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On the quantum thermodynamic origin of gravitational force by applying spacetime entanglement entropy and Unruh effect

Shujuan Liu

Department of Applied Physics, Zhejiang University of Technology, Hangzhou 31023, China

Hongwei Xiong*

Department of Applied Physics, Zhejiang University of Technology, Hangzhou 31023, China and Wilczek Quantum Center, Department of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China

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Abstract

We consider the quantum thermodynamic origin of the gravitational force of matter by applying the spacetime entanglement entropy and the Unruh effect originating from vacuum quantum fluctuations. By analyzing both the local thermal equilibrium and quasi-static processes of a system, we may get both the magnitude and direction of Newton's gravitational force in our theoretical model. Our work shows the possibility that the elusive Unruh effect has already shown its manifestation through gravitational force.

Keywords: spacetime entanglement entropy; Unruh effect; gravitational force; quantum thermodynamics; holographic principle

*Electronic address: `xionghw@zjut.edu.cn`

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I. INTRODUCTION

In the last decade, the investigation of spacetime entanglement [1–8] has given remarkable opportunities to consider the coalescence of quantum mechanics and gravitational force, although it is still unclear how to unify quantum mechanics and general relativity. Nevertheless, the concept of quantum entanglement has been found to connect closely with some fundamental properties of spacetime, such as vacuum quantum fluctuations [9–11], the holographic principle [12–14] and black hole [15–18].

Much clearer than the application to spacetime, the concept of quantum entanglement has already promoted our understanding of Boltzmann entropy and statistical thermodynamics [19–21]. For a thermodynamic system we want to study, if we consider the whole system including the external environment, the thermodynamic system is highly entangled with the external environment. In this case, the usual entropy of this thermodynamic system is in fact the entanglement entropy obtained for the reduced density matrix of this thermodynamic system.

In the present work, we apply the entanglement entropy of spacetime and the Unruh effect for an accelerating particle to consider the quantum thermodynamic origin of gravitational force. In particular, we use a quasi-static process to consider theoretically the direction of gravitational force, which has potential application for further studies of the gravitational force for dark energy [22, 23], black hole, and so on.

The paper is organized as follows. In Sec. II, we give a brief introduction to the entanglement entropy and the Unruh effect for the Minkowski spacetime. In Sec. III, we give the finite spacetime temperature distribution of matter from the consideration of spacetime entanglement entropy and statistical thermodynamics. In Sec. IV, based on the consideration of the local thermal equilibrium and a quasi-static process of a system, we give an interpretation to Newton's gravitational force and in particular the attractive characteristic. In the last section, we give a brief summary and discussion.

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II. SPACETIME ENTANGLEMENT ENTROPY AND UNRUH EFFECT FOR MINKOWSKI SPACETIME

The Minkowski spacetime can be specified by the distance between two nearby points in spacetime, given by

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2. \tag{1}$$

Even for this flat spacetime without considering the spacetime curvature of general relativity, the Minkowski spacetime has some remarkable properties when both spacetime and quantum mechanics are considered.

The confluence of special relativity and quantum mechanics will lead to nontrivial vacuum quantum fluctuations [9–11]. Although we do not know the exact property of the quantum vacuum, we may assume the existence of an extremely complex and time-dependent quantum vacuum state $|\Psi_{vacuum}\rangle$ for the quantum vacuum of the Minkowski spacetime.

We should distinguish two different vacuum quantum fluctuations as follows.

A. The vacuum quantum fluctuations due to zero-point energy

The usual vacuum quantum fluctuations are considered for the existence of the zero-point energy of various quantum fields. The Casimir effect [11] between two conducting metals is due to the presence of the zero-point energy of electromagnetic field. Although there are other types of zero-point energy, the conducting metals can only change the zero-point energy of electromagnetic field in a noticeable way. Hence, the Casimir effect is about the specified vacuum quantum fluctuations due to electromagnetic field.

In fact, the Unruh effect [17, 24, 25] is also due to the vacuum quantum fluctuations of various quantum fields. For an inertial frame of reference in the Minkowski spacetime, we consider an electrically charged particle with acceleration $\mathbf{a} = d^2\mathbf{r}/dt^2$. Because the particle is electrically charged, it should have a coupling with the zero-point energy of the electromagnetic field in quantum vacuum. This will lead to a finite temperature distribution for the electromagnetic field around this accelerating particle in this inertial frame of reference. The peak of this finite temperature distribution is called as the Unruh temperature [17] given by

$$T_U = \frac{\hbar a}{2\pi c k_B}. \tag{2}$$

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In this equation, k_B is the Boltzmann constant, $\hbar$ is the reduced Planck constant, and c is the speed of light. In addition, a is the amplitude of **a** in semi-relativistic approximation. Rigorously speaking, a should be the magnitude of the proper acceleration of a particle. Note that in the above equation, T_U is independent of the charge of the particle, which implies that T_U is a universal value when other types of charge are addressed.

We want to emphasis two properties of the Unruh effect as follows.

1. T_U in the above expression should be regarded as a peak value of a local temperature distribution in an inertial frame of reference.

2. Besides the case of electrically charged particle usually considered for Unruh effect, the particle may have other types of charges. Hence, T_U may also mean the temperature for other gauge fields, such as gravitational field. Because the gravitational field is universal for any particle, the similar expression of Eq. (2) can be applied to gravitational field. In this paper, the Unruh temperature is considered for gravitational field.

B. The spacetime quantum fluctuations

When the sum of all zero-point energies are considered, it is well known that the vacuum energy density ϵ_V is extremely high and even divergent as a result of a rough consideration. It is natural that this will lead to violent quantum fluctuations of the spacetime geometry [26, 27] at the microscopic scale of l_p. To distinguish the vacuum quantum fluctuations introduced above, we call it as spacetime quantum fluctuations in this paper.

To give a clear picture of spacetime quantum fluctuations, we consider f_{AB} defined by

$$f_{AB} = \sqrt{\frac{\langle\Psi_{vacuum}|\, d_{AB}^2 \,|\Psi_{vacuum}\rangle - |\langle\Psi_{vacuum}|\, d_{AB} \,|\Psi_{vacuum}\rangle|^2}{\langle\Psi_{vacuum}|\, d_{AB}^2 \,|\Psi_{vacuum}\rangle}}. \tag{3}$$

Here d_{AB} is an operator in an inertial frame of reference to measure the spatial distance between nearby points A and B in spacetime. It is clear that f_{AB} shows the fluctuations of spacetime geometry.

If A and B are macroscopically separated, it is expected that the fluctuation f_{AB} is negligible, while below or of the order of a microscopic distance l_p, there would be significant fluctuations in f_{AB}. At the present stage, we do not know the exact value of l_p. However, the

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existence of spacetime quantum fluctuations [26, 27] and the stable spacetime property at macroscopic scales means that there should be a distance l_p. We will give further discussion of l_p in the next section.

III. FINITE SPACETIME TEMPERATURE DISTRIBUTION DUE TO MATTER

We consider a sphere of radius R. The surface of this sphere divides the whole universe into two systems S_A and S_B, i.e., the interior of the sphere S_A and the external environment S_B. Without the presence of any other matter in the Minkowski spacetime, the entanglement entropy is [28]

$$S_{entangle} = -\mathrm{Tr}\left[\rho_A \log \rho_A\right]. \tag{4}$$

Here $\rho_A = \mathrm{Tr}_B(\rho)$ is the reduced density matrix with $\rho = |\Psi_{vacuum}\rangle\langle\Psi_{vacuum}|$ the density matrix for the pure state $|\Psi_{vacuum}\rangle$ of the Minkowski spacetime. It is easy to show that $S_{entangle} = -\mathrm{Tr}\left[\rho_B \log \rho_B\right]$ with $\rho_B = \mathrm{Tr}_A(\rho)$.

For the situation that $R >> l_p$, it is expected that the entanglement entropy $S_{entangle}$ depends only on the property of $|\Psi_{vacuum}\rangle$ in the region of a thin spherical shell with the width of the order of l_p. Hence, it seems reasonable to assume the following conjecture of the spacetime entanglement entropy

$$S_{entangle} \sim k_B \frac{A_{area}}{l_p^2}. \tag{5}$$

Here $A_{area} = 4\pi R^2$ is the area of the sphere. This is the so-called area laws for the entanglement entropy [29–31]. We have another way to understand this relation. From Eq. (5), we may also regard A_{area}/l_p^2 as the number of Planck areas on the spherical surface. It is worthwhile to point out that at the present stage, this formula does not mean directly the holographic principle because we do not consider the possible presence of matter distribution inside the sphere yet.

Now we consider the case that there is a classical particle with mass M inside the sphere. Of course, the coupling between this particle and spacetime will lead to a change of $|\Psi_{vacuum}\rangle$ on the spherical surface. Hence, the modified entanglement entropy for the sphere becomes

$$S^M_{entangle} \sim k_B \frac{A_{area}}{l_p^2} + \Delta S_M. \tag{6}$$

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For the usual case that the particle M only gives slight change to the curvature of the spacetime, it is expected that $\Delta S_M << S_{entangle}$. However, the presence of this particle will lead to an important effect by applying the holographic principle. The holographic principle [12–14] implies that the energy Mc^2 will show its effect on the spherical surface. Combined with the first law of thermodynamics, we have

$$k_B T_M(R) \times \frac{A_{area}}{l_p^2} \sim Mc^2. \tag{7}$$

$T_M(R)$ is the effective spacetime temperature on the spherical surface.

From the above equation, we have

$$T_M(R) \sim \frac{c^2 l_p^2}{4\pi k_B} \frac{M}{R^2}. \tag{8}$$

It is worthwhile to discuss the following properties of this effective spacetime temperature.

(1) In the usual case, this effective spacetime temperature is extremely small by noticing that there is a factor l_p^2 in the above equation. We will show that l_p is the Planck length in due course.

(2) This effective spacetime temperature is about spacetime and gravitational field, rather than electromagnetic field.

(3) Because this effective spacetime temperature originates from the entanglement entropy and the presence of M inside the sphere, its finite value does not mean that there would be various radiations spontaneously. We may notice these radiations only when we have appropriate means to experience the entanglement entropy. This is a little similar to the observation of Casimir effect [11]. We must have two conducting metals to show the Casimir effect through the coupling with the fluctuating electromagnetic field in the quantum vacuum.

It seems that it would be extremely challenging to observe this effective spacetime temperature. However, combined with the physical picture of the Unruh effect, we will show the possibility that the simultaneous considerations of this effective spacetime temperature and the Unruh effect just lead to gravitational force.

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IV. NEWTONIAN GRAVITATIONAL FORCE DERIVED BY THE CONSIDERATION OF LOCAL SPACETIME THERMAL EQUILIBRIUM

A. Spacetime thermal equilibrium

We consider another fictitious particle with mass m and assume that this particle does not have any other interaction in addition to gravitational force. The particle M establishes an effective vacuum temperature field $T_M(\mathbf{r})$ given by Eq. (8). Now we consider the case that the particle m is fixed at location $\mathbf{r}$, relative to M. Because there is no relative motion between M and m, the whole system has the chance to be in spacetime thermal equilibrium. For simplicity, we consider the case that $M >> m$. To be in spacetime thermal equilibrium, there should be another effective temperature T_m for m so that

$$T_M(\mathbf{r}) = T_m. \tag{9}$$

When the relative location between M and m is fixed, we know that in the local inertial frame of reference for m, m has a finite acceleration $\mathbf{a}$. It is clear that the Unruh temperature should be calculated in a local inertial frame of reference. Hence, the Unruh temperature for m is

$$T_m(\mathbf{a}) = \frac{\hbar\,|\mathbf{a}|}{2\pi c k_B}. \tag{10}$$

The spacetime thermal equilibrium condition (9) leads to

$$a = \alpha\frac{c^3 l_p^2}{2\hbar}\frac{M}{r^2}. \tag{11}$$

Here $a = |\mathbf{a}|$. The coefficient α can be absorbed in the definition of Newton's gravitational constant G. Compared with Newton's law of gravitational force, we have

$$l_p = \left(\frac{2\hbar G}{\alpha c^3}\right)^{1/2}. \tag{12}$$

We see that with the choice of $\alpha = 2$, we get the conventional gravitational constant G if we regard l_p as the Planck length. Here, we show the possibility that l_p is more fundamental than G in a sense.

In the units with $\hbar$=1 and $c = 1$, we have $G = l_p^2$. We see that G decreases with the decreasing of l_p. This is due to the fact that with the decreasing of l_p, the degree of freedom increases, and hence the effective spacetime temperature decreases on the spherical surface.

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Peer-reviewed version available at *Entropy* **2019**, *21*, 296; doi:10.3390/e21030296

The spacetime thermal equilibrium condition means that m has smaller acceleration, and equivalently smaller G.

B. Quasi-static process to determine the direction of gravitational force

The previous studies only give the magnitude of gravitational force. Now we turn to consider the direction of gravitational force. We consider a quasi-static process by an external force $\mathbf{F}_{ext}$ so that the system is always in quasi-thermal equilibrium. In addition, we consider the case that the particle m moves toward M in a quasi-static way. Because $T_M \sim 1/r^2$, we see that the particle m will exchange heat energy with spacetime during the quasi-static process, while the kinetic energy will not change. The first law of thermodynamics then gives

$$dU_m = \delta Q + \delta W = 0. \tag{13}$$

Here δQ is the heat energy absorbed from spacetime, while δW is the work by the external force on the particle m. δW is given by

$$\delta W = \mathbf{F}_{ext} \cdot d\mathbf{r}. \tag{14}$$

Hence, for the quasi-static process of the system with $dU_m = 0$, we have

$$\mathbf{F}_{ext} \cdot d\mathbf{r} = -\delta Q. \tag{15}$$

For simplicity, we consider the case that the particle m moves along the line connecting M and m. If their distance increases, $\delta Q < 0$, and we have

$$\mathbf{F}_{ext} \sim \frac{\mathbf{r}}{r^3}.$$

This determines the direction of the external force to maintain the thermal equilibrium or time-independent location of the particle m. We see that this is equivalent to the fact that the gravitational force is attractive.

If we consider the case that the particle m moves toward M along the line connecting M and m, we have $\delta Q > 0$. We will still have $\mathbf{F}_{ext} \sim \mathbf{r}\ /r^3$, and leads to the attractive characteristic of gravitational force too. Combined with Eq. (11), the gravitational force can be then written as

$$\mathbf{F}_g = -\frac{GMm}{r^3}\mathbf{r}. \tag{16}$$

C. Free fall motion

In a gravitational field, we know that the free fall motion has no acceleration at all, based on Einstein's general relativity. In this case, the Unruh temperature is zero for the particle m, while the spacetime temperature due to M is larger than zero. Hence, during the free fall motion, there is always temperature difference between $T_M(\mathbf{r})$ and the Unruh temperature T_m. Because of this temperature difference, the free fall motion is not a quasi-static process. This temperature difference leads to the possibility of energy exchange between spacetime and the particle m.

Similarly to the analysis of Joule expansion in thermodynamics, for the free fall motion from A to B, we may construct a quasi-static process from A to B by a fictitious external force, and then at the end of this quasi-static process, the work of the external force is given to the particle m. In this case, in the non-relativistic approximation, the work by the gravitational force on the particle m during the free fall motion is

$$\Delta W = \phi(r_1) - \phi(r_2), \tag{17}$$

with $\phi(r) = -GMm/r$ and ΔW the work done on the particle m by the gravitational field.

V. SUMMARY AND DISCUSSION

In summary, we consider the quantum thermodynamic origin of Newton's gravitational force by considering simultaneously the spacetime entanglement entropy, holographic principle, thermodynamics and the Unruh effect for an accelerating particle. Different from previous works on the thermodynamic origin of gravitational force [2, 32–34], in this work we emphasis on the quantum entanglement of spacetime. In the present paper, we do not use the assumption of the displacement entropy $\Delta S \sim d$ in Refs. [2, 33], which implies a more solid basis for the quantum thermodynamic origin of Newton's gravitational force.

Currently, the direct observation of the Unruh effect is still elusive. For an electronically charged particle, the Unruh temperature is too small to have observable electromagnetic radiation with current techniques. Most recently, a pioneering quantum simulation of coherent Unruh radiation [35] is observed based on ultra-cold atomic system. Of course, this observation does not show directly the original Unruh effect for spacetime. In the present

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work, however, we show the possibility that the original Unruh effect has already shown its manifestation through gravitational force.

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