# Further proofs for the 1-photon path Entanglement communications scheme 

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#### Abstract

The author had previously set out devices to communicate over space-like intervals, with a full proof for the 2 -photon device and only a partial proof for the 1-photon device. The 2-photon device exploits entangled pairs; the 1 -photon device utilises path-entanglement. The 1-photon device is fully analysed, then similarities (and differences) are drawn to the 2 -photon device to show the holes in the No-communications Theorem: the creation operators representing the sum of paths through the device can be mapped outside the device and quantum state reduction/measurement is a space-like operation. A common misconception on faux rank-3 systems made from rank-2 components is elucidated, avoiding the criticism and null result obtained by naively taking the partial trace.


Keywords: EPR, Bell's Theorem, Aspect-Zbinden Experiments, No-communications theorem

## 1. Introduction

Interest in space-like communication has been aroused by the EPR paradox, Bell[1-2], then Aspect, Grangier and Roger's[3], then Gisin and Zbinden's[4] experiments. The correlations that exist are much more than classical correlations (as proven by Bell), as they aren't predetermined and happen at the instant of measurement and appear to be a physical effect, though other interpretations exist[5-7]. Cosmic censorship-type theories[8-10] have been shown wanting by the author- indeed the author has corresponded with the said theoreticians, with one open-minded and the others shutting down the discussion. As regards noted experimenters in the field (such as listed by reference above), a similar situation exists and for the open-minded one, this paper hopes to address their concern regarding the 1 -photon setup [11], where they admitted modulation but were doubtful on the information being sent over a space-like separation.

We regard this project as being on a more secure footing for the hard experimental facts-of-the-matter[3-4] with related phenomena and the theoretical underpinning killing off the censorship theories[11-13], which show new ground to, perhaps, patch old systems of thought to the new phenomena[14]. This in contrast to experiment lead only claims (as Sagan said, "Extraordinary claims require extraordinary evidence") such as the celebrated 2011 CERN fast neutrino mistake[15], which in that case was down to a delay in a data line. However, this is not to dismiss any marginal, purely experimental claims[16] of superluminal effects, despite the results largely being asserted to be believed (rather than proven by first principles as a contradiction to existing theories or built on robust well acknowledged phenomena with agreed
interpretations). We note that, that experiment[16] would either need to produce an output going faster than Maxwell's equations will permit or for the output to somehow anticipate the inputs. It can probably be ignored, unlike the well-known EPR phenomenon.

The author first looked into a 2-photon communication device $[13,17$ ] (figure 1). This used two photons in HV polarisation in one of the Bell states, which were produced by a process of spontaneous parametric down-conversion. The source was in the middle with one photon being sent to "Alice" where she measured or not and the other photon was sent to "Bob's" interferometer. The act of Alice's measurement was discerned by Bob for the production of a mixed state. If she left her photon alone, Bob would perceive interference. Michael Hall's incredulous initial words (private correspondence) about this were "you don't believe that the state $|H\rangle|V\rangle+|V\rangle|H\rangle$ behaves like $|H\rangle+|V\rangle$ through the interferometer?" His view point, along with Ginacarlo Ghiradi's was that the mere act of looking at one particle in the pair would automatically cause the mixed state, the system wasn't factorisable. However the author found a flaw in the Nocommunications theorem (NCT): one has to consider the joint evolution[13] of both systems (through space and then the interferometer apparatus, say

$$
O_{1} O_{2}\left(\left|H_{1}\right\rangle\left|V_{2}\right\rangle+\left|V_{1}\right\rangle\left|H_{2}\right\rangle\right) \text { - indeed entanglement }
$$ wouldn't even exist if the particles couldn't travel through empty space, no trace operation here) and both acts were unitary; the system stayed entangled even after the interferometer and Bob could discern interference (or not) effects[13]. Interestingly the entanglement of the 2-photons was swapped to path entanglement of one photon of the pair (Bob's) as it went through the interferometer. It became a simple matter to show by state vector reduction or by using the density matrix form, that the collapse process was space-like, that is,



Figure 1-2-photon setup

| Measurement/Modulation at distant system and state of two photon system | State of distant system | State of local system | Local measurement by interferometer after modulation of distant system |
| :---: | :---: | :---: | :---: |
| No modulation: 'Binary 0 ' $\left.\frac{1}{\sqrt{2}}\|H\rangle_{1}\|V\rangle_{2}+\|H\rangle_{2}\|V\rangle_{1}\right)$ | Entangled => Pure state $\frac{1}{\sqrt{2}}\left(\|H\rangle_{1}+\|V\rangle_{1}\right)$ <br> Behaves as, see Appendix1: arxiv.org/abs/1 106.2258 | $\begin{gathered} \text { Entangled } \Rightarrow>\text { Pure state } \\ \frac{1}{\sqrt{2}}\left(\|V\rangle_{2}+\|H\rangle_{2}\right) \end{gathered}$ <br> Behaves as, see Appendix1: arxiv.org/abs/1106.2258 | Pure state results in interference <br> (Or at least some interference since source is not ideally pure) |
| $\begin{gathered} \text { Modulation: 'Binary 1' } \\ \frac{\|H\rangle_{1}\|V\rangle_{2}}{\sqrt{2}} \text { or } \frac{\|H\rangle_{2}\|V\rangle_{1}}{\sqrt{2}} \end{gathered}$ | Not entangled <=> <br> Mixed state $\frac{\|H\rangle_{1}}{\sqrt{2}} \text { or } \frac{\|V\rangle_{1}}{\sqrt{2}}$ | Not entangled $<=>$ <br> Mixed state $\frac{\|H\rangle_{2}}{\sqrt{2}}$ or $\frac{\|V\rangle_{2}}{\sqrt{2}}$ | Mixed state gives no interference |

A single photon source (SPS) is incident on a Mach-Zehnder type interferometer with 50:50 beamsplitters. Alice's measurements discerned over space-like separations by Bob at his detectors C (constructive) or D (destructive). Many single photons (a spot from a beam-expander is used with an attenuator on a laser source) are used to represent one bit.


This is the fundamental law of Quantum Mechanics:-
If the paths can be distinguished then add probabilities
else if the paths can't be, then add amplitudes before calculating probabilities
Thus when Aliec measures, both of Bob's paths to his detectors become distinguishable.

$$
\begin{aligned}
& \text { Alice sends } \\
& \hline \text { Binary 0: No measurement }
\end{aligned} \begin{aligned}
& \text { Binary 0: Min signal, destructive interference from pure state at } \mathrm{D} \\
& \text { Binary 1: Measurement }
\end{aligned} \begin{aligned}
& \text { Binary 1: Max signal from mixed state at D } \\
& \left.P(\text { Rob few photons, binary 0 Alice no measurement })=\left|\frac{i}{\sqrt{2}}\right|^{2}+\left|\frac{e^{i \theta}}{\sqrt{4}}\right|^{2}+2\left|\frac{i}{\sqrt{2}}\right| \frac{e^{i \theta}}{\sqrt{4}} \right\rvert\, \cos \theta \\
& =0.5+0.25-\frac{1}{\sqrt{2}} \cos \theta \\
& =0.75 \pm 0.707 \cos \theta \\
& =0.043 \text { minimum } \\
& P(\text { Bob lots of photons, binary 1 Alice measurement })=\left|\frac{i}{\sqrt{2}}\right|^{2}+\left|\frac{i}{\sqrt{4}}\right|^{2} \\
& =0.5+0.25 \\
& =0.75
\end{aligned}
$$

there appears to be no dynamics to the process[4] (no wave equation etc.) and all that mattered was the sequence in which the two operations were performed (Alice or Bob measures first).

Next in the said paper[13] the 1-photon system (figure 2) was re-appraised (originally presented in [11]) and stressed that the result obtained did not speak about sub-systems, tensor products and partial traces but just one particle, with the sum of paths/sum of amplitudes approach; this was seen as a further foil to NCT, which was couched in such terms. Hugo Zbinden pointed out (private correspondence) that the device was correct (as by the sum of paths approach) but he didn't think it would allow space-like communication. He is of course correct - the sum of path proof shows only modulation but it doesn't necessarily show spacelike communication. This then is the goal of this paper, to complete the proof and show state reduction/collapse by a similar method to the 2-photon considerations.

The key point to Zbinden's limiting belief was that Alice was close to the interferometer (figure 2) and her influence through measurement, propagated causally through the interferometer to Bob and of course this occurred at the speed of light.

Zbinden's mind-set is limited to the dimensions of the device (although figure 2 implied Alice and Bob were a long way from the interferometer, see figure 3 with its depiction of the wavefunction).


Figure 3 - Alice and Bob at a distance much greater than the dimensions of the device

The proof for the 2-photon setup (figure 1) didn't dwell on the dimensions of the interferometer because it was inferred automatically that both the protagonists were a long way from the source in the centre, which was equidistant from their
detectors/modulators. The analysis popped out fine and if state collapse is to be believed[4] deduced space-like communication.

So in a nutshell, to dispel Zbinden's concerns, our final proof for the 1-photon setup only has to show the creation operators at the first beam-splitter (figure 2) mapped outside the device and that there is a sum of upper (modulated by Alice) and lower path wavefunctions. If the implication of the state collapse procedure is correct, distance has no bearing on the matter.

## 2. Modelling the whole system considering each output as a sub-system - (a faux rank-3 system)

Let us concentrate on the modified MZ interferometer setup and label the inputs and outputs (figure 4). The letters in brackets means that that port is unused. For a 1:1 beamsplitter, the transfer function leads to the rule for mapping the creation operators to the output[18-20] in the Heisenberg evolution picture, thus:

$$
\hat{a}_{\text {input }}^{\dagger}|0\rangle|0\rangle \rightarrow \frac{1}{\sqrt{2}}\left(\hat{a}_{\text {transmited }}^{\dagger}+i \hat{a}_{\text {reflected }}^{\dagger}\right)|0\rangle|0\rangle \quad \text { eqn. } 1
$$

And so we can model the path of a single photon through the device:

$$
\begin{align*}
& \hat{a}_{a}^{\dagger}\left|0_{h}\right\rangle\left|0_{k}\right\rangle\left|0_{l}\right\rangle \\
& \rightarrow \frac{1}{\sqrt{2}}\left(\hat{a}_{c}^{\dagger}+i \hat{a}_{d}^{\dagger}\right)\left|0_{h}\right\rangle\left|0_{k}\right\rangle\left|0_{l}\right\rangle  \tag{eqn. 2}\\
& \rightarrow \frac{1}{\sqrt{2}}\left(\hat{a}_{c}^{\dagger}+\frac{i e^{i \theta_{1}}}{\sqrt{2}}\left(\hat{a}_{h}^{\dagger}+i \hat{a}_{g}^{\dagger}\right)\right)\left|0_{h}\right\rangle\left|0_{k}\right\rangle\left|0_{l}\right\rangle
\end{align*}
$$

The possible output states are shown as a tensor product. An arbitrary phase has been introduced $e^{i \theta_{1}}$ along the path from $d$ to $f$, and the output at $d$ becomes the input at $f$, which then is transformed by eqn. 1 to the outputs $h$ and $g$.

Continuing in the same vein for output $c$, off the mirror, through the delay to the last splitter and outputs $l$ and $k\left(e^{i \theta_{2}}\right)$, this is obtained,

$$
\begin{align*}
& \hat{a}_{a}^{\dagger}|0\rangle_{h}|0\rangle_{k}|0\rangle_{l} \\
& \rightarrow \frac{1}{\sqrt{2}}\left(\frac{i e^{i \theta_{2}}}{\sqrt{2}}\left(\hat{a}_{l}^{\dagger}+i \hat{a}_{k}^{\dagger}\right)+\frac{i e^{i \theta_{1}}}{\sqrt{2}}\left(\hat{a}_{h}^{\dagger}+i \hat{a}_{g}^{\dagger}\right)\right)|0\rangle_{h}|0\rangle_{k}|0\rangle_{l} \tag{eqn. 3}
\end{align*}
$$

And once again, finally, to change the output $g$ to an input at $i$ and then outputs at $l$ and $k$ (introducing another arbitrary phase $e^{i \theta_{3}}$ along the leg $g$ to $i$. This expression has mapped the creation operators all the way through to the other side of the device:-


Figure 4 - Ports of the 1-photon device enumerated


Figure 5 - The Creation Operators mapped to the other side of the interferometer

Note that the port $k$ is transmitted and $l$ is reflected in the final expression. This also shows the output wavefunction is a sum of upper and lower paths propagating away from the device:

$$
\psi_{\text {output }}=\psi_{\text {lower }}^{\vec{~}}+\psi_{\text {upper }}^{\vec{~}} \quad \text { eqn. } 5
$$

Most people would agree that eqn. 4 is sufficient to show not only the modulation scheme of figure 2 but that it is space-like too - the superimposed wavefunctions it represents coming from the upper and lower paths can be any distance away from the source or device (as shown in figure 3).

Tidying up,

$$
\begin{aligned}
|\psi\rangle= & +\frac{i e^{i \theta_{1}}}{2}\left|1_{h}\right\rangle\left|0_{k}\right\rangle\left|0_{l}\right\rangle \\
& -e^{i \theta_{2}}\left(\frac{1}{2}+\frac{1}{2 \sqrt{2}} e^{i \theta}\right)\left|0_{h}\right\rangle\left|1_{k}\right\rangle\left|0_{l}\right\rangle \\
& +i e^{i \theta_{2}}\left(\frac{1}{2}-\frac{1}{2 \sqrt{2}} e^{i \theta}\right)\left|0_{h}\right\rangle\left|0_{k}\right\rangle\left|1_{l}\right\rangle
\end{aligned}
$$

eqn. 6

The global phases have been left out in figure 6 (below), as they make no difference in the expectation values but the path phase difference is shown in the variable $\theta$. The effect of the glass plate delay can be seen at outputs $k$ and $l$ as a favouring of a particular output. The wavefunction moves through space as a superposition of these output states.

The result of the calculation leads to the wavefunction (figure 6) below (which clearly is entangled).


Figure 6 - Unmeasured by Alice case

The act of no measurement by Alice (call it binary 0 ) gives interference at Bob. The expectation value at Bob's detectors can be found (with the number operator) and tracing out the redundant states, i.e.
$\left\{\right.$ N.B. $\left.\theta=\theta_{1}+\theta_{3}-\theta_{2}\right\}$

## Bob's $k$ output, no measurement

$$
\begin{aligned}
& \left\langle 1_{k}\right| N^{\dagger}\left(\frac{1}{2}+\frac{1}{2 \sqrt{2}} e^{-i \theta}\right)\left(\frac{1}{2}+\frac{1}{2 \sqrt{2}} e^{i \theta}\right) N\left|1_{k}\right\rangle \\
& =\frac{3}{8}+\frac{\cos \theta}{2 \sqrt{2}} \quad \begin{array}{l}
\text { cf figure } 2 \text { with differential } \\
\begin{array}{l}
\text { output across } l \text { and } k \text { and sec. } 4 \\
\text { conclusion. }
\end{array}
\end{array}
\end{aligned}
$$

Which is the same as finding the expectation after tracing out outputs $h$ and $l$ (note the global phase is left out, as it makes no difference, since we just multiply a complex number by its conjugate in finding the expectation):

$$
\begin{align*}
& \operatorname{Tr}_{h l}(\psi)=\left\langle 0_{h}\right|\left\langle 0_{l}\right| \psi+\left\langle 1_{h}\right|\left\langle 1_{l}\right| \psi \\
& =\left(\frac{1}{2}+\frac{1}{2 \sqrt{2}} e^{i \theta}\right)\left|1_{k}\right\rangle \tag{eqn. 8}
\end{align*}
$$

Bob's $l$ output, no measurement

$$
\begin{aligned}
& \left\langle 1_{k}\right| N^{\dagger}\left(\frac{1}{2}-\frac{1}{2 \sqrt{2}} e^{-i \theta}\right)\left(\frac{1}{2}-\frac{1}{2 \sqrt{2}} e^{i \theta}\right) N\left|1_{k}\right\rangle \text { eqn. } 9 \\
& =\frac{3}{8}-\frac{\cos \theta}{2 \sqrt{2}} \quad \begin{array}{l}
\text { cf figure } 2 \text { with differential } \\
\begin{array}{l}
\text { output across } l \text { and } k \text { and sec. } 4 \\
\text { conclusion. }
\end{array}
\end{array} .
\end{aligned}
$$

Once again, this is the same as finding the expectation after tracing out outputs $h$ and k (global phase left out, once again):

$$
\begin{align*}
& \operatorname{Tr}_{h k}(\psi)=\left\langle 0_{h}\right|\left\langle 0_{k}\right| \psi+\left\langle 1_{h}\right|\left\langle 1_{k}\right| \psi \\
& =\left(\frac{1}{2}-\frac{1}{2 \sqrt{2}} e^{i \theta}\right)\left|1_{l}\right\rangle \tag{eqn. 10}
\end{align*}
$$

## 3. A Common Mistake in the process of taking the

 Partial Trace for this faux rank-3 systemTo illustrate the act of measurement by Alice (binary 1), we'll trace out system $h$ (this time we'll leave in the global phase factors as their contribution to the vacuum state is important) and a brusque analysis would have us believe that this happens:

$$
\begin{aligned}
\psi_{\text {measurred }}= & T r_{h}(\psi)=\left\langle 0_{h}\right| \psi+\left\langle 1_{h}\right| \psi \\
\psi_{\text {measurred }}= & +\frac{i e^{i \theta_{1}}}{2}\left|0_{k}\right\rangle\left|0_{l}\right\rangle \\
& -e^{i \theta_{2}}\left(\frac{1}{2}+\frac{1}{2 \sqrt{2}} e^{i \theta}\right)\left|1_{k}\right\rangle\left|0_{l}\right\rangle \\
& +i e^{i \theta_{2}}\left(\frac{1}{2}-\frac{1}{2 \sqrt{2}} e^{i \theta}\right)\left|0_{k}\right\rangle\left|1_{l}\right\rangle
\end{aligned}
$$

eqn. 11

Whereupon tracing out $l$ would give $\operatorname{Bob}(k$ output) and tracing out $k$ would give $\operatorname{Bob}(l$ output $)$ :

$$
\begin{aligned}
& \operatorname{Tr}_{l}\left(\psi_{\text {measured }}\right)=\left\langle 0_{h}\right|\left\langle 0_{l}\right| \psi+\left\langle 1_{h}\right|\left\langle 1_{l}\right| \psi \\
& =\left(\frac{1}{2}+\frac{1}{2 \sqrt{2}} e^{i \theta}\right)\left|1_{k}\right\rangle
\end{aligned}
$$

And

$$
\begin{aligned}
& \operatorname{Tr}_{k}\left(\psi_{\text {measured }}\right)=\left\langle 0_{h}\right|\left\langle 0_{k}\right| \psi+\left\langle 1_{h}\right|\left\langle 1_{k}\right| \psi \\
& =\left(\frac{1}{2}-\frac{1}{2 \sqrt{2}} e^{i \theta}\right)\left|1_{l}\right\rangle
\end{aligned}
$$

This is hardly surprising due to the cyclical permutatitive nature of the partial trace. So are we to believe that measurement would have no effect?

Clearly Quantum Mechanics and the analytical apparatus of it can't be wrong, so where is the mistake? The mistake is in the inappropriate use of the analysis in doing a partial trace on a faux rank-3 system which is really rank- 2 .

## 4. Resolution of the problem caused by trite analysis

The first part of figure 4 is really a 3 -way splitter formed from 2-way splitters (figure 7):


## Figure 7 - A faux 3-way splitter made from 2-way elements

 (magnitude of coefficients not important)The 2-way splitter takes as its input a "vector" and multiplies it by 1 or i. The "vectors" formed by the output are linearly independent (orthogonal too but this is not the issue). Thus when a single photon source (SPS) is input to a $50: 50$ beamsplitter the output is (eqn. 1): $\frac{1}{\sqrt{2}}(|1\rangle|0\rangle+i|0\rangle|1\rangle)$ and we might consider the individual outputs as 2 -dimensional vectors:


Figure 8 - The output of a 2-way splitter considered as vectors

Nesting 2-way splitters, as by figure 7, does not make a true 3-way splitter, whose outputs would be linearly independent (figure 9). In this case, measurement on one of the outputs would leave the other outputs entangled[11].


Figure 9 - The output of a true 3-way splitter considered as vectors

One may argue that a delay plate could be introduced and render all the output vectors the same; however such a transform is not possible on all the vectors. One can't argue that the real world is two-dimensional, as the x -axis can be rotated to the $y$-axis... and then really one-dimensional, because the $y$-axis could be rotated to the z -axis. One would need to be able to perform the same transform on all the axis.

Thus when we write the output of our faux 3-way system as (eqn. 6):

$$
\begin{aligned}
|\psi\rangle= & +\frac{i e^{i \theta_{1}}}{2}\left|1_{h}\right\rangle\left|0_{k}\right\rangle\left|0_{l}\right\rangle \\
& -e^{i \theta_{2}}\left(\frac{1}{2}+\frac{1}{2 \sqrt{2}} e^{i \theta}\right)\left|0_{h}\right\rangle\left|1_{k}\right\rangle\left|0_{l}\right\rangle \\
& +i e^{i \theta_{2}}\left(\frac{1}{2}-\frac{1}{2 \sqrt{2}} e^{i \theta}\right)\left|0_{h}\right\rangle\left|0_{k}\right\rangle\left|1_{l}\right\rangle
\end{aligned}
$$

$\left\{\right.$ N.B. $\left.\theta=\theta_{1}+\theta_{3}-\theta_{2}\right\}$
The matrix (rank $1 \times 3$ as a "bra" and $3 \times 1$ "ket") representing it is really singular/degenerate/linearly dependent. It is not correct to write the density matrix from the above as: (constants substituted)

$$
\left.\begin{array}{l}
\rho_{h k l}=|\psi\rangle\langle\psi| \\
=\begin{array}{l}
\left\langle 1_{h}\right\rangle\left|0_{k}\right\rangle\left|0_{l}\right\rangle \\
\left|0_{h}\right\rangle\left|1_{k}\right\rangle\left|0_{l}\right\rangle \\
\left|0_{h}\right\rangle\left|0_{k}\right\rangle\left|0_{k}\right|\left\langle 1_{h}\right|
\end{array} \\
\left\langle 0_{l}\right|\left\langle 1_{k}\right|\left\langle 0_{h}\right|
\end{array} \begin{array}{ccc}
\left\langle 1_{l}\right|\left\langle 0_{k}\right|\left\langle 0_{h}\right| \\
a^{2} & a b^{*} & a c^{*} \\
b a^{*} & b^{2} & b c^{*} \\
c a^{*} & c b^{*} & c^{2}
\end{array}\right] .
$$

And the tracing out of "system" $h$ as:

$$
\left.\begin{array}{l}
\rho_{k l}=\left\langle 0_{h}\right| \rho_{h k l}\left|0_{h}\right\rangle+\left\langle 1_{h}\right| \rho_{h k l}\left|1_{h}\right\rangle \\
\Rightarrow \\
\left.\left\langle 0_{k}\right\rangle\left|0_{l}\right\rangle\right\rangle \\
\left|1_{k}\right\rangle\left|0_{l}\right\rangle \\
\left|0_{k}\right\rangle\left|0_{l}\right| \\
\left.0_{l}\right\rangle
\end{array} \begin{array}{ccc}
a^{2} & \left\langle 0_{l}\right|\left\langle 1_{k}\right| & \left\langle 1_{l}\right|\left\langle 0_{k}\right| \\
0 & b^{2} & b c^{*} \\
0 & c b^{*} & c^{2}
\end{array}\right) .
$$

Incorrect reduced density matrix
This leads to the mistaken conclusion that systems $k$ and $l$ are still coherent after Alice measures but the system isn't really rank-3.

Physically this can be understood as the splitters being random particle sorters, where the vacuum state is injected from the unused port[19]:

$$
\rho=|\psi\rangle\langle\psi|=\frac{1}{2}|1\rangle|0\rangle\langle 0|\langle 1|+\frac{1}{2}|0\rangle|1\rangle\langle 1|\langle 0| \quad \text { eqn. } 12
$$

Regarding figures 4 and 7, upon measurement of the upper path (by Alice) random phases are introduced on the other two arms thus:

$$
\begin{align*}
& |\psi\rangle=\left|\psi_{2}\right\rangle+\left|\psi_{3}\right\rangle \\
& =\frac{e^{i \theta_{\text {Lomer }}}}{2}\left(\hat{a}_{c}^{\dagger}\right)|0\rangle_{c}|0\rangle_{g}-\frac{e^{i \theta_{U_{\text {pper }}}}}{2 \sqrt{2}}\left(\hat{a}_{g}^{\dagger}\right)|0\rangle_{c}|0\rangle_{g} \tag{eqn. 13}
\end{align*}
$$

These random phases result from two independent random processes and why would the vacuum at disparate locations be correlated (apart from their single photon outputs being correlated, by conservation of probability of the wavefunction, on measurement)? So, very rarely, it could lead to the state:

$$
\begin{aligned}
& |\psi\rangle=\left|\psi_{2}\right\rangle+\left|\psi_{3}\right\rangle \\
& =\frac{1}{2}|1\rangle_{c}|0\rangle_{g}+\frac{e^{i \text { constant }}}{2 \sqrt{2}}|0\rangle_{c}|1\rangle_{g}
\end{aligned}
$$

The autocorrelation function between random phases/delays $e^{i \theta_{\text {Loner }}}$ and $e^{i \theta_{\text {pperer }}}$ is essentially a Dirac distribution. Most likely the mixed state would result:

$$
\rho_{\text {Measured }}=\frac{1}{4}|1\rangle|0\rangle\langle 0|\langle 1|+\frac{1}{8}|0\rangle|1\rangle\langle 1|\langle 0| \quad \text { eqn. } 14
$$

This would give an expectation of $3 / 8$ at both of Bob's detectors and zero by a differential measurement.

Via eqn. 7 and eqn. 9 and a differential measurement across the "dark" and "light" ports, we fare the same as figure 2 (which doesn't assume dark and light ports) and the same difference in levels between binary 0 and 1 (see eqn. 14) - a minimum of 0 and a maximum of $\frac{\cos \theta}{\sqrt{2}}$ (so $\frac{1}{\sqrt{2}}$ at $\theta=0$ ) is achieved.

## 5. Against objection to idea that measurement can affect anything once out side the interferometer

Quantum Mechanics is linear and so the setup in figure 2 and the claim of figure 3 and eqn. 5, would imply that measurement by Alice outside the device (after the horse has bolted and the photon is no longer in the apparatus, so to speak) would have an effect at Bob's detectors, is correct. However for sceptics, there is an easy modification to turn the "external" version of the device in figure $2 / 3$ into an "internal" version; Bob's detectors are at the end of an elongated interferometer and the wavefunction must causally pass over the detectors. This gets around objections to the argument of eqn. $4 / 5$ and the "projection" of the creation operators outside the device.

Figure 10 - an "internal" version of figures $2 / 3$ where Bob's two detectors ( $\mathrm{Bob}_{1}$ and $\mathrm{Bob}_{2}$ ) are present at the end of an elongated interferometer.

## 6. Discussion and conclusion

The sum of paths/sum of amplitudes proof given in earlier papers for the 1-photon system (figure 2) was criticised as being a necessary but not a necessary and sufficient proof for superluminality - that is in some putative communication scheme, we must have modulation but that doesn't automatically imply superluminality. The proof, some believe, gives the impression of a photon wavefunction moving through the apparatus and traversing each component in a timelike fashion. We beg to differ: eqn. 4 shows the summation of the wavefunctions from the upper and lower legs after they have been through the final beamsplitter. The expectation values at Bob's detectors have a feed-through component from Alice's splitter and her influence collapses her wavefunction, which is summed at Bob outside the apparatus (figure 3).

Is it to be believed that the wave function propagating through the interferometer is really the issue? If Alice's measurement is near to the interferometer, it would just seem that her influence has to propagate through the apparatus until the final beamsplitter. We argue that the interferometer is merely the device for the correct setup of the rays emanating from the source to: go to Alice and then to Bob with some component from Alice. An overall global phase in the wavefunctions (representing the causal delay transiting the apparatus) does not appear in the expectation values and has no effect on it. What is relevant is her coherence or not on Bob's interference pattern.

Quantum Mechanics indicates the measurement/trace process is space-like. The absolute temporal sequence is important:-


Alice measures first then Bob performs his measurement for one or both of the outputs.

Alice doesn't measure first, Bob measures his outputs and observes interference.

What is intriguing is that the density matrix description of the system applies far away from the interferometer (it just sets up the rays from the source) and has no time element (no propagator), only the sequence in which the operations are performed matters. This implies space-like communication and corresponds to the notion that wavefunction collapse is instantaneous or near instantaneous.

## References

1. Bell, J.S., On the Einstein-Podolsky-Rosen Paradox. Physics Letters A, 1964. 1: p. 195-200.
2. Bell, J.S., Foundations of Quantum Mechanics. New York: Academic, 1971. 171.
3. Roger, A. Aspect P. Grangier G., Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A New Violation of Bell's Inequalities. Phys. Rev. Lett., 1982. 49(91).
4. Zbinden H., Gisin N., Testing the speed of 'spooky action at a distance'. Nature, 2008. 454.
5. Sheehan D. P. Quantum Retrocausation III. 2016. University of San Diego, C.A.: AIP.
6. Everett H., Relative State Formulation of Quantum Mechanics. Reviews of Modern Physics, 1957. 29: p. 454-462.
7. Zurek, W.H., Decoherence and the Transition from Quantum to Classical. Los Alamos Science, 2002. 27.
8. Hall, M.J.W., Imprecise Measurements and Non-Locality in Quantum Mechanics. Physics Letters A, 1987. 125(2,3): p. 89,91.
9. Ghirardi, G. C.; Rimini, A.; Weber, T., $A$ General Argument against Superluminal Transmission through the Quantum Mechanical Measurement Process. Lettere
al Nuovo Cimento, 1980, 8th March. 27(10): p. 293-298.
10. Ghirardi, G. C.; Rimini, A.; Weber, T., Experiments of the EPR Type Involving CPViolation do not allow Faster-than-Light Communications Between Distant Observers. Europhys. Lett., 1988. 6(2): p. 95-100.
11. Cornwall, R.O., Superluminal Signalling by Path Entanglement. 2015
(https://www.academia.edu/12881902/Disproof_of_the_ No-communication_Theorem_by_Decoherence_Theory).
12. Cornwall R. O., Disproof of the Nocommunication Theorem by Decoherence Theory. 2015
(https://www.academia.edu/12881902/Disproof_of_the_ No-communication_Theorem_by_Decoherence_Theory).
13. Cornwall R. O., The misuse of the Nocommunication Theorem. 2016
(https://www.academia.edu/29296558/The_misuse_of_th e_No-communication_Theorem).
14. Cornwall, R.O., A Mechanism for the effects of Relativity 2014
(https://www.academia.edu/11587819/A_Mechanism_for _the_effects_of_Relativity).
15. Wikipedia. Faster-than-light neutrino anomaly. 2019; Available from: https://en.wikipedia.org/wiki/Faster-thanlight_neutrino_anomaly.
16. Lett, Ryan T. Glasser; Ulrich Vogl; Paul D., Stimulated generation of superluminal light pulses via four-wave mixing. Arxiv preprint, 2018 (https://arxiv.org/abs/1204.0810).
17. Cornwall, R.O., Secure Quantum

Communication and Superluminal Signalling on the Bell Channel. Infinite Energy, 2006. 69 (https://www.academia.edu/11587674/Secure_Quantum Communication_and_Superluminal_Signalling_on_the_- $\bar{B}$ ell_Channel http://vixra.org/abs/1311.0074).
18. Ralph, Hans-A Bachor; Timothy C., A Guide to Experiments in Quantum Optics. 2004: Wiley-VCH.
19. Zeilinger, A., General properties of lossless beam splitters in interferometry. Am. J. Phys., 1981. 49(9).
20. Gerry, Christopher; Knight, Peter, Introductory Quantum Optics. 2004: CUP.

