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# Dynamic Urban Growth Model: Phases of Urban Development*

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## Abstract

This lecture aims to survey the existing literature on the dynamic urban growth. The application in this lecture is a small step in the long iterative process between the construction of a model and its use for practical purposes. In this lecture, we follow the notion of urban development and conduct an analysis of conceptual modeling phases of urban development by Paeliuck (1970).
**Keywords:** urban growth, urban development

## 1 Introduction

Urban models have a long history. During the last few decades, the development of urban models has grown rapidly. The variety of models makes the classifications rather diverse. A city is considered as a complex system. It consists of numerous interactive sub-systems and is affected by diverse factors including governmental land policies, population growth, transportation infrastructure, and market behavior. Land use and transportation systems are considered as the two most important subsystems determining urban form and structure in the long term. Urban growth is one of the most important topics in urban studies, and its main driving forces are population growth and transportation development. Modeling and simulation are believed to be powerful tools to explore the mechanisms of urban evolution and provide planning support in growth management.

Urban growth benefits human civilization in terms of society, economy and culture. Nowadays, cities control the world's economy and political decision power, manage the flows of financial, man-made and natural resources, and concentrate the knowledge of science, art and technology. However, rapid urban growth/sprawl also causes social, physical and environmental problems. Nuissl et al. (2009) pointed out that urbanization leads to an unnatural or spoiled landscape in spatial planning. Agricultural land or forest at the urban fringe is converted to residential and industrial areas. Such conversion weakens ecosystem services and landscape functions in different ways (De Groot et al.. 2002, Curran and de Sherbinin 2004). The increasing use of cars and increased energy consumption cause concentrate pollution, which both damages people's health and the



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natural environment. Urban sprawl in suburban requires more costs in terms of travel, and increases the inefficient investment of infrastructure and services (i.e. road, water supply, electricity, sewerage and drainage) in low density urban areas (Batty, 2008). In developing regions, rapid urban growth has led to enormous environmental loads and a growing problem of poverty.

Population growth is one of the most important driving forces of change in any urban system. If urban population swells, the city must expand upward or outward. Along with economic development and technologies (mainly transport and communication) revolution, rapid urban growth can be characterized by the development of suburban expansion and redevelopment in the city centre. Sprawl is a form of urban growth that happens through rapid suburbanization, especially in North America, with a fragmented form and low density in development (Gordon and Richardson 1997, Duany et al. 2001, Carruthers 2003).

## 2 Model

As an essential way to learn the urban growth phenomena, modeling and simulation is regarded as a efficient way to understand the mechanisms of urban dynamics, to evaluate current urban systems, and to provide planning support in urban growth management, e.g. land-use models may help to build future growth scenarios and to assess possible environmental impacts (Lambin et al., 2006).

Modeling can either be conceptual, symbolic or mathematical. This depends on the purposes of the specific application. Before carrying on the modeling, one has to figure out the driving forces behind urban growth phenomenon. It is well known that a city is a complex system. It contains various interactive subsystems and is thus affected by a variety of variables or factors. Bürgi et al. (2004) distinguished between five major types of driving forces: socioeconomic, political, technological, natural and cultural factors. They also classified these into the primary, secondary and tertiary driving forces, although many times it is not easy to differentiate between impacts and driving factors in reality (Verburg, 2006).

According to Miller et al. (2004), an integrated urban system model with a focus on transportation should include socio-demographic components, demographics, location choices of households and firms, economic variables, transportation and effects on land use and environment.

Acording to Paeliuck urban growth can be approached from three viewpoints: **empirical analysis of observed data (either through time, or cross-sectionally), theoretical analysis of mathematical models, and simulation by means of empirical model.**

A final synthesis would yield a full-fledged econometric analysis. Here the second approach is applied in order to discover certain properties of moderately disaggregated models of urban growth. Further disaggregation is, at least in principle, a straightforward matter. Attention will be confined to population growth and employment variables: other aspects of urban growth (production, value-added, wages, prices) will be left for further investigation.

The approach used here can be considered as lying somewhere between economic base analysis and input-output models. It differs from most models in that it is essentially dynamic.

Policy conclusions cannot be drawn at this stage but certain elementary reflections will appear in the conclusions of the paper.

For our purposes, we will distinguish the following variables?

$P$: total population;

$A$: active population;

$I$: inactive population;
$E_i$: population–induced employment;
$E_s$,: employment serving local private needs;
$E_a$: employment induced by association;
$E_e$: employment in the building sector;
$E_u$: employment in local public works and administration.

$$P_t = A_t + I_t \tag{1}$$

$$A_t = \alpha P_t + \gamma \tag{2}$$

$$E_{i,t+1} = a_{10} A_t + b_{10} \tag{3}$$

$$E_{i,t+1} = a_{10} A_t + b_{10} \tag{4}$$

Equations (2) through (4) can be solved for $P_t$ giving:

$$P = \left(\frac{\alpha a_{10}}{\alpha a_1}\right)^t P_0^* + \frac{\gamma(a_{10} - 1) + b_{10} + b_1}{\alpha(1 - a_{10}) - a_1} = a^{*t} P_0^* + b^* \tag{5}$$

The rate of growth, $r$, is equal to:

$$r = \frac{a^* - 1}{1 + \left(\frac{b^*}{1-a^*}\right) P_0^{*-1} a^{*1-t}} \tag{6}$$

## 2.1 Variable $E_e$

This variable probably enters into the model at an early stage of urban development since it is related to population changes (residential building) and changes in total employment (industrial construction). A supplementary equation is adequate for variable $E_e$.

$$E_{e,t+1} = \zeta \Delta' P_{t+1} + \theta \Delta' A_{t+1} + \eta = \zeta P_{t+1} - \zeta P_t + \theta A_{t+1} - \theta A_t + \eta. \tag{7}$$

A one-period lag is subsumed. The activities are interlinked in a complicated manner and the formal solution is

$$P_t = \left(\frac{\alpha(a_{10} - \theta) - \zeta}{\alpha(1 - \theta) - a_1 - \zeta}\right)^t P_0^* + \frac{\gamma(a_{10} - 1) = B_{10} + b_1 + \eta}{\alpha(1 - a_{10}) - a_1} \tag{8}$$

If the asymptotic growth factor, $1 + r$, of Equation (5) exceeds unity, the effect of introducing the building accelerator is, quite naturally, to increase the rate of growth of the urban economy.

## 2.2 Activities $E_a$, Input-Output and Female Labor

Initially a lag of one year is assumed in the reaction of $E_a$ to $E_i$

$$E_{a,t+1} = a_{21} E_{i,t} + b_{21} \tag{9}$$

and then an input-output link between $E_a$ and $E_i$ is added, giving

$$E_{a,t+1} = a_{21} E_{i,t} + a_{22} E_{i,t} + b_{21} \tag{10}$$

A diagram of the system would look almost as complicated as the urban economy itself.

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The solution of system of Equations (2) through (10), omiting Equations (5) and (8), is a second order linear non-homogeneous difference equation with constant coefficients. Removing Equation (7), the reduced system is solved as

$$P_{t+2}a^*P_{t-1} - b^*P_t = c^* \tag{11}$$

where

$$a^* = \frac{\alpha a_{10}(1+a_{22})}{\alpha - a_1} > 0 \tag{12}$$

and

$$b^* = \frac{a\alpha_{10}a_{21}}{\alpha - a_1} > 0 \tag{13}$$

giving an explicit solution

$$P_t = \alpha^*(1+r^*) + \alpha^{**}(1+r^{**})^t + c^{**} \tag{14}$$

with no cycles, given the sings of $a^*$ and $b^*$. The term $1+r^*$ and $1+r^{**}$ are conjugate roots of a quadratic, with one of the roots dominating asymptotically.

The availability of (cheap) female labor can be introduced by a slight modification of Equations (2) and (3):

$$A_t = \alpha P_t + \gamma$$

$$A_t = \beta P_t + \nu$$

$$E_{i,t+1} = a_{10}^{*}A_t + a_{10}^{**}I_t + b_{10}^{*} \tag{15}$$

A quick check, wich is not shown here, verified the fact that Equation (3) meraly affects positively the rate of growth of the urban system, and this is *a priori* evident.

### 2.3 Variable $E_u$ and External (Dis)economies

Urban facilities (administration, public works) and external economies could be added by the following equations:

$$E_{u,t+1} = d_{10}P_t + c_{10} \tag{16}$$

$$E_{i,t+1} = a_{10}^{0}(A_t - E_{a,t} - E_{u,t}) + a_{10}^{00}E_{a,t} + a_{01}^{000}E_{\nu,t} + a_{10}^{0000}I_t + b_{10}$$

This would probably simply increase the rate of growth of the system but this question has not been pursued. External diseconomies[1] are investigated by means of the following simple model.

$$A_t = \alpha P_t + \gamma$$

[1] *External diseconomies of scale are the result of outside factors beyond the control of a company increasing its total costs, as output in the rest of the industry increases. The increase in costs can be associated with market prices increasing for some or all of the factors of production.*

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### 2.4 The General Case

This case is presented as a problem within systems of difference equations. The system can be written as

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -\gamma & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_{t+1} \\ A_{t+1} \\ E_{t+1} \\ E_{i,t+1} \\ E_{s,t+1} \end{bmatrix} = \begin{bmatrix} 1\nu & -\nu & \nu & 0 & 0 \\ -\gamma & 1-\gamma & 0\ \gamma & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & -\alpha & 1+\alpha & 0 \\ \beta\beta & 0 & 0 & 0 & 1-\beta+\beta^{**} \end{bmatrix} \begin{bmatrix} P_t \\ A_t 1 \\ E_t \\ E_{i,t} \\ E_{s,t} \end{bmatrix}$$

or alternatively as

$$\begin{bmatrix} P_{t+1} \\ A_{t+1} \\ E_{t+1} \\ E_{i,t+1} \\ E_{s,t+1} \end{bmatrix} = \begin{bmatrix} 1+\nu^* & -\nu & \nu & 0 & 0 \\ \gamma\nu^* & 1-\gamma\nu-\gamma^* & \gamma\nu+\gamma^* & 0 & 0 \\ \beta\beta^* & \alpha & -\alpha & 1+\alpha^* & 1+\beta^{**}-\beta \\ 0 & \alpha & -\alpha & 1+\alpha^* & 0 \\ \beta\beta^* & 0 & 0 & 0 & 1-\beta+\beta^{**} \end{bmatrix} \begin{bmatrix} P_t \\ A_t 1 \\ E_t \\ E_{i,t} \\ E_{s,t} \end{bmatrix} \quad (18)$$

wich can be condensed into

$$Y_{t+1} = AY_t \quad (19)$$

The system has the solution

$$Y_t = A^t Y_0 \quad (20)$$

## 3 Phases of Urban Development

Investigation of simple dynamic urban processes has led to certain insights into the sensitivity of the urban growth process. Probably more so than a national expansion path, the urban development function is subject to built-in cyclical processes that can easily be triggered off, due to the interplay of certain strategic parameters. It is probably due to the lack of time-series data that these facts have not received as much attention as has classical business cycle behavior. However, the type of lags assumed in the model and their duration are not easily ascertained. It seems that there exists here a new field for research in urban economics that could lead to a different view of available data and different answers to current urban problems. It seems reasonable to simulate, urban development under different assumptions regarding the forces at work, and distinguishing the different phases of urban growth.

These phases have been more or less indicated already by the successive models developed above. Figure 1 indicates four possible phases. *Phase I* would start with population-induced industries, service-industries, and construction. *Phase II* would be the associated-industries era, together with input-output and complementary-labor attraction. External economies and urban services would characterize *Phase III* while Phase IV would be the policy-phase, with urban services offsetting external diseconomies. It would he desirable to reproduce plausible and better-observed patterns of population growth. Moreover, the equations of the model might even provide a breakdown by types of activity.

One could check whether this breakdown corresponds to given working hypotheses and observed data. The evolutions sketched have been expressed in terms of a very simplistic variable, urban population. Other variables which should be considered include

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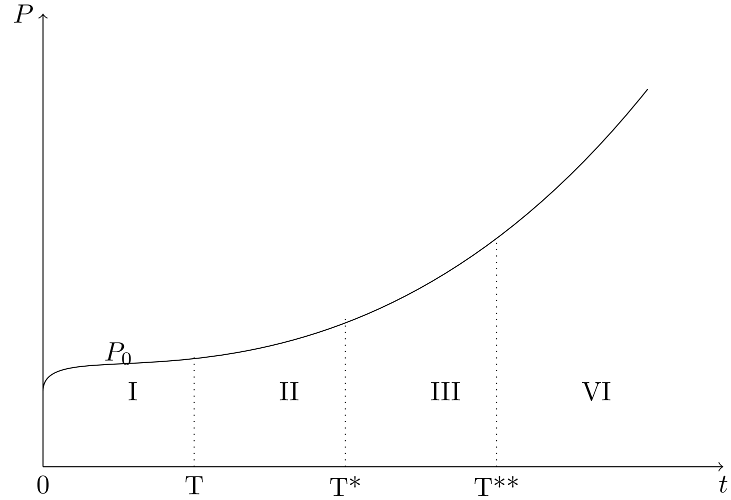


**Fig 1.** Phases of Urban Development. Source: Paeliuck (1970).

production, value-added, prices, wages. Respective elasticities of demand and supply should be considered and an attempt should be made to link urban development to national expansion patterns. This raises the question of the spatial element in the model which has been only partially considered. Demographic "supply" within an area was implicitly assumed to be infinitely elastic. Certain spatial aspects were introduced only intuitively, insofar as annexations occur with urban population growth, and insofar as certain activities that were distinguished locate in urban areas according to fairly well known patterns. Policy variables other than $E$, should be introduced, and special attention should be devoted in a dynamic setting to the correct timing of these measures. This is all the more necessary since we definitely appear to have arrived at *Phase IV*. A simple example will make this clear.

Assume a sub-model composed only of the following equations:

$$A_t = \alpha P_t + \gamma$$

$$E_{i,t\,+1} = a^*_{10} E_{u,t} - a^* *_{1\;0} P_t + b^*_{10}$$

$$E_{u,t+j} = d_{10} P_{t+j} + e_{10} \quad \text{for} \quad j = -1, 0, +1.$$

Solutions to this model are:

$$P_{t+2} - \frac{d_{10} - a^{**}_{10}}{\alpha} P_{t+1} - \frac{a^*_{10} d_{10}}{\alpha} P_t = C_1 \quad for \quad j = -1$$

$$P_{t+1} - \frac{(a^*_{10} d_{10} - a^{**}_{10})}{\alpha - d_{10}} P_t = C_2 \quad for \quad j = 0$$

$$P_{t+2} - \frac{d_{10} - a^*_{10} d_{10}}{d_{10}} P_{t+1} - \frac{a^{**}_{10}}{d_{10}} P_t = C_3 \quad for \quad j = 1$$

For $j = -1$, the insufficiency of $d_{10}$ could introduce alternation in development. For $j = 0$, the effect of $d_{10}$ on the rate of growth is straightforward, and the model selects a steady growth path. For $j = 1$, the effect of increasing $d_{10}$ might do more harm than good. It is therefore obvious that optimal urban policy remains a distant objective, and one which requires further preliminary dynamic analysis.

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