Allocating Exclusive and Intermittent Bus Lanes in Dynamic Traffic Networks

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Abstract: The urban transportation network design problem involving exclusive bus lanes (XBLs) has been widely discussed and analyzed in recent years. An improved and more flexible transit lane management strategy—intermittent bus lanes (IBLs)—prove to be potentially more efficient and car-friendly than XBLs. The common benefit of XBLs and IBLs arises from the fact that they separate the bus and car traffic and hence can eliminate the impacts of slowly moving buses on the car traffic. This paper proposes a cell transmission model for separate car and bus traffic (CTM-SCB) in a network with some dedicated roadway segments reserved for buses. By encapsulating the CTM-SCB model, an XBL-based network design problem and an IBL-based network design problem are then formulated and solved, respectively, where the former model statically sets bus lanes while the later one allows a dynamic allocation of bus lanes. A synthetic freeway-arterial network and a real-world urban street network (where the latter was extracted from the Harbin South New Industrial City) are used as test networks for evaluating the proposed models and methods. The numerical results show that both XBLs and IBLs enjoy significant operational efficiency benefits compared to the situation of no protected bus lanes. However, we believe that the expected improvement from XBLs to IBLs need further tests and validations.

Keywords: cell transmission model; dynamic traffic assignment; exclusive bus lanes; intermittent bus lanes; network design

1. Introduction

Exclusive bus lanes (XBLs), also called dedicated bus lanes (DBLs), have been implemented on urban arterials of many metropolises and achieved great success for public transit management. As shown in Figure 1, an XBL is reserved for buses, thus making buses operated in a congestion-free environment. This lane-based traffic management technique can significantly improve arrival reliability and riding comfortability of conventional bus services. However, the allocation of XBLs is meaningful only when it can improve the overall traffic efficiency. When the bus frequency and utilization are too low, XBLs probably become an uneconomical strategy.

To avoid this deficiency, Viegas and Lu [1,2] proposed the concept of intermittent bus lanes (IBLs), which were also referred to since then in the name of dynamic bus lanes (DyBLs). An IBL is often used on secondary arterials, where it is exclusively reserved for buses when needed but open to passenger cars when no bus is present. As shown in Figure 2, the IBL, which is located along the leftmost lane along the freeway, is closed for the ordinary traffic when a bus is approaching. Figure
3 shows an IBL indicator signal that is installed overhead to inform incoming car drivers before they enter the IBL-installed road segments. Compared to XBLs, IBLs not only ensure the driving priority of buses on the road, but also minimize the resulting negative impact on the car traffic, at least theoretically. Intuitively, IBLs can achieve a better balance between ensuring buses priority and causing less delay for cars.

Figure 1. XBLs in Brooklyn, NY (Source: https://en.wikipedia.org/wiki/bus_lanes_in_new_york_city).

Figure 2. IBLs in New York, NY (Source: https://ops.fhwa.dot.gov/freewaymgmt/publications/frwy_mgmt_handbook/chapter8_01.htm).

Figure 3. Signals used for IBLs in Washington, DC (Source: https://nacto.org/publication/transit-street-design-guide/transit-lanes-transitways/lane-elements/signs-signals/).

Chiabaut et al. [3] compared the capacity difference among three scenarios: Considering buses as moving bottlenecks, setting XBLs, and setting IBLs. Their work focuses on the route or link level,
which ignores the merging and diverging behaviors. To the best of our knowledge, a comparison on
the system performance of the above three scenarios in the network level rarely exists in the literature.

In the modeling paradigm of static traffic assignment or transit assignment, there is very few
research on network design problems involving IBLs. Static assignment tools cannot meet the needs
of evaluating the performance of dynamically allocated bus lanes. Based on the well-known cell
transmission model (CTM), this paper attempts to build a dynamic traffic assignment (DTA) model
for describing mixed car and bus traffic flows in a network with or without the bus lane setting and
evaluating the benefit brought from the implementation of XBLs and IBLs.

The remainder of this paper is structured as follows. Section 2 provides a literature review for
XBL- and IBL-related studies and cell-based network design problems, respectively. Section 3
presents a system-optimum (SO) CTM-based mathematical programming model that can be used to
comparatively evaluate XBLs and IBLs. Section 4 analyzes the numerical results and Section 5
concludes the paper.

2. Previous Research

The XBL-based network design has become a hot research topic, as can been in Mesbah et al. [4],
Miandoabchi et al. [5], Yu et al. [6] and Yao et al. [7]. For example, Mesbah et al. [4] designed a bi-
level model to evaluate the network performance brought by XBLs. In the upper level, the sum of
total travel time and vehicle operation cost is used as the objective function. In the lower level, the
travelers’ choice behavior is described as a combined mode and route choice in a deterministic user-
equilibrium (UE) model. The bi-level problem was solved by generalized Benders’ decomposition.
Miandoabchi et al. [5] considered the allocation of XBLs in urban road network design such as lane
addition, lane reversal, and so on. Yu et al. [6] combined the bus frequency and utilization of XBLs in
an integrated optimization manner. With or without setting bus lanes, Yao et al. [7] integrated the
stochastic link travel cost and bus waiting cost caused by road network degradation into a bi-level
XBL network design problem.

IBLs do not have a mandatory requirement of forcing ahead-of-bus passenger cars to leave the
bus lane whenever a bus gets into the bus lane. Thus, the signal timing of IBLs needs to be adjusted
to forbid the entering of passenger cars ahead of schedule and make sure that no queue is formed
when a bus runs into the bus lane. To maximize the utilization of road space, Viegas and Lu [1] firstly
designed a control system for IBLs to give priority to buses when they present. After the leaving of
buses, the lane was recovered to cars. The simulation result showed that IBLs can gain bigger
improvement for buses with less delay for other vehicle classes. Currie and Lai [8] pointed out that
the IBL implementation in Melbourne can improve the speed of trams between 1% and 10%. The
improvement was less than the case in Lisbon, which was between 15% and 25%. Based on cellular
automaton, Zhu [9] compared IBLs with XBLs in a two-lane traffic system. It appeared that these two
methods both have their applicable conditions according to the flow saturation level. Zyryanov and
Mironchuk [10] conducted experiments with the simulation software called AIMSUN\(^1\) and evaluated
the speed changes of different vehicle classes. Guler and Menendez [11] analyzed delays at signalized
intersection that provide priorities to buses. The intersections can intermittently close one car lanes
and reserve it for buses before their passing.

Different from IBLs, bus lanes with intermittent priority (BLIPs) use variable message signs to
make the cars in front of buses leave the bus lane. Eichler and Daganzo [12] applied kinematic wave
theory for analyzing the performance of BLIPs. The results showed that BLIP's perform the best when
the traffic flow rates reach the saturation rate. Based on the VISSIM\(^2\) simulation software, Carey et al.

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1 AIMSUN is short for Advanced Interactive Microscopic Simulator for Urban and Non-Urban Network. It is an integrated
modeling tool that provides transport solutions which includes the static and dynamic traffic assignment methods with
microscopic, mesoscopic and hybrid simulations. It also allows many operation from adding bus lanes to study a total area.

2 VISSIM stands for "Verkehr In Städten-SIMulationsmodell" (German for "Traffic in cities-simulation model"). It is a
microscopic multi-modal traffic flow simulation software package produced and distributed by the German company PTV.
[13] conducted a BLIP study for Eugene, Oregon, which shows that the implementation of BLIP can obtain improvement in the peak direction. Xie et al. [14] used the simulation software SymuVia to evaluate BLIPs and gain the key parameters that influenced the application most.

Allocating IBLs and BLIPs on the network level remains a challenge for researchers because of the lack of an appropriate traffic flow modeling method. Thus, it is very important to build a model, which can properly describe the behaviors of the mixed car and bus traffic for the IBL network design problem.

Although many theories and methods can be utilized to model this kind of mixed traffic, we hereby review and present two mostly used modeling diagrams, namely macroscopic fundamental diagram (MFD) and CTM, which are both close to the fundamental diagram of traffic flow.

The fundamental diagram of traffic flow is a graphical tool that depicts the relationship between traffic volume and traffic density. A macroscopic traffic model involving traffic volume, traffic density and traffic speed constitutes the basis of the fundamental diagram. It can be utilized to predict the capability of a road link, or its behavior when applying inflow regulation or speed limits. Treating buses as moving bottlenecks, the fundamental diagram has been used to study the mixed car-bus traffic flow for decades. The relevant works include Lebacque et al. [15], Lattanzio et al. [16], Delle Monache and Goatin [17].

MFD reveals the relationships between accumulation flow and average speed or flow, and also between average flow and outflow in the aggregate level (Daganzo and Geroliminis [18]). Moreover, MFD holds an intrinsic property of the network itself and remains invariant when demand changes (Chiabaut [19]). It is presented as a network modeling method that describes transportation system in macroscopic levels. MFD and its extend theories are often used in estimation of average trip times, description of network characteristics, perimeter flow control and so on.

Classically, MFD usually describes the relationship between accumulation vehicle flow, inflow, outflow and network-inside flow (Leclercq et al. [20]). When the outflow is bounded by the supply of the neighboring link, it also corresponds to the discretized version of the Lighthill-Whitham-Richards (LWR) model (Lighthill and Whitham [21], Richards [22]) that is also known as CTM (Daganzo [23,24]).

CTM is a mesoscopic traffic flow modeling method and has been developed as a practical DTA tool that can provide more detailed behaviors such as merging, diverging and spillback phenomenon than MFD. Different from the invariant of MFD in describing network flow characteristics, CTM presents variations of the density or vehicle accumulation flow in each cell that is highly correlated to the demand pattern of transportation network. CTM and modified CTMs were the primary research tools when investigating traffic signal control, evacuation management, dynamic traffic assignment and travel time prediction (Liu et al. [25]).

For a comprehensive review of MFD, the reader can refer to Ampountolas et al. [26]. For a comprehensive review of CTM, the reader can refer to Ngoduy et al. [27]. Moreover, the integrated work of MFD and CTM is referred to Zhang et al. [28].

By using MFD, Chiabaut and his research group conducted a series of studies on evaluating the IBL setting and controlling strategies. First, Chiabaut et al. [29] proposed an extended kinematic wave model and used it to evaluate the BLIP strategy. Although BLIPs reduce roadway capacity and increase bus travel time initially, they pointed out that BLIPs will improve the system behavior in a larger area. Then, Chiabaut [19] extended the original car-only MFD to the passenger bi-modal MFD. Furthermore, Chiabaut et al. [3] showed that, in most synthetic cases, the arterial capacity in the mixed traffic condition is higher than the one in the IBL allocation condition, and so are the arterial capacity in the IBL allocation condition higher than the one in the XBL allocation condition. But in a more realistic case with congested traffic, setting IBLs became more attractive in terms of capacity than setting XBLs and non-IBL or non-XBL setting.

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3 SymuVia is a dynamic microscopic software package that developed by LICIT (Laboratoire d'Ingénierie Circulation Transports). It produces each single vehicle trajectory based on the Lighthill-Whitham-Richards model.
In its basic linear version, CTM consists only one class of passengers—private car passengers (Ziliaskopoulos, [30]). To set up bus lanes, buses are the most important part of all vehicles which cannot be ignored. Recently, many studies focused on incorporating other vehicle classes into the CTM framework. Tuerprasert and Aswakul [31] developed a multi-class CTM model which divided a cell into a head cell and a tail cell to account for the speed difference. Mesa-Arango and Ukkusuri [32] incorporated trucks into CTM and proposed a car-truck interaction DTA model. To simulate the propagation of mixed traffic flows consisting of human-driven and autonomous vehicles, Levin and Boyles [33] developed a new multi-class CTM by assuming the same speed but different reaction time for different vehicle classes. The new model was used to evaluate the different network performance under different autonomous vehicle proportions. Liu et al. [25] integrated buses into CTM which derived the bus CTM model that can represent the moving bottleneck phenomenon of buses in the mixed traffic.

Based on CTM, many dynamic network design problems were formulated and solved such as roadway capacity expansion (Waller and Ziliaskopoulos [34], Ukkusuri and Waller [35]), infrastructure maintenance planning (Ng et al. [36]), emergency evacuation planning (Xie et al. [37], Guo et al. [38]), optimal signal control strategies (Zhang and Chang [39]), air traffic flow management (Wei et al. [40]), roadway capacity expansion and signal control optimization (Karoonsoontawong and Waller [41]) and variable speed limit control (Hadizuzzaman and Qiu [42]) among others.

Compared to the former studies, this paper first proposes an SO-oriented DTA model for mixed car and bus traffic. The model can be used to describe traffic flows in a network with or without setting up XBLs and IBLs. Rather than studying IBLs along a corridor such as Chiabaut et al. [3], this paper allocates the XBLs and IBLs on the network level.

We emphasize a number of particular reasons for which we choose the SO principle in this study for the proposed network design problems:

1. Under free-flow or light congestion situations, the network performance in the SO state is the approximately same as or close to that under the UE state in any traffic network.

2. Under congested conditions, while the SO result cannot approximate the UE one, the former provides a lower bound for the latter. This lower bound information proves to be useful in many traffic system optimization or evaluation cases.

3. In some traffic network design exercises (e.g., Poorzahedy and Turnquist [43]), the SO objective could be approximately replaced by the UE objective and vice versa in evaluating how the system performance is improved by adding (or removing) capacity to the system or changing the system operations, since our design concern is de facto the system performance improvement instead of the system performance itself.

4. Using the SO objective greatly simplifies the modeling and solution complexity. It makes the application of the proposed method for large transport networks much more tractable.

3. Problem Formulation

3.1 Daganzo’s CTM

CTM was introduced by Daganzo [23] in 1990s. As an approximation of the hydrodynamic traffic flow theory, CTM recreates the flow fluctuation in discrete cells and time intervals. The model is specified by the conservation and propagation laws as follows:

\[ n_{i+1}^{t+1} = n_i^t + y_i^t - y_{i+1}^{t+1} \]  \hspace{1cm} (1)

\[ y_i^t = \min \{ n_{i-1}^t, Q_i^t, \delta (N_i^t - n_i^t) \} \]  \hspace{1cm} (2)

where the subscripts \( i \) and \( i + 1 \) represent the indexes of two spatially consecutive cells and the superscripts \( t \) and \( t + 1 \) represent the indexes of two consecutive time intervals. By using these subscripts and superscripts, \( n_i^t \) and \( y_i^t \) are the number of vehicles in cell \( i \) at time interval \( t \) and the inflow of cell \( i \) at time interval \( t \), respectively; \( Q_i^t \) and \( N_i^t \) are the maximum propagation flow of
cell \( i \) and the maximum number of vehicles that can be held in cell \( i \). Finally, \( \delta \) is the ratio of the free-flow speed over backward propagation speed.

Equation (1) specifies the conservation rule, being used to update the number of vehicles in a cell over time. Equation (2) is the propagation rule, determining how much flow is transferred between two consecutive cells. Given an initial system status, the above equations can decide the evolution of traffic flow over a series of cells along a route and obtain the state of any cell in the route for any time interval.

Daganzo [24] further extended CTM from the route level to the network level by introducing the propagation rules for emerging cells and diverging cells. Ultimately, CTM became a simple but powerful dynamic flow modeling tool that can simulate and evaluate traffic flows on the network level.

3.2 Ziliaskopoulos’ CTM-based DTA model

Based on CTM (Daganzo, [23], [24]), Ziliaskopoulos [30] built a linear programming (LP) model to describe the evolution of dynamic network flows under the SO principle. This CTM-embedded LP model features a DTA tool with a single destination. The linear structure of the model provides advantages on both mathematical analysis and solution development.

3.3 Problem description

As we described earlier, the main difference between XBL and IBL is that the exclusive use of bus lanes in XBL is fixed over time while bus lanes in IBL are interchangeably used by cars and buses.

So in the XBL-based network design problem, the bus lane allocation variables are time-independent; in the IBL-based problem, the bus lane allocation variables are time dependent. Nevertheless, the common network flow modeling component underlying these two network design problems is a DTA model for mixed car and bus traffic.

In the following, we first propose a modified CTM model as the underlying DTA tool used in the above two network design problems. Then, based on this DTA tool, the XBL-based and the IBL-based network design problems are respectively formulated.

3.4 Notation

The notation used in this paper is shown as follows.

Sets

\[
M = \{m| \text{car, bus}\}
\]

\[
C = \text{Set of cells, which is a union of the set of general traffic cells } C^g \text{ and the set of bus lane cells } C^b, \text{i.e., } C^g \cup C^b = C,
\]

\[
T = \text{Set of discrete time intervals (excluding } T = 0) \text{ during the analysis period, where } T^{\text{car}} \text{ is the subset of } T \text{ that the travel demand can only be loaded into the original cells by the car mode, } T^{\text{car+bus}} \text{ is the subset of } T \text{ that the travel demand can be loaded into the original cells by both car and bus modes, i.e., } T^{\text{car}} \cup T^{\text{car+bus}} = T
\]

\[
E = \text{Set of connectors, i.e., } E = \{(i,j)\}, \text{where cells } i \text{ and } j \text{ are consecutive cells.}
\]

\[
\Gamma(i) = \text{Set of successor cells of cell } i
\]

\[
\Gamma^{-1}(i) = \text{Set of predecessor cells of cell } i
\]

Parameters

\[
a^m = \text{Passenger capacity of vehicle type } m
\]

\[
p_m = \text{Passenger car equivalent factor of vehicle type } m
\]

\[
d^i_t = \text{Total number of passengers originating from source cell } i \text{ at time interval } t
\]

\[
d^i_{m,t} = \text{Number of passengers in vehicle type } m \text{ originating from source cell } i \text{ at time interval } t
\]

\[
Q^i_{m,t} = \text{Maximum number of vehicles of type } m \text{ that can move into or move out of cell } i \text{ at time interval } t
\]
\( q_{i,m} \) Maximum number of vehicles of type \( m \) that can move into or move out of cell \( i \) at time interval \( t \) in one bus lane

\( N_i \) Maximum number of vehicles held in cell \( i \)

\( \theta \) Car capacity reduction per bus flow unit

\( \delta_i \) Ratio of the free-flow speed over backward propagation speed of cell \( i \) at time interval \( t \)

\( v_0^m \) Free-flow speed of vehicles of type \( m \)

\( \zeta_i^m \) Capacity of cell \( i \) for vehicles of type \( m \) at time interval \( t = 0 \)

\( \phi_i^t \) Sum of set-up and operational costs of a bus lane in cell \( i \) at time interval \( t \)

\( \psi \) Available total budget for setting up bus lanes

\( n_i \) Number of lanes in cell \( i \)

**Variables**

\( x_{i,m}^t \) Number of vehicles of type \( m \) for cell \( i \) at time interval \( t \)

\( y_{ij}^t \) Number of vehicles of type \( m \) propagating from cell \( i \) to cell \( j \) at time interval \( t \)

\( \phi_i^t \) Binary variable that indicates whether or not a bus lane in cell \( i \) at time interval \( t \) is open, where \( \phi_i^t = 1 \) means the opening of the bus lane and \( \phi_i^t = 0 \) means the closeness.

### 3.5 A bi-modal SO DTA model

Inspired by the work of Ziliaskopoulos [30] and Mesa-Arango and Ukkusuri [32], we revised the so-called cell transmission model with car-truck interactions (CTM-CTI) and proposed a new CTM named the cell transmission model for separate car and bus traffic (CTM-SCB). Different from the bus-based CTM (Liu et al. [25]) in which cars and buses share road capacity, road capacity is divided into separate car capacity and bus capacity in CTM-SCB. By separate here, we include two different capacity separation cases: Spatially separate and temporally separate. Spatially separate is used here in the case of XBLs, in which bus lanes are exclusively reserved for buses and cars are not allowed to enter these lanes all the time, while temporally separate is used in the case of IBLs in which bus lanes are only temporally reserved for buses when needed and these lanes are open to cars in the remaining time.

Mesa-Arango and Ukkusuri [32] constructed a so-called CTM-CTI model based on five extra assumptions compared to those for the original CTM. We simply inherited them and substituted trucks with buses. To make the model more tractable, the following three new specifications or assumptions are introduced to integrate a bus system into a cell-based network.

1. Multiple cell types. Since buses are operated only on the pre-specified routes and car and bus movements are treated in different ways under our settings, we categorize all cells into two types: general traffic cells and bus lane cells. General traffic cells are those cells not serving buses and bus lane cells are those cells on bus routes. The general traffic cells only store and propagate car flows and the bus lane cells can store and propagate bus flows only (for XBLs) or both car and bus flows (for IBLs).

2. Demand frequency and reservation. The frequency of buses is often fixed on the road segments which results in the intermittent arrivals of buses. Consequently, bus passengers have to wait for buses at bus stops and they can only board on buses every fixed time intervals. To simulate this behavior, the bus demand is injected every several time intervals. In constraints (4) and (5) of CTM-SCB, the total demand can only be assigned to the car mode in time set \( T^{car} \) and the total demand can be assigned to both the car mode and bus mode in time set \( T^{car+bus} \), where \( T^{car} \cup T^{car+bus} = T \). Also, the total demand between any O-D pair is fixed where the car demand and bus demand are variables dependent on passengers’ mode choice results. It is assumed that the implementation of XBLs or IBLs will not induce any new demand and the total demand is not a function of the travel time between this O-D pair.

3. One-destination networks. In this study, only one-destination traffic networks are used, for both car and bus traffic, which greatly enhance the solution tractability of the proposed models. Further work will extend the presented modeling and solution methods to more general multi-destination networks. It is hoped that the magnitude of the network efficiency benefits from
implementing XBLs and IBLs between one-destination networks and multi-destination networks are comparable.

Based on the above specifications or assumptions, our CTM-SCB model is presented here in the following form, which covers both the spatially and temporally separate cases defined above.

This is an example of an equation:

$$\min Z = \sum_{\forall m \in M} \sum_{\forall t \in T} \sum_{\forall i \in C_G} a^m x_i^{t,m}$$

subject to

$$d_i^{t-1} = d_i^{t-1,\text{car}}$$ \quad \forall i \in C_G, \forall t \in T^{\text{car}}$$

$$d_i^{t-1} = d_i^{t-1,\text{car}} + d_i^{t-1,\text{bus}}$$ \quad \forall i \in C_G, \forall t \in T^{\text{car+bus}}$$

$$x_i^{t,m} = x_i^{t-1,m} + p_m d_i^{t-1,m} - \sum_{\forall j \in \Gamma(t)} y_{ij}^{t-1,m}$$ \quad \forall i \in C_G, \forall t \in T; \forall m \in M$$

$$x_i^{t,m} = x_i^{t-1,m} + \sum_{\forall j \in \Gamma(t)} y_{ji}^{t-1,m}$$ \quad \forall i \in C_G, \forall t \in T; \forall m \in M$$

$$\sum_{\forall j \in \Gamma(t)} y_{ij}^{t,\text{car}} \leq x_i^{t,\text{car}}$$ \quad \forall i, j \in C^g, (i,j) \in E, \forall t \in T$$

$$\sum_{\forall j \in \Gamma(t)} y_{ij}^{t,\text{car}} \leq Q_i^{t,\text{car}} - \theta x_i^{t,\text{bus}}$$ \quad \forall i, j \in C^g, (i,j) \in E, \forall t \in T$$

$$\sum_{\forall j \in \Gamma^{-1}(i)} y_{ji}^{t,\text{car}} \leq Q_i^{t,\text{car}} - \theta x_i^{t,\text{bus}}$$ \quad \forall i, j \in C^g/c^g, (i,j) \in E, \forall t \in T$$

$$\sum_{\forall j \in \Gamma^{-1}(i)} y_{ji}^{t,\text{car}} \leq Q_i^{t,\text{car}} - \theta (x_i^{t,\text{bus}} - x_i^{t-1,\text{bus}})$$ \quad \forall i, j \in C^g, (i,j) \in E, \forall t \in T$$

$$\sum_{\forall j \in \Gamma^{-1}(i)} y_{ji}^{t,\text{car}} \leq \delta_i^{t} N_i^t - \delta_i^{t} (x_i^{t,\text{car}} + x_i^{t,\text{bus}})$$ \quad \forall i, j \in C^g, (i,j) \in E, \forall t \in T$$

$$\sum_{\forall j \in \Gamma^{-1}(i)} y_{ij}^{t,\text{bus}} \leq \left(\frac{v_{bus}}{v_{car}}\right) x_i^{t,\text{bus}}$$ \quad \forall i, j \in C^b, (i,j) \in E, \forall t \in T$$

$$\sum_{\forall j \in \Gamma^{-1}(i)} y_{ij}^{t,\text{bus}} \leq Q_i^{t,\text{bus}}$$ \quad \forall i, j \in C^b, (i,j) \in E, \forall t \in T$$

$$\sum_{\forall j \in \Gamma^{-1}(i)} y_{ji}^{t,\text{bus}} \leq Q_i^{t,\text{bus}}$$ \quad \forall i, j \in C^b, (i,j) \in E, \forall t \in T$$

$$\sum_{\forall j \in \Gamma^{-1}(i)} y_{ji}^{t,\text{bus}} \leq \delta_i^{t} N_i^t - \delta_i^{t} (x_i^{t,\text{car}} + x_i^{t,\text{bus}})$$ \quad \forall i, j \in C^b, (i,j) \in E, \forall t \in T$$

$$x_i^{0,m} = \zeta_i^m$$ \quad \forall i \in C; \forall m \in M$$

$$y_{ij}^{0,m} = 0$$ \quad \forall (i,j) \in E; \forall m \in M$$

$$x_i^{t,m} \geq 0$$ \quad \forall i \in C; \forall t \in T; \forall m \in M$$

$$y_{ij}^{t,m} \geq 0$$ \quad \forall (i,j) \in E; \forall t \in T; \forall m \in M$$
The objective function (3) is set to minimizing the total travel time, which is the sum of all passengers’ travel time, where \( \alpha^m x_{it}^m \) is equal to the number of passengers of vehicle type \( m \) for cell \( i \) at time interval \( t \). Note that the duration of time intervals is omitted because it is a constant. Equations (4) and (5) imply that the car mode can only be chosen in time interval \( T_{car} \) and both the car and bus modes can be chosen in time interval \( T_{car+bus} \). This special demand loading pattern on some degree reflects the waiting behavior of bus passengers in that the bus demand can be only loaded to the network in specific intervals \( T_{car+bus} \).

Constraints (6), (7) and (8) are flow conservations for source cells, sink cells and ordinary cells, respectively.

Constraints (9) to (13) regulate the flow propagation for cars. Constraint (9) indicates the flow propagation of cars in the free-flow speed condition. Constraints (10) to (12) represent that the passing car flow between consecutive cells should not exceed the maximum capacity minus capacity reduction caused by the bus moving bottlenecks. Constraint (10) is applied for the outgoing flow of the upstream cells. Constraint (11) describes the incoming flow of the downstream cells except sink cell (source cells and ordinary cells). Constraint (12) describes incoming flow of the downstream sink cell. Constraint (13) represents the incoming car flow to a cell must not be higher than its remaining capacity.

Constraints (14) to (17) regulate the flow propagation for buses. Constraint (14) indicates the flow propagation of buses in the free-flow speed condition. Without any priority implementations, the free-flow speed of buses is often slower than cars. We derive that the propagation flow of buses between consecutive cells in the free-flow condition cannot be higher than \( (y^b_{0}/u_{0}^{car})x_{it}^{bus} \). Constraint (15) represents that the outgoing bus flow from the upstream cells should not exceed the maximum capacity. Constraint (16) represents that the incoming bus flow to the downstream cells should not exceed the maximum capacity. Constraint (17) represents the incoming bus flow to a cell must not be higher than its remaining capacity.

Constraints (18) and (19) specify the initial number of vehicles in the cells and the flow propagating between consecutive cells in the first time interval.

Constraints (20) and (21) denote that the number of vehicles in any cells and the propagating flow between any consecutive cells must be non-negative.

### 3.6 An optimal XBL allocation model

The set-up and operational costs for bus lanes can be referred to in Danaher, Levinson, and Zimmerman [44] and Mahendra and Ang-Olson [45]. However, transit priority signals installed at intersections, as described by, for example, Guler and Menendez [11], is ignored and the resulting costs for setting transit priority signals at intersections are not considered. This simplification can make the model much easier to construct and solve.

Based on the CTM-SCB model proposed above, an optimal XBL allocation model is constructed below. In this network design model, the exclusive road priority is given to buses running on XBLs and cars are not allowed to use the XBLs all the time. Thus, the main decision of the model is a one-time allocation variable \( \phi_i \) for each candidate road segment, in which \( \phi_i \) is fixed in the whole analysis period. Some of the constraints in CTM-SCB are modified to incorporate the XBL setting into this model.

Specifically, constraints (10) to (12) are revised and reformulated as constraints (22) to (24) that are shown as follows.

\[
\sum_{j \in T_i} y_{ij}^{car} \leq (1 - \phi_i)(Q_i^{t,car} - \theta x_{i}^{t,bus}) + \phi_i \frac{n_i - 1}{n_i} Q_i^{t,car} \quad \forall i, j \in C_i, (i, j) \in E, \forall t \in T
\]

\[
\sum_{j \in T^{-1}_i} y_{ji}^{car} \leq (1 - \phi_i)(Q_i^{t,car} - \theta x_{i}^{t,bus}) + \phi_i \frac{n_i - 1}{n_i} Q_i^{t,car} \quad \forall i, j \in C_i/C_i, (i, j) \in E, \forall t \in T
\]
\[ \sum_{j \in T^{-1}(i)} y_{ji}^{t,\text{car}} \leq (1 - \phi_i)(Q_i^{t,\text{car}} - \theta(x_i^{t,\text{bus}} - x_i^{t-1,\text{bus}})) \quad \forall i, j \in C^g, (i, j) \in E, \forall t \in T \]  
\[ + \phi_i \frac{n_i - 1}{n_i} Q_i^{t,\text{car}} \]

The variable \( \phi_i \) is the indicator that decides whether or not cell \( i \) sets up an XBL. If an XBL is set up in cell \( i \), then \( \phi_i = 1 \); if not, then \( \phi_i = 0 \). Thus, constraint (22) implies: The capacity is all given to car traffic when no XBL is set in the cell; when an XBL is set in the cell, one lane is dedicated to buses and the capacity for car traffic is decreased by one lane. As similar to the formulation of the outgoing flow from the upstream cells in constraint (22), constraints (23) and (24) present a similar condition for the incoming flow to the downstream cells.

Constraints (14) to (16) are modified and reformulated as constraints (25) to (27) as follows.

\[ \sum_{j \in T(I)} y_{ij}^{l,\text{bus}} \leq (1 - \phi_i)(v_i^{l,\text{bus}}/v_i^{c,\text{car}}) x_i^{t,\text{bus}} + \phi_i x_i^{t,\text{bus}} \quad \forall i, j \in C^b, (i, j) \in E, \forall t \in T \]  
\[ \sum_{j \in T^{-1}(i)} y_{ij}^{l,\text{bus}} \leq (1 - \phi_i)Q_i^{l,\text{bus}} + \phi_i \bar{Q}_i^{l,\text{bus}} \quad \forall i, j \in C^b, (i, j) \in E, \forall t \in T \]  
\[ \sum_{j \in T^{-1}(i)} y_{ij}^{l,\text{bus}} \leq (1 - \phi_i)Q_i^{l,\text{bus}} + \phi_i \bar{Q}_i^{l,\text{bus}} \quad \forall i, j \in C^b, (i, j) \in E, \forall t \in T \]

Constraint (25) indicates the flow propagation of buses in the free-flow speed condition with or without setting XBLs. With setting XBLs, buses run at the same free-flow speed as cars. Constraint (26) represents the maximum outgoing bus flow from the upstream cells with or without setting XBLs. With setting XBLs, the outgoing bus flow from the upstream cells should not exceed the maximum capacity in one bus lane. Constraint (27) regulates the maximum incoming bus flow to the downstream cells with or without setting XBLs.

Constraints (28) and (29) shown below are the XBL setting constraints.

\[ \sum_{i \in C^b} \phi_i \phi_i \leq \Psi \]  
\[ \phi_i \in \{0, 1\} \quad \forall i \in C^b \]

Constraint (28) presents that the total cost for setting up XBLs must not be higher than the available budget. Constraint (29) defines that the XBL setting variable \( \phi_i \) is a binary variable.

In summary, the constraint set for the optimal XBL allocation model includes constraints (3) to (9), constraint (13), and constraints (17) to (29).

3.7 An optimal IBL allocation model

Based on CTM-SCB, an optimal IBL allocation model is also proposed. IBLs are exclusively used by buses when they enter the bus lanes and the bus lanes are then returned to cars when no bus appears. The bus lanes are intermittently assigned to buses or cars in terms of the presence of buses. As a result, the bus lane allocation variable \( \phi_i \) is integrated into the CTM-SCB and reflects the time-varying utilization of bus lanes.

For the IBL setting, constraints (10) to (12) are revised and reformulated as constraints (30) to (32) as follows.

\[ \sum_{j \in T(I)} y_{ij}^{l,\text{car}} \leq (1 - \phi_i)(Q_i^{l,\text{car}} - \theta(x_i^{l,\text{bus}})) + \phi_i \frac{n_i - 1}{n_i} Q_i^{l,\text{car}} \quad \forall i, j \in C^g, (i, j) \in E, \forall t \in T \]  
\[ \sum_{j \in T^{-1}(i)} y_{ij}^{l,\text{car}} \leq (1 - \phi_i)(Q_i^{l,\text{car}} - \theta(x_i^{l,\text{bus}})) + \phi_i \frac{n_i - 1}{n_i} Q_i^{l,\text{car}} \quad \forall i, j \in C^g/C^g_s, (i, j) \in E, \forall t \in T \]
\[ \sum_{j \in \Gamma(i)} y_{ji}^{t,car} \leq (1 - \phi_i^t) [Q_i^{t,car} - \theta(x_i^{t,bus} - x_i^{t-1,bus})] + \phi_i^t \frac{n_i - 1}{n_i} Q_i^{t,car} \quad \forall i, j \in C^b, (i, j) \in E, \forall t \in T \] (32)

Constraints (30) to (32) are similar to constraints (22) to (24). The differences are that the XBL setting variable \( \phi^t \) is replaced by the IBL setting variable \( \phi_i^t \).

And constraints (14) to (16) are modified and reformulated as constraints (33) to (35) as follows.

\[ \sum_{j \in \Gamma(i)} y_{ji}^{t,bus} \leq (1 - \phi_i^t) (v^b_{i,j}/v_o^{car}) x_i^{t,bus} + \phi_i^t x_i^{t,bus} \quad \forall i, j \in C^b, (i, j) \in E, \forall t \in T \] (33)

\[ \sum_{j \in \Gamma(i)} y_{ji}^{t,bus} \leq (1 - \phi_i^t) Q_i^{t,bus} + \phi_i^t \tilde{Q}_i^{t,bus} \quad \forall i, j \in C^b, (i, j) \in E, \forall t \in T \] (34)

\[ \sum_{j \in \Gamma(i)} y_{ji}^{t,bus} \leq (1 - \phi_i^t) Q_i^{t,bus} + \phi_i^t \tilde{Q}_i^{t,bus} \quad \forall i, j \in C^b, (i, j) \in E, \forall t \in T \] (35)

Constraints (33) to (35) are similar to constraints (25) to (27). The difference is also the replacement of bus lanes setting variable.

The variable \( \phi_i^t \) in constraints (28) and (29) are substituted by \( \phi_i^t \) and the new constraints (36) and (37) are shown as follows.

\[ \sum_{i \in C^b} \sum_{t \in T} \varphi_i^t \phi_i^t \leq \Psi \] (36)

\[ \phi_i^t \in \{0,1\} \quad \forall i \in C^b, \forall t \in T \] (37)

In summary, the constraint set for the optimal IBL allocation model includes constraints (3) to (9), constraint (13), constraints (17) to (21), and constraints (30) to (37).

### 4. Numerical Results

#### 4.1 Solution algorithm

It is obvious that the CTM-SCB problem is in the LP form while both the XBL and IBL network design problems are in the mixed-integer nonlinear programming (MINLP) form. It is well known that LP problems can be solved by many commercial solvers efficiently. However, MINLP problems are much more complex and harder to solve.

In this study, we use the DIscrrete and Continuous OPTimizer (DICOPT) solver developed by Viswanathan and Grossmann [46] to decompose and solve the resulting MINLP problems. DICOPT can solve problems where the convexity condition may not hold. However, it does not guarantee the solutions to be global optimum.

The MINLP model can be described as follows:

\[ \min Z = f(x, y, \phi) \] (38)

\[ f(x, y, \phi) \sim b \] (39)

\[ x \geq 0 \] (40)

\[ y \geq 0 \] (41)

\[ \phi \in \{0,1\} \] (42)

where \( x \) is a vector that represents flows in cells, \( y \) is a vector that represents flows propagating between cells, \( \phi \) is a vector that represents bus-lane-setting indicators. The symbol \( \sim \) represents the relational operators \( \leq, =, \geq \).
To solve the proposed MINLP problem, three key techniques are used: Outer approximation, equality relaxation and augmented penalty. During the solution process, it first solves a relaxed nonlinear programming (NLP) problem. If integer solutions do not exist, a series of iterations consisting of NLP and mixed integer linear programming (MILP) problems are solved then. The MILP master problem is solved using outer approximation and equality relaxation algorithms where an exact penalty function allows the existence of nonconvex constraints.

Above all, the optimization software of General Algebraic Modeling System (GAMS, [47]) is utilized as a solution tool which respectively implements CPLEX and DICOPT solvers to solve the proposed LP and MINLP models. In GAMS, the sets, parameters, variables, equations, models and solution statements are all programmed. All the computational results presented below were conducted on a Windows-based computer with a Core 2 Duo 2.60-GHz CPU and 4 GB RAM.

### 4.2 A synthetic network

The models are tested on an example synthetic network depicted in Figure 4 for various levels of travel demand and construction budget. It should be emphasized, however, that the experiments are presented primarily for demonstrating the simplicity and applicability of the proposed models. The results may not be generalized because they are produced from only one specific network topology; in addition, the parameter values used are not necessarily realistic but taken by convenience for demonstration. The focus of this analysis is on problem behavior rather than computational efficiency or policy recommendation.

![Figure 4. The synthetic freeway-arterial network: (a) Node-arc representation; (b) Cell representation.](image)

In its node-arc topology, the test network consists of 22 nodes and 30 arcs (see Figure 4(a)). The links vertically assigned along the middle line represent a freeway facility, while the outer and crossing links represent arterial roads. There are three origin nodes (node 1, 8, and 13) and a single destination node (node 7). There exist three bus lines currently being operated in this network. Bus line 1 starts from node 1 and ends at node 7 passing through nodes 2, 3, 4, 5 and 6. Bus line 2 starts from node 13 and ends at node 7 passing through nodes 12, 4, 14, 15, 19, 20, 21 and 22. Bus line 3 starts from node 8 and ends at node 7 passing through nodes 9, 10, 11, 12, 16, 17 and 18. The network characteristics are the same as mentioned by Waller et al. [48].
The corresponding cell-based network (see Figure 4(b)) was constructed based on the above geometric and road characteristics. The size of each time interval was set to 5 seconds and the partition of links into cells was done according to Daganzo’s CTM theory.

For a freeway link with the free-flow speed of 100 feet/sec, a free-flow-moving vehicle travels 500 feet in 5 seconds; therefore, the size of the cells on this link will be 500 feet. A similar process is repeated for the arterial links, arriving at a length of 250 feet. For this example network, the size of the cell and the number of lanes determine the value of the jam density for each cell. We assume that at jam density, a 250-feet freeway cell can hold at most 20 vehicles (including cars and buses) and a 250-feet arterial cell 10 vehicles.

The maximum number of vehicles that can move into or move out of cells depends on the saturation flow rate and the size of time intervals. For an arterial saturation flow rate of 2,880 vehicles per hour per lane (vphpl) and a 5-second time interval, at most 4 vehicles can be transmitted out of (or into) a one-lane cell and 8 vehicles out of (or into) a two-lane cell. For a freeway link with three lanes and the saturation flow rate of 2,880, this leads to 12 vehicles per lane per 5-second interval.

When $Q_i^{c,car}$ equals to 12, the saturation flow rate of buses $Q_i^{b,bus}$ is set up to 6 and $Q_i^{t,bus}$ is set up to 12. The rate of $v_i^{bus}/v_i^{car}$ is equal to 0.75. The capacity reduction parameter $\theta$ is set to be 0.75. The passenger car equivalent factor of cars is set as $p_{car} = 1$ and the factor of buses is set as $p_{bus} = 2$. The passenger capacity of cars and buses are set as $\alpha^{c,car} = 1$ and $\alpha^{bus} = 4$ separately. Finally, all $\delta$ values are taken as 1 for simplicity.

A total demand of 924 passengers in the initial 10 time intervals is loaded into source cells. The bus frequency for the three bus lines is set to be 5 time intervals for computation simplicity. The total simulation time is 60 time intervals.

The one-time XBL installation fee for each cell is US$250,000, and the total budget is US$17,500,000 over the entire network. In contrast, the IBL real-time operational fee for each cell in every time interval is US$4,300.

We first evaluated the network flow solution in the original network without setting up XBLs or IBLs. The total passenger travel time (TPTT) is 19,746. The total car passenger travel time (TCPTT) is 4,738 and the total bus passenger travel time (TBPTT) is 15,008. Then, we evaluated the network flow solution with XBLs. The TPTT, TCPTT and TBPTT are respectively 18,997, 4,591 and 14,406, respectively.

The optimal XBL setting shows that the cells along bus line 1 and part of bus line 2 and 3 are used for setting up XBLs. The XBLs prompt an early arrival of the buses at the sink cell. In the meantime, as a result, the traffic flow of cars that are occupied by XBLs is apparently reduced. Most of the cars shift their routes to the parallel paths from those paths with XBLs. Such a coordination between cars and buses makes the total travel time of the network better reduced.

To further test the effect of demand and budget on the system performance, we ran a sensitivity analysis. The travel demand is set as 5 levels that are from the light congestion to heavy congestion. The total budget used for XBLs is set from 0 to 1500 with an interval of 500, which results in 5 budget levels. The results are shown in Figure 5. In these figures, the Z-axis are represented by TPTT, TCPTT and TBPTT differently. From Figure 5, we can find: Increasing demand increases the total car/bus passenger travel time; the higher construction budget set for XBLs, the less total car/bus passenger travel times.

![Figure 5. Network performance evaluation with budget level and demand level: (a) TPTT; (b) TCPTT; (c) TBPTT.](image-url)
Then, we evaluated the network flow solutions resulted from IBLs. The TPTT, TCPTT and TBPTT are respectively 18,877, 4,271 and 14,606. As a result, bus lanes are opened to buses whenever they arrive at the corresponding cells. For other times, bus lanes are closed, which results in the return of one lane to cars. From cell 1 to cell 30, the car capacity is increased after buses leave and more cars can now use these paths. In other words, setting up IBLs improves the utilization efficiency of the road capacity and reduces the clear time of the network.

As mentioned above, without setting bus lanes, the TPTT value is 19,746. The optimal XBL setting saves 749 unit values in the objective function and improves the system efficiency by 3.9%. And the optimal IBL setting saves 869 unit values, which corresponds to a 4.6% reduction of TPTT. Obviously, both XBLs and IBLs can improve the system performance in a quite significant manner.

To further compare the performance between XBLs and IBLs, we depicted the above results in a comparative manner in Figure 6. Figure 6(a) is a 3-dimensional map that shows the improvement percentage of IBLs compare to XBLs, and Figure 6(b) simply shows the positive value area or negative value area of improvement percentage. From Figure 6, we can find that most of the scenarios have the results that IBLs improve the system behavior better than XBLs.

Figure 6. Performance improvement from XBLs to IBLs with budget level and demand level: (a) Three-dimensional result; (b) The projected result on the budget-demand plane.

The above evaluation result is consistent with that by Chiabaut et al. [3]: The appropriate condition of setting up IBLs that reduces TPTT and improve system performance rarely exists and is restricted by many factors.

The computation time of solving CTM-SCB is about 7 seconds. In the meantime, the computation time of solving the optimal XBL allocation problem and the optimal IBL allocation problem are 326 seconds and 753 seconds, respectively. As CTM-SCB is a LP problem, it can be solved efficiently. When we solve two bus lane allocation problems, the computation time increases to a much higher level. Moreover, the computation time of solving the IBL allocation problem is more than twice the time of solving the XBL allocation problem.

4.3 A real network

The road network with bus lines in part of the Harbin South New Industrial City was used for another numerical test in this paper. There were 19 nodes and 27 links in the test network. There exist two bus lines currently being operated in this network. Bus line 1 starts from node 2 and ends at node 15 passing through nodes 4, 5 and 12. Bus line 2 begins with node 8 and terminates at node 15 passing through nodes 9, 19, 18, 17 and 16. The area is divided into 9 traffic zones. Time-dependent travel demands are derived from the social and economic census data of the city. The network topology is shown in Figure 7. Table 1 records the detailed characteristics of road segments including road name, number of lanes, capacity and link length.
Figure 7. The node-arc topology of the real-world urban network used in the numerical analysis.

Table 1. The link attributes of the real-world urban network used in the numerical analysis

<table>
<thead>
<tr>
<th>Road Name</th>
<th>Lane Numbers (two-way)</th>
<th>One-way capacity (vehicles/hour)</th>
<th>Link length (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xinjiang Street</td>
<td>6</td>
<td>3,600</td>
<td>8.4</td>
</tr>
<tr>
<td>Youxie Street</td>
<td>4</td>
<td>2,400</td>
<td>4.5</td>
</tr>
<tr>
<td>8th Avenue</td>
<td>6</td>
<td>3,600</td>
<td>1.3</td>
</tr>
<tr>
<td>2nd Avenue</td>
<td>6</td>
<td>3,600</td>
<td>6.0</td>
</tr>
<tr>
<td>12th Avenue</td>
<td>6</td>
<td>3,600</td>
<td>4.5</td>
</tr>
<tr>
<td>11th Road</td>
<td>6</td>
<td>3,600</td>
<td>4.3</td>
</tr>
<tr>
<td>H-W Highway</td>
<td>4</td>
<td>2,400</td>
<td>1.8</td>
</tr>
<tr>
<td>Union Street</td>
<td>4</td>
<td>2,400</td>
<td>2.8</td>
</tr>
<tr>
<td>17th Road</td>
<td>4</td>
<td>2,400</td>
<td>3.7</td>
</tr>
<tr>
<td>Shuangyong Road</td>
<td>4</td>
<td>2,400</td>
<td>1.9</td>
</tr>
<tr>
<td>Songhua Road</td>
<td>6</td>
<td>3,600</td>
<td>9.7</td>
</tr>
</tbody>
</table>

The updating time interval is set as 1 minute. Thus the total analysis period of 2 hours contains 120 time intervals. The free-flow speed of cars in the network is set as 60 km/h. Then, the length of a single cell is calculated by $60 \times \frac{1}{60} = 1$ km. The above node-link network is then converted into a cell-based network, as shown in Figure 8.

In Figure 8, there are 67 cells, consisting of 2 source cells, 1 sink cell, 10 merging cells, 9 diverging cells and 45 normal cells. The total demand of all traffic zones will be aggregated and loaded into the source cells of cell 1 and cell 2. All the cars and buses coming out of the source cells will finish their trips at sink cell 67 eventually.
Figure 8. The cell-based topology of the real-world urban network used in the numerical analysis.

In the cell-based network, the free-flow speed of buses is set as 45 km/h. As the free-flow speed of cars is 60 km/h, then the rate of $v_{0,bus}/v_{0,car}$ is equal to 0.75. The jam density is set as 125 vehicles/km. According to the data in Table 1, the parameters in Xinjiang Street, 2nd Avenue, 8th Avenue, the 12th Avenue, 11th Road and Songhua Road are calculated and given as $Q^{t,car}_i = 60$, $Q^{t,bus}_i = 30$, $Q^{st,bus}_i = 60$, $N^i_t = 375$. The parameters in Youxie Street, H-W Highway, Union Street, 17th Road and Shuangyong Road are given as $Q^{t,car}_i = 40$, $Q^{t,bus}_i = 20$, $Q^{st,bus}_i = 40$, $N^i_t = 250$. The capacity reduction parameter $\theta$ is set to be 0.75. The passenger car equivalent factor of cars is set as $p_{car} = 1$ and the factor of buses is set as $p_{bus} = 2$. The passenger capacity of cars and buses are set as $a_{car} = 1$ and $a_{bus} = 2$ separately.

A total demand of 2,646 passengers in the initial 25 time intervals is loaded into cell 1. A total demand of 4,284 in the initial 25 time interval is loaded into cell 2. The bus frequencies for the two bus lines are set to be 5 minutes which results in that total passenger demands are transformed into vehicles of both modes for every 5 minutes. At the other time intervals, total passenger demands only transformed into car vehicles. This makes sense because that people can use their cars without waiting but they need to wait to board on a bus.

The one-time XBL installation fee for each cell is CNY 250,000, and the total budget is CNY 7,500,000 over the entire network. In contrast, the IBL real-time operational fee for each cell in every time interval is CNY 4,300.

We first evaluated the network flow solution in the original network without setting up XBLs or IBLs. The TPTT value is 133,749. Then, we evaluated the network flow solution in the network with allocating XBLs. The TPTT value is thus 105,754. The propagation and reservation of the car and bus flow patterns in the time-space diagram can be seen in Figure 9.
Figure 9. The spatiotemporal flow result of the optimal XBL allocation model in the numerical example: (a) Car traffic; (b) Bus traffic.

In Figure 9, the horizontal axis represents cell numbers and the vertical axis represents time intervals. The color of this heat map from blue to red shows the number of vehicles in the cell: The blue color indicates the minimum number of vehicles; the red color indicates the maximum number of vehicles.

The optimal XBL setting can be seen in Figure 10. From Figure 10, we can see that the cells along bus line 1 and 2 are used for setting up XBLs. In Figure 10, the results show that the XBLs prompt an early arrival of the buses at the sink cell. In the meantime, as a result, the traffic flow of cars in those cells that are occupied by XBLs is apparently reduced. Most of the cars shift their routes to parallel paths from those paths with XBLs. The coordination between cars and buses makes the total travel time of the network better optimized.

Figure 10. The optimal XBL allocation result in the numerical example.

Then, we evaluated the network flow solution in the network with dynamically setting up IBLs. The resulting TPTT value is 105,188. The propagation and reservation of the car and bus flow patterns can be seen in Figure 11. The optimal IBL setting can be seen in Figure 12.
Figure 11. The spatiotemporal flow result of the optimal IBL allocation model in the numerical example: (a) Car traffic; (b) Bus traffic.

As can be seen in Figure 12, bus lanes are opened to buses whenever they arrive at the corresponding cells. For other times, bus lanes are closed, which results in the return of one lane to cars. In Figure 11, the heat map shows a dynamic variation of car and bus flow patterns with setting up IBLs. From cell 22 to cell 28, the cars capacity is increased after buses finish their trips. Thus, more cars can now use this shortest path, where the capacity is reduced by the bus lanes. In other words, setting up IBLs improves the utilization efficiency of the road capacity and reduces the clear time of the network.

As mentioned above, without setting bus lanes, the TPTT value of the road network is 133,749. The optimal XBL setting saves 27,995 minutes and improve the system efficiency by 26.5%. And the optimal IBL setting saves 28,560 minutes, which correspond to a 27.2% reduction of the total travel time. Evidently, both XBLs and IBLs can improve the system performance in a significant manner.

However, if we compare the results of XBLs with IBLs, the improvement from XBLs to IBLs is only 566 minutes or a 0.54% reduction, which is quite marginal. It proves that IBLs are not that much superior than XBLs, at least in our numerical analysis. As a similar result was also demonstrated by Chiabaut et al. [3], the appropriate condition of setting up IBLs that reduce total travel time and improve system performance rarely exists and is restricted by many factors.

The computation time of solving CTM-SCB is about 6 seconds. In the meantime, the computation time of solving the optimal XBL allocation problem and the optimal IBL allocation problem are 422 seconds and 726 seconds, respectively. The computation time for case study 2 is almost the same as that for case study 1 because of their comparable network sizes.

Once again, the authors want to emphasize that the models proposed in this paper imply the SO principle and they may be more appropriately used to evaluate the system performance under some specific situations, such as evacuation planning, autonomous vehicular traffic and advanced traveler information systems, rather than commuting traffic networks.
5. Summary and Future Work

In summary, this paper focuses on the provision of assigning road use priority to buses on the network level. The emerging transit promotion strategy of using XBLs and IBLs were reviewed, modeled and numerically compared in the paper. The mixed car and bus traffic is modeled by a DTA tool developed from Daganzo’s original CTM and Ziliaskopoulos’ CTM-based DTA model. This tool becomes complex in that it can handle separate car and bus traffic and be used to accommodate the XBL and IBL settings. The DTA model was then integrated into two network design models, one of which designs the optimal XBL allocations and the other model offers the optimal IBL allocations. The DICOPT solver in GAMS was used to solve the resulting MINLP-type network design problems. The numerical results from modeling and optimizing a synthetic freeway-arterial network and a real-world urban street network (where the latter is extracted from the Harbin South New Industrial City) show that both XBLs and IBLs can significantly improve the network efficiency.

The methodology proposed in this paper is distinguished from other studies in that the former sets XBLs and IBLs on the network level rather than the route or link level. Although it is anticipated that IBLs distribute the priority of bus lanes dynamically and more flexibly to buses and hence can improve the system performance to a higher degree than XBLs, the numerical results showed that the improvement from XBLs to IBLs is quite marginal. Our numerical result confirms and extends the contributions of Chiabaut et al. [3]. Considering that IBLs are more complex to be implemented and operated and may cost more investment, it is still a question which bus lane priority strategy, XBLs or IBLs, should be chosen in real-world implementations.

In prospect, our future work will be conducted in the following aspects. First, the behavior mechanism assumption in this paper is the SO principle. The UE or other types of equilibrium principle can reflect the individual mode choice and route choice more realistically. Thus the transformation from the SO principle to an equilibrium principle can better reflect real traffic network conditions. Second, a more extensive experiment that includes more urban networks with different network topologies, demand levels, traffic control settings should be conducted to more thoroughly understand and compare the relative performance of XBLs and IBLs. Third, a very interesting question in using XBLs and IBLs is how the two are coordinated with other bus priority or exclusion techniques, such as bus priority signals and bus rapid systems, and how they are jointly used to improve the overall traffic network performance.

Author Contributions: The authors confirm contribution to the paper as follows: study conception and design: Haiyang Liu and Jian Wang; data collection: Haiyang Liu and Xiaolong Ma; analysis and interpretation of results: Haiyang Liu; draft manuscript preparation: Haiyang Liu. All authors reviewed the results and approved the final version of the manuscript. The authors confirm sole responsibility for the following: study conception and design, data collection, analysis and interpretation of results, and manuscript preparation.

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