Analytical calculations of quark confinement, linear interaction potential, and net spin in a proton using a quark rotational model

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Abstract
In this study, we describe quark confinement in terms of linear interaction potentials and solve the problem of the net spin of a proton. The three quarks in a proton are assumed to revolve around a common center, and their masses are determined assuming they are Dirac particles. On the basis of these assumptions, the magnetic moment of a proton can be derived. Moreover, the rotation of the quarks is considered, in which an electrical current induces a magnetic field. Thus, the scalar product of the magnetic moment and field describes the linear interaction potential between the quarks, and the mass of the proton can be obtained. The proton mass predicted by this physical model is consistent with experimental values, and no numerical or fitting calculations are required. Furthermore, using the newly derived spins and angular momentum of the three quarks, we derived the net spin of a proton. Additionally, we predicted the mass of a pi-meson from the same model, which agrees with the experimental values.

Keywords: quark, linear interactive potential, mass of a proton, spin, quark confinement, pi-meson

1 Introduction

Hadrons and quarks have been studied for decades. A proton is composed of three quarks [1-7], and its mass has been measured with good accuracy but cannot be predicted theoretically using analytic solutions. Numerical calculations for determining the proton mass are well known in quantum chromodynamics (QCD), where the mass of hadrons can be calculated using a supercomputer [8-12]. The QCD model is described by the SU(3) and Yang-Mills fields [13-15].

However, for lattice QCD, the calculation time needed is extremely long even when supercomputers are used, and the color charges used by lattice QCD are controversial because the color charge of the gluons that mediate the strong interaction has not yet been measured. Moreover, lattice QCD uses discrete distances and their interpolations, implying that lattice QCD is an essentially approximated method. Thus, there is a need for analytical calculations without the numerical or fitting method. Furthermore, currently, the pi-meson mass from quarks cannot be described using the QCD method. Recall, the color charge was initially introduced to avoid the violation of the expulsion principle [16]. However, because the spin of a quark is $1/6\hbar$ not $1/2\hbar$, the three quarks in a proton do not violate the expulsion principle with no color charges.

Moreover, the quark confinement problem or the question of why the proton persists is yet to be resolved because the numerical calculations for lattice QCD imply a so-called ideal experiment and the essentially approximated method, which does not reveal why quarks cannot be measured singly and why the proton is not destroyed permanently. This implies that lattice QCD does not provide a clear physical picture of the quark confinement.

Although lattice QCD calculations of the mass of hadrons have been conducted, based on experimental observations of Ω-, pi-and K-particles (i.e., the lattice QCD inputs these masses manually), the parameters have been determined such that the numerical calculations predict the masses of other unobserved hadrons [17]. Additionally, the lattice QCD inputs the mass of a quark

manually [17]. This result is meaningful to some extent, but the calculations cannot be performed without numerical and fitting modeling. In conclusion, the results cannot explain the basic physical picture of quark confinement.
Herein, we use more simple and basic calculations for a proton than the QCD approach. This physical model yields mathematically the linear interaction potential between quarks, the spins of both a quark and a proton, and the mass of a proton without numerical calculations.

First, we calculate the mass of a quark in a proton (i.e., up and down quark), the spin of a quark, the mass of a proton (i.e., the linear potential), and the net spin of a proton. To calculate the mass of a proton, we assume that three quarks have rotational moments, and the quark mass is determined from the energy gap in a quark's space–time. The rotation of the quarks is attributed to an electrical current that induces a magnetic field, which explains the magnetic moment of a proton. The scalar product of the magnetic moment and the magnetic field produces a linear interaction potential between quarks that predicts the mass of a proton.

Considering the background of the problem of the spin of a proton, thus far, it is considered that the summation of spins of quarks results in the spin of a proton. However, the spins of quarks' contributions reach only 30% of the total spins of a proton [18]. Also, in our previous study [19], although the value of spin of each quark is $\pm 1/6\hbar$, the summation is $1/6\hbar$, which is 32% of the spin of a proton ($1/2\hbar$). Herein, we consider this spin problem. The spin of a proton is the total angular momentum j, whose elements could be taken as a summation of spins of quarks and angular momentum l. This angular momentum l is naturally introduced because each quark has rotations to generate magnetic field energy.

Comparing these analytic results with experimental observations, the analytic model corresponds well with the observed mass and spin of a proton. The derived linear potential provides a mechanism that explains how a larger relative distance between quarks causes a stronger attractive force between them, implying that quarks cannot be measured in isolation. Although quark confinement has been numerically modeled [12,13], herein, we use analytical, mathematical expressions without numerical methods; thus, a purely physical model for quark confinement is obtained.

2 Theory

2.1 Proposed model

As shown in Fig. 1, circular coordinates with a radius r are considered. Each apex has a quark whose electric charge is $+1/3e$ because the net charge of a proton is $+e$. In this model, each quark revolves around the origin O, maintaining a constant relative distance between the three quarks. Fig. 1, therefore, shows a simple model of a proton.

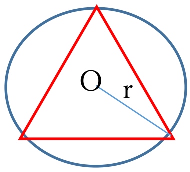

Fig. 1 Model of a proton. Three quarks are located at each apex of the equilateral triangle. Maintaining the shape of the triangle, the rotation along the origin O is introduced.

2.2 Mass and spin of a quark

Mass of a quark

The mass of a quark is derived by considering the coulombic force:

$$F_E = \frac{1}{3} eE = \frac{dp_q}{dt}, \tag{1}$$

where p_q is the momentum of a quark, which rotates.

On the basis of Eq. (1), the coulombic force from a quark is equal to that from an electron, as expressed as follows:

$$p_q = 3p, p = \frac{m_e v}{\sqrt{1-\frac{v^2}{c^2}}}, \quad (2)$$

where p and m_e are the momentum and mass of an electron, respectively.
Thus, the mass of a quark m_A is expressed as follows:

$$m_A = 3m_e. \quad (3)$$

Spin of a quark

Spin magnetic momentum is expressed as follows:

$$\mu_B = -g\frac{q}{m_A}s, \quad (4)$$

where q and s denote the charge of a quark and the spin angular momentum, respectively.
Herein, the g-factor in Eq. (4) is assumed to be equal to 3.

$$\mu_B = -3\frac{1/3e}{3m_e}s = -\frac{e}{m_e}(\frac{1}{3}s). \quad (5)$$

Because s in Eq. (5) is $\pm\frac{1}{2}\hbar$, the quark spin is,

$$s_q = \pm\frac{1}{6}\hbar. \quad (6)$$

This spin value is 30% for the net spin of a proton and thus agrees with experiments [18].

2.3 Mass of a proton and the linear potential

The rotation of the three quarks can be modeled as the current I if we consider the quarks as charged particles. Thus,

$$I = \frac{3\times(1/3)e}{T} = \frac{\omega e}{2\pi}, \quad (7)$$

where T, e, and ω denote the period of the rotation movement, the charge of an electron, and the angular frequency of the motion, respectively.
A magnetic moment is generally defined as

$$p = IS, \quad (8)$$

where p, I, and S denote the magnetic moment, current, and area, respectively.
The area S is simply expressed as

$$S = \pi r^2. \quad (9)$$

The following assumptions are introduced:

(1) The quark mass is determined by the energy gap of the vacuum in a quark space–time (i.e., it is a Dirac particle)
(2) The rotational angular frequency is determined by the energy gap from the quarks' vacuum
(3) This Dirac particle is described as

$$\frac{1}{2}\hbar\omega = 3m_A c^2, \quad (10)$$

where the left-hand side denotes the zero-point energy of the quark in a vacuum and the right-hand side is the sum of the energies of the three quarks at rest, with m_A being the mass of a single quark.
Notably, in a vacuum (not a quark vacuum), the following equation holds for an electron:

$$\frac{1}{2}\hbar\omega = m_e c^2, \quad (11)$$

where m_e denotes the mass of an electron.
Eq. (11) can also be considered a pair creation of an electron and a positron. Thus, Eq. (10) is also considered a pair creation of a proton and an antiproton (i.e., $\bar{u}\bar{u}\bar{d}$).
Considering the above Eq. (10), the magnetic moment of a proton p is given as

$$p = \frac{e}{2\pi}\frac{6m_A c^2}{\hbar}\pi r^2. \quad (12)$$

The magnetic field H induced by current I is expressed as follows:

$$H = \frac{I}{2r} = \frac{\omega e}{4\pi r} = \frac{e}{4\pi r}\frac{6m_A c^2}{\hbar}. \quad (13)$$

Thus, in combination with the magnetic moment p, the linear interaction potential is expressed as follows:

$$E = -p \cdot H = -e^2 \frac{1}{8\pi} (\frac{6m_A c^2}{\hbar})^2 r. \quad (14)$$

This potential yields the proton mass given as

$$|E| = m_p c^2, \quad (15)$$

where m_p denotes the proton mass.

2.4 Spin of a proton

Because the rotational period time T is divided by a quantized time t_c,

$$T = nt_c, \quad (16)$$

where t_c of the quantized space–time is derived from the zero-point energy for a quark [19]. For this quantized space–time, see the Appendix of this paper. In the quark space, the zero-point energy is defined as

$$\frac{1}{2}\hbar\omega = 3m_A c^2, \quad (10)$$

where m_A is the mass of a quark. Notably, the first generation of quark mass is three times that of an electron. Now, from Eq. (10), the quantized space–time is derived as

$$t_c = \frac{2\pi\hbar}{6m_A c^2}. \quad (17)$$

We assume that the angular momentum is quantized as

$$l = n\hbar, \quad (18)$$

where n is an integer.

Considering Eq. (10), the angular frequency ω is given as

$$\omega = \frac{6m_A c^2}{\hbar}. \quad (19)$$

As shown in the proposed quark model (Fig. 1), the triangle whose apex denotes each quark has a rotation with the period T,

$$2\pi a = Tv, \quad (20)$$

where v and a denote the velocity of the quarks' rotation and the constant radius of a proton, respectively.

Here, the angular momentum is expressed as

$$l = 3m_A \times 3v \times a = 9am_A v, \quad (21)$$

where the number of quarks is 3; thus, combining Eqs. (16), (17), and (20),

$$2\pi a = \mathrm{n}\frac{2\pi\hbar}{6m_A c^2} \cdot \frac{l}{9am_A}. \quad (22)$$

On the basis of the above equation, l can be expressed as follows:

$$l = 2\pi a \frac{6m_A c^2}{2\pi\hbar} \cdot 9am_A \frac{1}{n}. \quad (23)$$

Also, considering that

$$l = n\hbar, \quad (18)$$

then,

$$n^2 = 54m_A^2 c^2 a^2 \frac{1}{\hbar^2}. \quad (24)$$

Therefore,

$$l = \sqrt{54}\frac{m_A ac}{\hbar}\hbar \approx 0.336\hbar. \quad (25)$$

Here, $a = 0.8 \times 10^{-15}\ m$ (i.e., the radius of a proton), and m_A is three times the mass of an electron. In the previous section 2.2, we showed that the spin of each quark is given as

$$\mathrm{s} = \pm\frac{1}{6}\hbar. \quad (26)$$

Thus, considering *uud* in a proton, the total spin of quarks is given as

$$s_{\text{total}} = \frac{1}{6}\hbar, \quad (27)$$

and the total angular momentum j is

$$j = s_{\text{total}} + l = \frac{1}{6}\hbar + 0.336\hbar = 0.496\hbar \approx \frac{1}{2}\hbar. \quad (28)$$

Measuring the spin of a proton as the total angular momentum as above, the result will agree well with the measurements [18].

2.5 Mass of a meson derived from quarks

Please note that this section is the review section of our paper [19]. In the previous section, we described quarks' interaction as static magnetic field energy, with which we could obtain the mass of a proton and a linear potential. The salient point is that this interaction was also derived from the zero-point energy, and the linear attractive potential was obtained purely analytically (not numerically). Here we discuss the pi-meson, which acts between two nucleons as a strong attractive interaction. This force, which is the Lorentz force, is also derived from the zero-point energy.

Fig. 2 shows the schematic of two nucleons in which three quarks rotate. Each rotation of the two nucleons has the same angular frequency and rotational direction. Now, we consider the motions of quarks 1a and 1b.

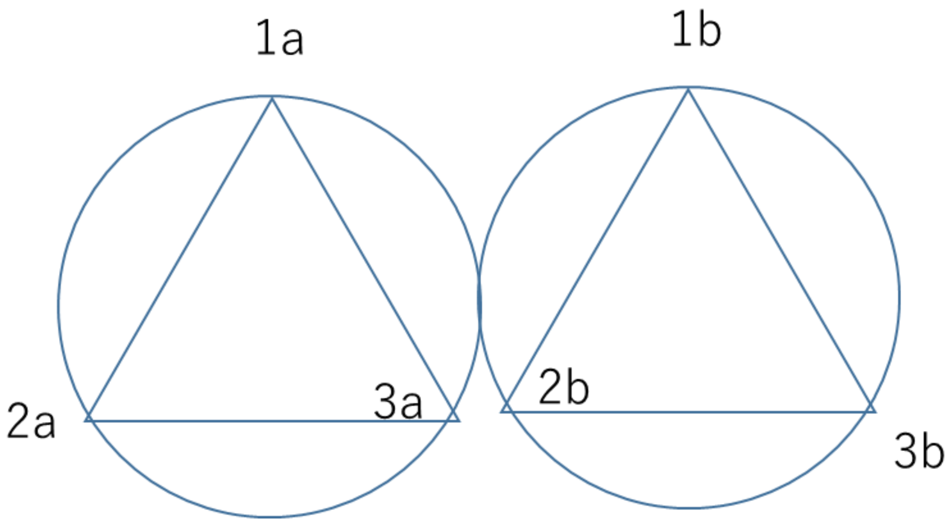

Fig. 2 Schematic of two combining nucleons. Each number indicates a quark. In this model, the two triangles have the same rotation, i.e., they maintain their relative distances, and each triangle rotates with the same angular frequency and in the same direction. Notably, the angular frequency is derived from the zero-point energy. These quarks maintain a constant distance; i.e., the radius r_c and rotational velocity remain the same. Thus, we can use the model in Fig. 3. In Fig. 3, quarks 1a and 1b move in the same direction with the same velocity. This model is analogized by two electric current leads, which exhibit an attractive force [20], which is the Lorentz force. Thus, the strong interaction between two nucleons is the Lorentz force, which is derived from the zero-point energy.

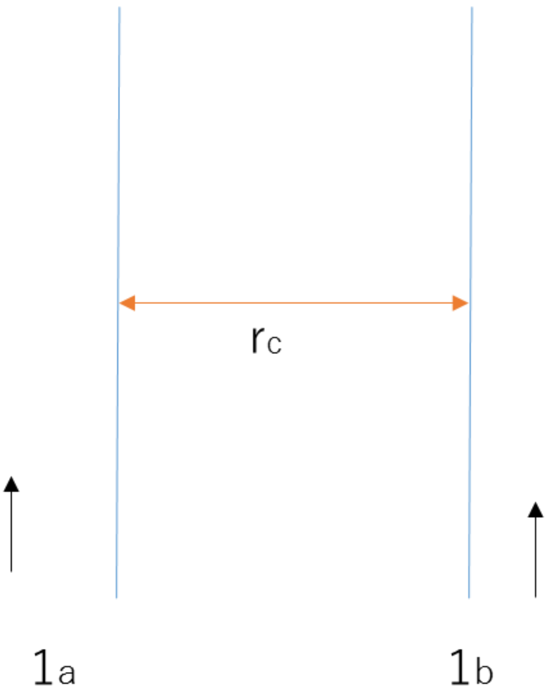

Fig. 3 Model of the previous figure. Quarks 1a and 1b rotate at the same relative distance r_c. Thus, this is a model of line motions. Therefore, each quark experiences the Lorentz attractive force from the analogy of two current leads experiencing an attractive magnetic force between each other.

Let us calculate the mass of a pi-meson considering this model.
First, the internal magnetic permeability of a nucleon is determined on the basis of its perfect ferromagnetism (i.e., $B = H$). Therefore, the Lorentz force is given as

$$F = \frac{1}{3}evH, \tag{29}$$

$$v = r_c\omega. \tag{30}$$

Here, the zero-point energy is expressed as follows:

$$\frac{1}{2}\hbar\omega = 3m_Ac^2. \tag{31}$$

Thus,

$$v = r_c\frac{6m_Ac^2}{\hbar}. \tag{32}$$

From [17], three quarks, each having a charge $1/3e$, rotates with an angular frequency ω, resulting in a current I and magnetic field H. Thus, the central magnetic field is given as

$$H = \frac{I}{2r_c}, \tag{33}$$

$$I = \frac{1/3e}{T}, \tag{34}$$

where T is the period of rotations of the quarks.
Thus, the current is expressed as

$$I = \frac{1}{3}e\frac{\omega}{2\pi} = \frac{1}{3}e\frac{1}{2\pi}\frac{6m_Ac^2}{\hbar}. \tag{35}$$

On the basis of this expression, the magnetic field is given as

$$H = \frac{1}{4\pi}\frac{1/3e}{r_c}\frac{6m_Ac^2}{\hbar}. \tag{36}$$

Thus, energy, which has an important, attractive linear potential, is gained:

$$u = -\int Fdr = -\frac{1}{3}evHr_c = -\frac{1}{36\pi}e^2r_c(\frac{6m_Ac^2}{\hbar})^2. \tag{37}$$

Substituting the physics constants,

$$|u| = 21.9 \times 10^{-12}\ \mathrm{J}. \tag{38}$$

Thus, we obtain the mass of pi-meson m_m as

$$m_m = 2.4 \times 10^{-28}\ \text{kg}. \tag{40}$$

This value agrees with the mass predicted by Yukawa [15].

3 Results

Here, we compare experimental observations with the value of the proton mass calculated using the proposed model. The physical parameters listed in Table 1 were used for the calculations.

Table 1. Parameters for hand calculations

Plank constant ħ	1.05×10^{-34} Js
Electric charge q	1.6×10^{-19}C
Quark mass $m_A = 3\ m_e$	$3 \times 9.1 \times 10^{-31}$ kg
Proton radius $r = a$	0.8×10^{-15} m

The results of the calculations are listed in Table 2. Experimental data are averaged from two sources [15,20]. As shown in Table 2, the theory of this study well predicts the mass of a proton.

Table 2. Comparison of the mass of a proton

Hand calculation	1.8×10^{-27} kg
Experiment	1.67×10^{-27}kg

Moreover, Fig. 4 shows the linear potential between protons based on the interaction of pi-mesons. The equation is as follows:

$$U = \frac{(3 \times \frac{1}{3}e)^2}{4\pi\varepsilon_0 r} + 3 \times \left[-e^2 \frac{1}{36\pi}\left(\frac{6m_A c^2}{\hbar}\right)^2 r\right]. \tag{41}$$

The first term indicates up or down quarks having the charge $+1/3e$ interact with each other by the Coulomb repulsive force, and the second term indicates the attractive linear force derived in this study (Eq. (37)). Notably, the coefficient 3 implies that, to satisfy the conservation of the charge, there must be three modes in Fig. 2 (i.e., 1a–1b, 2a–2b, and 3a–3b). In Fig. 4, at approximately 1 fm, the potential becomes 0.85 GeV, which agrees with the experimental values [21].

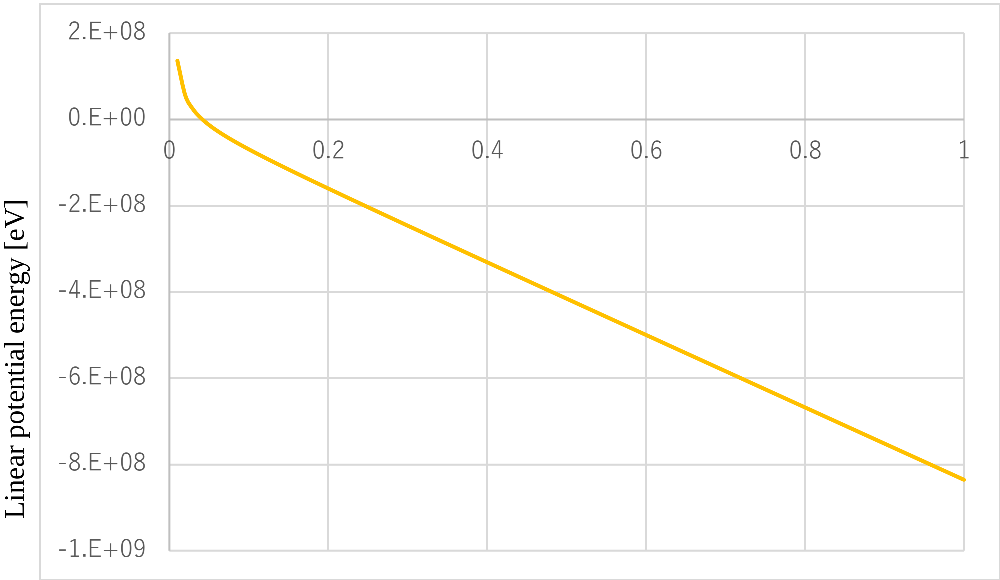

Fig. 4 Linear potential between protons. The attractive linear potential is dominant, but at the realms less than 0.4×10^{-16} m, it becomes a repulsive interaction, which is because the three quarks having the same electrical positive charge interact as a Coulomb force. Figure 4 is compared with an experiment result [21], which implies that the presented theory agrees with the experiment.

4 Discussion

4.1 Advantages and significances of the proposed model over QCD and its validity

Typically, strong force interactions are calculated using a lattice QCD. However, such calculations require numerical methods; thus, the resulting values are inaccurate, which describes why the quark is not measured singly and why the proton is not destroyed permanently. Notably, that lattice QCD employs discrete distances and their interpolations, which are insufficient for a continuous distance. Furthermore, the lattice QCD inputs the masses of quarks, K-, pi- and Ω-particles manually, such that the numetical calcaltion agree with the experimens [17].Thus, lattice QCD must be an essential approximation in addsition to the numerical and fitting calculations. As the approximated method, QCD is vital to some degree. However, currently, pi-meson has not been created from quarks using QCD. Finally, primitively, we cannot see the real, physical meaning, and existence of color charges, which have not yet been observed experimentally.

Here, after setting the mass and spin of a quark, we started with a simple physical model of three quarks forming an equilateral triangle and used basic physical relations to derive the magnetic moment and mass of the proton. In the process, we derived the linear interaction potential. Thus, the interaction potential increases along with the relative distance between quarks. This relationship explains why protons persist and single quarks have not been observed. Thus, the mass of a proton is explained using the aforementioned model. Moreover, we could create pi-meson from the quarks, in which an interactive linear potential was also obtained. Furthermore, our model correctly predicts the net spin of a proton, implying that our employed model is valid. The signifiance of our presented paper is that, compared with the lattice QCD, we have not used any manual parameters, i.e., the present paper calculates the facts analytically without any fitting.

4.2 Reasons color charges are not needed

To consider why we do not select the concept of a color charge, we recall that color charges were initially introduced so that the three quarks in a proton violate the expulsion principle because it was assumed that each quark has a 1/2ħ spin as that of an electron. As mentioned in the previous section, however, the total spin s_{total} of a proton (uud) is 1/6ħ. (These quarks form the orbital angular momentum.) This value of the spin does not dominate over the spin of an electron 1/2ħ. The case that violates the expulsion principle is that the absolute value of the total spin s_{total} is over 1/2ħ. For example, when there are two equal up-spin electrons at an identical state, the total spin becomes $\frac{1}{2}\hbar \times 2 = \hbar$, which dominates over the value of 1/2ħ and violates the expulsion principle. However, even when our three quarks are considered and because the total spin s_{total} is 1/6ħ less than 1/2ħ, the expulsion principle is not violated with no color charge.

5 Conclusions

In this study, the problem of quark confinement and the net spin of a proton were studied. The three quarks revolve around a common center and have a mass. This rotation produces a current and a magnetic field, from which the magnetic moment of the proton can be calculated. Moreover, the scalar product of the magnetic moment and the magnetic field result in a linear potential between the quarks, and the mass of the proton can be obtained. This derived linear potential between the quarks explains the problem of quark confinement since the potential increases along with the distance between quarks. This relationship between the quark spacing and the linear potential explains why quarks are not observed in isolation and why protons cohere.

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Additional information

This study does not have any competing interests, such as funding, employment, and personal financial interests. It also does not have any nonfinancial competing interests.

Appendix

Zero-point energy and the concept of a quantized space–time

The concept of quantized space–time, as well as the zero-point energy, can be elaborated in a vacuum condition. Herein, we describe each concept by solving the Dirac equation. The equation shows that, inside a vacuum, during the formation of electrons and positrons, the mass gap existing between them can be represented as follows:

$$\hbar\omega_0 = 2m_e c^2, \tag{1a}$$

where ω_0, m_e, and c are the angular frequency, electron mass, and speed of light, respectively.
Eq. (1a) can be expressed as follows:

$$\frac{1}{2}\hbar\omega_0 = m_e c^2. \tag{2a}$$

The left-hand side is identical to the expression of zero-point energy based on the Hamiltonian of the harmonic oscillation:

$$H = (n + \frac{1}{2})\hbar\omega_0. \tag{3a}$$

Notably, the first term in the expanded form of Eq. (3a) relates to time-dependent electromagnetic fields, which are calculated in depth using quantum mechanics, whereas the second term relates to time-independent electromagnetic fields. This expression for the zero-point energy is related to static electromagnetic fields. If the angular frequency ω_0 is constant, then the zero-point energy is the universal specific constant energy. Eq. (3a) also describes the energy gap of the vacuum based on the result of the Dirac equation. Therefore, the zero-point energy could be considered an expression of the basic energy of the vacuum. As discussed in our previous report [19], the mass of an electron is the most basic parameter; thus, Eq. (2a) provides a constant quantized space λ_0 and a quantized time t_0, which are defined as follows:

$$\lambda_0 = \frac{\hbar}{2m_e c}, \tag{4a}$$

$$t_0 = \frac{\hbar}{2m_e c^2}. \tag{5a}$$

Consequently, we can derive a more general equation of constant quantized space-time length and time as follows:

$$\lambda_c = \lambda_0\sqrt{1 - \frac{v^2}{c^2}}, \tag{6a}$$

$$t_c = t_0\sqrt{1 - \frac{v^2}{c^2}}. \tag{7a}$$

Fig. 1a shows a schematic of the quantized space–time. The gravitational or Lorentz force from the magnetic fields is because up- and down-spin electrons embedded in a quantized space–time λ_c form rotations and then combine as paired quantized space–time because of the attractive force from the Lorentz force or gravity. This paired quantized space–time behaves like a boson.

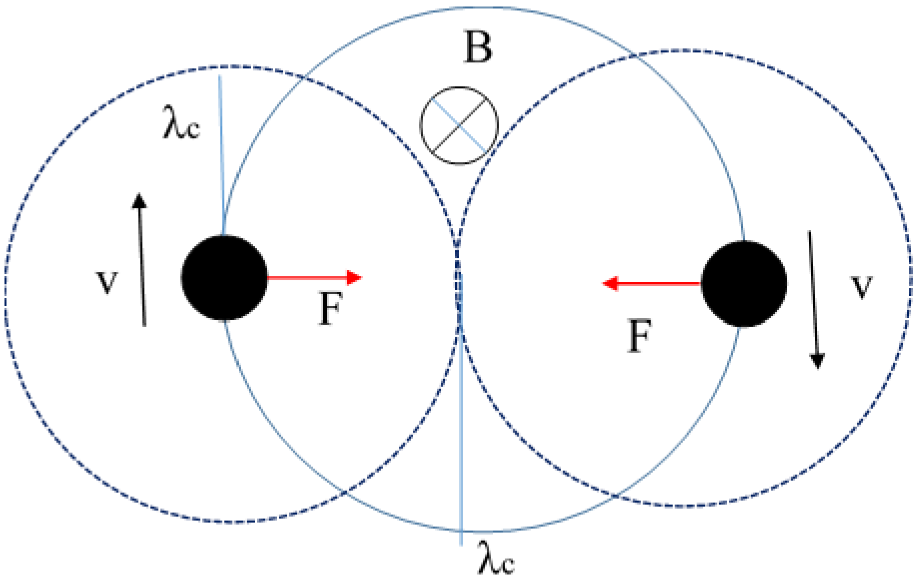

Fig. 1a. Schematic model of a quantized space–time. The up- and down-spin electrons do not have real bodies but are embedded in a quantized space. That is, each of the two quantized spaces rotates to create quantized magnetic field energy, producing the Lorentz force F, which equals the attractive gravitational force F. Note that v denotes the rotational velocity, and the magnetic flux is defined on the central circle.