Modeling and forecasting Daily stock Returns of Guaranty Trust Bank Nigeria Plc Using ARMA-GARCH Models, Persistence, Half-life Volatility and Backtesting

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Abstract

In financial time series modelling and forecasting, combining ARMA and GARCH models tend to produce superior and reliable models for volatility persistence, half-life volatility and backtesting (application of model in real life). In Nigeria, banking stocks are mostly traded because of its potential benefits to investors. This study modelled and forecasted the Guaranty Trust (GT) Bank daily stock returns from January 2nd 2001 to May 8th 2017 data set collected from a secondary source. The ARMA-GARCH models, persistence, half-life and backtesting were used to analysed the collected data using student t and skewed student t distributions, and the analyses are carried out R environment using rugard and performanceAnalytics Packages. The study revealed that using the lowest information criteria values only could be misleading rather we added the use of backtesting. The ARMA(1,1)-GARCH(1,1) models fitted exhibited high persistency in the daily stock returns while the days it takes for mean-reverting of the models ranges from 5 days to 100 days but unfortunately the models failed backtesting. The results further revealed ARMA(1,1)-eGARCH (2,2) model with student t distribution provides a suitable model for evaluating the GT bank stock returns among the competing models while it takes less than 30 days for the persistence volatility to return back to its average value of the stock returns. This study recommended that researchers should adopt backtesting approach while fitting GARCH models while GT bank stocks investor should be assured that no matter the fluctuations in the stock market, the GT bank stock returns has the ability to returns to its mean price return.

Keywords: Returns, Stocks, Guaranty Trust (GT) Bank, Generalized Autoregressive Conditional Heteroskedasticity (GARCH), Persistence, Half-life, Volatility, Backtesting.

1.0 Introduction

Autoregressive Moving Average (ARMA) and Autoregressive Integrated Moving Average (ARIMA) models are popular and excellent for modeling and forecasting univariate time series data as proposed by Box and Jenkins (1970), and its extension with exogenous variables as Autoregressive Integrated Moving Average with Explanatory Variable (ARIMAX) (Kongcharoen and Kruangpradit, 2013). These models are applied in almost all fields of endeavours such as engineering, geophysics, business, economics, finance, agriculture, medical sciences, social sciences, meteorology, quality control etc (Kirchgassner and Wolters, 2007; Adenomon, 2017a, Adenomon, 2017b; Cooray, 2008; Dobre and Alexandru, 2008; Gujarati, 2003; Adekeye and Aiyelabegan, 2006). The ARMA and ARIMA models are used to model conditional expectation of a process but in ARMA model, the conditional variance is constant. This means that ARMA model cannot capture process with time-varying conditional variance (volatility) which is mostly common with economic and financial data.

Actually, with economic and financial time series data, time-varying is more common than constant volatility, and accurate modeling of time volatility is of great importance in financial time series analysis (Ruppert, 2011). Financial time series contains uncertainty, volatility, excess kurtosis, high standard deviation, high skewness and sometimes non normality (Pedroni (2001); Grigoletto & Lisi (2009); Emenogu and Adenomon, 2018; Emenogu et al. 2018). To model and capture properly the characteristics of financial time series models such as Auto-Regressive Conditional Heteroscedastic (ARCH), Generalized Auto-Regressive Conditional Heteroscedastic (GARCH), multivariate GARCH, Stochastic volatility (SV) and various variants of the models have been proposed to handle these characteristics of financial time series (Lawrance, 2003).

From the foregoing, considering the flexibility and simplicity of the ARMA model and the capability of the GARCH model to capture volatility in financial time series, combining the ARMA model with the GARCH model for the innovations, yielding the so-called ARMA-GARCH model, provides the econometricians and financial analyst with a more flexible and yet tractable model that allows the model to capture the mean and variance components that is common with financial time series volatility (Lange, 2011; Panait and Slavescu, 2012) meaning that the ARMA-GARCH model will produced a more reliable estimates for financial analyst to

take a better decision. This paper therefore investigates the persistence, half-life volatility and forecasting (Backtesting that is providing real life model) of daily stock returns of Guaranty Trust Bank, Nigeria plc using ARMA-GARCH Models.

Guaranty Trust (GT) Bank plc was incorporated as a limited liability company licensed to provide commercial and other banking services to the Nigerian public in 1990. The Bank commenced operations in February 1991, and has since then grown to become one of the most respected and service focused banks in Nigeria. Also, In September 1996, Guaranty Trust Bank plc became a publicly quoted company and won the Nigerian Stock Exchange President's Merit award that same year, and subsequently in the years 2000, 2003, 2005, 2006, 2007, 2008 and 2009. In February 2002, the Bank was granted a universal banking license and later appointed a settlement bank by the Central Bank of Nigeria (CBN) in 2003. In May 2011, The Bank successfully launched a US\$500 million bond — which bring about the first non-sovereign benchmark bond offering from sub-Saharan Africa (outside South Africa), to the international community. The highly successful offering, which matures in 2016, went further to show that the international finance community's believe in the GT Bank brand. The Bank's culture is tied to eight guiding principles called the Orange Rules; Simplicity, Professionalism, Service, Friendliness, Excellence, Trustworthiness, Social Responsibility and Innovation. With these features of GT Bank, the bank stands suitable for customers and investor both in Nigeria and abroad where the bank has branches (The European Financial Review, 2012).

2.0 Brief literature on the persistence and half-life Volatility of Stocks Returns

The volatility of asset returns is a measure of how much the returns fluctuates around its means (Marra, 2015). In addition, volatility is the purest measure of risk in financial markets and by this, it has becomes the expected price of uncertainty. A good volatility model and forecast help impact the public confidence significantly and by extension on the broader global economy. What comes to mind again is the persistence and half-life volatility of any given stock.

The persistence of financial stock is the extent to which events today have an efficient influence on the whole future history of a stochastic process, and as such is a central issue in financial time series, macroeconomic theory and policy (Caporale and Pittis, 2001). In a stationary GARCH process, the persistence volatility returns back to its means at the long term horizon and it is a rate calculated by the sum of GARCH and ARCH coefficients. And in many

financial time series it is usually close to one (1) (Ahmed et al., 2018; Engle and Patton, 2001; Vosvrda, 2006). While on the other hand, the half-life of the volatility shocks measure the average time period for the volatility to return back to its mean value in the long run horizon (Ahmed et al., 2018; Sahai, 2016).

Engle and Patton (2001) examine the Dow Jones Industrial index from 23 August 1988 to 22 August 2000. Their result indicated that the volatility returns are quite persistent.

Magnus and Fosu (2006) modeled and forecasted the volatility of returns on Ghana Stock exchange using GARCH models. They found that presence of high level of persistence in the returns in the stock market.

Vosvrda (2006) compared empirical analysis of persistence and dependence patterns among capital market using univariate and multivariate measures. The results revealed that the univariate measure shows a low level of persistence while multivariate measure shows that the persistence change dependence on structure in different period of lags.

Panait and Slavescus (2012) investigated the volatility and persistence of seven Romanian companies traded on Bucharest Stock Exchange and three market indices from 1997-2012 using GARCH-in-Mean Models. They found out that persistency is more in the daily returns as compared to weekly and monthly series.

Emenike and Ani (2014) examined the nature of volatility of stock returns in the Nigerian banking sector using ARMA-GARCH models using data covering 3rd January to December 2012. Their results revealed volatility persistence was high for the sample period they considered.

Usman et al. (2017) examined the performance of eleven competing GARCH models for fitting the rate of returns of monthly observations on the index returns series of the market over a period of January 1996 to December 2015. The overall results revealed increased volatility of the market returns.

Chu et al (2017) provided the first GARCH modeling of the seven most popular cryptocurrencies using twelve GARCH models fitted for each cryptocurrencies. Their work concluded IGARCH and GJR-GARCH models provided the best fits in terms of modeling of the volatility in the most popular and largest cryptocurrencies.

Kuhe (2018) examined the volatility persistence and asymmetry with exogenous break in Nigerian stock market using data from 3rd July 1999 to 12th June 2017 using standard symmetric GARCH (1,1), asymmetric EGARCH (1,1) and GJR-GARCH (1,1) models. The study revealed among other results a high persistence of shocks in the return series for the estimated models.

Ahmed et al. (2018) examined and compared the mean reversion phenomenon in developed and emerging stock markets, they employed data from 1st January to 30th June 2016 using GARCH (1,1) model. Their results revealed that South Korean market has the slowest mean reversion and thus has the highest half-life period while Pakistan stock exhibited fastest reverting process.

3.0 Model Specification

This study focuses on the ARMA-GARCH models that are robust for forecasting the volatility of financial time series data; so ARMA-GARCH model and some of its extensions are presented in this section.

ARMA-GARCH specification is employed to model the conditional mean and conditional variance (volatility) of any financial time series because of its superiority in modelling such series. GARCH models model conditional variances much as the conditional expectation by an ARMA model (Ruppert, 2011). Therefore ARMA model can be combined to any form of GARCH model.

The ARMA (p,q)-GARCH (1,1) model can be specified as follows:

$$\left. \begin{aligned} r_t &= \sum_{i=1}^p \theta_i r_{t-i} + \sum_{j=1}^q \phi_j \varepsilon_{t-j} + \varepsilon_t \\ \varepsilon_t \sqrt{\sigma_t^2} Z_t, \quad Z_t &\sim D(0, \sigma_t^2) \\ \sigma_t^2 &= \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \end{aligned} \right\} \quad (1)$$

Where r_t is the daily rate of return, θ is the AR(p) term in the mean equation in order to account for time dependence in returns, ϕ is the MA(q) term in the mean equation, ε_t is the residual term in the mean equation, Z_t is the standardized residual sequence of iid random variable with mean zero and variance as one while D represents distribution of the shock returns (Moshiri and Foroutan, 2006; Chen et al., 2013; Meitz and Saikkonen, 2011; Francq and Zakoian, 2004; Ling and McAleer, 2003; Koul and Ling, 2006; Zhu and Ling, 2011).

Because of space we will discuss the following family GARCH models below:

3.1 Autoregressive Conditional Heteroskedasticity (ARCH) Family Model

Every ARCH or GARCH family model requires two distinct specifications, namely: the mean and the variance equations (Atoi, 2014). The mean equation for a conditional heteroskedasticity in a return series, y_t is given by

$$y_t = E_{t-1}(y_t) + \varepsilon_t \quad (2)$$

where

$$\varepsilon_t = \phi_t \sigma_t$$

The mean equation in equation (2) also applies to other GARCH family models. $E_{t-1}(\cdot)$ is the expected value conditional on information available at time $t-1$, while ε_t is the error generated from the mean equation at time t and ϕ_t is the sequence of independent and identically distributed random variables with zero mean and unit variance.

The variance equation for an ARCH(p) model is given by

$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \dots + \alpha_p a_{t-p}^2 \quad (3)$$

It can be seen in the equation that large values of the innovation of asset returns have bigger impact on the conditional variance because they are squared, which means that a large shock tends to follow another large shock and that is the same way the clusters of the volatility behave. So the ARCH(p) model becomes:

$$a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \dots + \alpha_p a_{t-p}^2 \quad (4)$$

Where $\varepsilon_t \sim N(0,1)$ iid, $\omega > 0$ and $\alpha_i \geq 0$ for $i > 0$. In practice, ε_t is assumed to follow the standard normal or a standardized student- t distribution or a generalized error distribution (Tsay 2005).

3.2 Asymmetric Power ARCH

According to Rossi (2004), the asymmetric power ARCH model proposed by Ding, Engle & Granger (1993) given below forms the basis for deriving the GARCH family models

Given that:

$$r = \mu + a_t,$$

$$\varepsilon_t = \sigma_t \varepsilon_t,$$

$$\varepsilon_t \sim N(0,1)$$

$$\sigma_t^\delta = \omega + \sum_{i=1}^p \alpha_i (|a_{t-i}| - \gamma_i a_{t-i})^\delta + \sum_{j=1}^q \beta_j \sigma_{t-j}^\delta, \quad (5)$$

where

$$\begin{aligned} \omega &> 0, & \delta &\geq 0, \\ \alpha_i &\geq 0 & i &= 1, 2, \dots, p \\ -1 < \gamma_i < 1 & i &= 1, 2, \dots, p \\ \beta_j &> 0 & j &= 1, 2, \dots, q \end{aligned}$$

This model imposes a Box-Cox transformation of the conditional standard deviation process and the asymmetric absolute residuals. The leverage effect is the asymmetric response of volatility to positive and negative “shocks”.

3.3 Standard GARCH(p, q) Model:

The mathematical model for the sGARCH(p,q) model is obtained from equation (5) by letting $\delta = 2$ and $\gamma_i = 0$, $i = 1, \dots, p$ to be:

$$a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i a_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \quad (6)$$

Where $a_t = r_t - \mu_t$ (r_t is the continuously compounded log return series), and

$\varepsilon_t \sim N(0,1)$ iid, the parameter α_i is the ARCH parameter and β_j is the GARCH parameter, and

$\omega > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, and $\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) < 1$, (Rossi, 2004; Tsay, 2005 and Jiang, 2012).

The restriction on ARCH and GARCH parameters (α_i, β_j) suggests that the volatility (a_i) is finite and that the conditional standard deviation (σ_i) increases. It can be observed that if $q = 0$, then the model GARCH parameter (β_j) becomes extinct and what is left is an ARCH(p) model.

To expatiate on the properties of GARCH models, the following representation is necessary:

Let $\eta_t = a_t^2 - \sigma_t^2$ so that $\sigma_t^2 = a_t^2 - \eta_t$. By substituting $\sigma_{t-i}^2 = a_{t-i}^2 - \eta_{t-i}$, ($i = 0, \dots, q$) into Eq. (4), the GARCH model can be rewritten as

$$a_t = \alpha_0 + \sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) a_{t-i}^2 + \eta_t - \sum_{j=1}^q \beta_j \eta_{t-j}, \quad (7)$$

It can be seen that $\{\eta_t\}$ is a martingale difference series (i.e., $E(\eta_t) = 0$ and

$\text{cov}(\eta_t, \eta_{t-j}) = 0$, for $j \geq 1$). However, $\{\eta_t\}$ in general is not an iid sequence.

A GARCH model can be regarded as an application of the ARMA idea to the squared series a_t^2 .

Using the unconditional mean of an ARMA model, results in this

$$E(a_t^2) = \frac{\alpha_0}{1 - \sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i)}$$

provided that the denominator of the prior fraction is positive. (Tsay, 2005)

When $p=1$ and $q=1$, we have GARCH(1, 1) model given by:

$$\begin{aligned} a_t &= \sigma_t \varepsilon_t, \\ \sigma_t^2 &= \omega + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \end{aligned} \quad (8)$$

3.4 GJR-GARCH(p, q) Model

The Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model, which is a model that attempts to address volatility clustering in an innovation process, is obtained by letting $\delta = 2$.

When $\delta = 2$ and $0 \leq \gamma_i < 1$,

$$\begin{aligned} \sigma_t^2 &= \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \\ &= \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}|^2 + \gamma_i^2 \varepsilon_{t-i}^2 - 2\gamma_i |\varepsilon_{t-i}| \varepsilon_{t-i}) + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \\ \sigma_t^2 &= \begin{cases} \omega + \sum_{i=1}^p \alpha_i^2 (1 + \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} < 0 \\ \omega + \sum_{i=1}^p \alpha_i (1 - \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} > 0 \end{cases} \end{aligned} \quad (9)$$

$$\text{i.e.;} \quad \sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i (1 - \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{i=1}^p \alpha_i \{(1 + \gamma_i)^2 - (1 - \gamma_i)^2\} S_i^- \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2$$

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i (1 - \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p 4\alpha_i \gamma_i S_i^- \varepsilon_{t-i}^2$$

where

$$S_i^- = \begin{cases} 1 & \text{if } \varepsilon_{t-i} < 0 \\ 0 & \text{if } \varepsilon_{t-i} \geq 0 \end{cases},$$

Now define

$$\alpha_i^* = \alpha_i(1 - \gamma_i)^2 \text{ and } \gamma_i^* = 4\alpha_i\gamma_i,$$

then

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i(1 - \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p \gamma_i^* S_i^- \varepsilon_{t-1}^2 \quad (10)$$

Which is the GJRGARCH model (Rossi, 2004).

But when $-1 \leq \gamma_i < 0$,

Then recall Eq. (9)

$$\begin{aligned} \sigma_t^2 &= \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \\ &= \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}|^2 + \gamma_i^2 \varepsilon_{t-i}^2 - 2\gamma_i |\varepsilon_{t-i}| \varepsilon_{t-i}) + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \\ \sigma_t^2 &= \begin{cases} \omega + \sum_{i=1}^p \alpha_i^2 (1 - \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} > 0 \\ \omega + \sum_{i=1}^p \alpha_i (1 + \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} < 0 \end{cases} \\ \sigma_t^2 &= \omega + \sum_{i=1}^p \alpha_i (1 + \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p \alpha_i \{(1 - \gamma_i)^2 - (1 + \gamma_i)^2\} S_i^+ \varepsilon_{t-i}^2 \\ &= \omega + \sum_{i=1}^p \alpha_i (1 + \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p \alpha_i \{1 + \gamma_i^2 - 2\gamma_i - 1 - \gamma_i^2 - 2\gamma_i\} S_i^+ \varepsilon_{t-i}^2 \end{aligned}$$

Where

$$S_i^+ = \begin{cases} 1 & \text{if } \varepsilon_{t-i} > 0 \\ 0 & \text{if } \varepsilon_{t-i} \leq 0 \end{cases}$$

also define

$$\alpha_i^* = \alpha_i(1 + \gamma_i)^2 \text{ and } \gamma_i^* = -4\alpha_i\gamma_i,$$

then

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i^* \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p \gamma_i^* S_i^+ \varepsilon_{t-1}^2 \quad (11)$$

which allows positive shocks to have a stronger effect on volatility than negative shocks (Rossi, 2004). But when $p = q = 1$, the GJRGARCH(1,1) model will be written as

$$\sigma_t^2 = \omega + \alpha \varepsilon_t^2 + \gamma \mathcal{S}_t \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2. \quad (12)$$

3.5 IGARCH(1, 1) Model

The integrated GARCH (IGARCH) models are unit- root GARCH models. The IGARCH (1, 1) model is specified in Tsay (2005) and Grek (2014) as

$$a_t = \sigma_t \varepsilon_t; \quad \sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + (1 - \beta_1) a_{t-1}^2 \quad (13)$$

Where $\varepsilon_t \sim N(0, 1)$ iid, and $0 < \beta_1 < 1$, Ali (2013) used α_i to denote $1 - \beta_i$.

The model is also an exponential smoothing model for the $\{a_t^2\}$ series. To see this, rewrite the model as

$$\begin{aligned} \sigma_t^2 &= (1 - \beta_1) a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \\ &= (1 - \beta_1) a_{t-1}^2 + \beta_1 [(1 - \beta) a_{t-2}^2 + \beta_1 \sigma_{t-2}^2] \\ &= (1 - \beta_1) a_{t-1}^2 + (1 - \beta_1) \beta_1 a_{t-2}^2 + \beta_1^2 \sigma_{t-2}^2. \end{aligned} \quad (14)$$

By repeated substitutions, we have

$$\sigma_t^2 = (1 - \beta_1)(a_{t-1}^2 + \beta_1 a_{t-2}^2 + \beta_1^2 a_{t-3}^2 + \dots), \quad (15)$$

which is the well-known exponential smoothing formation with β_1 being the discounting factor (Tsay, 2005).

3.6 TGARCH(p, q) Model

The Threshold GARCH model is another model used to handle leverage effects, and a TGARCH(p, q) model is given by the following:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p (\alpha_i + \gamma_i N_{t-i}) a_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, \quad (16)$$

where N_{t-i} is an indicator for negative a_{t-i} , that is,

$$N_{t-i} = \begin{cases} 1 & \text{if } a_{t-i} < 0, \\ 0 & \text{if } a_{t-i} \geq 0, \end{cases}$$

and α_i , γ_i , and β_j are nonnegative parameters satisfying conditions similar to those of GARCH models, (Tsay, 2005). When $p = 1, q = 1$, the TGARCH(1, 1) model becomes:

$$\sigma_t^2 = \omega + (\alpha + \gamma N_{t-1}) a_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (17)$$

3.7 NGARCH(p, q) Model

The Nonlinear Generalized Autoregressive Conditional Heteroskedasticity (NGARCH) Model has been presented variously in literature by the following scholars: Hsieh & Ritchken (2005), Lanne & Saikkonen (2005), Malecka (2014) and Kononovicius & Ruseckas (2015). The following model can be shown to represent all the presentations:

$$h_t = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \gamma_i \varepsilon_{t-i} + \sum_{j=1}^p \beta_j h_{t-j} \quad (18)$$

Where h_t is the conditional variance, and ω , β and α satisfy $\omega > 0$, $\beta \geq 0$ and $\alpha \geq 0$.

Which can also be written as

$$\sigma_t = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \gamma_i \varepsilon_{t-i} + \sum_{j=1}^p \beta_j \sigma_{t-j} \quad (19)$$

3.8 The Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH) Model

The EGARCH model was proposed by Nelson (1991) to overcome some weaknesses of the GARCH model in handling financial time series pointed out by Enocksson and Skoog(2012), In particular, to allow for asymmetric effects between positive and negative asset returns, he considered the weighted innovation

$$g(\varepsilon_t) = \theta \varepsilon_t + \gamma [|\varepsilon_t| - E(|\varepsilon_t|)], \quad (20)$$

where θ and γ are real constants. Both ε_t and $|\varepsilon_t| - E(|\varepsilon_t|)$ are zero-mean iid sequences with continuous distributions. Therefore, $E[g(\varepsilon_t)] = 0$. The asymmetry of $g(\varepsilon_t)$ can easily be seen by rewriting it as

$$g(\varepsilon_t) = \begin{cases} (\theta + \gamma)\varepsilon_t - \gamma E(|\varepsilon_t|) & \text{if } \varepsilon_t \geq 0, \\ (\theta - \gamma)\varepsilon_t - \gamma E(|\varepsilon_t|) & \text{if } \varepsilon_t < 0. \end{cases} \quad (21)$$

An EGARCH(m, s) model, according to Tsay (2005), Dhamija and Bhalla (2010), Jiang (2012), Ali (2013) and Grek (2014), can be written as

$$a_t = \sigma_t \varepsilon_t, \quad \ln(\sigma_t^2) = \omega + \sum_{i=1}^s \alpha_i \frac{|a_{t-i}| + \theta_i a_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^m \beta_j \ln(\sigma_{t-j}^2), \quad (22)$$

Which specifically results in EGARCH (1, 1) being written as

$$a_t = \sigma_t \varepsilon_t$$

$$\ln(\sigma_t^2) = \omega + \alpha(|a_{t-1}| - E(|a_{t-1}|)) + \theta a_{t-1} + \beta \ln(\sigma_{t-1}^2) \quad (23)$$

where $|a_{t-1}| - E(|a_{t-1}|)$ are iid and have mean zero. When the EGARCH model has a Gaussian distribution of error term, then $E(|\varepsilon_t|) = \sqrt{2/\pi}$, which gives:

$$\ln(\sigma_t^2) = \omega + \alpha(|a_{t-1}| - \sqrt{2/\pi}) + \theta a_{t-1} + \beta \ln(\sigma_{t-1}^2) \quad (24)$$

3.9 The Absolute Value GARCH (AVGARCH):

An asymmetric GARCH (AGARCH), according to Ali (2013) is simply

$$a_t = \sigma_t \varepsilon_t; \quad \sigma^2 = \omega + \sum_{i=1}^p \alpha_i |\varepsilon_{t-i} - b|^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, \quad (25)$$

While the absolute value generalized autoregressive conditional heteroskedasticity (AVGARCH) model is specified as:

$$a_t = \sigma_t \varepsilon_t; \quad \sigma^2 = \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i} + b| - c(\varepsilon_{t-i} + b))^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \quad (26)$$

3.10 Nonlinear (Asymmetric) GARCH, or N(A)GARCH or NAGARCH

NAGARCH plays key role in option pricing with stochastic volatility because, as we shall see later on, NAGARCH allows you to derive closed-form expressions for European option prices in spite of the rich volatility dynamics. Because a NAGARCH may be written as

$$\sigma_{t+1}^2 = \omega + \alpha \sigma_t^2 (z_t - \delta)^2 + \beta \sigma_t^2 \quad (27)$$

And if $z_t \sim IIDN(0,1)$, z_t is independent of σ_t^2 as σ_t^2 is only a function of an infinite number of past squared returns, it is possible to easily derive the long run, unconditional variance under NGARCH and the assumption of stationarity:

$$\begin{aligned} E[\sigma_{t+1}^2] &= \bar{\sigma}^2 = \omega + \alpha E[\sigma_t^2 (z_t - \delta)^2] + \beta E[\sigma_t^2] \\ &= \omega + \alpha E[\sigma_t^2] E(z_t^2 + \delta^2 - 2\delta z_t) + \beta E[\sigma_t^2] \\ &= \omega + \alpha \bar{\sigma}^2 (1 + \delta^2) + \beta \bar{\sigma}^2 \end{aligned} \quad (28)$$

Where $\bar{\sigma}^2 = E[\sigma_t^2]$ and $E[\sigma_t^2] = E[\sigma_{t+1}^2]$ because of stationary. Therefore

$$\bar{\sigma}^2 [1 - \alpha(1 + \delta^2) + \beta] = \omega \Rightarrow \bar{\sigma}^2 = \frac{\omega}{1 - \alpha(1 + \delta^2) + \beta} \quad (29)$$

Which exists and positive if and only if $\alpha(1 + \delta^2) + \beta < 1$. This has two implications:

- (i) The persistence index of a NAGARCH(1,1) is $\alpha(1 + \delta^2) + \beta$ and not simply $\alpha + \beta$;
- (ii) a NAGARCH(1,1) model is stationary if and only if $\alpha(1 + \delta^2) + \beta < 1$.

See details in Nelson (1991); Hall & Yao (2003); Enders (2004); Christoffersen, et al. (2008) and Engle & Rangel (2008).

3.11 Persistence, Half-life Volatility and Backtesting

3.11.1 Persistence

The low or high persistency in volatility exhibited by financial time series can be determined by the GARCH coefficients of a stationary GARCH model. The persistence of a GARCH model can be calculated as the sum of GARCH (β_1) and ARCH (α_1) coefficients that is $\alpha + \beta_1$. In most financial time series, it is very close to one (1) (Banerjee and Sarkar, 2006; Ahmed et al, 2018). Persistence could take the following conditions:

If $\alpha + \beta_1 < 1$: The model ensures positive conditional variance as well as stationary.

If $\alpha + \beta_1 = 1$: we have an exponential decay model, then the half-life becomes infinite. Meaning the model is strictly stationary.

If $\alpha + \beta_1 > 1$: The GARCH model is said to be non-stationary, meaning that the volatility ultimately detonates toward the infinitude (Ahmed et al, 2018). In addition, the model shows that the conditional variance is unstable, unpredicted and the process is non-stationary (Kuhe, 2018).

3.11.2 Half-Life Volatility

Half-life volatility measures the mean reverting speed (average time) of a stock price or returns. The mathematical expression of half-life volatility is given as

$$Half - Life = \frac{\ln(0.5)}{\ln(\alpha_1 + \beta_2)}$$

It can be noted that the value of $\alpha + \beta_1$ influences the mean reverting speed (Ahmed et al., 2018), which means that if the value of $\alpha + \beta_1$ is closer to one (1), then the volatility shocks of the half-life will be longer.

3.11.3 Backtesting

Financial risk model evaluation or backtesting is an important part of the internal model's approach to market risk management as put out by Basle Committee on Banking Supervision (Christoffersen & Pelletier, 2004). Backtesting is a statistical procedure where actual profits and losses are systematically compared to corresponding VaR estimates (Nieppola, 2009; Emenogu, et al., 2018). This study adopted Backtesting techniques of Christoffersen & Pelletier, (2004); The test was implemented in R using rugarch package and this test considered both the unconditional (Kupiec) and conditional (Christoffersen) coverage tests for the correct number of exceedances (see details in Christoffersen(1998) and Christoffersen et al. (2001)).

The unconditional (Kupiec) test also refer to as POF-test (Proportion of failure) with its null hypothesis given as

$$H_0 : p = \hat{p} = \frac{y}{T}$$

Here y is the number of exceptions and T is the number of observations and k is the confidence level. The test is given as

$$LR_{POF} = -2\ln\left(\frac{(1-p)^{T-y} p^y}{[1-(\frac{y}{T})^{T-y} (\frac{y}{T})^y]}\right).$$

Under the null hypothesis that the model is correct and LR_{POF} is asymptotically chi-squared (χ^2) distributed with degree of freedom as one (1). If the value of the LR_{POF} statistic is greater than the critical value (or p-value<0.01 for 1% level of significant or p-value<0.05 for 5% level of significant) the null hypothesis is rejected and the model then is inaccurate.

The Christoffersen's Interval Forecast Test combined the independence statistic with the Kupiec's POF test to obtained the joint test (Christoffersen, 1998; Nieppola, 2009). This test examined the properties of a good VaR model, the correct failure rate and independence of exceptions, that is condition coverage (cc). the conditional coverage (cc) is given as

$$LR_{cc} = LR_{POF} + LR_{ind}$$

Where

$$LR_{ind} = \sum_{i=2}^n \left[-2\ln\left(\frac{p(1-p)^{u_i-1}}{(\frac{1}{u_i})(1-\frac{1}{u_i})^{u_i-1}}\right) \right] - 2\ln\left(\frac{p(1-p)^{u-1}}{(\frac{1}{u})(1-\frac{1}{u})^{u-1}}\right)$$

Where u_i is the time between exceptions I and i-1 while u is the sum of u_i

If the value of the LR_{cc} statistic is greater than the critical value (or p-value<0.01 for 1% level of significant or p-value<0.05 for 5% level of significant) the null hypothesis is rejected and that leads to the rejection of the model.

3.12 Distributions of GARCH models

In this study we employed two innovations namely student t and skewed student t distributions they can account for excess kurtosis and non-normality in financial returns (Heracleous, 2003; Wilhelmsson, 2016; Kuhe, 2018).

The student t distribution is given as

$$f(y) = \frac{\Gamma(\frac{v+1}{2})}{\sqrt{v\pi}\Gamma(\frac{v}{2})} \left(1 + \frac{y^2}{v}\right)^{-\frac{(v+1)}{2}}; -\infty < y < \infty$$

The Skewed student t distribution is given as

$$f(y; \mu, \sigma, v, \lambda) = \begin{cases} bc \left(1 + \frac{1}{v-2} \left(\frac{b(\frac{y-\mu}{\sigma})+a}{1-\lambda}\right)^2\right)^{-\frac{v+1}{2}}, & \text{if } y < -\frac{a}{b} \\ bc \left(1 + \frac{1}{v-2} \left(\frac{b(\frac{y-\mu}{\sigma})+a}{1+\lambda}\right)^2\right)^{-\frac{v+1}{2}}, & \text{if } y \geq -\frac{a}{b} \end{cases}$$

Where v is the shape parameter with $2 < v < \infty$ and λ is the skewness parameter with $-1 < \lambda < 1$.

The constants a, b and c are given as

$$a = 4\lambda c \left(\frac{v-2}{v-1}\right); b = 1 + 3(\lambda)^2 - a^2; c = \frac{\Gamma(\frac{v+1}{2})}{\sqrt{\pi(v-2)}\Gamma(\frac{v}{2})}$$

Where μ and σ are the mean and the standard deviation of the skewed student t distribution respectively.

4.0 Materials and Methods

The data used in this study was collected from www.cashcraft.com under stock trend and analysis. Daily stock price for Guaranty Trust Bank Nigeria plc from January 2nd 2001 to May 8th 2017 (a total of 4017 observations) was collected from the website. The returns was calculated using the formula below

$$R_t = \ln P_t - \ln P_{t-1}. \quad (30)$$

Then a total observation becomes 4016.

5.0 Results

The analyses of this study was carried in R environment using rugarch package by Ghalanos (2018) and PerformanceAnalytics package by Peterson et al.(2018). The section begins with the descriptive statistics of the daily stock price of GT Bank Nigeria, plc. Figures 1, 2, 3 and 4 presents the plot of the daily actual price of GT bank stock, the plot of the log Transform of the actual price of GT bank stock, the plot of log transformed of stock returns of GT Bank daily stock price and the plot of cleansed log transform of stock returns of GT Bank respectively.

Fig. 1: Plot of the Actual price of GT bank stock

Figure 1 above presents the Actual price of the Guaranty Bank stock plc from January 2nd 2001 to May 8th 2017. The figure exhibited some trend.

Fig. 2: Plot of the log Transform of the Actual price of GT bank stock

Figure 2 above presents the log transform of the Actual price of the Guaranty Bank stock plc from January 2nd 2001 to May 8th 2017. The figure exhibited some pattern and achieved stability through transformation.

Fig 3: Plot of log transformed of stock returns of GT Bank

Figure 3 above presents the log transform of the stock returns of the Guaranty Bank stock plc from January 2nd 2001 to May 8th 2017. The figure actually exhibited the pattern of a typical financial time series that is volatility.

Fig 4: Plot of cleansed log transform of stock returns of GT Bank

Figure 4 above presents the cleansed log transform of the stock returns of the Guaranty Bank stock plc from January 2nd 2001 to May 8th 2017. This is done to remove the effects of possible outliers if any in the financial time series. The analysis of the financial time series in this study will be based on this cleansed series.

Table 1: Summary Statistics of Daily stock Returns of Guaranty Trust Bank Nigeria Plc

Statistics	Log of returns of Daily Stock price	Actual Daily Stock Price	Log transform of Daily Actual Stock price
Min	-2.28279	1.02	0.01980263
Max	2.28279	39.98	3.688379
Median	0	16.13	2.780681
Mean	0.0004655802	17.32804	2.71587
Estimated sd	0.059 99086	8.334726	0.5533098
Estimated skewness	-0.2103046	0.3325078	-0.5420392
Estimated kurtosis	1049.868	2.242218	2.418776
Jarque-Bera Normality Test	X-squared: 182929273.4134 p Value: < 2.2e-16	X-squared: 170.2176 p Value: < 2.2e-16	X-squared: 253.2493 p Value: < 2.2e-16
Number of Observations	4016	4017	4017
ARCH Test	Chi-squared = 830.2 p-value < 2.2e-16	Chi-squared = 3984.9 p-value < 2.2e-16	Chi-squared = 3978.3 p-value < 2.2e-16
ADF-first difference test	test-statistic is: -91.9653 p-value: < 2.2e-16	test-statistic is: -45.1483 p-value: < 2.2e-16	test-statistic is: -61.079 p-value: < 2.2e-16

Table 1 above examined the characteristics of the financial time series used in this study. The actual stock price, the log transform of the stock price and the log transform of the stock returns exhibited the characteristics of a typical financial time series (i.e evidence of volatility) (Abdulkareem & Abdulkareem, 2016). The series exhibited large standard deviation, skewness and kurtosis. The series further exhibited non-normality using Jarque-Bera Statistic (p-values<0.05) and shows the presence of ARCH effects (p-values<0.05), and all the type of series exhibited stationarity at first difference. In addition the averages of the stock series revealed positive values; this implies that the stock price is gaining. With this characteristics revealed above, GARCH and ARMA-GARCH models are appropriate in studying the volatility of the Guaranty Trust Bank stock returns.

Table 2: The Performance of the ARMA(1,1)-GARCH(1,1) Models using Information Criteria with respect to the distributions

Models	Information Criteria	Student t distribution	Skewed student t distribution
ARMA (1,1) -eGARCH (1,1)	Akaike	-4.8457	-4.8480
	Bayes	-4.8347	-4.8354
	Shibata	-4.8457	-4.8480
	Hannan-Quinn	-4.8418	-4.8435
ARMA (1,1) -TGARCH (1,1)	Akaike	-6.0027	-6.0064
	Bayes	-5.9917	-5.9939
	Shibata	-6.0027	-6.0064
	Hannan-Quinn	-5.9988	-6.0020
ARMA (1,1) -NAGARCH (1,1)	Akaike	-5.0575	-5.0519
	Bayes	-5.0466	-5.0393
	Shibata	-5.0575	-5.0519
	Hannan-Quinn	-5.0536	-5.0474
ARMA (1,1) -AVGARCH (1,1)	Akaike	-5.9990	-5.9566
	Bayes	-5.9864	-5.9425
	Shibata	-5.9990	-5.9566
	Hannan-Quinn	-5.9945	-5.9516

In table 2 above, four competing models are compared using student t distribution and skewed student t distribution. The following information criteria such as Akaike, Bayes, Shibata and Hannan-Quinn were used in selecting the preferred model. The results revealed ARMA(1,1)-TGARCH(1,1) as preferred model with the least values of the information criteria using student t and skewed student t distributions.

Table 3: The persistence and half-life volatility of the ARMA(1,1)-GARCH(1,1) models with respect to the distributions

Models	Distributions	Persistence	Half-life (Days)
ARMA (1,1) -eGARCH (1,1)	Student t distribution	0.8822875	5.534669
	Skewed student t distribution	0.8853582	5.692593
ARMA (1,1) -TGARCH (1,1)	Student t distribution	0.9515151	13.94671
	Skewed student t distribution	0.9503758	13.6184
ARMA (1,1) -NAGARCH (1,1)	Student t distribution	0.992533	92.48072
	Skewed student t distribution	0.9855705	47.68934
ARMA (1,1) -AVGARCH (1,1)	Student t distribution	0.9481713	13.02416
	Skewed student t distribution	0.939799	11.16372

Evidence from persistence and half-life volatility in table 3 above shows that the Guaranty trust stock returns can be modeled and predicted since all the persistence values are all less than 1 (one). ARMA(1,1)-NAGARCH(1,1) exhibited the highest persistence and half-life volatility values while ARMA(1,1)-eGARCH(1,1) exhibited the lowest persistence and half-life volatility values. For all the models, the days of mean-reverting ranges from 5 days to 95 days.

Table 4: Backtesting of the ARMA(1,1)-GARCH(1,1): GARCH Roll Forecast (Backtest Length: 1016)

Model	Distributions	Alpha	Expected Exceed	Actual VaR Exceed	Unconditional Coverage (Kupiec) H ₀ : Correct Exceedances	Conditional Coverage (Christoffersen) H ₀ : Correct Exceedances and independence of Failure
ARMA (1,1) -eGARCH (1,1)	Student t	1%	10.2	4	LR.uc Statistic: 4.9 LR.uc Critical: 6.635 LR.uc p-value: 0.027 Decision: Accept	LR.cc Statistic: 11.765 LR.cc Critical: 9.21 LR.cc p-value: 0.003 Decision: Reject
		5%	50.8	60	LR.uc Statistic: 1.662 LR.uc Critical: 3.841 LR.uc p-value: 0.197 Decision : Accept	LR.cc Statistic: 2.267 LR.cc Critical: 5.991 LR.cc p-value: 0.322 Decision: Accept
	Skewed student t	1%	10.2	6	LR.uc Statistic: 2.017 LR.uc Critical: 6.635 LR.uc p-value: 0.156 Decision: Accept	LR.cc Statistic: 2.088 LR.cc Critical: 9.21 LR.cc p-value: 0.352 Decision: Accept
		5%	50.8	55	LR.uc Statistic: 0.356 LR.uc Critical: 3.841 LR.uc p-value: 0.551 Decision: Accept	LR.cc Statistic: 0.356 LR.cc Critical: 5.991 LR.cc p-value: 0.837 Decision: Accept
	Student t	1%	10.2	38	LR.uc Statistic: 45.351 LR.uc Critical: 6.635 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 45.578 LR.cc Critical: 9.21 LR.cc p-value: 0 Decision: Reject
		5%	50.8	96	LR.uc Statistic: 33.949 LR.uc Critical: 3.841 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 34.06 LR.cc Critical: 5.991 LR.cc p-value: 0 Decision: Reject
ARMA (1,1) -TGARCH (1,1)	Student t	1%	10.2	38	LR.uc Statistic: 45.351 LR.uc Critical: 6.635 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 45.578 LR.cc Critical: 9.21 LR.cc p-value: 0 Decision: Reject
		5%	50.8	96	LR.uc Statistic: 33.949 LR.uc Critical: 3.841 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 34.06 LR.cc Critical: 5.991 LR.cc p-value: 0 Decision: Reject
	Skewed student t	1%	10.2	38	LR.uc Statistic: 45.351 LR.uc Critical: 6.635 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 45.578 LR.cc Critical: 9.21 LR.cc p-value: 0 Decision: Reject
		5%	50.8	96	LR.uc Statistic: 33.949 LR.uc Critical: 3.841 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 34.06 LR.cc Critical: 5.991 LR.cc p-value: 0 Decision: Reject
ARMA (1,1) -	Student t	1%	10.2	28	LR.uc Statistic: 21.408 LR.uc Critical: 6.635	LR.cc Statistic: 22.997 LR.cc Critical: 9.21

NAGARCH (1,1)					LR.uc p-value: 0 Decision: Reject	LR.cc p-value: 0 Decision: Reject	
		5%	50.8	90	LR.uc Statistic: 26.158 LR.uc Critical: 3.841 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 26.31 LR.cc Critical: 5.991 LR.cc p-value: 0 Decision: Reject	
	Skewed student t	1%	10.2	30	LR.uc Statistic: 25.678 LR.uc Critical: 6.635 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 27.506 LR.cc Critical: 9.21 LR.cc p-value: 0 Decision: Reject	
		5%	50.8	90	LR.uc Statistic: 26.158 LR.uc Critical: 3.841 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 26.31 LR.cc Critical: 5.991 LR.cc p-value: 0 Decision: Reject	
	ARMA (1,1) – AVGARCH (1,1)	Student t	1%	10.2	38	LR.uc Statistic: 45.351 LR.uc Critical: 6.635 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 45.578 LR.cc Critical: 9.21 LR.cc p-value: 0 Decision: Reject
			5%	50.8	96	LR.uc Statistic: 33.949 LR.uc Critical: 3.841 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 34.419 LR.cc Critical: 5.991 LR.cc p-value: 0 Decision: Reject
Skewed student t		1%	10.2	37	LR.uc Statistic: 42.685 LR.uc Critical: 6.635 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 42.982 LR.cc Critical: 9.21 LR.cc p-value: 0 Decision: Reject	
		5%	50.8	97	LR.uc Statistic: 35.33 LR.uc Critical: 3.841 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 35.398 LR.cc Critical: 5.991 LR.cc p-value: 0 Decision: Reject	

Backtesting approach is a means to select and use financial GARCH models for real life application. This approach revealed ARMA(1,1)-eGARCH(1,1) as good model in respective of the distribution but only failed at 1% alpha level in student t distribution, this is supported by figures 5 to 8 at the appendix B while other models failed the Backtesting (figures omitted because of space). Furthermore, coefficients of the ARMA(1,1)-eGARCH(1,1) model for both distributions (see Tables 8 and 9 at the appendix A) are more significant when compared to the other models (that is, ARMA(1,1)-TGARCH(1,1); ARMA(1,1)-NAGARCH(1,1) and ARMA(1,1)-AVGARCH(1,1)) (see Tables 10 to 15 at the appendix A). These results led us to consider higher order GARCH model as ARMA(1,1)-GARCH(2,2) models.

Table 5: The Performance of the ARMA(1,1)-GARCH(2,2) Models using Information Criteria with respect to the distributions

Models	Information Criteria	Student t distribution	Skewed student t distribution
ARMA (1,1) -eGARCH (2,2)	Akaike	-5.1904	-5.1334
	Bayes	-5.1748	-5.1162
	Shibata	-5.1904	-5.1335
	Hannan-Quinn	-5.1849	-5.1273
ARMA (1,1) -TGARCH (2,2)	Akaike	-5.9878	-5.9916
	Bayes	-5.9721	-5.9743
	Shibata	-5.9878	-5.9916
	Hannan-Quinn	-5.9822	-5.9855
ARMA (1,1) -NAGARCH (2,2)	Akaike	-5.0607	-5.0621
	Bayes	-5.0450	-5.0449
	Shibata	-5.0607	-5.0621
	Hannan-Quinn	-5.0551	-5.0560
ARMA (1,1) -AVGARCH (2,2)	Akaike	-6.0110	-5.9200
	Bayes	-5.9922	-5.8996
	Shibata	-6.0110	-5.9200
	Hannan-Quinn	-6.0043	-5.9127

In table 5 above, four competing models are compared with respect to student t distribution and skewed student t distribution. The following information criteria such as Akaike, Bayes, Shibata and Hannan-Quinn were used in selecting the preferred model. The results revealed ARMA(1,1)-AVGARCH(2,2) is preferred for student t distribution and ARMA(1,1)-TGARCH(2,2) model is preferred for skewed student t distribution.

Table 6: The persistence and half-life volatility of the ARMA(1,1)-GARCH(2,2) models with respect to the distributions

Models	Distributions	Persistence	Half-life (Days)
ARMA (1,1) -eGARCH (2,2)	Student t distribution	0.9745043	26.83874
	Skewed student t distribution	0.9576603	16.02202
ARMA (1,1) -TGARCH (2,2)	Student t distribution	0.9425471	11.71463
	Skewed student t distribution	0.9399941	11.20117
ARMA (1,1) -NAGARCH (2,2)	Student t distribution	0.9810724	36.27328
	Skewed student t distribution	0.986655	51.59337
ARMA (1,1) -AVGARCH (2,2)	Student t distribution	0.9876131	55.61095
	Skewed student t distribution	0.9537416	14.63495

Evidence from persistence and half-life volatility in table 6 above shows that the Guaranty trust stock returns can be modeled and predicted since all the persistence values are all less than 1 (one). ARMA(1,1)-AVGARCH(2,2) exhibited the highest persistence and half-life volatility values with respect to student t distribution while ARMA(1,1)-NAGARCH(2,2) exhibited the highest persistence and half-life volatility values with respect to skew student t distribution. The ARMA(1,1)-TGARCH(2,2) exhibited the lowest persistence and half-life volatility values for both distributions under consideration. For all the models, the days of mean-reverting ranges from 10 days to 60 days.

Table 7: Backtesting of the ARMA(1,1)-GARCH(2,2): GARCH Roll Forecast (Backtest Length: 1016)

Model	Distributions	Alpha	Expected Exceed	Actual VaR Exceed	Unconditional Coverage (Kupiec) H ₀ : Correct Exceedances	Conditional Coverage (Christoffersen) H ₀ : Correct Exceedances and independence of Failure
ARMA (1,1) – eGARCH (2,2)	Student t	1%	10.2	9	LR.uc Statistic: 0.139 LR.uc Critical: 6.635 LR.uc p-value: 0.709 Decision: Accept	LR.cc Statistic: 0.3 LR.cc Critical: 9.21 LR.cc p-value: 0.861 Decision: Accept
		5%	50.8	58	LR.uc Statistic: 1.029 LR.uc Critical: 3.841 LR.uc p-value: 0.31 Decision: Accept	LR.cc Statistic: 1.878 LR.cc Critical: 5.991 LR.cc p-value: 0.391 Decision: Accept
	Skewed student t	1%	10.2	7	LR.uc Statistic: 1.114 LR.uc Critical: 6.635 LR.uc p-value: 0.291 Decision: Accept	LR.cc Statistic: 1.211 LR.cc Critical: 9.21 LR.cc p-value: 0.546 Decision: Accept
		5%	50.8	66	LR.uc Statistic: 4.393 LR.uc Critical: 3.841 LR.uc p-value: 0.036 Decision: Reject	LR.cc Statistic: 6.075 LR.cc Critical: 5.991 LR.cc p-value: 0.048 Decision: Reject
	Student t	1%	10.2	31	LR.uc Statistic: 27.918 LR.uc Critical: 6.635 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 27.921 LR.cc Critical: 9.21 LR.cc p-value: 0 Decision: Reject
		5%	50.8	89	LR.uc Statistic: 24.944 LR.uc Critical: 3.841 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 25.637 LR.cc Critical: 5.991 LR.cc p-value: 0 Decision: Reject
	Skewed student t	1%	10.2	34	LR.uc Statistic: 35.027 LR.uc Critical: 6.635 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 35.046 LR.cc Critical: 9.21 LR.cc p-value: 0 Decision: Reject
		5%	50.8	90	LR.uc Statistic: 26.158 LR.uc Critical: 3.841 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 26.736 LR.cc Critical: 5.991 LR.cc p-value: 0 Decision: Reject
ARMA (1,1) –	Student t	1%	10.2	41	LR.uc Statistic: 53.675 LR.uc Critical: 6.635	LR.cc Statistic: 53.748 LR.cc Critical: 9.21

NAGARCH (2, 2)					LR.uc p-value: 0 Reject Null: YES	LR.cc p-value: 0 Decision: Reject
		5%	50.8	109	LR.uc Statistic: 53.615 LR.uc Critical: 3.841 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 53.624 LR.cc Critical: 5.991 LR.cc p-value: 0 Decision: Reject
	Skewed student t	1%	10.2	30	LR.uc Statistic: 25.678 LR.uc Critical: 6.635 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 25.693 LR.cc Critical: 9.21 LR.cc p-value: 0 Decision: Reject
		5%	50.8	92	LR.uc Statistic: 28.66 LR.uc Critical: 3.841 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 30.415 LR.cc Critical: 5.991 LR.cc p-value: 0 Decision: Reject
	Student t	1%	10.2	34	LR.uc Statistic: 35.027 LR.uc Critical: 6.635 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 35.603 LR.cc Critical: 9.21 LR.cc p-value: 0 Decision: Reject
		5%	50.8	86	LR.uc Statistic: 21.449 LR.uc Critical: 3.841 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 22.551 LR.cc Critical: 5.991 LR.cc p-value: 0 Decision: Reject
ARMA (1, 1) – AVGARCH (2, 2)	Skewed student t	1%	10.2	36	LR.uc Statistic: 40.074 LR.uc Critical: 6.635 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 40.143 LR.cc Critical: 9.21 LR.cc p-value: 0 Decision: Reject
		5%	50.8	89	LR.uc Statistic: 24.944 LR.uc Critical: 3.841 LR.uc p-value: 0 Decision: Reject	LR.cc Statistic: 26.371 LR.cc Critical: 5.991 LR.cc p-value: 0 Decision: Reject

Backtesting approach revealed ARMA(1,1)-eGARCH(2,2) as good model in respective of the distribution at 1% and 5% alpha levels, this is supported by figures 9 to 12 at the appendix B while other models failed the Backtesting (figures omitted because of space). Furthermore, coefficients of the ARMA(1,1)-eGARCH(2,2) model for both distributions are more significant (see Tables 16 and 17 at appendix A) when compared to the other models (that is, ARMA(1,1)-TGARCH(2,2); ARMA(1,1)-NAGARCH(2,2) and ARMA(1,1)-AVGARCH(2,2)) see Tables 18 to 23.

5.1 Discussion of Results

The log transform of the Guaranty Trust Bank stock returns exhibited the characteristics of a typical financial time series (i.e evidence of volatility) (Abdulkareem & Abdulkareem, 2016) as shown in Table 1. The series exhibited large standard deviation, skewness and kurtosis. The series further exhibited non-normality using Jarque-Bera Statistic (p-values<0.05), shows the presence of ARCH effects (p-values<0.05) and the series exhibited stationarity at first difference. In addition the average value of the returns revealed a positive value which implies that the stock price is gaining (Kuhe, 2018). With these characteristics of the stock returns, the GARCH and ARMA-GARCH models are appropriate in studying the volatility of the Guaranty Trust Bank stock returns (Emenike & Ani, 2014; Ahmed et al, 2018).

In table 2, the four competing models were compared using student t distribution and skewed student t distribution. The following information criteria such as Akaike, Bayes, Shibata and Hannan-Quinn were used to select the preferred model. The results revealed ARMA(1,1)-TGARCH(1,1) as preferred model with the least values of the information criteria for both student t and skew student t distributions.

Evidence from persistence and half-life volatility in table 3 shows that the Guaranty trust stock returns can be modeled and predicted since all the persistence values are all less than 1 (one). This also means that the models ensures positive conditional variance as well as stationary (Banerjee and Sarkar, 2006; Ahmed et al, 2018). The ARMA(1,1)-NAGARCH(1,1) exhibited the highest persistence and half-life volatility values while ARMA(1,1)-eGARCH(1,1) exhibited the lowest persistence and half-life volatility values for both distributions. For all the models, the days of mean-reverting ranges from 5 days to 95 days (that is within three (3) months).

Backtesting approach is a means to select and use financial GARCH models for real life application. This approach revealed ARMA(1,1)-eGARCH(1,1) as good model for both distributions but only failed the Conditional Coverage (Christoffersen), this is Correct Exceedances and independence of Failure at 1% alpha level in student t distribution, this is supported by figures 5 to 8 at the appendix B. This contradicts the results from the information criteria that selected ARMA(1,1)-TGARCH(1,1) as the preferred model. This suggests model should not be selected by information criteria only but should be selected in addition by how significant is the coefficients of the model and possibly by backtesting approach (Christoffersen, 1998; Nieppola, 2009). The other models under considerations failed the Backtesting (figures omitted because of space). Furthermore, coefficients of the ARMA(1,1)-eGARCH(1,1) model for both distributions (see Tables 8 and 9 in Appendix A) are more significant when compared to the other models (that is, ARMA(1,1)-TGARCH(1,1); ARMA(1,1)-NAGARCH(1,1) and ARMA(1,1)-AVGARCH(1,1)) (see Tables 10 to 15 in Appendix). These results led us to consider higher order GARCH model as ARMA(1,1)-GARCH(2,2) models which follows the works Starica (2003) and, Hansen and Lunde (2005) that revealed that the GARCH(1,1) was clearly inferior to models that can accommodate a leverage effect. But our results contradicts the work of Namugaya et al (2014) that GARCH(1,1) outperformed the higher order of GARCH models, this could be because their work did not consider how good is their model.

In table 5, the four competing models of higher order were compared with respect to student t distribution and skewed student t distribution. The following information criteria such as Akaike, Bayes, Shibata and Hannan-Quinn were employed to select the preferred model. The results revealed ARMA(1,1)-AVGARCH(2,2) is preferred for student t distribution and ARMA(1,1)-TGARCH(2,2) model is preferred for skew student t distribution.

Evidence from persistence and half-life volatility in table 6 shows that the Guaranty trust stock returns can be modeled and predicted since all the persistence values are all less than 1 (one). This also means that the models ensures positive conditional variance as well as stationary (Banerjee and Sarkar, 2006; Ahmed et al, 2018). ARMA(1,1)-AVGARCH(2,2) exhibited the highest persistence and half-life volatility values with respect to student t distribution while ARMA(1,1)-NAGARCH(2,2) exhibited the highest persistence and half-life volatility values with respect to skewed student t distribution. The ARMA(1,1)-TGARCH(2,2)

exhibited the lowest persistence and half-life volatility values for both distributions under consideration. For all the models, the days of mean-reverting ranges from 10 days to 60 days. Backtesting approach revealed ARMA(1,1)-eGARCH(2,2) in Table 7 as good model in respective of the distribution at 1% and 5% alpha levels, this is supported by figures 9 to 12 at the appendix B while other models failed the Backtesting (figures omitted because of space). Furthermore, coefficients of the ARMA(1,1)-eGARCH(2,2) model for both distributions (see Tables 16 and 17 in Appendix A) are more significant when compared to the other models (that is, ARMA(1,1)-TGARCH(2,2); ARMA(1,1)-NAGARCH(2,2) and ARMA(1,1)-AVGARCH(2,2)) (see Tables 18 to 23 in Appendix A). As mention earlier, ARMA(1,1)-eGARCH(2,2) was selected because it completely passed the backtesting though ARMA(1,1)-AVGARCH(2,2) was selected by information criteria. This suggests model should not be selected by information criteria only but should be selected in addition by how significant is the coefficients of the model and possibly by backtesting approach(Christoffersen, 1998; Nieppola, 2009). Lastly, in all the models considered, there were no ARCH effects in the residuals of the estimated models.

6.0 Conclusion and recommendations

The Guaranty Trust (GT) Bank's culture is tied to eight guiding principles called the Orange Rules; Simplicity, Professionalism, Service, Friendliness, Excellence, Trustworthiness, Social Responsibility and Innovation. With these features of GT Bank, the bank stands suitable for customers and investor both in Nigeria and abroad where the bank has branches. In fact, the US Embassy used GT Bank as a means paying for Visa application in Nigeria. The GT bank stock investors and customers should be assured as GT bank is a stock in emerging market to invest in (Ahmed et al., 2018). This is true because all the models considered ensures positive conditional variance as well as stationary (Banerjee and Sarkar, 2006; Ahmed et al, 2018).

The study revealed that using the lowest information criteria values only could not be enough to select preferred GARCH model rather we should add the use of backtesting. The models fitted exhibited high persistency in the daily stock returns while the days it takes for mean-reverting of the models ranges from 5 days to 100 days. The results further revealed ARMA(1,1)-GARCH (2,2) model with student t distribution provides a suitable model for evaluating the GT bank

stock returns among the competing models while it takes less than 30 days for the persistence volatility to return back to its average value of the stock returns.

This study recommended that researchers should adopt backtesting approach while fitting GARCH models while GT bank stocks investor should be assured that no matter the fluctuations in the stock market, the GT bank stock returns has the ability to returns to its mean price returns.

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Appendix A

Table 8: Estimates of ARMA(1,1)-eGARCH(1,1) with std

-----*				
* GARCH Model Fit *				

Conditional Variance Dynamics				

GARCH Model	: eGARCH(1,1)			
Mean Model	: ARFIMA(1,0,1)			
Distribution	: std			

Optimal Parameters				

	Estimate	Std. Error	t value	Pr(> t)
arl	0.090426	0.135239	0.66864	0.503724
mal	0.015825	0.133828	0.11825	0.905871
omega	-0.638445	0.020334	-31.39719	0.000000
alpha1	0.192669	0.052765	3.65144	0.000261
beta1	0.882287	0.001346	655.31371	0.000000
gamma1	1.831176	0.019753	92.70358	0.000000
shape	2.100000	0.009205	228.12482	0.000000

Robust Standard Errors:

	Estimate	Std. Error	t value	Pr(> t)
arl	0.090426	0.143390	0.63063	0.52828
mal	0.015825	0.149513	0.10584	0.91571
omega	-0.638445	0.081165	-7.86604	0.00000
alpha1	0.192669	0.171535	1.12321	0.26135
beta1	0.882287	0.002625	336.12545	0.00000
gamma1	1.831176	0.357599	5.12075	0.00000
shape	2.100000	0.049998	42.00152	0.00000

LogLikelihood : 9737.114

Information Criteria

Akaike	-4.8457
Bayes	-4.8347
Shibata	-4.8457
Hannan-Quinn	-4.8418

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.06633	0.7968
Lag[2* (p+q) + (p+q) -1] [5]	0.08466	1.0000
Lag[4* (p+q) + (p+q) -1] [9]	0.23001	1.0000
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.0004513	0.9831
Lag[2* (p+q) + (p+q) -1] [5]	0.0013750	1.0000
Lag[4* (p+q) + (p+q) -1] [9]	0.0022635	1.0000
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.0004649	0.500	2.000	0.9828
ARCH Lag[5]	0.0011178	1.440	1.667	1.0000
ARCH Lag[7]	0.0016103	2.315	1.543	1.0000

Nyblom stability test

Joint Statistic:	10.9421
Individual Statistics:	
arl	1.2313
mal	1.2554
omega	2.7533
alpha1	0.8187
beta1	2.1565
gamma1	0.6688
shape	0.2326

Asymptotic Critical Values (10% 5% 1%)
 Joint Statistic: 1.69 1.9 2.35
 Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob sig
Sign Bias	0.98272	0.3258
Negative Sign Bias	0.33742	0.7358
Positive Sign Bias	0.02935	0.9766
Joint Effect	1.04044	0.7915

Adjusted Pearson Goodness-of-Fit Test:

	group	statistic	p-value (g-1)
1	20	852	1.903e-168
2	30	1131	1.637e-219
3	40	1420	7.300e-273
4	50	1694	7.905e-323

Table 9: Estimates of ARMA(1,1)-eGARCH(1,1) with sstd

-----*

* GARCH Model Fit *

Conditional Variance Dynamics

GARCH Model : eGARCH(1,1)

Mean Model : ARFIMA(1,0,1)

Distribution : sstd

Optimal Parameters

	Estimate	Std. Error	t value	Pr(> t)
arl	0.090252	0.059767	1.51006	0.131029
mal	0.013462	0.058206	0.23129	0.817091
omega	-0.355651	0.021578	-16.48217	0.000000
alpha1	0.617628	0.165270	3.73708	0.000186
beta1	0.885358	0.004641	190.76635	0.000000
gamma1	5.501824	0.189529	29.02895	0.000000
skew	1.000633	0.009679	103.38505	0.000000
shape	2.010000	0.000613	3278.52434	0.000000

Robust Standard Errors:

	Estimate	Std. Error	t value	Pr(> t)
arl	0.090252	0.038433	2.34831	0.018859
mal	0.013462	0.025677	0.52431	0.600066
omega	-0.355651	0.091076	-3.90499	0.000094
alpha1	0.617628	0.518456	1.19128	0.233542
beta1	0.885358	0.026923	32.88446	0.000000
gamma1	5.501824	0.598899	9.18656	0.000000
skew	1.000633	0.007893	126.77568	0.000000
shape	2.010000	0.001837	1094.18647	0.000000

LogLikelihood : 9742.685

Information Criteria

Akaike	-4.8480
Bayes	-4.8354
Shibata	-4.8480
Hannan-Quinn	-4.8435

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.07050	0.7906
Lag[2* (p+q) + (p+q) -1] [5]	0.08858	1.0000
Lag[4* (p+q) + (p+q) -1] [9]	0.23150	1.0000
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

-------	--	--

```

                statistic p-value
Lag[1]          0.0004545   0.983
Lag[2*(p+q)+(p+q)-1][5] 0.0013845 1.000
Lag[4*(p+q)+(p+q)-1][9] 0.0022795 1.000
d.o.f=2

```

Weighted ARCH LM Tests

```

-----
Statistic Shape Scale P-Value
ARCH Lag[3] 0.0004681 0.500 2.000 0.9827
ARCH Lag[5] 0.0011254 1.440 1.667 1.0000
ARCH Lag[7] 0.0016218 2.315 1.543 1.0000

```

Nyblom stability test

Joint Statistic: 12.5781

Individual Statistics:

```

arl      1.2117
mal      1.2345
omega    2.6897
alpha1   0.9021
beta1    1.7730
gamma1   0.6292
skew     0.1230
shape    0.2369

```

Asymptotic Critical Values (10% 5% 1%)

```

Joint Statistic:      1.89 2.11 2.59
Individual Statistic: 0.35 0.47 0.75

```

Sign Bias Test

```

-----
t-value  prob sig
Sign Bias      0.98086 0.3267
Negative Sign Bias 0.33726 0.7359
Positive Sign Bias 0.03006 0.9760
Joint Effect    1.03727 0.7922

```

Adjusted Pearson Goodness-of-Fit Test:

```

-----
group statistic p-value(g-1)
1    20      961.4    9.289e-192
2    30     1308.9    2.714e-257
3    40     1647.1    7.026e-321
4    50     1970.2    0.000e+00

```

Table 10: Estimates of ARMA(1,1)-TGARCH(1,1) with std

```

*-----*
*          GARCH Model Fit          *
*-----*

```

Conditional Variance Dynamics

```

-----
GARCH Model      : fGARCH(1,1)
fGARCH Sub-Model : TGARCH
Mean Model       : ARFIMA(1,0,1)
Distribution      : std

```

Optimal Parameters

```

-----
Estimate Std. Error t value Pr(>|t|)
arl      0.260521   0.018296 14.23931 0.00000
mal      -0.111401  0.018403 -6.05358 0.00000
omega    0.000000   0.000000  0.18102 0.85635
alpha1   0.695885   0.015610 44.58006 0.00000
beta1    0.499255   0.008670 57.58337 0.00000
etal1    -0.005608  0.021908 -0.25598 0.79797
shape    3.117434   0.057788 53.94586 0.00000

```

Robust Standard Errors:

```

Estimate Std. Error t value Pr(>|t|)
arl      0.260521   0.327748  0.794880 0.426683
mal      -0.111401  0.423033 -0.263339 0.792289
omega    0.000000   0.000065  0.000551 0.999560
alpha1   0.695885   2.953122  0.235644 0.813709
beta1    0.499255   2.233770  0.223503 0.823144

```

```

etal1    -0.005608   0.246792 -0.022723 0.981871
shape    3.117434   1.074083  2.902415 0.003703

```

LogLikelihood : 12060.42

Information Criteria

```

-----
Akaike      -6.0027
Bayes       -5.9917
Shibata     -6.0027
Hannan-Quinn -5.9988

```

Weighted Ljung-Box Test on Standardized Residuals

```

-----
                statistic p-value
Lag[1]          2.709e-10      1
Lag[2*(p+q)+(p+q)-1][5] 2.575e-08 1
Lag[4*(p+q)+(p+q)-1][9] 5.878e-08 1
d.o.f=2

```

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

```

-----
                statistic p-value
Lag[1]          0.002273   0.962
Lag[2*(p+q)+(p+q)-1][5] 0.006825 1.000
Lag[4*(p+q)+(p+q)-1][9] 0.011387 1.000
d.o.f=2

```

Weighted ARCH LM Tests

```

-----
Statistic Shape Scale P-Value
ARCH Lag[3] 0.002273 0.500 2.000 0.9620
ARCH Lag[5] 0.005458 1.440 1.667 0.9998
ARCH Lag[7] 0.008126 2.315 1.543 1.0000

```

Nyblom stability test

Joint Statistic: 278.5786

Individual Statistics:

```

arl      0.2703
mal      0.1850
omega    127.6935
alpha1   65.3278
beta1    8.8435
etal1    1.1637
shape    3.4515

```

Asymptotic Critical Values (10% 5% 1%)

```

Joint Statistic:      1.69 1.9 2.35
Individual Statistic: 0.35 0.47 0.75

```

Sign Bias Test

```

-----
t-value  prob sig
Sign Bias      0.1211 0.9036
Negative Sign Bias 0.3729 0.7093
Positive Sign Bias 0.4161 0.6774
Joint Effect    0.3281 0.9547

```

Adjusted Pearson Goodness-of-Fit Test:

```

-----
group statistic p-value(g-1)
1    20      855.3    3.746e-169
2    30     1208.4    6.256e-236
3    40     1450.8    2.785e-279
4    50     1752.6    0.000e+00

```

Table 11: Estimates of ARMA(1,1)-TGARCH(1,1) with sstd

<

alpha1 68.81920
 beta1 10.30766
 eta11 1.58122
 skew 0.05679
 shape 3.51394

Asymptotic Critical Values (10% 5% 1%)
 Joint Statistic: 1.89 2.11 2.59
 Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob sig
Sign Bias	0.1977	0.8433
Negative Sign Bias	0.3610	0.7181
Positive Sign Bias	0.4341	0.6643
Joint Effect	0.3590	0.9486

Adjusted Pearson Goodness-of-Fit Test:

group	statistic	p-value (g-1)
1	20	960 1.798e-191
2	30	1290 3.095e-253
3	40	1638 5.095e-319
4	50	1936 0.000e+00

Table 12: Estimates of ARMA(1,1)-NAGARCH(1,1) with std

```

*-----*
*               GARCH Model Fit               *
*-----*

Conditional Variance Dynamics
-----
GARCH Model       : fGARCH(1,1)
fGARCH Sub-Model  : NAGARCH
Mean Model        : ARFIMA(1,0,1)
Distribution       : std

Optimal Parameters
-----
      Estimate      Std. Error    t value    Pr(>|t|)
arl      0.202254      0.161208      1.25462    0.20962
mal     -0.136256      0.166396     -0.81887    0.41286
omega     0.000000      0.000000      0.11911    0.90519
alpha1    0.348048      0.012211     28.50355    0.00000
beta1     0.643824      0.009687     66.46287    0.00000
eta21     0.043594      0.089369      0.48779    0.62570
shape     3.715735      0.102281     36.32859    0.00000

Robust Standard Errors:
      Estimate      Std. Error    t value    Pr(>|t|)
arl      0.202254      0.414763      0.48764    0.625807
mal     -0.136256      0.515821     -0.26415    0.791661
omega     0.000000      0.000073      0.00049    0.999609
alpha1    0.348048      1.863233      0.18680    0.851819
beta1     0.643824      1.833290      0.35119    0.725450
eta21     0.043594      0.332024      0.13130    0.895540
shape     3.715735      1.523823      2.43843    0.014751

LogLikelihood : 10162.54

Information Criteria
-----

Akaike          -5.0575
Bayes           -5.0466
Shibata         -5.0575
Hannan-Quinn   -5.0536

Weighted Ljung-Box Test on Standardized Residuals
-----
                                statistic p-value
Lag[1]                                0.03067  0.861
Lag[2*(p+q)+(p+q)-1][5]             0.03263  1.000
Lag[4*(p+q)+(p+q)-1][9]             0.05507  1.000
d.o.f=2

```

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	Statistic	p-value
Lag[1]	0.001840	0.9658
Lag[2* (p+q) + (p+q) - 1] [5]	0.006133	1.0000
Lag[4* (p+q) + (p+q) - 1] [9]	0.010398	1.0000
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.002144	0.500	2.000	0.9631
ARCH Lag[5]	0.005173	1.440	1.667	0.9998
ARCH Lag[7]	0.007636	2.315	1.543	1.0000

Nyblom stability test

Joint Statistic: 230.4385

Individual Statistics:

arl	0.3625
mal	0.3992
omega	101.8067
alpha1	51.9765
beta1	7.8753
eta21	1.3598
shape	4.1795

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.69 1.9 2.35

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob sig
Sign Bias	0.9102	0.3628
Negative Sign Bias	0.4771	0.6333
Positive Sign Bias	0.1879	0.8510
Joint Effect	1.0520	0.7887

Adjusted Pearson Goodness-of-Fit Test:

group	Statistic	p-value (g-1)
1	20	1161 2.433e-234
2	30	1491 4.173e-296
3	40	1744 0.000e+00
4	50	1987 0.000e+00

Table 13: Estimates of ARMA(1,1)-NAGARCH(1,1) with sstd

* GARCH Model Fit *

Conditional Variance Dynamics

GARCH Model : fGARCH(1,1)

fGARCH Sub-Model : NAGARCH

Mean Model : ARFIMA(1,0,1)

Distribution : sstd

Optimal Parameters

	Estimate	Std. Error	t value	Pr(> t)
arl	0.21159	0.166371	1.27179	0.20345
mal	-0.14934	0.172436	-0.86605	0.38646
omega	0.00000	0.000000	0.11943	0.90494
alpha1	0.34296	0.011964	28.66595	0.00000
beta1	0.64255	0.009524	67.46838	0.00000
eta21	0.01319	0.112685	0.11705	0.90682
skew	1.00799	0.012574	80.16590	0.00000
shape	3.73914	0.102750	36.39075	0.00000

Robust Standard Errors:

	Estimate	Std. Error	t value	Pr(> t)
arl	0.21159	0.251290	0.842011	0.399782
mal	-0.14934	0.358117	-0.417011	0.676671
omega	0.00000	0.000073	0.000492	0.999607
alpha1	0.34296	1.829657	0.187445	0.851312
beta1	0.64255	1.838481	0.349501	0.726713
eta21	0.01319	1.457099	0.009052	0.992778
skew	1.00799	0.041364	24.368965	0.000000
shape	3.73914	1.625545	2.300239	0.021435

LogLikelihood : 10152.18

Information Criteria

Akaike	-5.0519
Bayes	-5.0393
Shibata	-5.0519
Hannan-Quinn	-5.0474

Weighted Ljung-Box Test on Standardized Residuals

	Statistic	p-value
Lag[1]	0.03295	0.856
Lag[2* (p+q) + (p+q) - 1] [5]	0.03493	1.000
Lag[4* (p+q) + (p+q) - 1] [9]	0.05748	1.000
d.o.f=2		

H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	Statistic	p-value
Lag[1]	0.001834	0.9658
Lag[2* (p+q) + (p+q) - 1] [5]	0.006111	1.0000
Lag[4* (p+q) + (p+q) - 1] [9]	0.010360	1.0000
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.002136	0.500	2.000	0.9631
ARCH Lag[5]	0.005153	1.440	1.667	0.9998
ARCH Lag[7]	0.007607	2.315	1.543	1.0000

Nyblom stability test

Joint Statistic: 232.8581

Individual Statistics:

arl	0.37484
mal	0.41837
omega	101.51981
alpha1	55.11488
beta1	8.72212
eta21	0.65460
skew	0.07016
shape	4.15469

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 1.89 2.11 2.59

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

	t-value	prob sig
Sign Bias	0.9112	0.3622
Negative Sign Bias	0.4772	0.6333
Positive Sign Bias	0.1882	0.8508
Joint Effect	1.0544	0.7881

Adjusted Pearson Goodness-of-Fit Test:

group	Statistic	p-value (g-1)
1	20	1255 1.619e-254
2	30	1659 0.000e+00
3	40	1917 0.000e+00
4	50	2194 0.000e+00

Table 14: Estimates of ARMA(1,1)-AVGARCH(1,1) with std

```

*-----*
*          GARCH Model Fit          *
*-----*

```

Conditional Variance Dynamics

```

-----
GARCH Model      : fGARCH(1,1)
fGARCH Sub-Model : AVGARCH
Mean Model       : ARFIMA(1,0,1)
Distribution      : std

```

Optimal Parameters

```

-----
      Estimate Std. Error t value Pr(>|t|)
ar1      0.216232   0.020440  10.57901 0.000000
ma1     -0.091422   0.022696  -4.02803 0.000056
omega    0.000000   0.000000   0.17852 0.858313
alpha1   0.722956   0.015914  45.42756 0.000000
beta1    0.476527   0.008804  54.12823 0.000000
eta11   -0.026953   0.021307  -1.26499 0.205876
eta21    0.000193   0.001045   0.18441 0.853692
shape    3.141671   0.058494  53.70923 0.000000

```

Robust Standard Errors:

```

      Estimate Std. Error t value Pr(>|t|)
ar1      0.216232   0.228626   0.945789 0.344256
ma1     -0.091422   0.543642  -0.168165 0.866453
omega    0.000000   0.000068   0.000531 0.999576
alpha1   0.722956   2.763034   0.261653 0.793589
beta1    0.476527   2.080736   0.229018 0.818855
eta11   -0.026953   0.287347  -0.093800 0.925268
eta21    0.000193   0.002573   0.074930 0.940270
shape    3.141671   1.430933   2.195539 0.028125

```

LogLikelihood : 12053.98

Information Criteria

```

-----
Akaike      -5.9990
Bayes       -5.9864
Shibata     -5.9990
Hannan-Quinn -5.9945

```

Weighted Ljung-Box Test on Standardized Residuals

```

-----
              statistic p-value
Lag[1]        3.257e-08  0.9999
Lag[2*(p+q)+(p+q)-1][5] 1.757e-07  1.0000
Lag[4*(p+q)+(p+q)-1][9] 3.368e-07  1.0000
d.o.f=2
H0 : No serial correlation

```

Weighted Ljung-Box Test on Standardized Squared Residuals

```

-----
              statistic p-value
Lag[1]        0.002416  0.9608
Lag[2*(p+q)+(p+q)-1][5] 0.007254  1.0000
Lag[4*(p+q)+(p+q)-1][9] 0.012102  1.0000
d.o.f=2

```

Weighted ARCH LM Tests

```

-----
      Statistic Shape Scale P-Value
ARCH Lag[3]  0.002416 0.500 2.000 0.9608
ARCH Lag[5]  0.005801 1.440 1.667 0.9998
ARCH Lag[7]  0.008636 2.315 1.543 1.0000

```

Nyblom stability test

```

-----
Joint Statistic: 281.1214
Individual Statistics:
ar1      0.2856
ma1      0.4489

```

```

omega 130.6555
alpha1 69.7212
beta1  11.1180
eta11  0.5819
eta21  0.3538
shape  3.9707

```

```

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:      1.89 2.11 2.59
Individual Statistic: 0.35 0.47 0.75

```

Sign Bias Test

```

-----
              t-value prob sig
Sign Bias      0.1974 0.8435
Negative Sign Bias 0.3617 0.7176
Positive Sign Bias 0.4444 0.6568
Joint Effect    0.3668 0.9470

```

Adjusted Pearson Goodness-of-Fit Test:

```

-----
      group statistic p-value(g-1)
1      20          961    1.130e-191
2      30         1315    1.142e-258
3      40         1572    5.360e-305
4      50         1898    0.000e+00

```

Table 15: Estimates of ARMA(1,1)-AVGARCH(1,1) with sstd

```

*-----*
*          GARCH Model Fit          *
*-----*

```

Conditional Variance Dynamics

```

-----
GARCH Model      : fGARCH(1,1)
fGARCH Sub-Model : AVGARCH
Mean Model       : ARFIMA(1,0,1)
Distribution      : sstd

```

Optimal Parameters

```

-----
      Estimate Std. Error t value Pr(>|t|)
ar1      0.186919   0.052008  3.59404 0.000326
ma1     -0.044629   0.054695  -0.81596 0.414520
omega    0.000000   0.000000   0.18133 0.856108
alpha1   0.603595   0.013305  45.36510 0.000000
beta1    0.535856   0.008274  64.76511 0.000000
eta11   -0.023806   0.021919  -1.08608 0.277441
eta21    0.000161   0.001017   0.15863 0.873957
skew     1.008536   0.012038  83.78155 0.000000
shape    3.330057   0.069111  48.18388 0.000000

```

Robust Standard Errors:

```

      Estimate Std. Error t value Pr(>|t|)
ar1      0.186919   0.754665   0.247684 0.80438
ma1     -0.044629   0.299470  -0.149028 0.88153
omega    0.000000   0.000064   0.000561 0.99955
alpha1   0.603595   2.862902   0.210833 0.83302
beta1    0.535856   2.346030   0.228410 0.81933
eta11   -0.023806   0.347020  -0.068602 0.94531
eta21    0.000161   0.000145   1.112047 0.26612
skew     1.008536   0.025408  39.693201 0.00000
shape    3.330057   2.369263   1.405524 0.15987

```

LogLikelihood : 11969.9

Information Criteria

```

-----
Akaike      -5.9566
Bayes       -5.9425
Shibata     -5.9566
Hannan-Quinn -5.9516

```

Weighted Ljung-Box Test on Standardized Residuals

```

-----
              statistic p-value

```


Lag[1] 6.605e-09 0.9999
Lag[2*(p+q)+(p+q)-1][5] 1.011e-08 1.0000
Lag[4*(p+q)+(p+q)-1][9] 1.728e-08 1.0000
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.001817	0.966
Lag[2*(p+q)+(p+q)-1][5]	0.005456	1.000
Lag[4*(p+q)+(p+q)-1][9]	0.009103	1.000
d.o.f=2		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[3]	0.001817	0.500	2.000	0.9660
ARCH Lag[5]	0.004364	1.440	1.667	0.9999
ARCH Lag[7]	0.006496	2.315	1.543	1.0000

Nyblom stability test

Joint Statistic: 284.2378	
Individual Statistics:	
arl	0.32177
mal	0.17936
omega	119.81614
alpha1	71.63048
beta1	9.27011
eta11	0.99530
eta21	0.63851
skew	0.05967
shape	4.18341

Asymptotic Critical Values (10% 5% 1%)			
Joint Statistic:	2.1	2.32	2.82
Individual Statistic:	0.35	0.47	0.75

Sign Bias Test

	t-value	prob sig
Sign Bias	0.04364	0.9652
Negative Sign Bias	0.38816	0.6979
Positive Sign Bias	0.36206	0.7173
Joint Effect	0.28283	0.9632

Adjusted Pearson Goodness-of-Fit Test:

group	statistic	p-value (g-1)
1	20	1035 1.779e-207
2	30	1371 2.107e-270
3	40	1672 0.000e+00
4	50	2011 0.000e+00

Table 16: Estimates of ARMA(1,1)-eGARCH(2,2) with std

* GARCH Model Fit *

Conditional Variance Dynamics

GARCH Model : eGARCH(2,2)
Mean Model : ARFIMA(1,0,1)
Distribution : std

Optimal Parameters

	Estimate	Std. Error	t value	Pr(> t)
arl	0.68786	0.014697	46.8036	0
mal	-0.53668	0.015623	-34.3512	0
omega	-0.25233	0.001059	-238.2264	0
alpha1	0.39354	0.053218	7.3948	0
alpha2	-0.52520	0.002786	-188.5413	0

beta1	0.73683	0.000159	4631.2482	0
beta2	0.23768	0.000247	963.5852	0
gamma1	3.69469	0.006275	588.8134	0
gamma2	0.44676	0.002619	170.5717	0
shape	2.10000	0.000751	2797.4873	0

Robust Standard Errors:

	Estimate	Std. Error	t value	Pr(> t)
arl	0.68786	0.048823	14.089	0.00000
mal	-0.53668	0.040005	-13.415	0.00000
omega	-0.25233	0.004415	-57.147	0.00000
alpha1	0.39354	0.241286	1.631	0.10289
alpha2	-0.52520	0.003159	-166.235	0.00000
beta1	0.73683	0.002479	297.207	0.00000
beta2	0.23768	0.002884	82.418	0.00000
gamma1	3.69469	0.073306	50.401	0.00000
gamma2	0.44676	0.004863	91.866	0.00000
shape	2.10000	0.003104	676.648	0.00000

LogLikelihood : 10432.39

Information Criteria

Akaike	-5.1904
Bayes	-5.1748
Shibata	-5.1904
Hannan-Quinn	-5.1849

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.0006631	0.9795
Lag[2*(p+q)+(p+q)-1][5]	0.0019914	1.0000
Lag[4*(p+q)+(p+q)-1][9]	0.0033223	1.0000
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.0003884	0.9843
Lag[2*(p+q)+(p+q)-1][11]	0.0023365	1.0000
Lag[4*(p+q)+(p+q)-1][19]	0.0039019	1.0000
d.o.f=4		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[5]	0.0003889	0.500	2.000	0.9843
ARCH Lag[7]	0.0010005	1.473	1.746	1.0000
ARCH Lag[9]	0.0015138	2.402	1.619	1.0000

Nyblom stability test

Joint Statistic: 192.3745	
Individual Statistics:	
arl	1.17129
mal	1.21514
omega	13.94563
alpha1	1.66821
alpha2	0.05803
beta1	8.09970
beta2	6.96569
gamma1	13.50861
gamma2	3.15886
shape	9.79736

Asymptotic Critical Values (10% 5% 1%)			
Joint Statistic:	2.29	2.54	3.05
Individual Statistic:	0.35	0.47	0.75

Sign Bias Test

	t-value	prob sig
Sign Bias	0.95871	0.3378
Negative Sign Bias	0.48252	0.6295
Positive Sign Bias	0.05428	0.9567

Joint Effect 1.05095 0.7889

Adjusted Pearson Goodness-of-Fit Test:

group	statistic	p-value (g-1)
1	20	870.7 1.918e-172
2	30	1210.9 1.861e-236
3	40	1417.3 3.413e-272
4	50	1678.0 1.790e-319

Table 17: Estimates of ARMA(1,1)-eGARCH(2,2) with sstd

* GARCH Model Fit *

Conditional Variance Dynamics

GARCH Model : eGARCH(2,2)
Mean Model : ARFIMA(1,0,1)
Distribution : sstd

Optimal Parameters

	Estimate	Std. Error	t value	Pr(> t)
arl	0.311104	0.018626	16.7028	0.000000
mal	0.031624	0.013750	2.2999	0.021455
omega	-0.428565	0.023080	-18.5689	0.000000
alpha1	0.859577	0.148896	5.7730	0.000000
alpha2	1.059502	0.134331	7.8872	0.000000
beta1	0.031654	0.001147	27.5924	0.000000
beta2	0.926006	0.001006	920.5848	0.000000
gamma1	10.000000	0.022874	437.1748	0.000000
gamma2	9.399735	0.072643	129.3967	0.000000
skew	1.005109	0.009240	108.7818	0.000000
shape	2.010257	0.000173	11597.2175	0.000000

Robust Standard Errors:

	Estimate	Std. Error	t value	Pr(> t)
arl	0.311104	0.049329	6.3068	0.000000
mal	0.031624	0.020087	1.5743	0.11542
omega	-0.428565	0.150406	-2.8494	0.00438
alpha1	0.859577	0.774569	1.1097	0.26711
alpha2	1.059502	0.747112	1.4181	0.15615
beta1	0.031654	0.004750	6.6637	0.000000
beta2	0.926006	0.002727	339.5184	0.000000
gamma1	10.000000	1.405909	7.1128	0.000000
gamma2	9.399735	1.282506	7.3292	0.000000
skew	1.005109	0.006673	150.6302	0.000000
shape	2.010257	0.000824	2438.2258	0.000000

LogLikelihood : 10318.96

Information Criteria

Akaike	-5.1334
Bayes	-5.1162
Shibata	-5.1335
Hannan-Quinn	-5.1273

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.0006631	0.9795
Lag[2*(p+q)+(p+q)-1][5]	0.0019914	1.0000
Lag[4*(p+q)+(p+q)-1][9]	0.0033223	1.0000
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.0003884	0.9843
Lag[2*(p+q)+(p+q)-1][11]	0.0023365	1.0000

Lag[4*(p+q)+(p+q)-1][19] 0.0039019 1.0000
d.o.f=4

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[5]	0.0003889	0.500	2.000	0.9843
ARCH Lag[7]	0.0010005	1.473	1.746	1.0000
ARCH Lag[9]	0.0015138	2.402	1.619	1.0000

Nyblom stability test

Joint Statistic: 87.344

Individual Statistics:

arl	0.6185
mal	0.8204
omega	31.2178
alpha1	2.0947
alpha2	2.4273
beta1	8.3024
beta2	7.4301
gamma1	15.1495
gamma2	8.0161
skew	0.1114
shape	26.7576

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic:	2.49	2.75	3.27
Individual Statistic:	0.35	0.47	0.75

Sign Bias Test

	t-value	prob sig
Sign Bias	0.8896	0.3738
Negative Sign Bias	0.0580	0.9538
Positive Sign Bias	0.3984	0.6904
Joint Effect	0.9086	0.8234

Adjusted Pearson Goodness-of-Fit Test:

group	statistic	p-value (g-1)
1	20	1139 1.296e-229
2	30	1730 0.000e+00
3	40	2211 0.000e+00
4	50	2707 0.000e+00

Table 18: Estimates of ARMA(1,1)-TGARCH(2,2) with std

* GARCH Model Fit *

Conditional Variance Dynamics

GARCH Model : fGARCH(2,2)
fGARCH Sub-Model : TGARCH
Mean Model : ARFIMA(1,0,1)
Distribution : std

Optimal Parameters

	Estimate	Std. Error	t value	Pr(> t)
arl	0.188006	0.019754	9.51725	0.000000
mal	-0.044100	0.016926	-2.60537	0.009178
omega	0.000000	0.000000	0.56731	0.570503
alpha1	0.737207	0.022620	32.59082	0.000000
alpha2	0.007113	0.000221	32.23497	0.000000
beta1	0.391203	0.048539	8.05958	0.000000
beta2	0.067454	0.031730	2.12587	0.033514
eta11	-0.021401	0.022010	-0.97233	0.330887
eta12	-0.571818	0.013305	-42.97626	0.000000
shape	3.119393	0.058240	53.56058	0.000000

Robust Standard Errors:

	Estimate	Std. Error	t value	Pr(> t)
arl	0.188006	0.361081	0.520676	0.602593
mal	-0.044100	0.019349	-2.279187	0.022656

```

omega    0.000000    0.000007    0.005365    0.995720
alpha1   0.737207    2.342134    0.314759    0.752945
alpha2   0.007113    0.006765    1.051482    0.293037
beta1    0.391203    14.595151    0.026804    0.978616
beta2    0.067454    11.374815    0.005930    0.995268
eta11    -0.021401    0.712753    -0.030025    0.976047
eta12    -0.571818    3.397238    -0.168319    0.866333
shape    3.119393    5.171723    0.603163    0.546400

```

LogLikelihood : 12033.5

Information Criteria

```

-----
Akaike      -5.9878
Bayes       -5.9721
Shibata     -5.9878
Hannan-Quinn -5.9822

```

Weighted Ljung-Box Test on Standardized Residuals

```

-----
                        statistic p-value
Lag[1]                6.448e-09  0.9999
Lag[2*(p+q)+(p+q)-1][5] 6.551e-09  1.0000
Lag[4*(p+q)+(p+q)-1][9] 8.576e-09  1.0000
d.o.f=2
H0 : No serial correlation

```

Weighted Ljung-Box Test on Standardized Squared Residuals

```

-----
                        statistic p-value
Lag[1]                0.001951  0.9648
Lag[2*(p+q)+(p+q)-1][11] 0.011736  1.0000
Lag[4*(p+q)+(p+q)-1][19] 0.019599  1.0000
d.o.f=4

```

Weighted ARCH LM Tests

```

-----
Statistic Shape Scale P-Value
ARCH Lag[5]    0.001954 0.500 2.000 0.9647
ARCH Lag[7]    0.005026 1.473 1.746 0.9999
ARCH Lag[9]    0.007604 2.402 1.619 1.0000

```

Nyblom stability test

Joint Statistic: 287.706

Individual Statistics:

```

ar1      0.2268
ma1      0.1519
omega    124.3815
alpha1   69.1607
alpha2    7.1574
beta1    11.0448
beta2     6.4155
eta11     0.8532
eta12     6.4544
shape     3.2976

```

Asymptotic Critical Values (10% 5% 1%)

Joint Statistic: 2.29 2.54 3.05

Individual Statistic: 0.35 0.47 0.75

Sign Bias Test

```

-----
t-value prob sig
Sign Bias      0.002147 0.9983
Negative Sign Bias 0.384569 0.7006
Positive Sign Bias 0.377895 0.7055
Joint Effect    0.290821 0.9617

```

Adjusted Pearson Goodness-of-Fit Test:

```

-----
group statistic p-value(g-1)
1    20      926.9    2.054e-184
2    30     1253.4    1.733e-245
3    40     1511.5    3.960e-292
4    50     1814.4    0.000e+00

```

Table 19: Estimates of ARMA(1,1)-TGARCH(2,2) with sstd

```

*-----*
*              GARCH Model Fit              *
*-----*

Conditional Variance Dynamics
-----
GARCH Model      : fGARCH(2,2)
fGARCH Sub-Model : TGARCH
Mean Model       : ARFIMA(1,0,1)
Distribution      : sstd

Optimal Parameters
-----
Estimate Std. Error t value Pr(>|t|)
ar1      0.180819    0.027211    6.64515 0.000000
ma1      -0.040789    0.027805   -1.46695 0.142391
omega     0.000000    0.000000    0.25422 0.799323
alpha1    0.751149    0.019581   38.36156 0.000000
alpha2    0.021083    0.000863   24.42460 0.000000
beta1     0.338048    0.034700    9.74188 0.000000
beta2     0.099881    0.023224    4.30084 0.000017
eta11     -0.032566    0.022312   -1.45958 0.144405
eta12     -0.009352    0.036362   -0.25720 0.797028
skew      1.008252    0.011963   84.28353 0.000000
shape     3.119786    0.059674   52.28027 0.000000

```

Robust Standard Errors:

```

Estimate Std. Error t value Pr(>|t|)
ar1      0.180819    0.400435    0.451555 0.65159
ma1      -0.040789    0.065709   -0.620747 0.53477
omega     0.000000    0.000032    0.001112 0.99911
alpha1    0.751149    0.749862    1.001716 0.31648
alpha2    0.021083    0.006765    3.116443 0.00183
beta1     0.338048    6.615345    0.051101 0.95925
beta2     0.099881    6.442248    0.015504 0.98763
eta11     -0.032566    0.527177   -0.061775 0.95074
eta12     -0.009352    1.470815   -0.006359 0.99493
skew      1.008252    0.063847   15.791770 0.000000
shape     3.119786    3.512696    0.888146 0.37446

```

LogLikelihood : 12042.07

Information Criteria

```

-----
Akaike      -5.9916
Bayes       -5.9743
Shibata     -5.9916
Hannan-Quinn -5.9855

```

Weighted Ljung-Box Test on Standardized Residuals

```

-----
                        statistic p-value
Lag[1]                1.325e-07  0.9997
Lag[2*(p+q)+(p+q)-1][5] 5.397e-07  1.0000
Lag[4*(p+q)+(p+q)-1][9] 9.579e-07  1.0000
d.o.f=2
H0 : No serial correlation

```

Weighted Ljung-Box Test on Standardized Squared Residuals

```

-----
                        statistic p-value
Lag[1]                0.001474  0.9694
Lag[2*(p+q)+(p+q)-1][11] 0.008864  1.0000
Lag[4*(p+q)+(p+q)-1][19] 0.014803  1.0000
d.o.f=4

```

Weighted ARCH LM Tests

```

-----
Statistic Shape Scale P-Value
ARCH Lag[5]    0.001475 0.500 2.000 0.9694
ARCH Lag[7]    0.003796 1.473 1.746 0.9999
ARCH Lag[9]    0.005743 2.402 1.619 1.0000

```

Nyblom stability test

```

-----
Joint Statistic: 275.0765
Individual Statistics:
arl      0.2426
mal      0.1458
omega    120.3569
alpha1   59.1178
alpha2   5.9049
beta1    8.7332
beta2    5.8779
eta11    1.0777
eta12    6.0758
skew     0.1025
shape    3.6370

```

```

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:      2.49 2.75 3.27
Individual Statistic: 0.35 0.47 0.75

```

Sign Bias Test

```

-----
t-value  prob sig
Sign Bias      0.1533 0.8782
Negative Sign Bias 0.3937 0.6938
Positive Sign Bias 0.3207 0.7484
Joint Effect    0.2768 0.9643

```

Adjusted Pearson Goodness-of-Fit Test:

```

-----
group statistic p-value(g-1)
1    20      957.9   5.097e-191
2    30     1265.1   5.619e-248
3    40     1599.4   9.439e-311
4    50     1889.7   0.000e+00

```

Table 20: Estimates of ARMA(1,1)-NAGARCH(1,1) with std

```

*-----*
*          GARCH Model Fit          *
*-----*

```

Conditional Variance Dynamics

```

-----
GARCH Model      : fGARCH(2,2)
fGARCH Sub-Model : NAGARCH
Mean Model       : ARFIMA(1,0,1)
Distribution      : std

```

Optimal Parameters

```

-----
Estimate Std. Error t value Pr(>|t|)
arl      0.267628   0.170384  1.57074 0.116244
mal      -0.204195   0.180250 -1.13284 0.257281
omega    0.000000   0.000000  0.12858 0.897688
alpha1   0.361676   0.018067 20.01888 0.000000
alpha2   0.027698   0.009210  3.00738 0.002635
beta1    0.370490   0.081133  4.56642 0.000005
beta2    0.216297   0.058472  3.69914 0.000216
eta21    0.056881   0.058199  0.97736 0.328392
eta22    0.367374   0.021812 16.84251 0.000000
shape    3.725628   0.107154 34.76898 0.000000

```

Robust Standard Errors:

```

Estimate Std. Error t value Pr(>|t|)
arl      0.267628   0.652530  0.410138 0.68170
mal      -0.204195   0.564150 -0.361951 0.71739
omega    0.000000   0.000060  0.000595 0.99952
alpha1   0.361676   0.726879  0.497574 0.61878
alpha2   0.027698   0.068116  0.406631 0.68428
beta1    0.370490   7.019245  0.052782 0.95791
beta2    0.216297   6.860580  0.031528 0.97485
eta21    0.056881   0.198935  0.285929 0.77493
eta22    0.367374   1.733692  0.211903 0.83218
shape    3.725628   3.399272  1.096008 0.27308

```

LogLikelihood : 10171.79

Information Criteria

```

-----
Akaike      -5.0607
Bayes       -5.0450
Shibata     -5.0607
Hannan-Quinn -5.0551

```

Weighted Ljung-Box Test on Standardized Residuals

```

-----
statistic p-value
Lag[1]      0.03455 0.8525
Lag[2* (p+q)+(p+q)-1][5] 0.04001 1.0000
Lag[4* (p+q)+(p+q)-1][9] 0.06476 1.0000
d.o.f=2
H0 : No serial correlation

```

Weighted Ljung-Box Test on Standardized Squared Residuals

```

-----
statistic p-value
Lag[1]      0.002135 0.9631
Lag[2* (p+q)+(p+q)-1][11] 0.014612 1.0000
Lag[4* (p+q)+(p+q)-1][19] 0.024386 1.0000
d.o.f=4

```

Weighted ARCH LM Tests

```

-----
Statistic Shape Scale P-Value
ARCH Lag[5] 0.002531 0.500 2.000 0.9599
ARCH Lag[7] 0.006381 1.473 1.746 0.9998
ARCH Lag[9] 0.009663 2.402 1.619 1.0000

```

Nyblom stability test

```

-----
Joint Statistic: 228.1006
Individual Statistics:
arl      0.3208
mal      0.3712
omega    93.0616
alpha1   48.2432
alpha2   20.2380
beta1    6.3303
beta2    5.2483
eta21    2.1330
eta22    4.9865
shape    4.1481

```

```

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:      2.29 2.54 3.05
Individual Statistic: 0.35 0.47 0.75

```

Sign Bias Test

```

-----
t-value  prob sig
Sign Bias      0.9311 0.3519
Negative Sign Bias 0.5031 0.6149
Positive Sign Bias 0.1984 0.8427
Joint Effect    1.1151 0.7734

```

Adjusted Pearson Goodness-of-Fit Test:

```

-----
group statistic p-value(g-1)
1    20      1136   5.322e-229
2    30      1467   6.164e-291
3    40      1702   0.000e+00
4    50      1873   0.000e+00

```

Table 21: Estimates of ARMA(1,1)-NAGARCH(2,2) with sstd

```

*-----*
*          GARCH Model Fit          *
*-----*

```

Conditional Variance Dynamics

```

GARCH Model      : fGARCH(2,2)
fGARCH Sub-Model : NAGARCH
Mean Model       : ARFIMA(1,0,1)
Distribution      : sstd

```

Optimal Parameters

	Estimate	Std. Error	t value	Pr(> t)
arl	0.27314	0.164993	1.65545	0.097834
mal	-0.21735	0.177319	-1.22578	0.220283
omega	0.00000	0.000000	0.12625	0.899532
alpha1	0.39866	0.019042	20.93536	0.000000
alpha2	0.07167	0.009134	7.84656	0.000000
beta1	0.27848	0.077779	3.58034	0.000343
beta2	0.19095	0.051386	3.71606	0.000202
eta21	-0.11927	0.064471	-1.84992	0.064325
eta22	-0.75844	0.008760	-86.58119	0.000000
skew	1.00208	0.012026	83.32973	0.000000
shape	3.45518	0.083425	41.41662	0.000000

Robust Standard Errors:

	Estimate	Std. Error	t value	Pr(> t)
arl	0.27314	0.277317	0.984928	0.324660
mal	-0.21735	0.312482	-0.695573	0.486696
omega	0.00000	0.000064	0.000558	0.999554
alpha1	0.39866	1.439615	0.276921	0.781841
alpha2	0.07167	0.085600	0.837260	0.402447
beta1	0.27848	6.335548	0.043955	0.964941
beta2	0.19095	6.104333	0.031281	0.975045
eta21	-0.11927	5.516330	-0.021621	0.982751
eta22	-0.75844	5.670235	-0.133759	0.893593
skew	1.00208	0.113344	8.841085	0.000000
shape	3.45518	1.131894	3.052565	0.002269

LogLikelihood : 10175.73

Information Criteria

Akaike	-5.0621
Bayes	-5.0449
Shibata	-5.0621
Hannan-Quinn	-5.0560

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	0.05674	0.8117
Lag[2* (p+q) + (p+q) - 1] [5]	0.06030	1.0000
Lag[4* (p+q) + (p+q) - 1] [9]	0.08085	1.0000
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.003088	0.9557
Lag[2* (p+q) + (p+q) - 1] [11]	0.020713	1.0000
Lag[4* (p+q) + (p+q) - 1] [19]	0.034493	1.0000
d.o.f=4		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[5]	0.003576	0.500	2.000	0.9523
ARCH Lag[7]	0.009064	1.473	1.746	0.9997
ARCH Lag[9]	0.013686	2.402	1.619	1.0000

Nyblom stability test

Joint Statistic:	245.1376
Individual Statistics:	
arl	0.2838
mal	0.3408
omega	104.3987
alpha1	54.9018

alpha2	8.7199
beta1	8.7570
beta2	6.9370
eta21	0.7783
eta22	3.9323
skew	0.1141
shape	3.4639

Asymptotic Critical Values (10% 5% 1%)			
Joint Statistic:	2.49	2.75	3.27
Individual Statistic:	0.35	0.47	0.75

Sign Bias Test

	t-value	prob sig
Sign Bias	0.7966	0.4257
Negative Sign Bias	0.5196	0.6034
Positive Sign Bias	0.2699	0.7873
Joint Effect	0.9490	0.8136

Adjusted Pearson Goodness-of-Fit Test:

group	statistic	p-value(g-1)
1	20	1244
2	30	1611
3	40	1900
4	50	2164

Table 22: Estimates of ARMA(1,1)-AVGARCH(2,2) with std

```

*-----*
*               GARCH Model Fit               *
*-----*

```

Conditional Variance Dynamics

```

GARCH Model      : fGARCH(2,2)
fGARCH Sub-Model : AVGARCH
Mean Model       : ARFIMA(1,0,1)
Distribution      : std

```

Optimal Parameters

	Estimate	Std. Error	t value	Pr(> t)
arl	0.158970	0.015930	9.97959	0.000000
mal	-0.137234	0.019347	-7.09344	0.000000
omega	0.000000	0.000000	0.32907	0.742103
alpha1	0.736912	0.022551	32.67807	0.000000
alpha2	0.005087	0.000510	9.97569	0.000000
beta1	0.374727	0.041856	8.95270	0.000000
beta2	0.063603	0.026411	2.40821	0.016031
eta11	-0.045385	0.024043	-1.88765	0.059073
eta12	0.709412	0.171149	4.14499	0.000034
eta21	0.000162	0.000961	0.16819	0.866435
eta22	8.347500	0.822924	10.14371	0.000000
shape	3.088752	0.070739	43.66432	0.000000

Robust Standard Errors:

	Estimate	Std. Error	t value	Pr(> t)
arl	0.158970	0.099143	1.603451	0.108835
mal	-0.137234	0.077725	-1.765629	0.077458
omega	0.000000	0.000019	0.001899	0.998485
alpha1	0.736912	0.239248	3.080123	0.002069
alpha2	0.005087	0.000033	153.551376	0.000000
beta1	0.374727	10.078103	0.037182	0.970340
beta2	0.063603	8.950449	0.007106	0.994330
eta11	-0.045385	0.877183	-0.051739	0.958736
eta12	0.709412	0.067197	10.557231	0.000000
eta21	0.000162	0.001038	0.155686	0.876280
eta22	8.347500	0.913383	9.139099	0.000000
shape	3.088752	5.353848	0.576922	0.563992

LogLikelihood : 12082.03

Information Criteria

Akaike -6.0110
Bayes -5.9922
Shibata -6.0110
Hannan-Quinn -6.0043

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	2.408e-06	0.9988
Lag[2*(p+q)+(p+q)-1][5]	7.945e-06	1.0000
Lag[4*(p+q)+(p+q)-1][9]	1.349e-05	1.0000
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.001238	0.9719
Lag[2*(p+q)+(p+q)-1][11]	0.007444	1.0000
Lag[4*(p+q)+(p+q)-1][19]	0.012432	1.0000
d.o.f=4		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[5]	0.001239	0.500	2.000	0.9719
ARCH Lag[7]	0.003188	1.473	1.746	0.9999
ARCH Lag[9]	0.004823	2.402	1.619	1.0000

Nyblom stability test

Joint Statistic:	-1051.369
Individual Statistics:	
arl	1.4224
mal	1.5537
omega	113.9610
alpha1	28.8623
alpha2	7.2632
beta1	5.1249
beta2	7.1273
eta11	1.9999
eta12	7.2619
eta21	0.5528
eta22	7.1282
shape	6.1527

Asymptotic Critical Values (10% 5% 1%)	
Joint Statistic:	2.69 2.96 3.51
Individual Statistic:	0.35 0.47 0.75

Sign Bias Test

	t-value	prob	sig
Sign Bias	0.2294	0.8186	
Negative Sign Bias	0.3973	0.6911	
Positive Sign Bias	0.3141	0.7535	
Joint Effect	0.3056	0.9590	

Adjusted Pearson Goodness-of-Fit Test:

group	statistic	p-value(g-1)
1	20	1.372e-208
2	30	1.494e-310
3	40	2.088 0.000e+00
4	50	2.603 0.000e+00

Table 23: Estimates of ARMA(1,1)-AVGARCH(2,2) with sstd

* GARCH Model Fit *

Conditional Variance Dynamics

GARCH Model : fGARCH(2,2)
fGARCH Sub-Model : AVGARCH
Mean Model : ARFIMA(1,0,1)
Distribution : sstd

Optimal Parameters

	Estimate	Std. Error	t value	Pr(> t)
arl	0.101731	0.024077	4.225306	0.000024
mal	-0.001814	0.032550	-0.055728	0.955558
omega	0.000000	0.000000	0.213259	0.831125
alpha1	0.799645	0.018756	42.635197	0.000000
alpha2	0.008552	0.000802	10.663941	0.000000
beta1	0.273720	0.028337	9.659585	0.000000
beta2	0.046177	0.007158	6.450902	0.000000
eta11	-0.052875	0.024251	-2.180361	0.029231
eta12	0.710317	0.160919	4.414136	0.000010
eta21	0.158922	0.007346	21.634544	0.000000
eta22	9.417915	0.800430	11.766066	0.000000
skew	1.003885	0.011379	88.224025	0.000000
shape	2.822039	0.041927	67.308817	0.000000

Robust Standard Errors:

	Estimate	Std. Error	t value	Pr(> t)
arl	0.101731	1.067858	0.095266	0.924103
mal	-0.001814	1.465060	-0.001238	0.999012
omega	0.000000	0.000042	0.000854	0.999319
alpha1	0.799645	1.406233	0.568643	0.569598
alpha2	0.008552	0.000516	16.584686	0.000000
beta1	0.273720	4.509151	0.060703	0.951596
beta2	0.046177	4.998391	0.009238	0.992629
eta11	-0.052875	0.604364	-0.087489	0.930283
eta12	0.710317	1.239174	0.573218	0.566497
eta21	0.158922	0.556172	0.285742	0.775076
eta22	9.417915	2.795129	3.369403	0.000753
skew	1.003885	0.044910	22.353023	0.000000
shape	2.822039	1.009785	2.794693	0.005195

LogLikelihood : 11900.28

Information Criteria

Akaike -5.9200
Bayes -5.8996
Shibata -5.9200
Hannan-Quinn -5.9127

Weighted Ljung-Box Test on Standardized Residuals

	statistic	p-value
Lag[1]	5.637e-06	0.9981
Lag[2*(p+q)+(p+q)-1][5]	1.794e-05	1.0000
Lag[4*(p+q)+(p+q)-1][9]	3.022e-05	1.0000
d.o.f=2		
H0 : No serial correlation		

Weighted Ljung-Box Test on Standardized Squared Residuals

	statistic	p-value
Lag[1]	0.001205	0.9723
Lag[2*(p+q)+(p+q)-1][11]	0.007248	1.0000
Lag[4*(p+q)+(p+q)-1][19]	0.012105	1.0000
d.o.f=4		

Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag[5]	0.001207	0.500	2.000	0.9723
ARCH Lag[7]	0.003104	1.473	1.746	0.9999
ARCH Lag[9]	0.004696	2.402	1.619	1.0000

Nyblom stability test

Joint Statistic: -1109.483
Individual Statistics:

arl	1.5266	3	40	1826	0.000e+00
mal	1.3519	4	50	2153	0.000e+00
omega	103.5352				
alpha1	16.0413				
alpha2	11.4688				
beta1	7.7769				
beta2	11.1323				
eta11	23.9089				
eta12	11.4688				
eta21	56.5608				
eta22	11.1304				
skew	0.1852				
shape	3.4749				
Asymptotic Critical Values (10% 5% 1%)					
Joint Statistic:	2.89 3.15 3.69				
Individual Statistic:	0.35 0.47 0.75				
Sign Bias Test					

	t-value	prob	sig		
Sign Bias	0.2267	0.8207			
Negative Sign Bias	0.3927	0.6945			
Positive Sign Bias	0.2951	0.7679			
Joint Effect	0.2865	0.9625			
Adjusted Pearson Goodness-of-Fit Test:					

group	statistic	p-value	(g-1)		
1	20	1142	2.338e-230		
2	30	1506	3.365e-299		

Appendix B

i

ii

iii

iv

v

Figure 6: Rolling Forecast Plots from ARMA(1,1)-eGARCH(1,1) at 95% confidence level (with std)

Key:

- i. Density Forecast
- ii. Sigma Forecast
- iii. Series Forecast
- iv. VaR Forecast
- v. Fit Coefficients (with s.e band)

std: student t distribution

i

ii

iii

iv

v

Figure 7: Rolling Forecast Plots from ARMA(1,1)-eGARCH(1,1) at 99% confidence level (with sstd)

- Key:
- i. Density Forecast
 - ii. Sigma Forecast
 - iii. Series Forecast
 - iv. VaR Forecast
 - v. Fit Coefficients (with s.e band)
- sstd: skewed student t distribution

i

ii

iii

iv

v

Figure 8: Rolling Forecast Plots from ARMA(1,1)-eGARCH(1,1) at 95% confidence level (with sstd)

Key:

- i. Density Forecast
- ii. Sigma Forecast
- iii. Series Forecast
- iv. VaR Forecast
- v. Fit Coefficients (with s.e band)

sstd: skewed student t distribution

i

ii

iii

iv

v

Figure 9: Rolling Forecast Plots from ARMA(1,1)-eGARCH(2,2) at 99% confidence level (with std)

Key:
i. Density Forecast
ii. Sigma Forecast
iii. Series Forecast
iv. VaR Forecast
v. Fit Coefficients (with s.e band)
std: student t distribution

i

ii

iii

iv

v

Figure 10: Rolling Forecast Plots from ARMA(1,1)-eGARCH(2,2)at 95% confidence level(with std)

Key:

- i. Density Forecast
- ii. Sigma Forecast
- iii. Series Forecast
- iv. VaR Forecast
- v. Fit Coefficients (with s.e band)

std: student t distribution

i

ii

iii

iv

v

Figure 11: Rolling Forecast Plots from ARMA(1,1)-eGARCH(2,2) at 99% confidence level (with sstd)

Key:
i. Density Forecast
ii. Sigma Forecast
iii. Series Forecast
iv. VaR Forecast
v. Fit Coefficients (with s.e band)
sstd: skewed student t distribution

Figure 12: Rolling Forecast Plots from ARMA(1,1)-eGARCH(2,2) at 95% confidence level (with sstd)

Key:

- i. Density Forecast
- ii. Sigma Forecast
- iii. Series Forecast
- iv. VaR Forecast
- v. Fit Coefficients (with s.e band)

sstd: skewed student t distribution