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On the Dynamics of Rotationally Supported Galaxies

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Abstract

A recent finding, based on empirical data of 153 rotationally supported galaxies, with very different morphologies, masses, sizes, and gas fractions, revealed that the baryonic and the dark matter in galaxies are strongly coupled, such that, if the first is known, the second follows and vice versa. Here, we propose a completely theoretical analysis of the dynamics of rotationally supported galaxies, which results in the same conclusion. We find that the relationship between baryonic and dark matter densities at any radius r is governed by the law, $\rho(r)_M + \rho(r)_{DM} = \rho_0$, where $\rho(r)_M$, and $\rho(r)_{DM}$ are, respectively, the densities of matter and dark matter at radius r, and ρ_0 is the density at the galaxy's center. Strikingly, we also found that the radius r_s, at which the rotation velocity is equal to half of its maximal value, constitutes a vivid signature of the galaxy, in the sense that it reveals rich information about the galaxy's dynamics, including the distribution of its matter and dark matter and their total amounts in the galaxy.

Keywords: Dark matter; galaxies; galaxy rotation curve; radial acceleration relation.

1. Introduction

It is well-established observationally that a galaxy's rotation curves typically rise from the galactic center in a small core and then bend down to reach an approximately constant value, extending to the galactic periphery [1] [2] [3] [4]. The flattening of a galaxy's rotation curves is compelling evidence for the existence of large "missing" masses, conjectured to be the masses of dark matter halos (e.g., [5] [6] [7] [8] [9]). Recently, it was found, based only on data of 153 rotationally supported galaxies, with very different morphologies, masses, sizes, and gas fractions, that "when g_{bar} (the rotational acceleration due to baryons) is observed, g_{obs} (the observed rotational acceleration) follows, and vice-versa" [10]. The discovered *radial acceleration relation* is described as "tantamount to a natural law for rotating galaxies" [10]. In the framework of the dark matter paradigm, this discovery implies that the dark matter contribution to a galaxy's mass is fully specified by that of the baryons and vice versa. The discovered law poses serious difficulty for the standard cosmological ΛCDM paradigm because simulations of galaxy formation based on this model, which assume dark matter halos in which galaxies form, do not naturally lead to realistic galaxies [10] [11] [12]. It also points to the possibility of a "new 'dark sector' physics," in which "the dark matter needs to respond to the distribution of baryons (or vice versa) in order to give the observed relation" ([10], p. 5).

Here, we respond to this challenge by demonstrating that an axiom-free and parameter-free relativizing of Newtonian dynamics, which gives rise naturally to a dark matter component, yields results that are compatible with the radial acceleration law. Assuming circular symmetry, the derived relationship prescribes that the sum of baryonic matter and dark matter densities at any distance *r* from a galaxy's center is equal to the Newtonian matter density ρ_0. Moreover, quite surprisingly, we find that a single measurement extracted from a galaxy's rotation curve, namely, the radius at which the rotation velocity equals its maximal (flattening) value or, alternatively, the galaxy's core radius) is sufficient for constructing the distributions of matter and dark matter and of predicting their total amounts in a galaxy.

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2. The Model

Our model is based on a recently proposed relativistic modification of Newtonian mechanics, accomplished only by accounting for the fact that any information carrier, including light waves, takes time to travel between two points in the 3D space [13]. Incorporating this fact into Newton's dynamics "relativizes" classical physics, without adding any axioms or free parameters. The rationale behind our correction of classical physics is simple and straightforward. If point B moves *away from* point A, then the delay of a light signal traveling from B to A becomes continuously longer and longer. In contrast, if B moves *toward* A, then the time delay of the signal's arrival from B to A becomes continuously shorter and shorter. In [13], we utilized the aforementioned modification to make inferences about the intergalactic dark matter, which is ill understood as some kind of negative vacuum "dark energy." Here, we extend the model to the case of a homogeneous, purely gravitational rotating disk, although similar extensions for spheroidal and elliptical rotating bodies are also tractable. For the discussed case, the derived densities of matter and dark matter as functions of the rotation velocity (see Section A of the Supporting Information) are given by

$$\frac{\rho(r)_M}{\rho_0} = \frac{1-\beta(r)}{1+\beta(r)}, \tag{1}$$

$$\frac{\rho(r)_{DM}}{\rho_0} = \frac{2\,\beta(r)}{1+\beta(r)}, \tag{2}$$

where r is the radius, $\beta(r) = \frac{v(r)}{c}$, and ρ_0 is the Newtonian density, which equals the baryonic density at the galaxy's center at which $\beta(0) = 0$. Note that

$$\rho(r)_M + \rho(r)_{DM} = \rho_0. \tag{3}$$

The theoretical relationship in Eq. (3) is concordant with the empirical radial acceleration law. Not only does it support the law, it also specifies the exact dependence of the density of baryonic matter in a galaxy on its dark matter and vice versa.

The energy densities (see Section A of the Supporting Information) are given by

$$\frac{e_M(r)}{e_0} = \frac{1-\beta(r)}{1+\beta(r)}\,\beta^2(r), \tag{4}$$

$$\frac{e_{DM}(r)}{e_0} = \frac{2\,\beta^3(r)}{1+\beta(r)}, \tag{5}$$

where e_0 is the Newtonian energy density: $e_0 = \frac{1}{2}\rho_0 c^2$. The total energy density at a given radius r is equal to:

$$e_{tot}(r) = e_M(r) + e_{DM}(r) = e_0\,\beta^2(r) = \frac{1}{2}\rho_0 c^2\,\beta^2(r) = \frac{1}{2}\rho_0\,v^2(r), \tag{6}$$

which, if solved for $v(r)$, yields

$$v(r) = \sqrt{\frac{2}{\rho_0} e_{tot}(r)}\ . \tag{7}$$

A useful method for investigating a galaxy's structure is to approximate the measured rotation curves by idealized mathematical curves. One idealization, proposed in [14], is the exponential function

$$\frac{v(r)}{V_{max}} = (1 - e^{-\frac{r}{r_S}}), \tag{8}$$

where r is the distance from the galaxy's center, V_{max} is the maximum orbital velocity, and r_s is a scale radius. The function in Eq. (8) resembles measured rotation curves. It rises at short distances and flattens at large distances. A larger value of r_s leads to a more gradual rise of the rotation velocity and vice versa. Despite the appeal of the above exponential function and its flexibility for approximating observed rotation curves, the scale radius r_s, which determines the curve steepness, does not represent a meaningful physical parameter. As example, $v(r_s) \approx 0.63212055$, which we have no simple physical

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interpretation for; the same applies to the functions derivatives with respects to r. Here, we propose another idealization of measured rotation curves, which shares similar features with the function in Eq. (8) but has a nice physical symmetry. The alternative function we propose is the power function

$$\frac{v(r)}{V_{max}} = (1 - 2^{-\frac{r}{r_s}}). \tag{9}$$

A nice symmetry of the proposed idealization is

$$v(r = r_s) = \frac{V_{max}}{2}. \tag{10}$$

The promise of the beauty of Eq. (10) in its utilization in the proposed model will be evidenced hereafter. For simplicity, we normalize the rotation curve by assuming $V_{max} = 1$. Substituting the normalized velocity term from Eq. (9) in Eqs. (1), (2), (4), and (5) yields

$$\frac{\rho_M}{\rho_0}(\tilde{r}) = \frac{1 - (1 - 2^{-\tilde{r}})}{1 + (1 - 2^{-\tilde{r}})}, \tag{11}$$

$$\frac{\rho_{DM}}{\rho_0}(\tilde{r}) = \frac{2\ (1 - 2^{-\tilde{r}})}{1 + (1 - 2^{-\tilde{r}})}, \tag{12}$$

$$\frac{e_M}{e_0}(\tilde{r}) = \frac{1 - (1 - 2^{-\tilde{r}})}{1 + (1 - 2^{-\tilde{r}})}\ (1 - 2^{-\tilde{r}})^2, \tag{13}$$

$$\frac{e_{DM}}{e_0}(\tilde{r}) = \frac{2\ (1 - 2^{-\tilde{r}})^3}{1 + (1 - 2^{-\tilde{r}})}, \tag{14}$$

where $\tilde{r} = \frac{r}{r_s}$. Figure 1 depicts the relative densities of matter and dark matter, as well as the normalized velocity curve, as functions of the normalized radius $\tilde{r}$. The normalized matter and dark matter energy densities are depicted in Figure 2.

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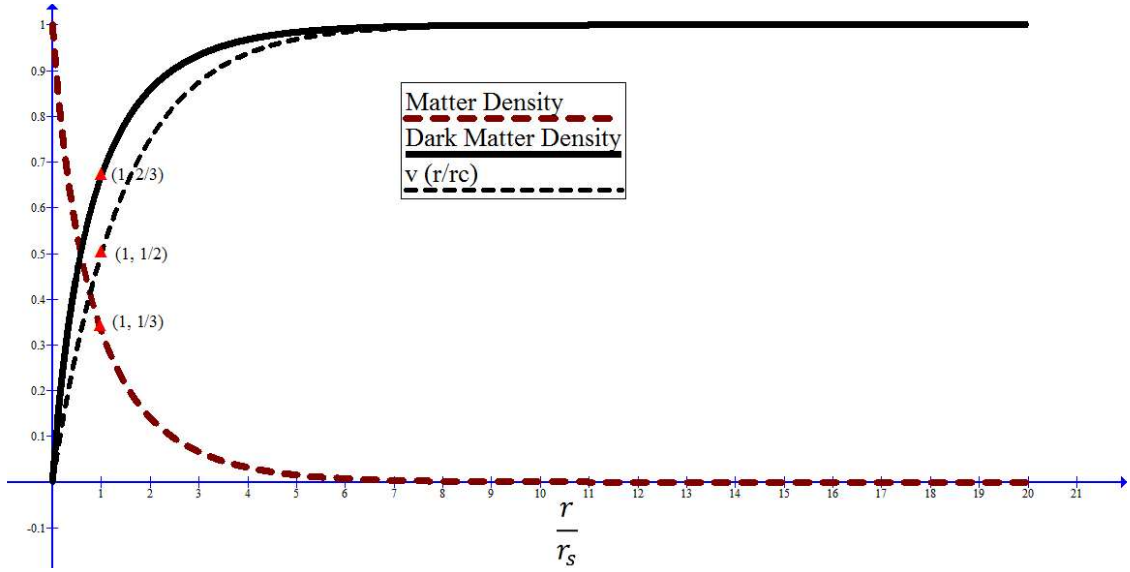

Figure 1. Normalized velocity, matter, and dark matter densities as a function of $\frac{r}{r_s}$.

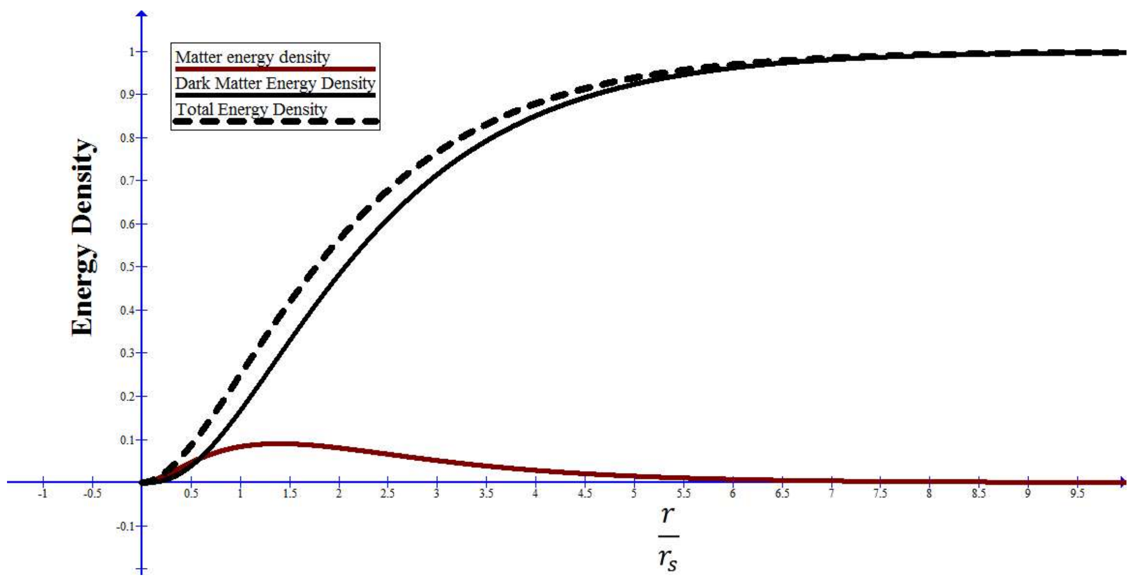

Figure 2. Matter and dark matter energy densities as functions of $\frac{r}{r_s}$.

Equations (12) to (14) (and Figures 1 and 2) have some interesting symmetries. For $r = r_s$, the relative matter and dark matter densities are given, respectively, by

$$\frac{\rho_M}{\rho_0}(r = r_s) = \frac{1-(1-2^{-1})}{1+(1-2^{-1})} = \frac{1-(1-\frac{1}{2})}{1+(1-\frac{1}{2})} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}, \tag{15}$$

and

$$\frac{\rho_{DM}}{\rho_0}(r = r_s) = \frac{2\,(1-2^{-1})}{1+(1-2^{-1})} = \frac{2\,.\,\frac{1}{2}}{\frac{3}{2}} = \frac{2}{3}. \tag{16}$$

The total amount of matter within radius r is given by

$$\mathrm{M}\,(\leq r) = \rho_0 \int_0^r \frac{1-(1-2^{-\frac{r}{r_s}})}{1+(1-2^{-\frac{r}{r_s}})}\, dr = \rho_0\; r_s \int_0^{\tilde{r}} \frac{1-(1-2^{-\tilde{r}})}{1+(1-2^{-\tilde{r}})}\, d\tilde{r}$$

$$= \rho_0\; r_s \frac{ln\left|\, 2-2^{-\frac{r}{r_s}}\right|}{ln}, \tag{17}$$

which, for $r \to \infty$, yields a total amount of matter equaling

$$M_M = \lim_{r\to\infty} \mathrm{M}\,(\leq r) = \rho_0\; r_s\,. \tag{18}$$

The total amount of dark matter within radius r is given by

$$\mathrm{DM}\,(\leq r) = \int_0^r \rho_{DM}\; dr = \int_0^r (\rho_0 - \rho_M)\, d\,r = \rho_0\; r - \int_0^r \rho_M\; d\,r$$

$$= \rho_0\; r - \rho_0\; r_s \frac{ln\left|\, 2-2^{-\frac{r}{r_s}}\right|}{ln2} = \rho_0\,(\,r - r_s \frac{ln\left|\, 2-2^{-\frac{r}{r_s}}\right|}{ln}\,)\,. \tag{19}$$

For galaxies with well-defined rotation curves, the radius r_s could be easily extracted by fitting a theoretical curve from the family of functions $\{1-2^{-\frac{r}{r_s}}\}$ to the measured rotation curve. A simpler method is to calculate r_s based on one of the reported scale radii. The most frequently reported scale radius is the "core" radius r_c, which refers to the distance from the galaxy center at which its matter density is one half of the central density ρ_0, which coincides in our model with the distance at which the densities of matter and dark matter are equal. From Eq. (11), we have

$$\frac{\rho_M}{\rho_0}(r = r_c) = \frac{1-\left(1-2^{-\frac{r_c}{r_s}}\right)}{1+\left(1-2^{-\frac{r_c}{r_s}}\right)} = \frac{1}{2}\,, \tag{20}$$

which solves for

$$r_s = \frac{\ln(2)}{\ln(3)-\ln(2)}\, r_c \approx 1.709511\, r_c\,. \tag{21}$$

Interestingly the radius at which the energy densities of matter and dark matter are predicted to be equal, is r_c. This surprising result could be easily verified by solving for r^*, which satisfies $\frac{e_M}{e_0}(r^*) = \frac{e_{DM}}{e_0}(r^*)$, or

$$\frac{1-\left(1-2^{-\frac{r^*}{r_s}}\right)}{1+\left(1-2^{-\frac{r^*}{r_s}}\right)}\left(1-2^{-\frac{r}{r_s}}\right)^2 = \frac{2\left(1-2^{-\frac{r^*}{r_s}}\right)^3}{1+\left(1-2^{-\frac{r^*}{r_s}}\right)}\,, \tag{22}$$

which solves for

$$r^* = \frac{\ln(3)-\ln(2)}{\ln(2)}\, r_s = \frac{\ln(3)-\ln(2)}{\ln(2)}\,\frac{\ln(2)}{\ln(3)-\ln(2)}\, r_c = r_c\,. \tag{23}$$

Thus, independently from measurements, we find that the radius at which the energy of matter is predicted to be half of the total energy is the famous core radius r_c.

3. Model Testing

We tested the derived predictions using the data of the three large spiral galaxies in the local group: The **Milky Way**, **Andromeda** (M31), and **Triangulum** (M33). The mass of the **Milky Way** bulge, disk, and halo is estimated to be M_b = 1.652 x 10^{10} $M_\odot$, M_d = 3.41 x 10^{10} $M_\odot$, and M_h = 6.524 x $10^{11} M_\odot$ [15], respectively, with a total mass equaling $M_{tot} \approx$ 7.03 $10^{11} M_\odot$. The galaxy's disk core radius and maximal rotation velocity are estimated to be r_c= 3.19 kpc and V_{max}= 238 km sec^{-1}, respectively. **Andromeda** is the largest galaxy in the local groups. Its bulge, disk, and halo masses are about M_b = 3.4 x 10^{10} $M_\odot$, M_d =

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6.9 x 10^{10} $M_{\odot}$, and M_h =1.5 x 10^{12} $M_{\odot}$, respectively [15]. The diameter of Andromeda at the widest point is estimated to be 67.45 ± 0.92 kpc. Andromeda's core radius equals r_c = 1.0 ± 0.2 kpc, and its maximal rotation velocity is about V_{max}= 260 km sec^{-1} [16] [17]. **Triangulum (M33)** has an estimated disk mass of (3–6) x 10^9 $M_{\odot}$ and a gas component of about 3.2 × 10^9 $M_{\odot}$, amounting together to a baryonic mass of (6.2 - 9.2) x 10^9 10^{10} $M_{\odot}$. The contribution of the dark matter component up to a radius of 17 kpc is about 5 × 10^{10} $M_{\odot}$ [18]. Triangulum's core radius equals r_{50} = 1.39 ± 0.04kpc, and its maximal rotation velocity is about V_{max}= 105.4 ± 6. km sec^{-1} [19] [20].

According to the above-quoted measurements, the percentages of the total dark matter in the Milky Way and Andromeda are, respectively, about 92.8%, and 99.3%; for Triangulum up to 17 kpc, the percentage of dark matter is about 86.7%. Figures 3 depicts the theoretical rotation curves of the three galaxies as functions of the distance r from their centers. Figures 4 and 5 depict, respectively, the matter and dark matter densities and cumulative distributions as functions of r.

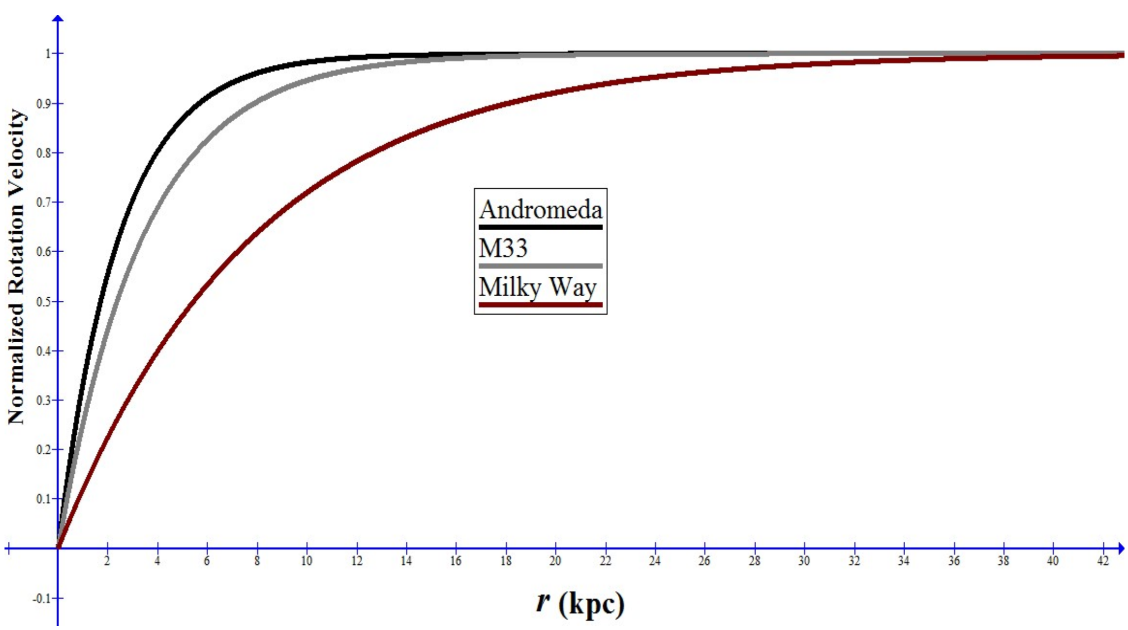

Figure 3. Theoretical rotation curves for three galaxies.

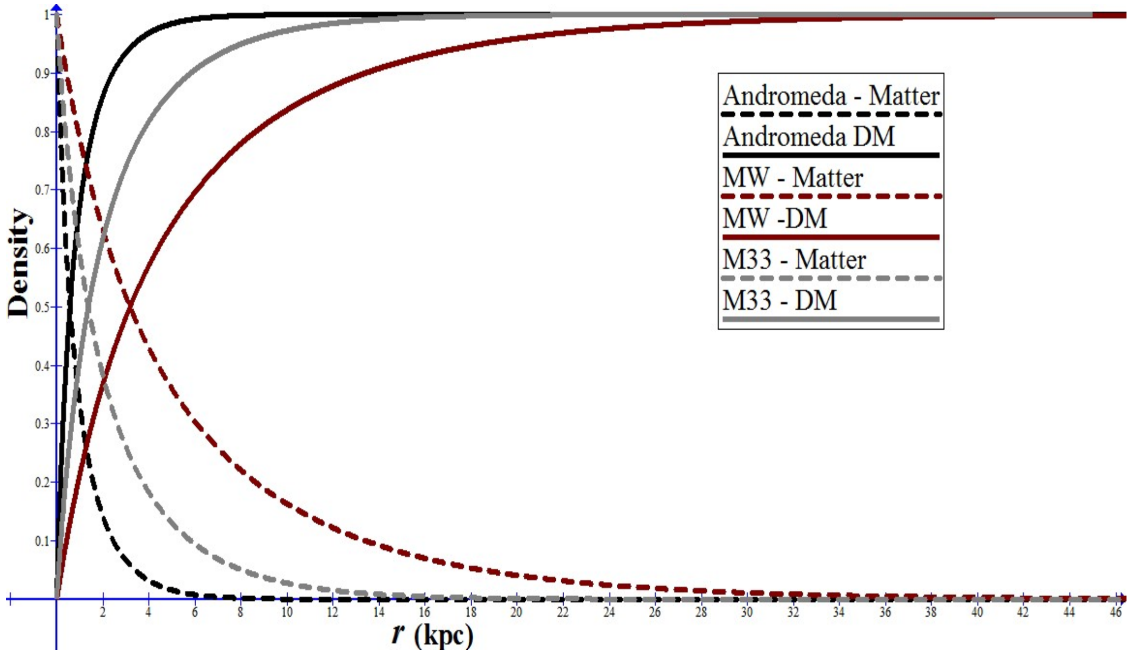

Figure 4. Matter and dark matter density distributions in three galaxies as functions of r.

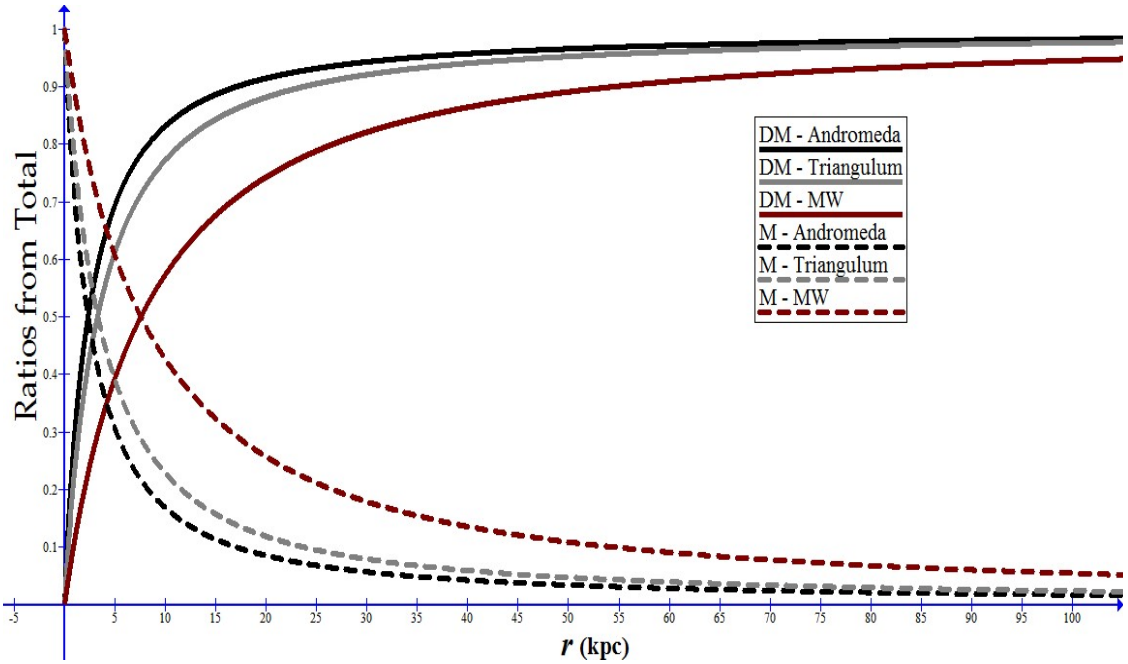

Figure 5. Cumulative distributions of matter and dark matter in three galaxies as functions of r.

Table 1 depicts the relevant parameters for the three investigated galaxies, together with the prediction of the percentages of dark matter out of the total mass in each galaxy within given radii. As shown in the right-hand column of the table, the absolute prediction errors for all three galaxies are small, with a mean error of ≈ 0.01. The dependence of the predicted percentage of dark matter, on the scale radius r_s, is captured by the simple linear equation:

$$\% \text{ DM} = 100 - r_s \; . \tag{24}$$

Table 1

Comparison between Measured and Predicted Proportions of Dark Matter

Galaxy	r_c (kpc)	r_s (kpc)	r_{max} (kpc)	V_{max} (kms-1)	Measured Matter Mass (in $M_\odot$)	Measured DM Mass (in $M_\odot$)	Measured % DM	Predicted % DM	Relative Absolute Error $\frac{Obs-Pred.}{Obs}$
Milky Way	3.19	5.45	100	238	5.062 x 10^{10}	6.524 x 10^{11}	92.8%	94.6	0.014
Andromeda	1	1.71	100	260	10.3 x 10^{10}	1.5 x 10^{12}	99.3%	98.3	0.01
Triangulum	1.39	2.38	17	105.4	4.5 x 10^9	5 x 10^{10}	86.7	86.1*	0.007

* For r = 100kpc the model's prediction for Triangulum is ≈ 97.6%

4. Concluding Remarks

It was recently shown, based on data of a large number of rotationally supported galaxies, with very different morphologies, masses, sizes, and gas fractions, that the rotational acceleration due to baryons and the observed rotational acceleration are strongly correlated [10]. In the framework of the dark matter paradigm, the discovered relationship means that the contribution of dark matter to the total mass of a galaxy is completely determined by observing its baryonic mass and vice versa. Our model provides theoretical proof for the correctness of the empirically derived rotational acceleration relationship. It does so by specifying the exact relationship type, as captured in Eq. (3). The emerging dependence between the densities of baryonic matter and dark matter in galaxies turned out to be linear and beautifully simple.

An important feature of the proposed model, in addition to being a completely theoretical, axiom-free, and parameter-free model, is that the predicted dark matter component in galaxies is emergent from the model and not presupposed in it. This feature is diametrically opposed to the assumption of the standard dark matter paradigm that galaxies form within a presupposed dark matter halo. Our model prescribes that dark matter emerges from baryonic matter due to its rotation velocity. As shown by Eqs. (11) and (12) (see also Figure 1), the density of dark matter as a function of r is a tracer of the rotation velocity $v(r)$. This feature of provides a plausible explanation for why the bulk of dark matter in galaxies resides at the galaxies halos, where the rotational velocity reaches its maximal value.

An intriguing testable prediction arising from the theoretical analysis detailed in the previous section is that one measurement, namely, the scale radius r_s, at which the rotation velocity becomes equal to $\frac{V_{max}}{2}$ (or, alternatively r_c, at which the density of baryons is equal to $\frac{\rho_0}{2}$) is sufficient for revealing very rich information about the galaxy, including the distributions of matter and dark matter in the galaxy as functions of the distance from its center, and the proportions of matter and dark matter, out of the total amount of matter in the galaxy. Our test described in Section X gives strong support to this prediction. However, a more convincing test, using a significantly larger number of rotationally supported galaxies, with different morphologies, sizes, etc., is called for.

A comparison between the proposed model and MOND [21] [22] should leave no doubt regarding the superiority of the first. First, unlike MOND, our model does not make any arbitrary change in Newton's laws. It merely relativizes them by accounting for the observation that waves, or any other information carrier, takes time to travel between two points in three-dimensional space. Second, our model is concordant with the standard cosmological model.

Finally, the simple and beautiful symmetries pertaining to the relationships between matter and dark matter are difficult to ignore. If supported by empirical evidence, they could stand out as another significant example of the simplicity and beauty by which nature organizes itself.

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Supporting Information

Section S1. Derivation of the Expressions of Matter and Dark Matter Densities for a Rotating Thin Disk

. To probe the dynamics of matter/dark matter of a rotating disk, assume a uniform disk with rest mass density ρ_0 and rest radius of R_0. To avoid unnecessary complexities, we assume that the disk is placed on the (x, y) plane and rotates around the z axis with constant angular velocity (see Fig. 1).

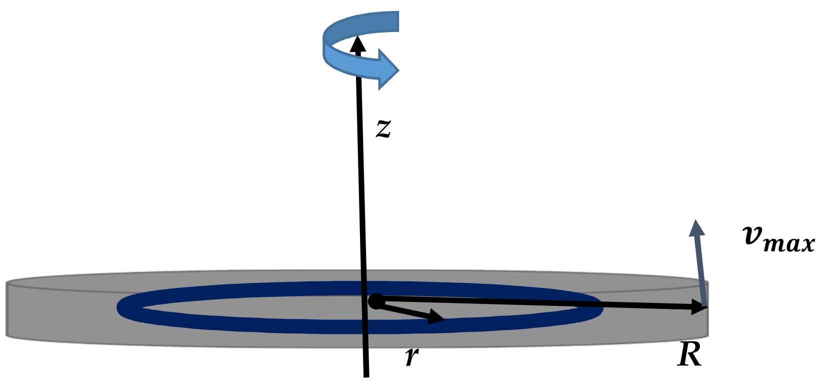

Figure S1. A uniform rotating disk.

In a previous article [S1], we applied our relativistic extension of Newtonian dynamics to the case of a rectilinearly receding body (e.g., a galaxy) along the line-of-sight of a stationary observer. The analysis gave rise naturally to a component of dark matter, which exists in duality to the receding body's observable matter. We derived transformations for time, length, and matter and dark matter densities as well as their respective energy densities, as functions of the body's recession velocity relative to a stationary observer. Here, we extend the model to infer about the dynamics of matter/dark matter at galactic scales. Without compromising the generality of our approach, for the sake of simplicity, we analyzed the relativistic dynamics of a homogeneous, purely gravitational rotating disk. However, the model could be applied without much difficulty to rotating bodies with different morphologies. To derive the dynamical equations of matter/dark matter duality for the case of a rotating disk, we utilize a theoretical result indicating that that the rectilinear and rotational types of motion are dynamically equivalent [S2] [S3]. Thus, from the standpoint of a nonrotating frame of reference, the equations of motion of a rotating body of mass with constant radial velocity ω, could be written directly from the equivalent equations derived above for rectilinear motion with constant velocity v. Utilizing this equivalence, the relativistic expressions for relevant physical variables for the case of circular motion are depicted in Table S1.

Table S1

Relativistic Expressions for an Inertial Rotational Motion

Variable	Transformation in Terms of Velocity
Time interval $\frac{\Delta t}{\Delta t_0}$	$\frac{1}{1-\beta(r)}$
Length $\frac{l}{l_0}$	$\frac{1+\beta(r)}{1-\beta(r)}$
Matter density $\frac{\rho_M}{\rho_0}$	$\frac{1-\beta(r)}{1+\beta(r)}$
Matter energy density $\frac{e_M}{e_0}$	$\frac{1-\beta(r)}{1+\beta(r)}\,\beta^2(r)$
Dark matter density $\frac{\rho_{DM}}{\rho_0}$	$\frac{2\,\beta(r)}{1+\beta(r)}$
Dark matter energy density $\frac{e_{DM}}{e_0}$	$\frac{2\,\beta^3(r)}{1+\beta(r)}$

In the table, ρ_0 is the Newtonian (rest) mass density, i.e., the density at the disk center where $v(r) = 0$, $\beta(r) = \frac{v(r)}{c,}$ and is the Newtonian kinetic energy density $(e_0 = \frac{1}{2}\rho_0 c^2)$.

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Section S2. Comparison between Two Possible Idealizations for Galactic Rotation Velocities

Figure S1 depicts two idealizations for galactic rotation curves for some hypothetical core radii: The exponential (power of e) function suggested in [S4] (dashed lines), and the power of 2 function proposed by us (full lines). As shown in the figures, the two functions behave quite similarly, i.e., they rise at lower r values and flatten at higher r values. A comparison between the figures in the four panels shows that the difference between the two functions is larger for large r_c values and quite insignificant for small ones.

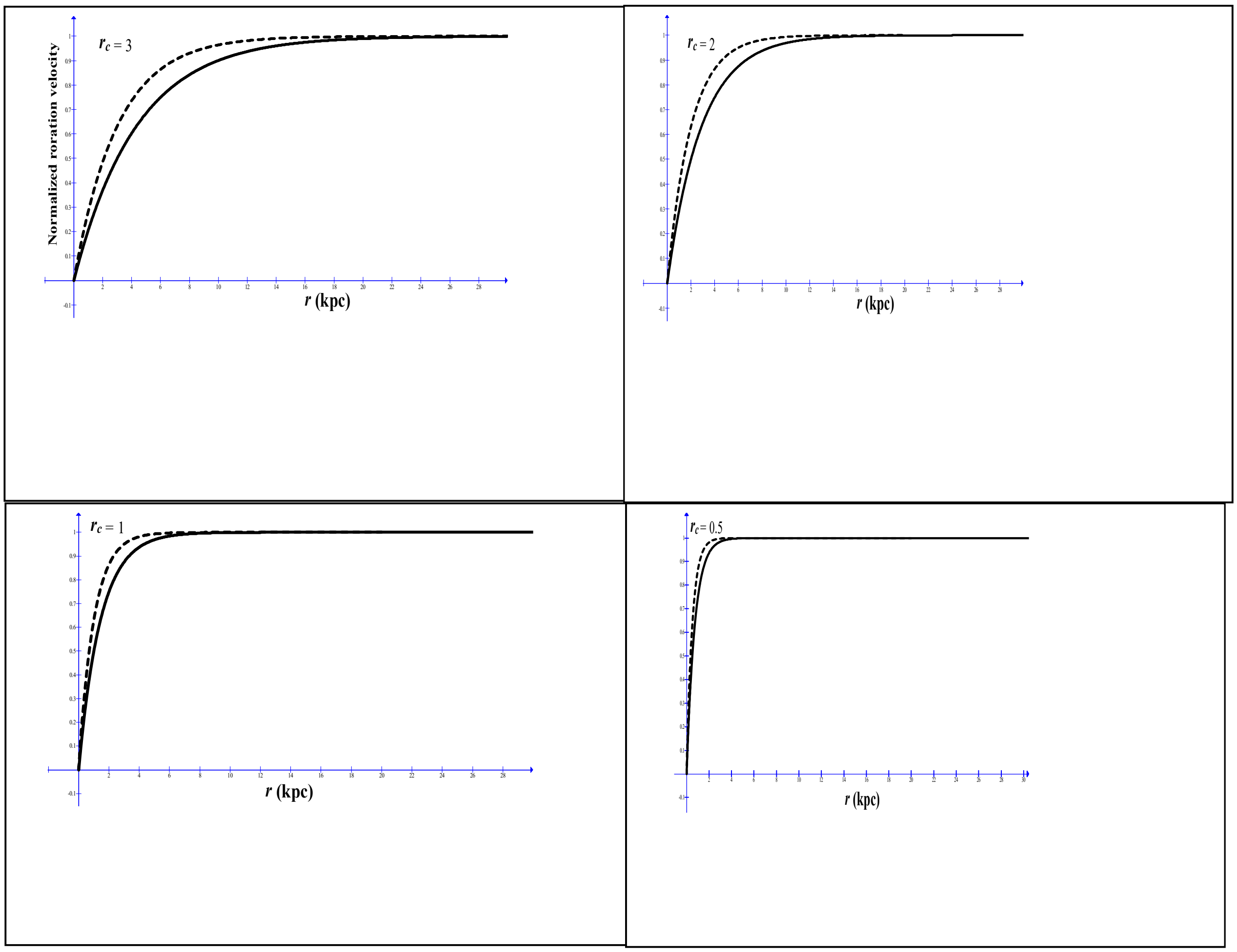

Figure 1. Comparison between two theoretical approximations of a galaxy's rotation curve.

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