Type of the Paper (Article)

A Recap of Voltage Stability Indices in the Past Three Decades

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Abstract: Increasing demand for electricity and modernization of power systems within competitive markets induce power systems to operate close to their stability limits. Therefore, a power system continuous monitoring and control by using voltage stability indices are known exigence. This is the first-ever effort, examined more than 40 voltage stability indices thorough an exhaustive study based in on their formulation, application, performance, and assessment measures. Which is conducted based on a logical and chronological order considering the most recent worldwide applied voltage stability indices. However, the generalizability of such an immense diversity of these indices in term of multivariable objectives subjects to a certain limitation to summate into a deciphered outcome. Despite all these obstacles, this study systematically reviews available indices in the literature within the past three decades to set out an integrated knowledge with an insight up-to-date exposition. That followed with a comparative analysis in term of their similarity, functionality, applicability, formulation, merit, demerit, and overall performances. Also, a broad categorization of voltage stability indices is addressed. This study with an immediate implication in practice as an exhaustive roadmap can be counted a planning and operation reference in the context of voltage stability for students, researchers, scholars, and practitioners.

Keywords: voltage stability; stability index; voltage stability indicator; power system stability, power system monitoring, voltage collapse; blackout, voltage stability classification

1. Introduction

Blackout phenomenon in a power system rises along with the initial stages of electricity generation and consumption. The history of the world tremendous blackouts due to voltage instability in power systems come back to Tokyo blackout on 23 July 1987; the United Kingdom, Swedish, Canada, Denmark, Italy, and the United States blackouts in 2003 [1,2]. In 1987, Tokyo experienced a blackout for more than 3 hours, in which 2.8 million consumers were affected by electricity outage [3]. Among various reasons that can cause a blackout at different stages with various mechanism, voltage instability is known as an intense player [4–6].

Adequate information and proper prediction of the blackout phenomenon due to voltage instability still remains a critical challenge around the world, in which continues researches have been tried to sum up with methodological approaches for blackout prediction and prevention in a

Peer-reviewed version available at *Energies* **2019**, *12*, 1544; doi:10.3390/en12081544

power system [7,8]. Voltage instability due to blackout can supersede a local-area or a wide-area stability in a power system that can lead entire system to voltage collapse. Therefore, a power system continues monitoring and its instability prediction are known exigent. There are many approaches in the literature to overcome instability phenomenon in a power system, which the univocal solutions are reactive power compensation, network loadability improvement, network re-configuration, optimally distributed generation, and etc. [9–15].

Since the 1920s, a power system stability and reliability statuses have been continuously monitored by the help of voltage stability indices, in term of a simple statistic to interwoven transient stability analysis [16]. Over the past decades, various methods and mechanism were hired to propose tools and techniques for voltage stability analysis in a power system. The earliest-worldwide applied methods are singular value decomposition [17], energy function [18], continues power flow [19], sensitivity analysis methods [20], bifurcations theory [21], minimum eigenvalue [22], modal analysis [22,23], integrated transmission line transfer index (ITLTI) [24] and etc. These approaches methodologies are elucidated in the next section in term of voltage stability indices along with indices' formulation and stability margin boundaries.

2. Voltage Stability Indices

In a power system, voltage stability indices pretend a profound function to situate a power system present operation, predict the system changing nature in the future, and evaluate a long-run development trend within predefined circumstances. Herein, the theoretical analysis and foundation of voltage stability indices of the first two classes (except the eight indices of online application) are discussed in chronological order.

2.1. L Index

In [25], an index is delineated with special characteristics of identification of a power system vulnerability, real power quantitative measure, reorganization of week bus or area, and forecasting of voltage collapse due to contingencies and generation and transmission losses to predict instability in an electric power system. The concept of L index is formulated based on an analytical analysis of line mode of 2-bus system as given in Equation (1):

$$L = \underset{j \in \alpha_L}{MAX} \left| 1 - \frac{\sum_{i \in \alpha_G} \bar{F}_{ji} \bar{V}_i}{\bar{V}_j} \right| \tag{1}$$

where, where, α_L is the set of consumer nodes; and α_G is the set of generator nodes.

2.2. Power Stability Index (PSI)

In [26], PSI is proposed to realize distributed generation (DG) optimum placement within critical sensitive bus close to voltage collapse. This index is proposed based on 2-bus system with less than unity margin for a voltage stable operation as given in Equation (2):

$$PSI = \frac{4 r_{ij} (P_L - P_G)}{[|V_i| \cos(\theta - \delta)]^2} \leq 1 \tag{2}$$

where, r_{ij} is the line resistance; P_L is real power at load bus; and P_G is injected real power to the system.

2.3. Voltage Deviation Index (VDI)

In [27], VDI is proposed that defines an absolute value of a bus voltage deviation compared one per unit. This index is generalized in term of N-bus system based on the sum of N voltages deviation, separately from each bus for all the system as given in Equation (3), and generalized in Equation (4):

$$VDI_j = |1 - V_j| \tag{3}$$

$$VDI_T = \sum_{j=1}^{N} \left| 1 - V_j \right| \tag{4}$$

where, N is the number of buses under study; and V_j is the targeted for index calculation.

2.4. Stability Index (SI)

In [28], a new index for radial distribution topology is proposed, which is driven by considering the composite load modeling and power flow analysis. The smallest magnitude of the index at any bus indicates the most sensitivity to the voltage collapse as given in Equation (5).

$$|V(m2)| = 0.707\left[b(jj) + \{b^2(jj) - 4.0c(jj)\}^{1/2}\right]^{1/2} \tag{5}$$

where, jj is the branch number. After some simplifications and substitutions of Equation (5), the SI is given by Equation (6) as follows:

$$\begin{aligned} SI(m2) = \left\{|V(m1)|^4 - 4.0\{P(m2)x(jj) - Q(m2)r(jj)\}\right\}^2 \\ - 4.0\{P(m2)r(jj) + Q(m2)x(jj)\}|V(m1)|^2 \end{aligned} \tag{6}$$

2.5. Voltage Collapse Prediction Index (VCPI)

In [29], VCPI is formulated to predict voltage collapse in a power system. This index is derived based on the system variables such as bus voltage magnitude, voltage angle information and system admittance matrix. The merit of this index is its capability to employ for online application. An N-bus system is considered that the index is given in Equation (7):

$$VCPI_{kth\ bus} = 1 - \frac{\sum_{\substack{m=1 \\ m \neq k}}^{N} |V'_m|}{V_k} \tag{7}$$

were, V_k and V_m are the voltage phasor at bus k and bus m. The threshold for VCPI is zero and 1 that the index value near to zero states more voltage stable bus. Furthermore, some features of this index are listed as follows:

- Prediction of voltage collapse in the system for every bus.
- The technique includes the effect of the load changes in the system on the particular bus voltage collapse estimation.
- This technique needs a modest amount of calculations for estimating the VCPI.
- This technique can be used for recognition of the weak bus in the system.
- This technique can be used for both online and offline application.

2.6. Sensitivity Analysis (SA)

The set of sensitivity indices are composed of various index within different formulation approaches [30–35]. These indices are employed to measure active and reactive power changes ($\Delta V_i / \Delta Q_i$, $\Delta V_i / \Delta P_i$) in a system with respect to bus voltage variation. This index is appropriate for sensitivity analysis and weak bus identification. However, sensitivity index alone will not be sufficient to identify weak buses especially in an interconnected system [9,36]. Beyond the other methods, sensitivity analysis plays important role in prediction of critical nodes in the system. It is important to investigate how this critical point is affected by changing the system conditions [37]. Also, sensitivity analysis can be a useful tool for determining, weak bus, active and reactive power losses, and the reactive power margin (Mvar distant to voltage collapse point) [37].

2.7. Bus participation factor (BPF)

In [19,38], authors are denoted BPF in view of the voltage collapse mechanism. BPF can be used for weak bus identification in a power system, which includes several indices based on this approach

[39,40]. The bus participation factor is employed as one of the indicators for identification of the weakest node in the system.

2.8. Voltage Stability Index (VSI)

In [41], VSI is proposed using a simple method based on the system variable parameters or local information such as bus voltage and current magnitudes in order to determine the distance to the voltage collapse from current operating point. The driven index is called VSI, given in Equation (8). The formulation of this index relies on power flow (power transfer) equations.

$$VSI_i = \left[1 + \left(\frac{I_i}{V_i}\right)\left(\frac{\Delta V_i}{\Delta I_i}\right)\right]^{\alpha} \tag{8}$$

At no load, VCI equals unity and at the voltage collapse point its value is zero. The VSI is similar to the voltage collapse indicator in which there is a petty change in their formulations. The evaluation of VSI is simple, and it only requires the magnitude of bus voltage and loads current at two different operating points. It is raised to the power of $(\alpha > 1)$ in order to give more or less linear characteristic to the index. The value of α depends on the system parameters.

2.9. Equivalent Node Voltage Collapse Index (ENVCI)

In [42], ENVCI is introduced based on equivalent system model (ESM) as given in Equation (9). According to [42], this index associated with some advantages such as its affectivity from both the local network and system outside the local network, real-time applications, identification of voltage collapse point when it is near to zero. Also, it is demonstrated as a prediction and monitoring tool to prevent the system from collapse. In this context, the authors put forward with futures of ENVCI in details as follows:

- Accuracy in index modeling, because the system outside effect to the local network, is considered.
- Distinguish in internal and external impedance consideration the impedance of the local network.
- Easiest in calculations with less computation time, compared to the customary methods based on power flow.

It can be counted an emergency remedial action scheme to protect the system from voltage collapse due to its functionality of identification the weak bus at near zero when the system is approaching its voltage collapse point.

$$ENVCI = 2(e_k e_n + f_k f_n) - (e_k^2 + e_k^2) \tag{9}$$

where, $\theta_{kn} = \theta_k - \theta_n$, more details are given in [42].

2.10. Voltage Collapse Index (VCI)

In [41], VCI is proposed based on the change of apparent power in the system as given in Equation (10). The Taylor's theorem is applied. Power flow calculation is applied to expand the index based on increase and decrease of the current.

$$VCI_i = \left[1 + \left(\frac{I_i \Delta V_i}{V_i \Delta I_i}\right)\right]^{\alpha} \tag{10}$$

where, $VCI_i \geq 0$ is used to linearize the trend of the index at the collapse point.

2.11. Improved Voltage Stability Index (IVSI)

In [27], an improved voltage stability index for voltage stability enhancement is proposed. The index formulation is based on power flow variables as given in Equation (11).

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$$IVSI_T = \sum_{i=1}^{N} IVSI_i \tag{11}$$

This index is introduced under bus voltage stability indices category. It can be used for both radial and interconnected power systems. The voltage stability margin is measured between 0 and 1. The index magnitude close to zero indicates a stable system, whereas, as long as the index for any bus in the system is closed to 1 indicates the unstable system. At this study, authors are generalized the applicability of the aforesaid bus index for overall system stability evaluation. The aim of the total voltage stability index has been optimizing the setting of compensation devices for N-Bus system.

2.12. Voltage Stability Factor (VSF)

In [43], VSF based on the power flow analysis is proposed. The proposed index is driven from 2-bus system as a part of the distribution system as given in Equation (12). The voltage collapse threshold for the index is defined zero. When VSF becomes zero the voltage collapse may occur, in this situation the receiving end bus voltage magnitude, becomes half of sending end bus voltage magnitude.

$$VSF_{total} = \sum_{m=1}^{k-1} (2V_{m+1} - V_m) \tag{12}$$

where, k is the total number of buses in the system and V_m is the magnitude of substation voltage. The higher value of VSF_{total} indicates more voltage stable operation.

2.13. Line Stability Index (L_{mn})

In [44], L_{mn} index is proposed using power transfer concepts in a single line power transmission network. The proposed index is given by Equation (13):

$$L_{mn} = \frac{4Qrx}{[|V_s| \sin(\theta - \delta)]^2} \tag{13}$$

where, V_s is the sending end voltage; θ is the line impedance angle; δ is the angle difference between the supply and the receiving end voltages; x is the line reactance; Qr is the reactive power at the receiving end.

2.14. Line Stability Factor (LQP)

In [45], LQP is proposed based on the power flow equation in a single line network as given in Equation (14):

$$LQP = 4\left(\frac{X}{V_i^2}\right)\left(\frac{X}{V_i^2}P_i^2 + Q_j\right) \tag{14}$$

where, V_i is the sending end voltage; P_i is the sending end real power; Q_i is the receiving end reactive power; X is the line reactance. For stable operation, the LQP must keep less than 1.

2.15. L Index

In [46], an index is discussed that ranked all contingencies in the system to find out the voltage unstable buses in the system. For the purpose of index extraction (L index), first the 2-bus system with its associated parameters are considered. Then tried to derive the equations that characterize the behavior of the single line diagram of the 2-bus system as given by Equations (15-18).

$$L = 4[(xP_L - rQ_L)^2 + xQ_L + rP_L] \tag{15}$$

For $L < 1.0$. The generalized form of the index is given for the reduced network as bellow:

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$$L = 4\left[\left(x_{eg}P_{leg} - r_{eg}Q_{leg}\right)^2 + x_{eg}Q_L + r_{eg}P_{leg}\right] \tag{16}$$

$$r_{eg} = \frac{R_{eg}}{(P^2 + Q^2)} \tag{17}$$

$$x_{eg} = \frac{X_{eg}}{(P^2 + Q^2)} \tag{18}$$

where, P_L and Q_L are real and reactive loads; P and Q are injection of real and reactive power in the system; r and x are line resistance and reactance; r_{eg} and x_{eg} are the equivalent resistance and reactance of single line; P_{leg} and Q_{leg} are the distribution network total real and reactive loads. When the network is loaded beyond its critical limit, the power becomes imaginary and it is the collapse point.

2.16. Voltage Collapse Proximity Indicator (VCPI)

In [47], a line index for online application is initiated based on maximum power transfer concept. This index is derived from 2-bus system with considering the $Z_r/Z_s = 1$ (the impedance ratio is lower than 1 in stable operation) as a voltage collapse predictor. A set of voltage collapse prediction indicators in view of the allowable maximum power transfer limits are given by Equations (19-22):

$$CPI(1) = \frac{P_r}{P_r(\max)} \tag{19}$$

$$VCPI(2) = \frac{Q_r}{Q_r(\max)} \tag{20}$$

$$VCPI(3) = \frac{P_l}{P_l(\max)} \tag{21}$$

$$VCPI(4) = \frac{Q_l}{Q_l(\max)} \tag{22}$$

where, P_r and Q_r are real and reactive power transferred to the receiving end; P_l and Q_l are real and reactive power losses in line. The results show some similarity, hence instead of considering four indicators, either real or reactive terms can be found. In the critical situation, both the power and loss indicators are approaches to become 1.

2.17. Novel Line Stability Index (NLSI)

In [48], NLSI is derived as given by Equation (23). Fundamentally, it is initiated from load flow equations of 2-bus system and authors claim it effectiveness for point of voltage collapse, weak bus identification and most critical line in an inter-connected system.

$$NLSI_{ij} = \frac{R_{ij}P_j + X_{ij}Q_j}{0.25V_i^2} \tag{23}$$

where, V_i is voltage at sending bus; P_j and Q_j are active and reactive power at receiving end bus; R_{ij} and X_{ij} are line resistance and reactance between sending end and receiving end buses, respectively. The voltage stability tolerance is considered less than 1 in order to ensure the line stability.

2.18. Fast Voltage Stability Index (FVSI)

In [49], an index is proposed that symbolized by FVSI. This index is derived based on the voltage collapse occurrence in contingencies as given by Equation (24):

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$$FVSI_{ij} = \frac{4Z^2 Q_j}{V_i^2 x} \tag{24}$$

where, Z and x are line impedance and reactance; Q_j is reactive power at the receiving end; V_i is sending end voltage. In order to ensure a stable power system, FVSI magnitude must be less than 1.

2.19. Critical Voltage (V_{cr})

In [50], a simple index from a single load and infinite bus power system using load flow equations and eigenvalue theorem is derived as given in Equation (25):

$$V_{cr} = \frac{E}{\sqrt{2(1+\cos(\alpha-\phi))}} \tag{25}$$

Equation (25) can be simply in term of Equation (26):

$$V_{cr} = \frac{E}{2\cos\theta} \tag{26}$$

where, V_{cr} is the critical voltage at receiving end; E is the infinite bus voltage; α is the line impedance angle; ϕ is the power factor angle ($PF = \cos\phi$); θ is the receiving end voltage angle.

2.20. Power Transfer Stability Index (PTSI)

In [51], PTSI is proposed, which is derived from 2-bus system based on the Thevenin equivalent system as given by Equation (27):

$$PTSI = \frac{2S_L Z_{Thev}(1+\cos(\beta-\alpha))}{E_{Thev}^2} \tag{27}$$

where, α is the phase angle of the load impedance; β is the phase angle of the Thevenin impedance. The threshold of the PTSI values is 0 and 1 (voltage collapse).

2.21. Line Voltage Stability Index (LVSI)

In [52], Line Voltage Stability Index (LVSI) is proposed that initiated from the relationship between line reactive power and the sending end voltage. This index is given by

$$LVSI = \frac{4rP_r}{V_s\cos(\theta-\delta)^2} \tag{28}$$

where, V_s is the sending end voltage; P_r is the active power at the receiving end; θ is the line impedance angle; δ is the phase angle; r is the line resistance. The condition of having a stable system, the value of LVSI must satisfy $LVSI \le 1$.

2.22. Impedance Ration Indicator

In [53], an index based on impedance ratio as a voltage collapse proximity indicator is proposed. This index is driven from 2-bus system by employing the Thevenin theorem as given by Equations (29-31):

$$\frac{Z_{ii}}{Z_i} \le 1 \tag{29}$$

$$Z_{ii}\angle\beta_i = i^{th} \quad diagonal\ element\ of\ [\mathbf{Z}] \tag{30}$$

$$[\mathbf{Z}] = [\mathbf{Y}]^{-1} \tag{31}$$

where, $Z_{ii}\angle\beta_i$ is the Thevenin's equivalent impedance; $Z_i\angle\phi_i$ is the impedance of the load. The aim of this indicator is the assessment of the validity and robustness of indicator over the operating range.

2.23. Minimum Eigenvalue and Right Eigenvector (RE) Method

In [19], a model analysis-based index is introduced. The smallest eigenvalue associated with right eigenvectors is used in this technique as expressed by Equation (32):

$$\Delta V = \sum_i \frac{\xi_i \, \eta_i}{\lambda_i} \Delta Q \tag{32}$$

where, ΔV indicates the deviation in voltage magnitudes; ΔQ indicates the deviation in injected reactive power; ξ_i is the i^{th} column right eigenvector; η_i is the i^{th} row left eigenvector of Reduced Jacobian matrix; λ is the diagonal eigenvalue matrix of Reduced Jacobian matrix.

The above equation obviously shows the relationship between involved parameters, in which the changes in reactive power, eigenvalue and eigenvectors have direct effect on ΔV. A system is voltage stability if all eigenvalues are positive. The real part of the eigenvalue indicates that the system is unstable. In this study, the RE at the minimum eigenvalue is applied as voltage stability indicator.

2.24. Singular Value Indicator

In [17], a static voltage stability index based on a singular value decomposition of power flow Jacobian matrix is formulated. This index aims to proximate the voltage instability and identify the critical nodes in the system. Supposed that the matrix $\mathbf{A}$ is and $n \times n$ quadratic (real) matrix as given by Equation (33):

$$\mathbf{A} = \mathbf{U}\sum\mathbf{V}^T = \sum_{i=1}^{n} \boldsymbol{\sigma_i}\, \boldsymbol{u_i}\, \boldsymbol{v_i^T} \tag{33}$$

where, $\mathbf{U}$ and $\mathbf{V}$ are $n \times n$ orthonormal matrices; $\boldsymbol{v_i}$ and $\boldsymbol{u_i}$ are singular vectors and the columns of the $\mathbf{U}$ and $\mathbf{V}$ matrices; $\sum$ is a diagonal matrix given by Equation (34):

$$\sum(\mathbf{A}) = \boldsymbol{diag}\,\{\boldsymbol{\sigma_i}(\mathbf{A})\} \tag{34}$$

where, $\boldsymbol{i} = 1, 2, \cdots, n$; where $\boldsymbol{\sigma_i} \geq 0$ for all $\boldsymbol{i}$.

The order of the diagonal matrix is $\boldsymbol{\sigma_1} \geq \boldsymbol{\sigma_2} \geq \cdots \geq \boldsymbol{\sigma_n} \geq 0$. With considering the power flow Jacobian matrix the result is yielded by Equation (35):

$$\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = \mathbf{V}\sum^{-1}\mathbf{U}^T \begin{bmatrix} \Delta F \\ \Delta G \end{bmatrix} \tag{35}$$

From singular value decomposition of the power flow Jacobian matrix, these points are observed:

- The smallest singular value ($\boldsymbol{\sigma_n}$) can be used as steady-state stability limit indicator;
- The right singular vector ($\boldsymbol{v_n}$) corresponding to smallest singular value ($\boldsymbol{\sigma_n}$) indicated sensitive voltage and angles;
- The left singular vector ($\boldsymbol{u_n}$) corresponding to smallest singular value ($\boldsymbol{\sigma_n}$) indicated the most sensitive direction for changes of the active and reactive power injections.

2.25. Predicting voltage collapse index (V/V_o)

In [54], a simple ($\mathrm{V/V_o}$) index is proposed; thus, the voltage magnitude (V) is obtained from load flow for the operating point of the system. Where, V_o (no load voltage) is known new values of voltage when the system all loads set to zero. This index indicates an overall picture of the system stability state. The smallest index value indicates the most sensitive weak bus in the system. This index can be used for on-line and off-line applications. While, with respect to change in loading parameters, this index shows a nonlinear profile. In [55,56], authors have argued that the $\mathrm{V/V_o}$ index is poor in all three computational cost, accuracy of collapse point prediction and adequacy to nonlinearity performances.

2.26. Test Function

In [57], the Test Function index is illustrated based on the quadratic shape of the proposed model. The test function index is more reliable than another Jacobian matrix-based method especially, eigenvalue and singular value methods as given by Equation (36).

$$t_{lk} = |e_l^T J J_{lk}^{-1} e_l| \tag{36}$$

where, J is the Jacobian matrix of the system; e_l is the l^{th} unit vector, i.e.; a vector with all entries zero except the l^{th} row, and J_{lk} is defined by Equation (37):

$$J_{lk} = (I - e_l e_l^T)J + e_l e_k^T \tag{37}$$

By rearranging the Jacobian matrix with the l^{th} row removed and replaced by row e_l^T. If $l = k = c$, the function shows critical test function as expressed by Equation (38):

$$t_{cc} = |e_c^T J J_{cc}^{-1} e_c| \tag{38}$$

It is found that the test function can be used to proximate the voltage collapse in a system, but it will not be able to identify the critical bus.

2.27. Tangent Vector Index (TVI_i)

In [58], $\mathrm{TVI_i}$ is derived the tangent vector index, based on the load change and corresponding tangent vector concept. This index directly measures the effect of load changes on the vector elements such as bus voltage magnitudes and angles. Therefore, it can be counted as a good approach to assess how far away the system is operating from the collapse point. This index is given by Equation (39):

$$TVI_i = \left|\frac{dV_i}{d\lambda}\right|^{-1} \tag{39}$$

where, V_i is the voltage at bus i; λ is the load. When the value of the derivative tends to infinity, then $TVI_i \rightarrow 0$.

2.28. Second Order Index (i Index)

In [59], a voltage stability index, named index i (or second order index) is presented. This index is driven based on maximum singular value concept and its derivative. The aim of this index is to overcome the deficiencies of the previous indices such as minimum singular value index, which are inadequate for a non-linear condition. This index is considered in respect of the system total load and maximum singular value of the inverse Jacobian matrix changes as given in Equation (40):

$$i = \frac{1}{i_0} \frac{\sigma_{max}}{d\sigma_{max}/d\lambda_{total}} \tag{40}$$

where, σ_{max} is the maximum singular value of the Jacobian inverse matrix; λ_{total} is the system total load; i_0 is the value of $\frac{\sigma_{max}}{d\sigma_{max}/d\lambda_{total}}$ at the initial operating point. The range for this index is defined 1 at stable condition and zero, when the system tends to collapse.

2.29. Critical Boundary Index (CBI)

In [60], CBI is introduced with some novelties based on active and reactive power change in a system. However, a system stability analysis using this index is time-consuming, but still, it is preferred due to its high accuracy of prediction as given in Equations (41-43). A transmission line is encounter worse case, when index is approaching to zero.

$$CBI_{ik} = \sqrt{\Delta P_{ik}^2 + \Delta Q_{ik}^2} \tag{41}$$

$$\Delta P_{ik} = X - P_o \tag{42}$$

$$\Delta Q_{ik} = Y - Q_o \tag{43}$$

where, i is the sending end bus number; k is the receiving end bus number.

2.30. Line Voltage Stability Index (LVSI)

In [61], LVSI is proposed using a methodological approach to evaluate parameter-based stability as given in Equations (44 and 45). The index value is between 1 and 2, in which lines close to value 1 are the most critical lines in a system.

$$LVSI = max\left(LVSI_j\right) \quad \forall\, j = 1, 2, 3, \dots l \tag{44}$$

$$LVSI > 1 \tag{45}$$

2.31. Integrated Transmission Line Transfer Index (ITLTI)

In [24], an integrated transmission line transfer index (ITLTI) based on radial topology is introduced as given in Equation (46). That known suitable for power transferring situation including leading, lagging, or unity power factor.

$$P_R = -\frac{AV_R^2}{B}\cos(\beta - \alpha) + \frac{V_S V_R}{B}\cos(\beta - \alpha) \tag{46}$$

where, P_R and V_R are active power and voltage at sending end bus; V_S is the sending end bus constant voltage; A and B are the line parameters.

2.32. Miscellaneous Indices

In [62], authors evaluated a power system operating condition beyond the collapse point. Also, in [1,37,63], authors are investigated various voltage stability indices using load shedding and optimum storage technologies placement techniques. In [64], authors compared three voltage stability indices, testing in a real power system of the Italian HV transmission grid with a focus on the functionality of the indices.

3. Voltage Stability Indices Categorization

In Table 1, in addition to the voltage stability indices, an exhaustive categorization of indices is pointed out that put forward with a novel approach and classification.

Table 1. An exhaustive representation and classification of voltage stability indices.

Type	Index	Abbreviation	Calculation	Stability Threshold	Reference
System parameters	L Index	L	$L = \underset{j \in \alpha_L}{MAX}\left\lvert 1 - \frac{\sum_{i \in \alpha_G} \bar{F}_{ji}\bar{V}_i}{\bar{V}_j}\right\rvert$	$L < 1$	[25]
	Power Stability Index	PSI	$PSI = \frac{4r_{ij}(P_L - P_G)}{[\lvert V_i\rvert \cos(\theta - \delta)]^2}$	$PSI \leq 1$	[48]
	Voltage Deviation Index	VDI	$VDI_j = \lvert 1 - V_j\rvert$	Details are given in the reference	[49]

	Stability Index	SI	$SI(m2) = \{\lvert V(m1)\rvert^4 - 4.0\{P(m2)x(jj) - Q(m2)r(jj)\}\}^2 - 4.0\{P(m2)r(jj) + Q(m2)x(jj)\}\lvert V(m1)\rvert^2$	The smallest magnitude is most sensitive to voltage collapses	[52]
	Voltage Collapse Prediction Index	$VCPI_{kth\ bus}$	$VCPI_{kth\ bus} = 1 - \frac{\sum_{\substack{m=1\\m\neq k}}^{N} \lvert V'_m\rvert}{V_k}$	$VCPI_{kth\ bus} < 1$	[54]
	Sensitivity Analysis	SA	$\Delta V_i/\Delta Q_i$ $\Delta V_i/\Delta P_i$	Details are given in the reference	[37]
	Bus Participation Factor	BPF	Details are given in [38]	Using power system simulation tool	[38]
	Voltage Stability Index	VSI	$VSI_i = \left[1 + \left(\frac{I_i}{V_i}\right)\left(\frac{\Delta V_i}{\Delta I_i}\right)\right]^{\alpha}$	$VSI_i \geq 0$	[57]
	Equivalent Node Voltage Collapse Index	$ENVCI$	$ENVCI = 2(e_k e_n + f_k f_n) - \left(e_k^2 + e_k^2\right)$	$ENVCI > 0$	[58]
	Voltage Collapse Index	VCI	$VCI_i = \left[1 + \left(\frac{I_i \Delta V_i}{V_i \Delta I_i}\right)\right]^{\alpha}$	$VCI_i \geq 0$	[57]
	Improved Voltage Stability Index	$IVSI$	$\frac{-4\sum_{j=0}^{n}\left(G_{ij} - B_{ij}\right)(P_i + Q_i) \quad (IVSI \leq 1)}{\left[\sum_{j=1}^{n} \lvert V_j\rvert \left[G_{ij}\left(\cos\delta_{ij} + \sin\delta_{ij}\right) - B_{ij}\left(\cos\delta_{ij} + \sin\delta_{ij}\right)\right]\right]^2}$		[49]
	Voltage Stability Factor	VSF	$VSF_{total} = \sum_{m=1}^{k-1}(2V_{m+1} - V_m)$	The greatest magnitude is more stable	[23]
For Line	Lmn Index	L_{mn}	$L_{mn} = \frac{4Qrx}{[\lvert V_s\rvert \sin(\theta - \delta)]^2}$	$L_{mn} < 1$	[59]
	Line Voltage Factor	LQP	$LQP = 4\left(\frac{X}{V_i^2}\right)\left(\frac{X}{V_i^2}P_i^2 + Q_j\right)$	$LQP < 1$	[43]
	Line Index	L	$L = 4\left[\left(x_{eg}P_{leg} - r_{eg}Q_{leg}\right)^2 + x_{eg}Q_L + r_{eg}P_{leg}\right]$	$L < 1$	[65]
	Voltage Collapse Proximity Indicator	$VCPI$	$VCPI(1) = \frac{P_r}{P_r(\max)}$ $VCPI(2) = \frac{Q_r}{Q_r(\max)}$ $VCPI(3) = \frac{P_l}{P_l(\max)}$ $VCPI(4) = \frac{Q_l}{Q_l(\max)}$	$VCPI < 1$	[66]
	Novel Line Stability Index	$NLSI$	$NLSI_{ij} = \frac{R_{ij}P_j + X_{ij}Q_j}{0.25V_i^2}$	$NLSI_{ij} < 1$	[67]
	Fast Voltage Stability Index	$FVSI$	$FVSI_{ij} = \frac{4Z^2 Q_j}{V_i^2 x}$	$FVSI_{ij} < 1$	[68]
	Critical Voltage	V_{cr}	$V_{cr} = \frac{E}{2\cos\theta}$	The critical voltage value	[50]

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		Power Transfer Stability Index	$PTSI$	$PTSI = \frac{2S_L Z_{Thev}(1+\cos(\beta-\alpha))}{E_{Thev}^2}$	$PTSI < 1$	[51]
		Line Voltage Stability Index	$LVSI$	$LVSI = \frac{4rP_r}{V_s\cos(\theta-\delta)^2}$	$LVSI \leq 1$	[1]
		Critical Boundary Index	CBI	$CBI_{ik} = \sqrt{\Delta P_{ik}^2 + \Delta Q_{ik}^2}$	$CBI \leq 1$	[60]
		Line Voltage Stability Index	$LVSI$	$LVSI = max(LVSI_j) \quad \forall\, j = 1,2,3,\dots l$	$LVSI > 1$	[61]
		Integrated Transmission Line Transfer Index	$ITLTI$	$P_R = -\frac{AV_R^2}{B}\cos(\beta-\alpha) + \frac{V_S V_R}{B}\cos(\beta-\alpha)$	Details are given in the reference	[24]
Jacobian matrix-based		Impedance Ratio Indicator		$\frac{Z_{ii}}{Z_i}$	$\frac{Z_{ii}}{Z_i} \leq 1$	[53]
		Minimum Eigenvalue and Right eigenvector method	RE	$\Delta V = \sum_i \frac{\xi_i \eta_i}{\lambda_i}\Delta Q$	All eigenvalues should be positive	[19]
		Minimum Singular value		$\begin{bmatrix}\Delta\theta\\ \Delta V\end{bmatrix} = \mathbf{V}\Sigma^{-1}\mathbf{U}^T\begin{bmatrix}\Delta F\\ \Delta G\end{bmatrix}$	Details are given in the reference	[17]
		Predicting Voltage Collapse		$\frac{V}{V_0}$	The smallest index value	[1]
		Test Function		$t_{cc} = \lvert e_c^T J J_{cc}^{-1} e_c \rvert$	Details are given in the reference	[57]
		Tangent Vector Index	TVI	$TVI_i = \left\lvert\frac{dV_i}{d\lambda}\right\rvert^{-1}$	Depends on load increase	[58]
		Second Order Index	i	$i = \frac{1}{i_0}\frac{\sigma_{max}}{d\sigma_{max}/d\lambda_{total}}$	$i > 0$	[59]
Phasor Measurement Units (PMU)-based		Recursive Least Square	RLS	$x_k = x_{k-1} + G_k(y_k - H_k^T x_{k-1})$ $G_k = P_{k-1}H_k(\lambda I + H_k^T P_{k-1} H_k)^{-1}$ $P_k = \frac{1}{\lambda}(I - G_k H_k^T)P_{k-1}$	Details are given in the reference	[66]
	Local Measurement-based	Voltage Instability Predictor	VIP	$\Delta S = \frac{(V_k - Z_{Th}I_k)^2}{4Z_{Th}}$	Details are given in the reference	[65]
		Voltage Stability Load Bus Index	$VSLBI$	$VSLBI_k = \frac{\lvert V_i(k)\rvert}{\lvert\Delta V_i(k)\rvert}$	Details are given in the reference	[62]
		Approximate Approach		$V_{Li} = E_{eq,i} - Z_{eq}I_{Li}$ $Z_{eq} = Z_{LLii}$	Details are given in the reference	[67]
		Simplified Voltage Stability Index	$SVSI$	$SVSI_i = \frac{\Delta V_i}{\beta V_i}$	$SVSI_i < 1$	[16]

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Observability-based	Voltage Collapse Proximity Indicator	$VCPI$	$VCPI_{kth\ bus} = \left\|1 - \frac{\sum_{m=1,\, m\neq k}^{N} V'_m}{V_k}\right\|$	$VCPI_{kth\ bus} < 1$	[29]
	Margin Voltage Stability Index	$MVSI$	$VSI = min\left(\frac{P_{margine}}{P_{max}}\ \frac{Q_{margine}}{Q_{max}}\ \frac{S_{margine}}{S_{max}}\right)$	Details are given in the reference	[68]
	Sensitivity Related Eigenvalue		$S_{Qgq} = -g_q^T (g_x^T)^{-1} \Delta_x Q_g$	Details are given in the reference	[69]

4. Results and Discussion

The first set of analyses and indices evaluation was conducted on IEEE 14-bus and 30-bus systems [26,27] using MATLAB® and PowerWorld® education and business simulation tools. The results obtained from these simulations aim to distinguish indices in term of foundation, performance, application, merit, demerit and overall behavior. Although, conducting a comparative analysis of almost all 40 indices are not workable, therefore indices are generalized in consensus groups considering their wide applicability and worldwide acceptability. Simulation of some indices, using for real-time online monitoring are ignored. Because verification and testing of these indices require special hardware and software tools.

In general, simulation results shape indices into various types for different purpose of application. Such as bus and line indices, which are used particularly for sensitive-critical nodal and line identification in a system for specific network topology (radial, interconnected, mesh, and so on). Also, simulation findings (Table 2-5) depict a diction-making approach to select proper method (variable-based or Jacobian matrix-based) and index for a desired application (power transfer capability, DG optimum placement, reactive power compensation, system reinforcement, optimal load shedding and etc. for any type of topology) within a constrained limit of certainty. Apparently, all indices are in a general consensus at first and second levels. Since an index performance is mainly correlative to active and reactive power change in a system, and partially to many other factors. Therefore, at this stage, a particularized discussion for the obvious numerical simulation results would lead to insufficient analysis. While, the simulation findings put forward with a general roadmap of decision-making considering prediction adequacy limit, type of system topology, purpose of application, and many more options. As a quick conclusion, it can be drawn that line indices are affected by system topology, especially in a multinetwork configuration. Some numerical zero values are approached close to zero (0.000001) in order to overcome the power flow matrix singularity.

Relying on the simulation results (Table 2-5) and authors previous studies, this study can be briefed as follows: Almost indices in the same category of formulation with the same theoretical foundation mechanism, and application resembling their accuracy and limitations. While performances of these indices are in disagreement with their principle of formulation and application. Therefore, behavior-based conditional variable is known exigence to be considered for selecting an index for monitoring or prediction of voltage stability in a power system.

It seems that VSF and PSI (nodal indices), also V_{cr} (line index) are in disagreement in performance with the rest of indices in their categories. Contrariwise, performances of indices at collapse point are quite in a significant variation. In which, some indices fail to predict stability margin at collapse point. As well as, due to power flow limitation (singularity), most of these indices run out to cover beyond the collapse point.

Meanwhile, applying the driven indices based on 2-bus system for an interconnected topology concussion an overall accuracy. So, these types of indices should be used for limited configuration, or to be generalized in term of multiconfiguration scheme. In addition to the above-mentioned discussion, each index is fit for special application scenario such as static, dynamic, semi-dynamic, and transient stability analysis.

Table 2. The obtained indices' magnitude for critical branch identification by each index (IEEE 14-bus system).

Branch		NLSI [48]	VCPI [47]		FVSI [49]	Lmn [59]	LQP [45]	L [46]	Vcr [50]	LVSI [52]
From	To		P	Q						
1	2	0.041723	0.05211165	0.052112	0.029621675	0.031626	0.026752	0.527999	0.527999	0.097140945
1	5	0.027323	0.053664748	0.053665	0.013449821	0.0129961	0.012704	0.527983	0.527983	0.651156176
2	3	0.299916	0.603222227	0.603222	0.145540502	0.158298	0.137778	0.509657	0.509657	1.250791857
2	4	0.076555	0.256248432	0.256248	-0.027923844	-0.029982	-0.025188	0.514064	0.514064	0.636275
2	5	0.026044	0.045083448	0.045083	0.0112837	0.0118415	0.010191	0.516391	0.516391	0.1137122
3	4	0.099444	0.277198618	0.277199	-0.030169995	-0.029247	-0.026155	0.496846	0.496846	1.187916261
4	5	0.006521	0.011445692	0.011446	0.002863807	0.0028176	0.002602	0.502886	0.502886	0.051265482
4	7	0	0		0	0	0	0.495065	0.495065	0
4	9	0.356591	0.627703555	0.627704	0.356590282	0.3589291	0.35659	0.491635	0.491635	0.000174855
5	6	0.07274	0.115325447	0.115325	0.072739998	0.0734022	0.07274	0.494136	0.494136	4.77773E-05
6	11	0.024123	0.028720079	0.02872	0.015360637	0.0155092	0.012508	0.517274	0.517274	0.060026116
6	12	0.040494	0.056754618	0.056755	0.01760098	0.0178617	0.0143	0.516576	0.516576	0.131357137
6	13	0.057598	0.070516198	0.070516	0.03320431	0.0337737	0.026398	0.516382	0.516382	0.14285981
7	8	0	0		0	0	0	0.516396	0.516396	0
7	9	0.064826	0.114111626	0.114112	0.064825202	0.0648746	0.064825	0.512818	0.512818	0.001376514
9	10	0.027853	0.032929845	0.03293	0.020073907	0.0201163	0.017582	0.509736	0.509736	0.081531112
9	14	0.116444	0.154906644	0.154907	0.059217931	0.0603135	0.048499	0.507436	0.507436	0.346865422
10	11	0.022919	0.028003345	0.028003	0.014804447	0.0147367	0.01252	0.508084	0.508084	0.069127813
12	13	0.148792	0.15512846	0.155128	0.092525866	0.0928123	0.041648	0.509234	0.509234	0.194360274
13	14	0.155423	0.203896609	0.203897	0.078305626	0.0795052	0.063087	0.504769	0.504769	0.447143333

Table 3. The obtained indices' magnitude for critical branch identification by each index (IEEE 30-bus system)

Branch		NLSI [48]	VCPI [47]		FVSI [49]	Lmn [59]	LQP [45]	L [46]	Vcr [50]	LVSI [52]
From	To		P	Q						
1	2	0.040829	0.050792427	0.050792	0.028895385	0.0310744	0.025997	0.098337212	0.52767	0.090772386
1	3	0.010919	0.014938477	0.014938	0.007585635	0.0083082	0.007057	0.012527058	0.52543	0.025650757
2	4	0.026048	0.045052501	0.045053	0.011276206	0.0118532	0.01018	0.028476133	0.51566	0.112409942
2	5	0.30087	0.604439542	0.60444	0.145826669	0.160899	0.138008	0.328598596	0.50665	1.144735735
2	6	0	0	0	0	0	0	0	0.5128	0
3	4	0.006176	0.01041434	0.010414	0.002609098	0.002668	0.002327	0.007126896	0.50382	0.03010354
4	6	0	0	0	0	0	0	0	0.49661	0
4	12	0.07499	0.118891841	0.118892	0.074989455	0.0757245	0.074989	0.085385582	0.48891	4.50637E-05
5	7	0.090705	0.115125046	0.115125	0.057375984	0.0563883	0.049579	0.119787331	0.49235	0.340584256
6	7	0.058802	0.07838679	0.078387	0.0386867	0.0395283	0.034978	0.09272165	0.49284	0.206965597
6	8	0.063398	0.0702055	0.070206	0.053334474	0.0537404	0.049309	0.39519504	0.49482	0.170861069
6	9	0	0	0	0	0	0	0	0.49027	0
6	10	0.043518	0.108695808	0.108696	0.043517353	9.9869167	0.043517	0.045080286	0.06397	2.27972E-07
6	28	0	0	0	0	0	0	0	0.49504	0
8	28	0	0	0	0	0	0	0	0.49455	0
9	10	0.007967	0.019898883	0.019899	0.007966678	0.5637133	0.007967	0.00954225	0.0665	2.13041E-07
9	11	0	0	0	0	0	0	0	0.50967	0
10	17	0.028633	0.033734145	0.033734	0.02059127	0.0737685	0.017952	0.041426624	0.50263	0.01411637
10	20	0.012902	0.01767585	0.017676	0.00643366	0.0065182	0.005359	0.014111878	0.50097	0.04240821
10	21	0.053035	0.060349918	0.06035	0.037360687	0.0376328	0.030728	0.097034106	0.50193	0.121570373

10	22	0	0	0	0	0	0	0	0.50197	0
12	13	0	0		0	0	0	0	0.51065	0
12	14	0.041984	0.059037913	0.059038	0.018051019	0.0183289	0.014659	0.046906472	0.50847	0.136465701
12	15	0.031106	0.041296468	0.041296	0.014679599	0.0149431	0.011672	0.035572384	0.50824	0.088769537
12	16	0.024647	0.02937138	0.029371	0.015701355	0.0158558	0.012805	0.028024412	0.50924	0.061544138
14	15	0.084992	0.09163833	0.091638	0.040839478	0.0409835	0.018357	0.092755535	0.50151	0.120696655
15	18	0.020048	0.027428989	0.027429	0.009061192	0.0091584	0.007301	0.021616528	0.49755	0.062840994
15	23	0.023879	0.028382608	0.028383	0.014939354	0.0150415	0.011999	0.026091741	0.49812	0.058727501
16	17	0.058128	0.072986174	0.072986	0.043887473	0.0440292	0.040854	0.070101281	0.50263	0.239539698
18	19	0.039604	0.051041895	0.051042	0.020694215	0.0207567	0.016627	0.043741276	0.49231	0.115489427
19	20	0.004651	0.006144183	0.006144	0.002260905	0.0022532	0.001809	0.005017176	0.49186	0.014409058
21	22	0	0		0	0	0	0	0.4962	0
22	24	0.0823	0.086120355	0.08612	0.06338884	0.0639164	0.044869	0.098570945	0.49575	0.125606794
23	24	0.112158	0.121864537	0.121865	0.085002814	0.0852595	0.068605	0.12586848	0.4924	0.222962198
24	25	0	0		0	0	0	0	0.49107	0
25	26	0.068102	0.072452792	0.072453	0.048854314	0.0493389	0.033735	0.07095236	0.4881	0.108666623
25	27	0	0		0	0	0	0	0.49042	0
28	27	0	0		0	0	0	0	0.48512	0
27	29	0.034381	0.042956211	0.042956	0.018252083	0.0186823	0.014258	0.036055342	0.49025	0.084977094
27	30	0.173159	0.252141728	0.252142	0.056013018	0.0583534	0.043683	0.189629084	0.48792	0.515014545
29	30	0.135086	0.197026775	0.197027	0.043749241	0.0444823	0.034177	0.139543964	0.47839	0.435559311

Table 4. The obtained indices' magnitude for weak bus identification by each index (IEEE 14-bus system).

Bus	**Branch**		**VSF**	**PSI**	**Vj/Vo**	**BPF**	**RE**	**S**
	From	**To**						
4	2	4	1.080872	1.93368064	0.94908	0.0139	0.119854	0.044
	3	4	1.004157	5.533106601	0.94908			
5	1	5	1.107938	0.263729008	0.954419	0.0064	0.080149	0.0427
	2	5	1.074834	0.306090238	0.954419			
	4	5	1.016568	0.244039621	0.954419			
7	4	7	0.976958	0	0.949303	0.1616	0.401572	0.1417
9	4	9	0.986545	3.55711E-05	0.929802	0.2256	0.476716	0.1377
	7	9	1.067908	1.96145E-05	0.929802			
10	9	10	1.060878	1.045255773	0.925681	0.2333	0.48392	0.1621
11	6	11	1.083193	0.302194356	0.93142	0.93142	0.0926	0.48392
	10	11	1.045101	0.532718621	0.93142			
12	6	12	1.085044	0.665506183	0.930355	0.0095	0.096489	0.1377
13	6	13	1.089898	0.645425171	0.926024	0.0198	0.138994	0.0872
	12	13	1.060002	0.36546909	0.926024			
14	9	14	1.076698	2.150384317	0.912321	0.2374	0.48619	0.2233
	13	14	1.065465	2.537073855	0.912321			

Table 5. The obtained indices' magnitude for three weakest buses identification by each index (IEEE 30-bus system).

Bus	**From**	**To**	**VSF**	**PSI**	**Vj/Vo**	**BPF**	**RE**	**S**
26	25	26	1.036053	0.369924625	0.93985	0.174031	0.414541	0.7299
29	27	29	1.044453	0.439171371	0.945386	0.157802	0.399068	0.6733
30	29	30	1.016233	2.707961113	0.934087	0.156804	0.394602	0.6024

5. Conclusions

This is the first-ever research effort that virtually delineates almost 40 voltage stability indices over the past 3 decades, along with their detailed exposition of foundation, formulation, performance behavior, optimal application, effectuation approaches, and overall in term of merit and demerit within the particularized circumstances. Also, with an in-depth review of the literature, a comprehensive categorization of most worldwide used indices are proposed that can be a novel framework for voltage stability distinguish within specific classes. Moreover, this research evaluates the emerging role of stability assessment in a power system within an in-depth analysis of voltage stability indices' formulation, application, performance, and assessment. In conclusion, this study exposes a brief thesis on inconsistency among voltage stability indices due to discrepancy behaviors. These divergences are attributed to a rather contradictory result for a specific application. Hence, these observations demonstrate with several contributions to the current literature, as well as can be counted an asset of information (an exhaustive roadmap) for researchers, scholars, operators, and engineers in the context of a power system voltage stability monitoring, and instability prediction and prevention.

Author Contributions: Conceptualization, M.S.S.D. and T.S.; methodology, M.S.S.D. and N.R.S; software, S.M.S.D.; validation, K.N., S.M.S.D. and P.M.; formal analysis, S.M.S.D.; investigation, M.S.S.D.; resources, N.R.S.; data curation, S.M.S.D.; writing—original draft preparation, M.S.S.D.; writing—review and editing, M.S.S.D., K.N. and T.S.; visualization, M.S.S.D.; supervision, T.S.; project administration, M.S.S.D., K.N. and P.M.; funding acquisition, M.S.S.D and T.S.

Acknowledgments: This study is a part of the research efforts, which is financially and technically supported by the Strategic Research Projects Center, University of the Ryukyus(Senbaru 1, Nishihara, Okinawa, 903-0213, Japan) under the research budget of JFY 2018 (98394640).

Conflicts of Interest: The authors declare no conflict of interest

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