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Time Energy Uncertainty Principle and Vacuum Fluctuations

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Abstract:
Phenomenon of vacuum fluctuations of virtual particles in high energy physics has been mathematically modeled by a third order differential equation. Aim of present theory is to unravel underlying mathematical equation capable of explaining microscopic phenomenon of vacuum fluctuations. Solution of such a differential equation after applying appropriate boundary conditions gives an acceptable wave function i.e. $\psi = 2\nu^{3/2} t \exp(-\nu t)$. Operation of energy and time operator on this wave function proves that these operators do not hold commutative property. Time energy uncertainty principle has been derived by calculating variance of energy and time for the same wave function.

Key Words:
time energy uncertainty principle, vacuum fluctuations, wave function, non commutator.

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1. Introduction:

Time energy uncertainty states that simultaneous measurement of time and energy is not possible. The time in the uncertainty relation defines time during which the system remains unperturbed or life time of the state. Time and energy operators do not commute each other was first introduced by Heisenberg [1]. The existence of time operator has been challenged by stating it as a parameter describing the evolution of the system [2-9]. This confusion had existed since last century because mathematical equation that can explain those microscopic natural phenomena which are direct consequence of time energy uncertainty was not on the screen. Aim of present paper is to propose a mathematical equation whose solution can give a wave function which can describe state of virtual particles. For the validity of time energy uncertainty relationship operation of energy and time operator on such wave function should show that time and energy operators should not commute with each other. As vacuum fluctuations of virtual particles in high energy physics is direct consequences of time energy uncertainty relationship [10-11].

2. Theory:

Phenomenon of vacuum fluctuations of virtual particles can be mathematically modeled by following linear homogenous third order differential equation,

$$\frac{1}{\nu^3}\frac{d^3\psi(t)}{dt^3}+\frac{1}{\nu^2}\frac{d^2\psi(t)}{dt^2}-\frac{1}{\nu}\frac{d\psi(t)}{dt}-\psi(t)=0 \tag{1}$$

In Eq.1 $\psi(t)$ is the dependent variables which is function of time while time is independent variable and ν is a constant have dimensions of frequency which makes Eq.1 dimensionally consistent. On analytically solving Eq.1, following solution is obtained,

$$\psi(t)=A\exp(\nu t)+B\exp(-\nu t)+Ct\exp(-\nu t) \tag{2}$$

For extremely short time in order to exist virtual particles exist due to energy fluctuations allowed by time energy uncertainty principle but they must also disappear quickly because of the same principle [11-12]. So appropriate boundary conditions for Eq.1 can be defined as,

$$\psi(0) = 0 \tag{3}$$

$$\psi(\infty) = 0 \tag{4}$$

Applying these two boundary conditions on Eq. (2) proves constants A and B zero while constant C to be non zero and Eq.2 reduces to,

$$\psi(t) = Ct\exp(-\nu t) \tag{5}$$

On normalizing wave function defined by Eq.5 i.e.

$$\int_0^\infty \psi^*(t)\psi(t)dt = CC^*\int_0^\infty t^2\exp(-2\nu t)dt = \frac{CC^*}{4\nu^3} = 1 \tag{6}$$

Value of normalization constant *C* can be evaluated from Eq. 6 and well behaved wave function for virtual particles can be written as,

$$\psi(t) = 2\nu^{3/2}t\exp(-\nu t) \tag{7}$$

Well behave wave function expressed by Eq.7 can be graphically represented as,

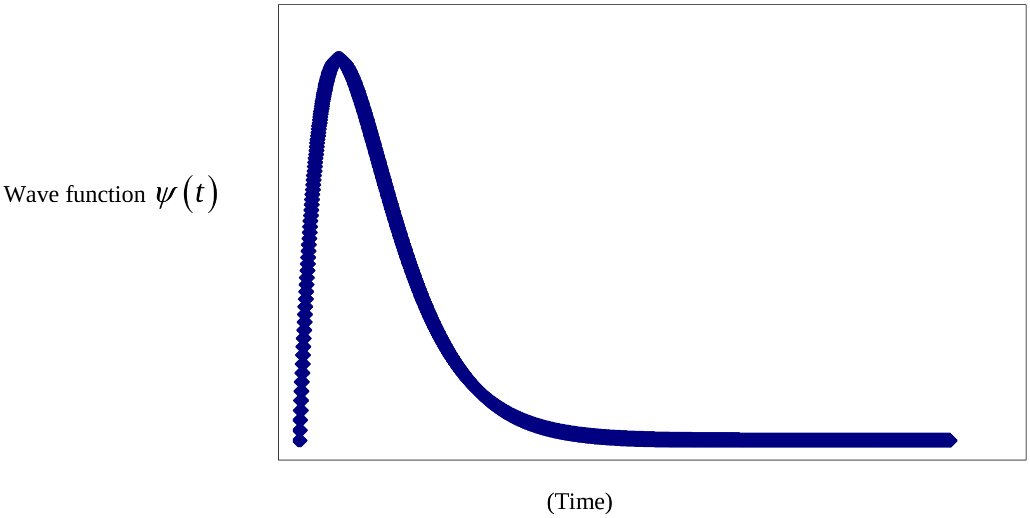

Fig. 2

Time energy operators do not hold commutative property for wave function given by Eq.7 i.e. [11]

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$$\left[\hat{E},\hat{t}\right]2\nu^{3/2}t\exp(-\nu t)=\left[i\hbar\partial/\partial t,\hat{t}\right]2\nu^{3/2}t\exp(-\nu t)=i\hbar 2\nu^{3/2}t\exp(-\nu t) \quad (8)$$

So according to Eq.8 time and energy operator do not hold commutative property for wave function described by Eq.5 and thus time and energy operators are non commutator i.e.

$$\left[\hat{E},\hat{t}\right]=\left[i\hbar\partial/\partial t,\hat{t}\right]=i\hbar \quad (9)$$

Since wave function described by Eq.7 is neither an Eigen function of energy operator nor of time. So we evaluate expectation values of energy and time operators. Expectation value of $\langle E^2\rangle$ for wave function described by Eq.7 can be evaluated as,

$$\langle E^2\rangle=\int_0^\infty\left(2\nu^{3/2}t\exp(-\nu t)\right)^*(i\hbar\,\partial/\partial t)^2\left(2\nu^{3/2}t\exp(-\nu t)\right)dt=\hbar^2\nu^2 \quad (10)$$

Expectation value of $\langle E\rangle$ for wave function described by Eq.7 can be evaluated as,

$$\langle E\rangle=\int_0^\infty\left(2\nu^{3/2}t\exp(-\nu t)\right)^*(i\hbar\,\partial/\partial t)\left(2\nu^{3/2}t\exp(-\nu t)\right)dt=0 \quad (11)$$

Expectation value of $\langle t^2\rangle$ for wave function described by Eq.7 can be evaluated as,

$$\langle t^2\rangle=\int_0^\infty\left(2\nu^{3/2}t\exp(-\nu t)\right)^*\hat{t}^2\left(2\nu^{3/2}t\exp(-\nu t)\right)dt=3/\nu^2 \quad (12)$$

Expectation value of $\langle t\rangle$ for wave function described by Eq.7 can be evaluated as,

$$\langle t\rangle=\int_0^\infty\left(2\nu^{3/2}t\exp(-t)\right)^*\hat{t}\left(2\nu^{3/2}t\exp(-t)\right)dt=3/2\nu \quad (13)$$

Variance of time and energy can be evaluated by following formulae,

$$\sigma_E=\sqrt{\langle E^2\rangle-\langle E\rangle^2}=\hbar\nu \quad (14)$$

$$\sigma_t=\sqrt{\langle t^2\rangle-\langle t\rangle^2}=\sqrt{3}/2\nu \quad (15)$$

So time energy uncertainty relation can be evaluated by product of variance of time and energy i.e.

$$\sigma_E\sigma_t=\Delta E\Delta t\geq\sqrt{3}/2\hbar \quad (16)$$

So Eq. 16 is time energy relationship according to which time and energy can be simultaneously measured with an accuracy of order of $0.866\hbar$.

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3. Conclusion:

Wave functions of virtual particles can be considered to be confined in a box with its width possessing dimension of time and ranging from zero to infinity. On moving along the dimension of time wave function starts from zero and then rises to finite value and at infinity again decays to zero. Simultaneous operations of time and energy operator on wave function render time and energy as conjugate variables. Expectation value of energy zero unravels that matter and antimatter particle produced in empty space have equal amount of energy with opposite sign thus their average energy is zero. Thus simultaneous measurement of time and energy of a system is not due to poor experimental technique but is a fundamental property of the act of measurement itself.

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