F}_0 \)-compact set in \( \mathcal{F}_0, \Omega \), then \( \pi_0(\mathcal{S}_{\mathcal{F}_0, \Omega}) \subset \mathcal{F}_0, \Sigma \) is also a locally \( g\)-\( \mathcal{F}_0 \)-compact set in \( \mathcal{F}_0, \Sigma \).

**Proof.** Let \( \pi_0 \in g\mathcal{C}[\mathcal{F}_0, \Omega : \mathcal{F}_0, \Sigma] \), where \( \mathcal{F}_0, \Omega = (\Omega, T_0, \mathcal{O}) \) and \( \mathcal{F}_0, \Sigma = (\Omega, T_0, \mathcal{O}) \) are \( T_0 \)-spaces, and suppose \( \mathcal{S}_{\mathcal{F}_0, \Omega} \subset \mathcal{F}_0, \Omega \) be locally \( g\)-\( \mathcal{F}_0 \)-compact in \( \mathcal{F}_0, \Omega \). Since \( \mathcal{S}_{\mathcal{F}_0, \Omega} \) is locally \( g\)-\( \mathcal{F}_0 \)-compact, for any given \( \xi \in \mathcal{S}_{\mathcal{F}_0, \Omega} \subset \mathcal{F}_0, \Omega \times g\mathcal{N}[\xi] \), there is a \( g\)-\( \mathcal{F}_0 \)-neighborhood \( \hat{N}_{\xi, \mathcal{F}_0} \in g\mathcal{N}[\xi] \) of \( \xi \) such that \( \hat{N}_{\xi, \mathcal{F}_0} \subset \mathcal{F}_0, \Omega \) and \( \der g(\hat{N}_{\xi, \mathcal{F}_0}) \subset \mathcal{F}_0, \Sigma \). Consequently, \( \xi \in \hat{N}_{\xi, \mathcal{F}_0} \subset \mathcal{F}_0, \Omega \) and \( \der g(\hat{N}_{\xi, \mathcal{F}_0}) \subset \mathcal{F}_0, \Sigma \). By hypothesis, \( \pi_0 \in g\mathcal{C}[\mathcal{F}_0, \Omega : \mathcal{F}_0, \Sigma] \). Therefore, \( \pi_0(\xi) \subset \pi_0(\hat{N}_{\xi, \mathcal{F}_0}) \subset \pi_0(\hat{N}_{\xi, \mathcal{F}_0} \cup \der g(\hat{N}_{\xi, \mathcal{F}_0})) \subset \pi_0(\hat{N}_{\xi, \mathcal{F}_0}) \). Since \( \hat{N}_{\xi, \mathcal{F}_0} \subset \mathcal{F}_0, \Omega \) is a \( g\)-\( \mathcal{F}_0 \)-neighborhood in \( \mathcal{F}_0, \Omega \) containing \( \xi \in \mathcal{S}_{\mathcal{F}_0, \Omega} \subset \mathcal{F}_0, \Omega \), \( \pi_0(\hat{N}_{\xi, \mathcal{F}_0}) \subset \mathcal{F}_0, \Sigma \) is a \( g\)-\( \mathcal{F}_0 \)-neighborhood in \( \mathcal{F}_0, \Sigma \) containing \( \pi_0(\xi) \subset \pi_0(\mathcal{S}_{\mathcal{F}_0, \Omega}) \subset \mathcal{F}_0, \Sigma \). Hence, \( \pi_0(\mathcal{S}_{\mathcal{F}_0, \Omega}) \subset \mathcal{F}_0, \Sigma \) is locally \( g\)-\( \mathcal{F}_0 \)-compact in \( \mathcal{F}_0, \Sigma \). Q.E.D.
The categorical classifications of $\mathsf{U}$-compactness and $\mathsf{gU}$-compactness in the $\mathcal{T}$-space $\mathcal{U} \subset \mathcal{U}$, and $\mathcal{U}_{\mathfrak{g}}$-compactness and $\mathcal{U}_{\mathfrak{gU}}$-compactness in the $\mathcal{T}_{\mathfrak{g}}$-space $\mathcal{U}_{\mathfrak{g}}$ are discussed and diagrammed on this basis in the next sections.

4. Discussion

4.1. Categorical Classifications. Having adopted a categorical approach in the classification of the $\mathcal{T}_{\mathfrak{g}}$-property, called $\mathcal{U}_{\mathfrak{gU}}$-compactness in the $\mathcal{T}_{\mathfrak{g}}$-space $\mathcal{U}_{\mathfrak{g}}$, the dual purposes of the this section are firstly, to establish the various relationships amongst the elements of the sequences $\langle \mathcal{U}_{\mathfrak{g}^\nu}-\mathcal{U}\rangle = \langle \Omega, \mathcal{U}_{\mathfrak{g}^\nu}\mathcal{T}_{\mathfrak{g}} \rangle$ of implications: $\langle \mathcal{U}_{\mathfrak{g}^\tau}-\mathcal{U}\rangle = \langle \Omega, \mathcal{U}_{\mathfrak{g}^\tau}\mathcal{T}_{\mathfrak{g}} \rangle$, where $\mathcal{E} \in \{A, CA, SA, LA\}$, and secondly, to illustrate them through diagrams.

Let $\mathcal{O}_\mathfrak{g} \subset \mathcal{U}_{\mathfrak{g}}$ be any $\mathfrak{g}$-open set in a $\mathcal{T}_{\mathfrak{g}}$-space $\mathcal{U}_{\mathfrak{g}} = (\Omega, \mathcal{T}_{\mathfrak{g}})$, and, for every $\nu \in \mathcal{I}_{\mathfrak{g}}$, let there exist a $\mu \in \mathcal{I}_{\mathfrak{g}}$ such that the relation $\mu \mathcal{O}_{\mathfrak{g}} \subset \mathcal{O}_{\mathfrak{g}}$ holds. Then, since $\mathcal{O}_{\mathfrak{g}} \subset \mathcal{O}_{\mathfrak{g}}$, it follows that $\mathcal{U}_{\mathfrak{g}}$-openness implies $\mathcal{U}_{\mathfrak{g}}$-openness and the latter, in turn, implies $\mathcal{U}_{\mathfrak{g}}$-openness. But since the statement that $\mathcal{U}_{\mathfrak{g}}$-compactness implies $\mathcal{U}_{\mathfrak{g}}$-compactness is a consequence of the statement that $\mathcal{U}_{\mathfrak{g}}$-openness implies $\mathcal{U}_{\mathfrak{g}}$-openness, it evidently follows that, $\mathcal{U}_{\mathfrak{g}}$-openness of category 0 implies $\mathcal{U}_{\mathfrak{g}}$-openness of category 1, it results that, $\mathcal{U}_{\mathfrak{g}}$-compactness of category 0, from the statement that, $\mathcal{U}_{\mathfrak{g}}$-openness of category 1 implies $\mathcal{U}_{\mathfrak{g}}$-openness of category 3, it results that, $\mathcal{U}_{\mathfrak{g}}$-compactness of category 3 implies $\mathcal{U}_{\mathfrak{g}}$-compactness of category 1; from the statement that, $\mathcal{U}_{\mathfrak{g}}$-openness of category 2 implies $\mathcal{U}_{\mathfrak{g}}$-openness of category 3, it results that, $\mathcal{U}_{\mathfrak{g}}$-compactness of category 3 implies $\mathcal{U}_{\mathfrak{g}}$-compactness of category 1. Thus, if $\mathcal{U}_{\mathfrak{g}} \subset \mathcal{U}_{\mathfrak{g}}$ is a $\mathcal{U}_{\mathfrak{g}}$-open set then, with respect to $\mathcal{U}_{\mathfrak{g}}$-openness, the following system of implications holds:

$$U_{\mathfrak{g}} \in \mathcal{U}_{\mathfrak{g}}^{0-}\mathcal{U}[\mathcal{U}_{\mathfrak{g}}] \implies U_{\mathfrak{g}} \in \mathcal{U}_{\mathfrak{g}}^{1-}\mathcal{U}[\mathcal{U}_{\mathfrak{g}}]$$

$$U_{\mathfrak{g}} \in \mathcal{U}_{\mathfrak{g}}^{2-}\mathcal{U}[\mathcal{U}_{\mathfrak{g}}] \implies U_{\mathfrak{g}} \in \mathcal{U}_{\mathfrak{g}}^{3-}\mathcal{U}[\mathcal{U}_{\mathfrak{g}}].$$

Such system with respect to $\mathcal{U}_{\mathfrak{g}}$-compactness, in turn, implies the following system of implications:

$$S_{\mathfrak{g}} \in \mathcal{U}_{\mathfrak{g}}^{0-}\mathcal{U}[\mathcal{U}_{\mathfrak{g}}] \iff S_{\mathfrak{g}} \in \mathcal{U}_{\mathfrak{g}}^{1-}\mathcal{U}[\mathcal{U}_{\mathfrak{g}}]$$

$$S_{\mathfrak{g}} \in \mathcal{U}_{\mathfrak{g}}^{2-}\mathcal{U}[\mathcal{U}_{\mathfrak{g}}] \iff S_{\mathfrak{g}} \in \mathcal{U}_{\mathfrak{g}}^{3-}\mathcal{U}[\mathcal{U}_{\mathfrak{g}}].$$

For visualization, a so-called categorical compactness diagram, expressing the various relationships amongst the classes of $\mathcal{U}_{\mathfrak{g}}$-compact and $\mathcal{U}_{\mathfrak{gU}}$-compact sets, is presented in FIG. 1. According to the previous section, it is plain that, $\mathcal{U}_{\mathfrak{g}}$-compactness in the ordinary sense implies both countable $\mathcal{U}_{\mathfrak{g}}$-compactness in the
ordinary sense and local countable $\mathcal{T}_g$-compactness in the ordinary sense; sequential $\mathcal{T}_g$-compactness in the ordinary sense implies countable $\mathcal{T}_g$-compactness in the ordinary sense. Moreover, the following implications also hold: $g\mathcal{T}_{g}^{LA} \leftarrow g\mathcal{T}_{g}^{A}$, $g\mathcal{T}_{g}^{CA} \leftarrow g\mathcal{T}_{g}^{A}$, and $g\mathcal{T}_{g}^{CA} \leftarrow g\mathcal{T}_{g}^{SA}$. Since the relation $\mathcal{T}_{g}^{E} \leftarrow g\mathcal{T}_{g}^{E}$ holds for every $E \in \{A, CA, SA, LA\}$, taking this last statement together with those preceding it into account, another compactness diagram is obtained. The diagram presented in Fig. 2 illustrates the various relationships amongst the elements of $\langle g\mathcal{T}_{g}^{[E]} \rangle_{E \in \Lambda}$ and $\langle \mathcal{T}_{g}^{E} \rangle_{E \in \Lambda}$, where $\Lambda = \{A, CA, SA, LA\}$. It is interesting to present a third compactness diagram illustrating both the implications and the categorical classifications of the elements of $\langle g\mathcal{T}_{g}^{[E]} \rangle_{E \in \Lambda}$, where $\nu \in \mathcal{I}^0_3$, and, obviously, $\Lambda = \{A, CA, SA, LA\}$.

For each $\nu \in \mathcal{I}^0_3$, it is immediate that these implications hold: $g\mathcal{T}_{g}^{[\nu]\nu_{\nu}} \leftarrow g\mathcal{T}_{g}^{[\nu]\nu_{\nu_{\nu}}}$, $g\mathcal{T}_{g}^{[\nu]\nu_{\nu_{\nu}} \cap \nu} \leftarrow g\mathcal{T}_{g}^{[\nu]\nu_{\nu_{\nu}} \cap \nu_{\nu}}$, and $g\mathcal{T}_{g}^{[\nu]\nu_{\nu_{\nu}} \cap \nu_{\nu_{\nu}}} \leftarrow g\mathcal{T}_{g}^{[\nu]\nu_{\nu_{\nu}} \cap \nu_{\nu_{\nu}}}$. Furthermore, for each $E \in \Lambda = \{A, CA, SA, LA\}$, it is also plain that these implications hold: $g\mathcal{T}_{g}^{[E]} \leftarrow g\mathcal{T}_{g}^{[E]} \cap \nu$, $g\mathcal{T}_{g}^{[E]} \leftarrow g\mathcal{T}_{g}^{[E]} \cap \nu_{\nu}$, and $g\mathcal{T}_{g}^{[E]} \leftarrow g\mathcal{T}_{g}^{[E]} \cap \nu_{\nu_{\nu}}$. When all these implications are taken into consideration, the resulting compactness diagram so obtained is that presented in Fig. 3. It is reasonably correct to call them $g\mathcal{T}_{g}^{[E]}$-spaces of type $E$ and of category $\nu$, where $(\nu, E) \in \mathcal{I}^0_3 \times \{A, CA, SA, LA\}$. As in the papers of [16], [18], and [34], among others, the manner we have positioned the arrows is solely to stress that, in general, none of the implications in Figs 1, 2 and 3 is reversible.
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In order to exemplify the notion of $\mathfrak{g}$-$\mathcal{T}_0$-spaces of type $E$ and of category $\nu$, where $(\nu, E) \in I_0 \times \{A, CA, SA, LA\}$, a nice application is presented in the following section.

4.2. A NICE APPLICATION. Focusing on basic concepts from the standpoint of the theory of $\mathfrak{g}$-$\mathcal{T}_0$-compactness, we shall now present a nice application comprising of some interesting cases.

Let $\mathcal{T}_0 : \mathcal{P}(\Omega) \rightarrow \mathcal{P}(\Omega)$ be the $\mathfrak{g}$-topology on $\Omega = (0, 1) \subset \mathbb{R}$ (set of real numbers) generated by $\mathcal{T}_0$-open and $\mathcal{T}_0$-closed sets belonging to:

$$\mathcal{T}_0 \overset{\text{def}}{=} \left\{ O_{\mathfrak{g}, \mu} : \forall \mu \in I_\infty \setminus I_2 \left( [O_{\mathfrak{g}, \mu} = \emptyset] \vee \left[ O_{\mathfrak{g}, \mu} = \left( \frac{1}{\mu}, 1 - \frac{1}{\mu} \right) \right] \right) \right\};$$

where $
u > 3$, the relation $\frac{1}{\sigma} \in \bigcup_{\mu \in I_2} O_{\mathfrak{g}, \mu} = \left( \frac{1}{\sigma} - 1 - \frac{1}{\sigma} \right)$.

Hence, from every $\mathfrak{g}$-$\mathcal{T}_0$-open subcovering $\langle O_{\mathfrak{g}, \mu} \rangle_{\alpha \in I_\infty \setminus I_2}$, for every $\mu \in I_\infty \setminus I_2$. On the other hand, for each $\sigma > 3$, the relation $\frac{1}{\sigma} \in \bigcup_{\mu \in I_2} O_{\mathfrak{g}, \mu} = \left( \frac{1}{\sigma} - 1 - \frac{1}{\sigma} \right)$.

Hence, from every $\mathfrak{g}$-$\mathcal{T}_0$-open subcovering $\langle O_{\mathfrak{g}, \mu} \rangle_{\alpha \in I_\infty \setminus I_2}$, for every $\mu \in I_\infty \setminus I_2$. On the other hand, for each $\sigma > 3$, the relation $\frac{1}{\sigma} \in \bigcup_{\mu \in I_2} O_{\mathfrak{g}, \mu} = \left( \frac{1}{\sigma} - 1 - \frac{1}{\sigma} \right)$.
$J_{\theta}^* (\infty) = I_{\theta}^* \setminus I_2^*$, the union $\bigcup_{(\alpha, \delta(\alpha)) \in J_{\delta} \times J_{\theta}^*} \mathcal{O}_{\theta, \delta(\alpha)}$ must fail to contain some point of $\Omega$ and, hence, there exist no finite $g_{-}\mathcal{T}_{\theta}$-open subcovering of $\langle \mathcal{O}_{\theta, \alpha} \rangle_{\alpha \in I_{\theta}^*} \setminus I_2^*$. This proves that $\mathcal{T}_{\theta} = (\mathcal{T}_{\theta}, \Omega)$, where $\Omega = (0, 1)$, is not a $\mathcal{T}_{\theta}[A]$-space. Since $g_{-}\mathcal{T}_{\theta}$-compactness implies $\mathcal{T}_{\theta}$-compactness, it follows, consequently, that it is also not a $g_{-}\mathcal{T}_{\theta}[A]$-space. Finally, from this case, it results that, not every $\mathcal{T}_{\theta}$-set of a $g_{-}\mathcal{T}_{\theta}[A]$-space is itself $g_{-}\mathcal{T}_{\theta}$-compact.

Let $\mathcal{T}_{\theta} : \mathcal{P}(\Omega) \rightarrow \mathcal{P}(\Omega)$ be the $g$-topology on $\Omega = \mathbb{N}$ (set of positive integers) generated by $\mathcal{T}_{\theta}$-open and $\mathcal{T}_{\theta}$-closed sets belonging to:

$$\mathcal{T}_{\theta} \ \overset{\text{def}}{=} \ \{ \mathcal{O}_{g, \{2\mu - 1, 2\mu\}} : (\forall \mu \in I_{\infty}) \left( (\mathcal{O}_{g, \{2\mu - 1, 2\mu\}} = \emptyset) \right.\left.\lor \left(\mathcal{O}_{g, \{2\mu - 1, 2\mu\}} = \{2\mu - 1, 2\mu\} \right) \right) \};$$

$$\neg \mathcal{T}_{\theta} \ \overset{\text{def}}{=} \ \{ \mathcal{K}_{g, \{2\mu - 1, 2\mu\}} : (\forall \mu \in I_{\infty}) \left( \mathcal{K}_{g, \{2\mu - 1, 2\mu\}} = \emptyset \right.\left.\lor \left(\mathcal{K}_{g, \{2\mu - 1, 2\mu\}} = \{2\mu - 1, 2\mu\} \right) \right) \};$$

respectively. As in the above case, it results that $\mathcal{T}_{\theta} : \mathcal{P}(\Omega) \rightarrow \mathcal{P}(\Omega)$ satisfies the relations $\mathcal{T}_{\theta}(\emptyset) = \emptyset$, $\mathcal{T}_{\theta}(\mathcal{O}_{g, \{2\mu - 1, 2\mu\}}) \subseteq \{2\mu - 1, 2\mu\}$ and, the relation $\mathcal{T}_{\theta}(\bigcap_{\mu \in I_{\theta}^*} \mathcal{O}_{g, \{2\mu - 1, 2\mu\}}) = \bigcap_{\mu \in I_{\theta}^*} \mathcal{T}_{\theta}(\mathcal{O}_{g, \{2\mu - 1, 2\mu\}})$ as well as the relation $\mathcal{T}_{\theta}(\bigcup_{\mu \in I_{\theta}^*} \mathcal{O}_{g, \{2\mu - 1, 2\mu\}}) = \bigcup_{\mu \in I_{\theta}^*} \mathcal{T}_{\theta}(\mathcal{O}_{g, \{2\mu - 1, 2\mu\}})$, since the two relations $\bigcap_{\mu \in I_{\theta}^*} \mathcal{O}_{g, \{2\mu - 1, 2\mu\}} = \emptyset \in \mathcal{T}_{\theta}$ and $\bigcup_{\mu \in I_{\theta}^*} \mathcal{O}_{g, \{2\mu - 1, 2\mu\}} = \Omega \in \mathcal{T}_{\theta}$, respectively, hold. Therefore, $\mathcal{T}_{\theta} = (\mathcal{T}_{\theta}, \Omega)$ is a $\mathcal{T}_{\theta}$-space and, moreover, since the relation $\mathcal{T}_{\theta} = (\mathcal{T}_{\theta}, \Omega) = (\mathcal{S}, \mathcal{T})$ holds, it is also a $\mathcal{T}$-space. Notice that $\langle \mathcal{O}_{g, \{2\alpha - 1, 2\alpha\}} \rangle_{\alpha \in I_{\infty}^*}$ is a $\mathcal{T}_{\theta}$-open covering of $\Omega$, since $\mathcal{O}_{g, \{2\alpha - 1, 2\alpha\}} \in O[\mathcal{T}_{\theta}]$ for every $\alpha \in I_{\infty}^*$ and furthermore, it is also a $g_{-}\mathcal{T}_{\theta}$-open covering of $\Omega$, since $\mathcal{O}_{g, \{2\alpha - 1, 2\alpha\}} \subseteq \op_{g}[\mathcal{O}_{g, \{2\alpha - 1, 2\alpha\}}] \in g-O[\mathcal{T}_{\theta}]$ for every $\alpha \in I_{\infty}^*$. However, $\mathcal{T}_{\theta} = (\mathcal{T}_{\theta}, \Omega)$, where $\Omega = \mathbb{N}$, is not a $\mathcal{T}_{\theta}[A]$-space because $\langle \mathcal{O}_{g, \{2\alpha - 1, 2\alpha\}} \rangle_{\alpha \in I_{\infty}^*}$ is a $\mathcal{T}_{\theta}$-open covering of $\Omega$ with no finite $\mathcal{T}_{\theta}$-open subcovering.

As stated above, since $g_{-}\mathcal{T}_{\theta}$-compactness implies $\mathcal{T}_{\theta}$-compactness, it follows, obviously, that it is also not a $g_{-}\mathcal{T}_{\theta}[A]$-space. On the other hand, the $\mathcal{T}_{\theta} = (\mathcal{T}_{\theta}, \Omega)$, where $\Omega = \mathbb{N}$, is also not a sequentially $g_{-}\mathcal{T}_{\theta}$-compact $\mathcal{T}_{\theta}$-space for the simple reason that sequence $\langle \xi_{\alpha} = \alpha \in \Omega \rangle_{\alpha \in I_{\infty}^*}$ in $\mathcal{T}_{\theta}$ contains no subsequence of the type $\langle \xi_{\delta(\alpha)} \rangle_{\alpha, \delta(\alpha) \in I_{\infty}^* \times I_{\infty}^*} \langle \xi_{\alpha} \rangle_{\alpha \in I_{\infty}^*}$ which converges to a point $\xi \in \Omega$. Hence, $\mathcal{T}_{\theta}$ is not a $g_{-}\mathcal{T}_{\theta}[A]$-space which, then, implies that it is also not a $\mathcal{T}_{\theta}[A]$-space.

Let $\mathcal{S}_{\theta} \subset \mathcal{T}_{\theta}$ be a non-empty $\mathcal{T}_{\theta}$-set in $\mathcal{T}_{\theta}$. Then, it is no error to express it as $\mathcal{S}_{\theta} = \mathcal{S}_{\theta}^{even} \cup \mathcal{S}_{\theta}^{odd}$, where $\mathcal{S}_{\theta}^{even} = \{ \mu : (\forall \alpha \in I_{\infty}^*) [\mu = 2\alpha] \}$ and $\mathcal{S}_{\theta}^{odd} = \{ \mu : (\forall \alpha \in I_{\infty}^*) [\mu = 2\alpha - 1] \}$. Since $\mathcal{S}_{\theta} \neq \emptyset$, consider an arbitrary point $\xi \in \mathcal{S}_{\theta}$. If $\xi \in \mathcal{S}_{\theta}^{even}$ then, for every $\mathcal{T}_{\theta}$-open set $U_{g, \xi} \in O[\mathcal{T}_{\theta}]$ containing $\xi$, $\mathcal{S}_{\theta}^{even} \cap (U_{g, \xi} \setminus \{\xi\}) = \emptyset$ and $\mathcal{S}_{\theta}^{odd} \cap (U_{g, \xi} \setminus \{\xi\}) \neq \emptyset$. But, if $\xi \in \mathcal{S}_{\theta}^{odd}$ then, for every $\mathcal{T}_{\theta}$-open set $U_{g, \xi} \in O[\mathcal{T}_{\theta}]$ containing $\xi$, $\mathcal{S}_{\theta}^{even} \cap (U_{g, \xi} \setminus \{\xi\}) \neq \emptyset$ and $\mathcal{S}_{\theta}^{odd} \cap (U_{g, \xi} \setminus \{\xi\}) = \emptyset$. In either case, it follows, then, that $\mathcal{S}_{\theta}$ have at least one $\mathcal{T}_{\theta}$-accumulation point. Accordingly, $\mathcal{T}_{\theta}$ is a $\mathcal{T}_{\theta}[CA]$-space. For every $\alpha \in I_{\infty}^*$, set $U_{g, 2\alpha - 1} = \{2\alpha - 1\}$ and $U_{g, 2\alpha} = \{2\alpha\}$. Accordingly, $U_{g, 2\alpha - 1}, U_{g, 2\alpha} \in g-O[\mathcal{T}_{\theta}]$ since, $U_{g, 2\alpha - 1}, U_{g, 2\alpha} \subseteq \op_{g}[\mathcal{O}_{g, \{2\alpha - 1, 2\alpha\}}] \in g-O[\mathcal{T}_{\theta}]$ for every $\alpha \in I_{\infty}^*$. Observe that, $\mathcal{S}_{\theta} \cap (U_{g, 2\alpha - 1} \setminus \{2\alpha - 1\}) = \emptyset$ and $\mathcal{S}_{\theta} \cap (U_{g, 2\alpha} \setminus \{2\alpha\}) = \emptyset$ for every $\alpha \in I_{\infty}^*$. This proves the existence of an infinite
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$g^{-}$-set $R_{\theta} \subset \mathcal{T}_\theta$ with no $g^{-}[\mathcal{CA}]-$accumulation point and, hence, $\mathcal{T}_\theta$ is not a $g^{-}[\mathcal{CA}]-space$.

Further $\mathcal{T}_\theta$-properties amongst the $g^{-}[\mathcal{A}]-spaces$ $g^{-}[\mathcal{A}]= (\Omega, g^{-}[\mathcal{A})$, $g^{-}[\mathcal{CA}]= (\Omega, g^{-}[\mathcal{CA})$, $g^{-}[\mathcal{SA}]= (\Omega, g^{-}[\mathcal{SA})$, and $g^{-}[\mathcal{LA}]= (\Omega, g^{-}[\mathcal{LA})$ called, respectively, $g^{-}[\mathcal{A}]-$space, countably $g^{-}[\mathcal{A}]-$space, sequentially $g^{-}[\mathcal{A}]-$space, and locally $g^{-}[\mathcal{A}]-space$, can be discussed in a similar way by slight modifications of some $\mathcal{T}_\theta$-properties found in those cases. In the following section, concluding remarks and future directions of the theory of $g^{-}$-$\mathcal{T}_\theta$-compactness are presented.

4.3. Concluding Remarks. In this paper, a new theory called Theory of $g^{-}$-$\mathcal{T}_\theta$-Compactness has been presented, the foundation of which was based on the theories of $g^{-}$-$\mathcal{T}_\theta$-sets, $g^{-}$-$\mathcal{T}_\theta$-maps and $g^{-}$-$\mathcal{T}_\theta$-separation axioms. A careful perusal of the Mathematical developments of the earlier sections will reveal that the proposed theory has, in its own rights, several advantages. The very first advantage is that the theory holds equally well when $(\Omega, \mathcal{T}_\theta)= (\Omega, \mathcal{T})$, and other characteristics adapted on this ground, in which case it might be called Theory of $g^{-}$-$\mathcal{T}_\theta$-Connectedness.

Thus, in a $\mathcal{T}_\theta$-space the theoretical framework categorises such statements as $g^{-}$-$\mathcal{T}_\theta$-compactness in terms of relatively open $\mathcal{T}_\theta$-sets, $g^{-}$-$\mathcal{T}_\theta$-compactness in terms of relatively semi-open $\mathcal{T}_\theta$-sets, $g^{-}$-$\mathcal{T}_\theta$-compactness in terms of relatively preopen $\mathcal{T}_\theta$-sets, and $g^{-}$-$\mathcal{T}_\theta$-compactness in terms of relatively semi-preopen $\mathcal{T}_\theta$-sets as $g^{-}$-$\mathcal{T}_\theta$-compactness of categories 0, 1, 2, and 3, respectively, and theorises the concepts in a unified way; in a $\mathcal{T}$-space it categorises such statements as $g^{-}$-$\mathcal{T}$-compactness in terms of relatively open $\mathcal{T}$-sets, $g^{-}$-$\mathcal{T}$-compactness in terms of relatively semi-open $\mathcal{T}$-sets, $g^{-}$-$\mathcal{T}$-compactness in terms of relatively preopen $\mathcal{T}$-sets, and $g^{-}$-$\mathcal{T}$-compactness in terms of relatively semi-preopen $\mathcal{T}$-sets as $g^{-}$-$\mathcal{T}$-compactness of categories 0, 1, 2, and 3, respectively, and theorises the concepts in a unified way.

It is an interesting topic for future research to develop the theory of $g^{-}$-$\mathcal{T}_\theta$-sets of mixed categories. More precisely, for some pair $(\nu, \mu) \in I_0 \times I_0$ such that $\nu \neq \mu$, to develop the theory of $g^{-}$-$\mathcal{T}_\theta$-compactness in terms of relatively $g^{-}$-$\mathcal{T}_\theta$-open sets belonging to the class $\{U_\theta = U_{\nu, \theta} \cup U_{\mu, \theta} : (U_{\nu, \theta}, U_{\mu, \theta}) \in g^{-}[\mathcal{O}(\mathcal{T}_\theta) \times g^{-}[\mathcal{O}(\mathcal{T}_\theta)]\}$ in a $\mathcal{T}_\theta$-space $\mathcal{T}_\theta$, as [1] and [9] developed the theory of $b$-open and $b$-closed sets in a $\mathcal{T}$-space $\mathcal{T}$. Such a theory is what we thought would certainly be worth considering, and the discussion of this paper ends here.

References

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