

Article

Interpolation of small datasets in the sandstone hydrocarbon reservoirs, case study from the Sava Depression, Croatia

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Abstract:

Here are analysed data taken in two hydrocarbon fields ("A" and "B"), located in the western part of Sava Depression (North Croatia). They are in the secondary phase of production. The selected reservoirs "L" (in the "A" Field) and "K" ("B") are of the Lower Pontian (Upper Miocene) age and belong to Kloštar-Ivanić Formation. Due to strong tectonics, there are numerous tectonic block, relatively rarely sampled with well and laboratory tests. Here are selected two variables for interpolation - reservoirs permeabilities and the injected volumes of field water. The following interpolation methods are described, compared and applied: Nearest Neighbourhood, Natural Neighbour (the first time in the Sava Depression) and Inverse Distance Weighting. The last one has been proven as the most appropriate for datasets with size lower than 20 points.

Keywords: interpolation, permeability, injected water, Inverse Distance Weighting, Sava Depression, Miocene, Croatia

1. Introduction

The secondary hydrocarbon recovery methods (water injection) has been applied in the Sava Depression (the Northern Croatia) from the 80s of the 20th century. Re-injection of the formation water is the most widely used method of supporting formation pressure. Here are analysed the fields "A" and "B", where re-injection of field (formation) water is applied. Because of the relatively small hydrodynamic units, i.e. small number of production and injection wells per unit, the reservoir mapping could be done only using simpler interpolation method. i.e. methods designed for a small number of input data, without complex spatial analysis (like variogram or co-variance).

Previous studies of the Miocene hydrocarbon reservoirs in the Sava Depression defined some rules for interpolations of different datasets. According to [1] and [2] the minimum data for geostatistic mapping is set on 20 or more "hard" values, thus defining the boundary of the small input set. Also, [3] and [4] defined a set of 15 input data as sufficient for the application of interpolation methods like Inverse Distance Weighting (abbr. IDW) and Nearest Neighbourhood (abbr. NN) in the Croatian part of the Pannonian Basin System (abbr. CPBS). Application of the IDW method in the CPBS had been documented in the Beničanci and Stari Gradec Fields ([5], [6], [7]), both located in the Drava depression. The NN method was applied also in the Kloštar field ([8]), located in the Sava Depression. Moreover, [9] applied the IDW method on the Dardevey iron ore deposit (NE Iran), and compared with the Ordinary Kriging technique. [10] compared the Ordinary Kriging with IDW on data from the East-Parvadeh coal deposit (Iran). [11] applied the IDW and calculated cross-validation for soil depth in the Medinipur Block area (West Bengal, India). Those examples offered the kind of

analysis and algorithms how to test applicability of such interpolations in different subsurface volumes and similar problems of spatial distributions. Consequently, here are applied the Nearest Neighborhood, Inverse Distances Weighting and Natural Neighbour, in the selected hydrocarbon reservoir, i.e. hydrodynamic unit. The quality subsurface maps could support much longer production in the analysed volumes, without regarding relatively small number of data per reservoir variable.

2. Basic geological settings of the Sava Depression (western part)

The analysed "A" and "B" Fields are located in the CPBS, i.e. in the western part of Sava Depression. They are about 90 km southeast of the Croatian capital of Zagreb (**Figure 1**).



Figure 1. Geographic position of "A" and "B" Fields within Sava depression (Northern Croatia)

Analysed reservoirs are of the Lower Pontian age. Those are "L" reservoir in the "A" and "K" in the "B" Field. Lithostratigraphically, they are part of the Kloštar- Ivanić Formation (**Figure 2**). The reservoir quality is shortly described with the following values:

- a) "L" reservoir - porosity 19.7 %, permeability $17.5 \cdot 10^{-3} \mu\text{m}^2$, thickness 17.5 m;
- b) „K“ reservoir - porosity 22.7 %, permeability $75.4 \cdot 10^{-3} \mu\text{m}^2$, thickness 10 m.

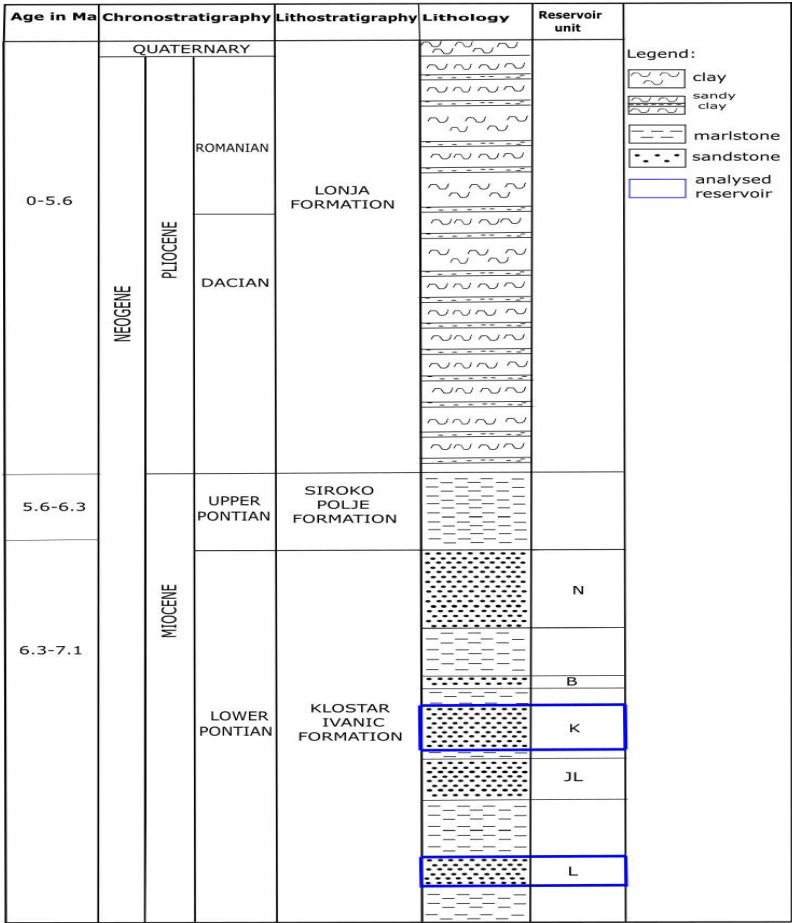


Figure 2. Typical geological column for the western part of the Sava Depression, with (litho)stratigraphic position of analysed reservoirs (“L” in “A” Field, “K” in “B” Field)

Hydrocarbon reservoirs discovered in Lower Pontian sandstones are of turbiditic origin. Lacustrine marls, from the same period, represent sediments from “calm” environment. Such a lacustrine environment with periodically active turbidites, characterised the entire CPBS during Upper Miocene (e.g.,[12]). The entire area of the CPBS from the Late Pannonian until the Late Pontian period is considered as a dominant clastic environment, with enormously large volumes of sandy and silty detritus deposited from turbidites. Chronostratigraphically, many authors accepted Pontian as a valid stage in the entire Pannonian Basin System (e.g., [13], [14], [15]). However, recently some authors published some new depositional models of the Upper Miocene period that rejected the Pontian as a stage applied in the CPBS (e.g., [16]). In this analysis, the Pontian age has been used as valid unit for description of reservoir stratigraphy, origin and age.

Thickness map of the “L” reservoir, with the largest hydrodynamic unit where is pressure supported with water injection, is shown at **Figure 3**. The injection started in 1984 within 3 wells. Today such process is maintained in 10 wells.



93
94
95

93
94
95



Figure 4. Thickness (isopach) map of the reservoir "K"

3. Short theory of applied interpolation methods

Here are described interpolation methods for mapping applied in presented subsurface analyses. Those are: Inverse Distance Weighting, Nearest Neighbourhood and Natural Neighbour.

3.1. Inverse Distance Weighting method

The Inverse Distance Weighting method is a mathematically simple interpolation method, where the unknown value of the variable is estimated from the measured values included into the searching circle (Figure 5) or ellipsoid using Equation 1, based on simple weighting method using power of distances.

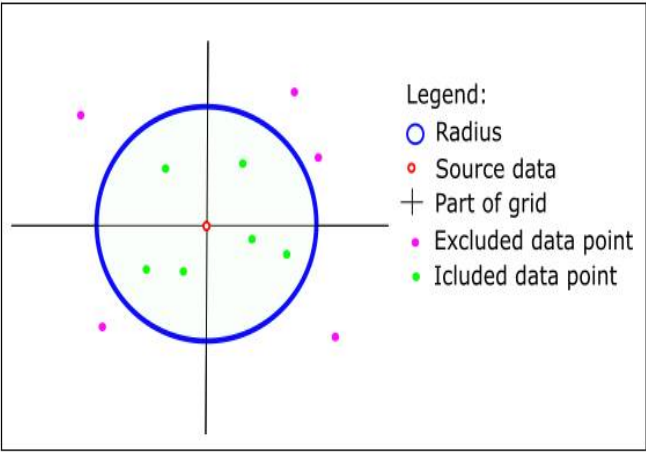


Figure 5. Searching radius for assessing the Inverse Distance Weighting value

$$z_{IU} = \frac{\frac{z_1}{d_1^p} + \frac{z_2}{d_2^p} + \dots + \frac{z_n}{d_n^p}}{\frac{1}{d_1^p} + \frac{1}{d_2^p} + \dots + \frac{1}{d_n^p}} \tag{1}$$

where:

- z_{IU} - estimated value;
- $d_1 \dots d_n$ - distance to locations 1...n;
- p - power of distance;
- $z_1 \dots z_n$ - real values at location 1 ... n.

The interpolation result depends exclusively on the distance, weighted by power exponent that is commonly selected with values between 1 and 3. Usually in the subsurface of the CPBS the value 2 is recommended, based on empirical tests performed in the subsurface geological mapping (e.g., [17], [18]).

3.2. Nearest Neighbourhood method

The Nearest Neighbourhood method is the simple interpolation method (Figure 6) that adds the value to location E, taking into account the value of the nearest adjacent data (e.g., A, K, G, P on Figure 6). The result is the zonal map, i.e., map filled by polygons (e.g., [19]).

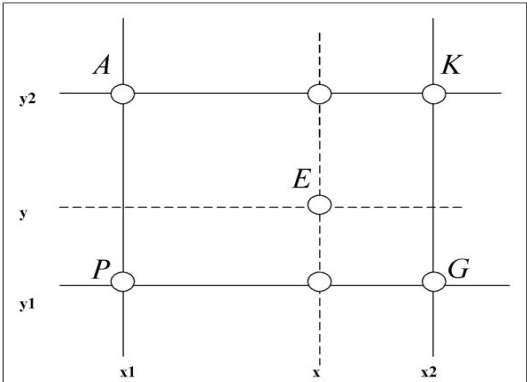


Figure 6. Estimated value of the point E with regard to points A, K, P and G ([20])

Space distance is calculated according to the expression for Euclid's distance (Equation 2):

$$d(E, G) = \sqrt{(E - G)^2} \tag{2}$$

where:

d - distance;

E and G - selected points (the closest ones) in space.

3.3. Natural Neighbour method

The method of Natural Neighborhood is a simple interpolation method based on Voronoi's polygons. Unknown value "X" has been determined from, e.g., the four neighbouring values "A₁₋₄", like it is shown in Figure 7.

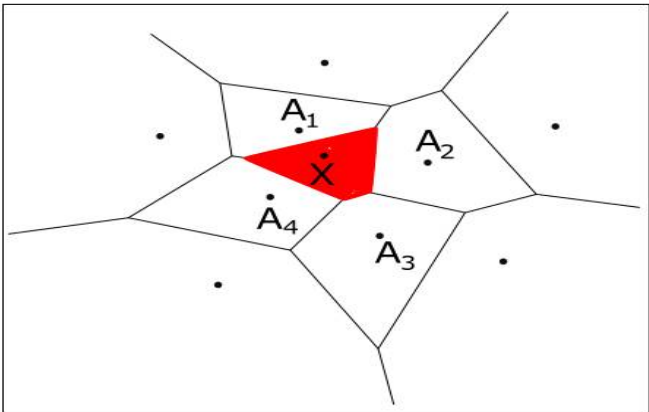


Figure 7. Estimated value of the point X with regard to points A₁₋₄ ([21], [22])

Mathematical expression for estimated natural neighbour values can be expressed with, e.g., Equation 3 (e.g., [23], [24], [25]):

$$X(x,y) = \sum_{i=1}^n (w_i A(x_i, y_i)) \tag{3}$$

where:

$X(x,y)$ - estimated point;

$A(x,y)$ - measured value in neighbouring points;

w_i - proportion of analysed polygon regarding total area of all constructed adjacent polygons.

4. Interpolation in reservoirs "L" and "K" – injected volumes and permeabilities

The interpolated variables, in both selected fields, were the permeability of the reservoir and the volumes of injected water. Those variables are crucial for interpretation and planning of the secondary hydrocarbon recovery methods, i.e. field’s waters re-injection.

Permeabilities of reservoirs "L" and "K" is determined in wells from laboratory measurements and previously uniformly extrapolated in particular blocks. **Table 1** shows the available data for permeability and injected volumes in the "L" and "K" reservoirs.

Table 1. Available permeability and injected volumes data in the "L" and "K" reservoirs

Wells in reservoir "L"		Permeability (10 ⁻³ μm ²)	
L-27, L-87, L- 160		24.2	
L-57, L-62, L-156		27.0	
L-4, L-37, L-65, L-68		23.2	
Wells in reservoir "K"		Permeability (10 ⁻³ μm ²)	
J-25,J-101,J-102,J-148, J-149, J-162,J-166, J-167, J-168, J-169, J-173, J-174		121.2	
J-120, J-158, J-170, J-171, J-172, J-175		29.6	
Reservoir "K"		Reservoir "L"	
Well	Injected volumes (m ³)	Well	Injected volumes (m ³)
J-166	992045	L-4	132116
J-172	593591	L-33	420251
J-173	273788	L-34	167108
		L-63	440031
		L-79	132352
		L-122	535171
		L-139	241085
		L-154	565872
		L-160	467987
		L-161	376438

Interpolated maps in “L” reservoir, using data from Table 1, has been shown in Figure 8 sequentially for all three methods – IDW, NN and NaN (from top to bottom). The cross-validation results are given in Table 2.

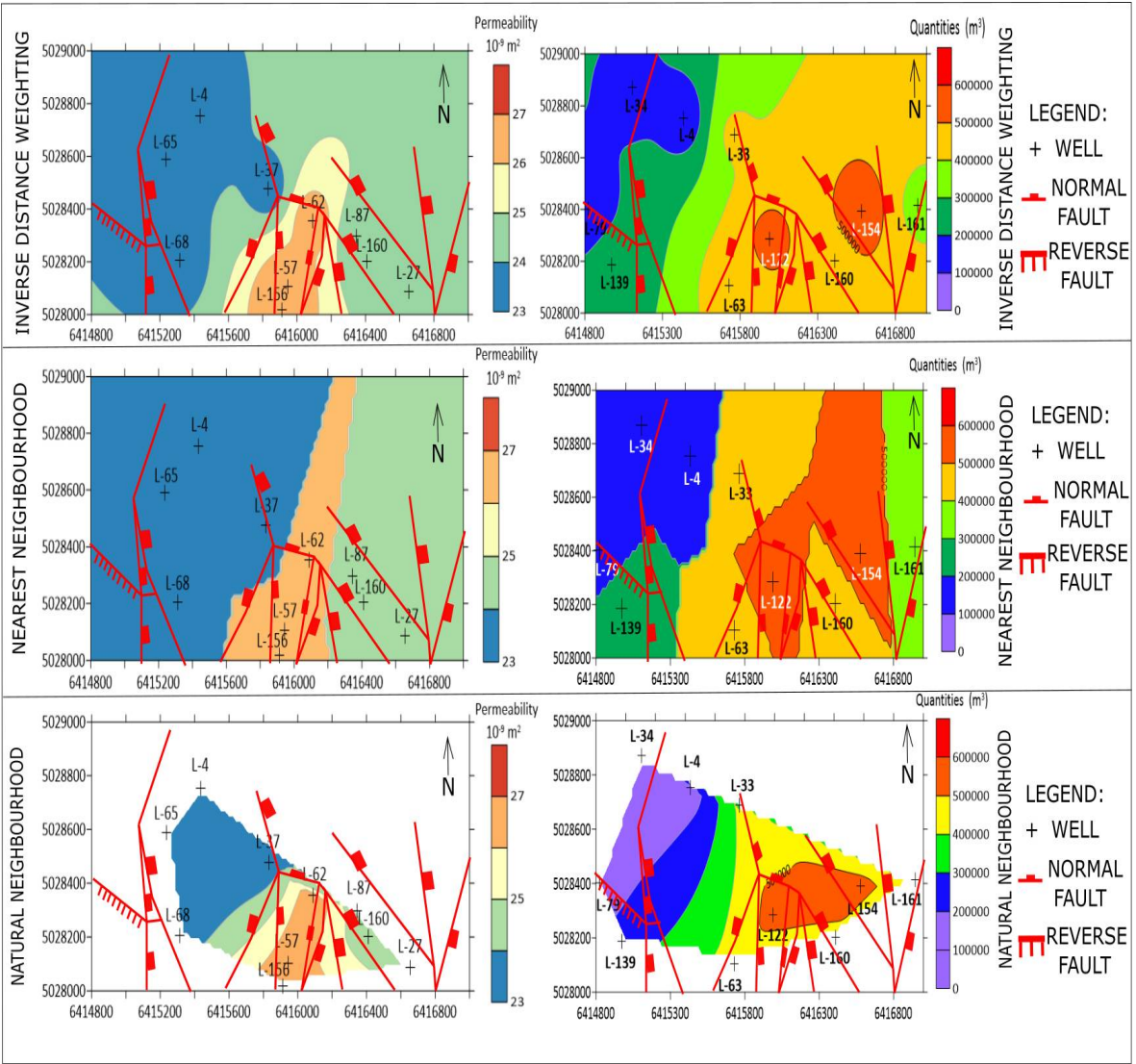


Figure 8. Results of IDW, NN and NaN methods (from top to bottom) or the permeability (left) and injected volumes (right) in the “L” reservoir

Table 2. Cross-validation values in the “L” reservoir

Variable	Number of data	Values of cross-validation		
		Inverse Distance Weighting	Nearest Neighbourhood	Natural Neighbour
Injected volumes	10	$1.21 \cdot 10^{10}$	$2.64 \cdot 10^{10}$	$2.36 \cdot 10^{10}$
Permeability	10	1.41	2.22	3.48

The interpolation results for the same variables in the "K" reservoir are shown in Figure 9. The cross-validation results for applied interpolation methods are shown in Table 3.

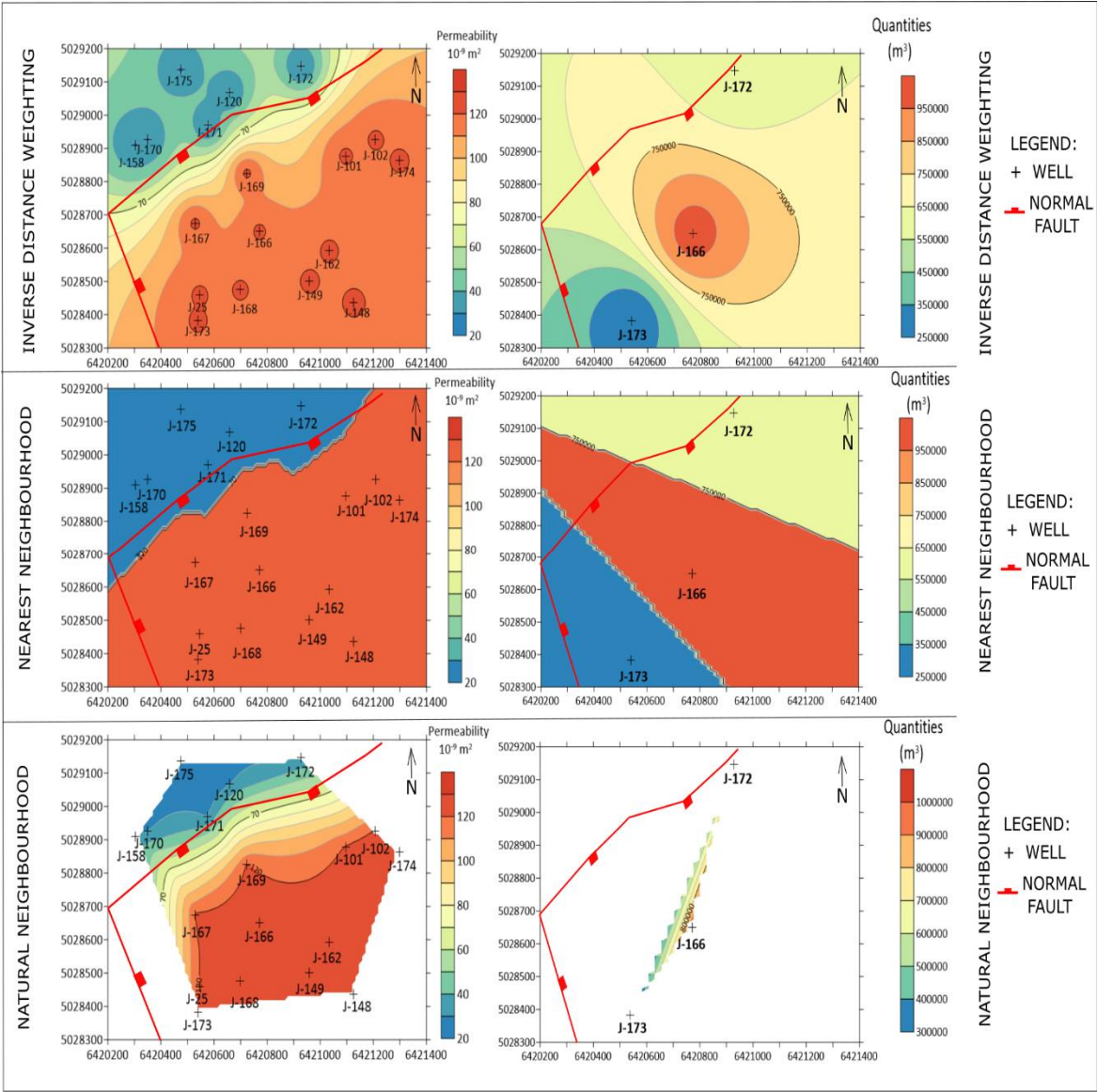


Figure 9. Results of IDW, NN and NaN methods (from top to bottom) for the permeability (left) and injected volumes (right) in the “K” reservoir

Table 3. Cross-validation values in the “K” reservoir

Variable	Number of data	Value of Cross-Validation		
		Inverse Distance Weighting	Nearest Neighbourhood	Natural Neighbour
Injected volumes	3	$2.86 \cdot 10^{11}$	$3.96 \cdot 10^{11}$	-
Permeability	18	480.8	1397.4	1044.7

5. Discussion and conclusion

The Inverse Distance Weighting maps showed a clear transitional zone, especially for the injected volumes variable. Such zones can somewhere be influenced by faults, e.g., like it influenced permeability distribution in the “L” reservoir (Figure 8, fault edges defined by points 6415600, 5028000 – 6415680, 5028780), or in the “K” reservoir (Figure 9, fault edges defined by 6420200, 5028700

– 6421220, 5029200). The similar can be observed for the injected volumes in the **Figure 8**. Furthermore, the interpolation results for the “K” reservoir (**Figure 9**) showed, so called, “bull-eye” or “butterfly” effect. Such features are usually observed in the small datasets, and need to be carefully interpreted, neglected or re-calculated (e.g., [26]). The disadvantage of the IDW could be mapping of the large transition zones with linear scaling, what could happen only in the almost totally homogeneous reservoir, without faulting.

The Nearest Neighbourhood maps are characterized with the polygons (zones). That make possible to get quick insight in the general shape of the waterfront (the “L” reservoir in **Figure 8**) and “K” reservoir in **Figure 9**). The disadvantages are the absence of a transition zone between individual wells. However, it can help to eliminate the “bull-eyes” effect in interpretation of the fault role, e.g., permeability map on **Figure 9**, where fault with SW-NE strike influenced the permeability distribution, i.e. had been older than reservoir. The similar can be concluded for the **Figure 8**, where the central (down) set of the faults approx. defined the permeability zone, i.e. had been older than reservoir.

The Natural Neighbour has one crucial difference. The method does not use extrapolation, i.e. interpolation has been done among the marginal points (**Figures 8 and 9**). Inside the interpolation area it uses transitional plotting, like the IDW. Generally, the method is unusable for the dataset with approx. less than 5 points, because interpolated area is too small, compared with the margins of the selected area (reservoir, hydrodynamic unit or similar).

Regarding analysed variables, the cumulative volumes of injected water, and indirectly the moving of the water front, can be followed the easiest at the IDW maps (**Figures 8 and 9**). Some large transitional zones can be observed between the J-166 and J-173 wells (**Figure 9**) or the L-4 and L-122 wells (**Figure 8**). Based on production history, the transitional zones, in this analysis, are defined as areas where the differences of injected volumes in two adjacent wells is larger than 50,000 m³.

All given examples are maps interpolated with the small datasets, defined as set with less than 20 points (inputs, “hard data”) that cannot be spatially analysed with advanced methods like Kriging. Moreover, the analysed dataset has been divided into three classes: (a) 1-5, (b) 6-10 and (c) 11-19 points. **The class (a)** cannot be analysed with the NaN method because it is often not possible to calculate the cross-validation and interpolated area is very small regarding unit margins. In the **class (b)** all three methods can give results. Also, in **the class (c)** the same conclusion had been given. The main selection criteria could be the cross-validation. In the all cases, the IDW had the smallest value (**Tables 2 and 3**). However, this method sometimes created inside transitional zone numerous “bull-eyes” or “half-butterfly” features. The applicability of tested method is summarised in the **Table 4**.

Table 4. Recommended interpolation methods for small input data set

Number of data	Applicability of interpolation method		
	Inverse Distance Weighting	Nearest neighbourhood	Natural neighbour
1-5	Yes	Yes	No
6-10	Yes	Yes	Yes
11-19	Yes	Yes	Yes

It could be concluded that analysed reservoirs could be mapped using he IDW, for any volume or area that includes less than 20 points (“hard” data). However, if there is observed large number of “bull-eyes” or “butterfly” features in the contours, it is recommended additionally to perform the NN, overly both maps and joint interpretation.

Author Contributions: Tomislav Malvić led the research. Josip Ivšinović made the interpolation method analyses and collected data. Josipa Velić checked regional geology and hydrocarbon settings. Rajna Rajić checked the mathematical consistency of applied equations and methods.

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