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Wealth accumulation in rotation forestry – failure of the Net Present Value optimization?

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Abstract

We investigate wealth accumulation in forestry, assuming that revenues are re-invested. Three different optimization criteria are compared, two of which are based on cash flows, the third financially grounded. Direct optimization of wealth appreciation rate always yields best results. Procedures gained by maximizing internal rate of return are only slightly inferior. With external discounting interest rate, the maximization of net present value yields arbitrary results, with at worst devastating financial consequences.

Keywords

Capital return; real estate; capitalization

1. Introduction

We are interested in the accumulation of wealth within the business of forestry. In other words, we are interested in forestry as a financial business. For financial purposes, a variety of economical optimization approaches can be used.

Most commonly, a discounting interest rate is applied in order to compute the present value of future incomes and expenses (1, 2, 3, 4, 5, 6, 7, 8, 9, 10). The discounting interest may vary over time (11, 12, 13, 14). It has been stated that uncertainty induces declining discount rates along with time (15, 11, 16, 17); however it can be shown that there is an opposite effect on prolongation interest. Risk of destructive events has been considered as a premium to discount interest (18, 19). Evolution of prices, as well as fluctuations in growth and prices, may be added (12, 20). Taxation does contribute, as well as personal financies (21, 22, 23). Imperfect capital market has been addressed (24). A Hamiltonian formulation is available (25, 26). Regardless of other details, financial soundness of any operation significantly depends on the choice of the discounting interest rate.

We are aware of one method in the determination of profitability in multiannual growth without an external discount rate (27, 28). Sound management is supposed to maximize internal discounting interest (or internal rate of return), harvesting income being discounted to cover initial investments (27, 28). There are a few problems related to this approach: it cannot be applied in the case of zero initial investment, and it does not consider the shape of the yield curve in any way. The latter deficiency is related to the fact that being established on cash-flow basis, the internal interest discounting cannot account for any time-variable capitalization effect.

The above-mentioned economical optimization criteria have been established on cash-flow basis. Recently, an optimization criterion has been established on financial grounds (29, 30). A representative capital return rate has been produced by integrating a momentary capital return, multiplied by a partition function of capitalization, along time-consequent path within a state space (29). The capitalization has been distributed to operative and non-operative capitalization, and it has been found financially sound forestry practices are dictated by the non-operative capitalization, along with its appreciation rate (30). It is important to note that capitalization can be changed any time by either investing, harvesting, or by selling the real estate, completely or partially. Regarding harvestings, thinning from above may reduce capitalization quickly, as delaying thinnings may increase it. In other words, forest properties can be liquidized at any time, partially or completely, as any other financial asset.

In this paper, we investigate the effect of objective function on wealth accumulation, assuming that revenues are re-invested. We first present a more general formulation, but later focus in stationary rotation forestry. Stationary rotation forestry here means that the distribution of stand ages does not evolve. This is practically possible if the stand ages are evenly distributed, and the same duration of rotation cycle is applied to stands. Stationary forestry also implies that prices and expenses do not evolve. We are naturally discussing prices and expenses in real terms.

We apply three different growth models, and three different sets of boundary conditions, the boundary conditions primarily relating to bare land value. We first introduce a basic notation of wealth accumulation, along with an application to rotation forestry. Then, we introduce the above-outlined three different economic criteria (objective functions) for the optimization of forest management. Thirdly, we introduce the three experimental growth models, and investigate wealth accumulation while any of the three economic criteria are followed. Finally, we compare the three different forest economics approaches from the viewpoint of wealth accumulation.

2. Methods

21. Wealth accumulation

Let us first introduce a very basic notation of wealth accumulation:

$$W(T_2) = W(T_1) + \int_{T_1}^{T_2} \frac{dW}{dt} dt = W(T_1) + \int_{T_1}^{T_2} W(t) r(t) dt \quad (1),$$

where $W(T_1)$ is wealth at time T_1, $W(T_2)$ is wealth at time T_2, and r is a relative wealth increment rate. Provided the time is expressed in years, r becomes the relative annual wealth increment rate. In the absence of withdravals, Eq. (1) can be rewritten

$$W(T_2) = W(T_1)\exp\left[\int_{T_1}^{T_2} r(t)dt\right] \qquad (2).$$

The above equations describe the total amount of wealth. It possibly should be constituted from stand-level capitalizations, given per unit area. By definition, the expected value capitalization per unit area is

$$\langle \kappa \rangle = \int_0^{\infty} p(\kappa)\kappa d\kappa \qquad (3),$$

where $p(\kappa)$ is the probability density function of capitalization κ. By change of variables we get

$$\langle \kappa \rangle = \int_0^{\tau} p(\kappa)\kappa \frac{d\kappa}{da} da = \int_0^{\tau} p(a)\kappa(a)da \qquad (4),$$

where a is stand age, and τ is rotation age. The expected value of the increment rate of capitalization is

$$\left\langle \frac{d\kappa}{dt} \right\rangle = \int_0^{\tau} p(a)\frac{d\kappa(a)}{dt}da \qquad (5).$$

Correspondingly, the momentary expected rate of relative capital return is

$$\langle r(t) \rangle = \frac{\left\langle \frac{d\kappa}{dt} \right\rangle}{\langle \kappa \rangle} = \frac{\int_0^{\tau} p(a)\frac{d\kappa(a,t)}{dt}da}{\int_0^{\tau} p(a)\kappa(a,t)da} = \frac{\int_0^{\tau} p(a)\kappa(a,t)r(a,t)da}{\int_0^{\tau} p(a)\kappa(a,t)da} \qquad (6),$$

which can be readily substituted to Eq. (2).

It is worth noting that in Eq. (6), there is a notation of stand age (a) and a notation of time (t). The probability density of stand age $p(a)$ could vary as a function of time. However, that would result as a treatment too complicated for the present purposes. Constancy of the stand age distribution is a real possibility if there is an even distribution of stand ages, varying between zero and the rotation age τ.

It is indicated in Eq. (6) that the capitalization $\kappa(a,t)$, as well as the momentary capital return rate, $r(a,t)$ are functions of time, in addition to stand age. This would be the case if there would be an appreciation of real estate values, for example, and an eventual appreciation of real estate values might considerably contribute to financial sustainability (30). However, in this paper we discuss forestry in terms of stationary capitalization per unit area. This corresponds to assuming that wood stumpage prices, as well as real estate prices, develop along with general development of prices and expenses. Then, the capital return rate $r(a)$ corresponds to real return, excluding eventual inflation of prices.

Now, in the case of constant capitalization per unit area, Eq. (2) becomes

$$\left(\frac{W(T_2)}{W(T_1)}\right)^{\frac{1}{T_2-T_1}} = e^{\langle r\rangle} \quad (7).$$

In other words, the expected value of the relative wealth increment rate $\langle r\rangle$ is a unique measure of wealth accumulation, regardless of the time horizon, and it is given by

$$\langle r\rangle = \frac{\int_0^{\tau} \frac{d\kappa(a)}{dt} da}{\int_0^{\tau} \kappa(a) da} \quad (8).$$

22. Objective functions to be compared

Probably the most commonly used objective function used in forestry optimizations is maximization of net present value of future revenues and expenses (1, 2, 3, 4, 5, 6, 7, 8, 9, 10):

$$NPV_{t=0} = \int_0^{\tau} R(t)e^{-it} dt \frac{1}{1-e^{-i\tau}} \quad (9),$$

where *R(t)* corresponds to net proceeds at time t, i is discounting interest rate, and τ again is rotation age. The last factor after the integral corresponds to discounting of further growth cycles, each of rotation age τ.

A significant problem in Eq. (9) is that it contains the external discount rate i. The discount rate is external in the sense that it is unrelated to the capital return within the growth process, and consequently subject to arbitrary changes. Another issue in Eq. (9) is that it does not consider the shape of the yield curve in any way. In other words, Eq. (9) discusses revenue on cash basis, instead of financial grounds.

The problem of arbitrary external interest has been resolved by maximizing an internal rate of return. As introduced by Newman (28, 27), it is determined for a period of duration τ according to the criterion

$$\int_0^\tau R(t)e^{-ot}dt = 0 \qquad (10).$$

The general form of Eq. (10) does not appear to make sense at first glance. However, the Equation makes complete sense provided negative net proceeds appear in the beginning of the rotation, and positive net proceeds are generated by the end of the growth cycle. It is sometimes stated that maximizing the internal rate of return would not consider any bare land value. Such a statement however is incorrect. Eq. (10) considers one single rotation, whereas Eq. (9) refers to maximization of long-term discounted proceeds. Optimization of one single rotation possibly should consider purchase of bare land at the beginning, and sales of it at the end. Even if actual real estate transaction would not be implemented, the purchase price should be considered as an opportunity cost of not selling the bare land in the beginning. By the end of the single rotation to be optimized, the bare land constitutes a saleable asset.

Evidently, a third objective function to be compared from the viewpoint of wealth accumulation should be maximization of the expected value of the relative wealth increment rate $\langle r \rangle$, given in Eq. (8). Such an objective function has been recently derived in terms of a state-space approach (29). It might be considered trivial that this criterion is superior from the viewpoint of wealth accumulation, considering that it has been derived as the unique measure of wealth accumulation rate at Eqs. (1…7). However, recent discussion has indicated that this is considered far from trivial. Regardless of the eventual triviality, it is of interest how much inferior objective functions (9) and (10) might be, in comparison to (8).

23. Forest growth models applied

231. Volumetric growth model for pine

As the first practical example, we consider a recently introduced (9) yield function, applicable to average pine stands in Northern Sweden. A volumetric growth function is

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$$V(t) = 580.14 * \left(1 - 6.3582^{-t/95}\right)^{2.8967} \quad (11).$$

The application introduced by Gong and Löfgren [9] assumes a volumetric stumpage price of 250 SEK/m^3 and an initial investment of 6000 SEK/ha. The maximum sustained volumetric yield rotation is 95 years, corresponding to that duration of time that gives the greatest average annual growth [9]. A 3% discount interest applied in Eq. (9) yields an optimal rotation age of 52 years (Gong and Löfgren 2016).

232. Value growth model for pine

Eq. (11) does not consider any variation of the volumetric stumpage price. In other words, the value growth corresponds to volumetric growth, multiplied by a constant. That may be an unrealistic assumption, for a variety of reasons, including harvesting expenses as well as industrial use of the crop. In order to release this assumption, Gong and Löfgren [9] established an age-dependent price function

$$p(t) = 104.63 * (t - 29)^{0.2602} \quad (12).$$

We here apply Eq. (12), for t>29, in addition to Eq. (11), in order to establish another version of the practical forestry example. Maximum sustainable value yield is gained at 130 years of rotation. A 3% discount interest applied in Eq. (9) yields an optimal rotation age of 62 years [9].

233. Value growth model for spruce

The third empirical model describes fertile spruce forests with natural regeneration. We will again assume a constant stand age distribution, as in Eq. (8), but now the stand age does not refer to the age of trees, but to time elapsed after the latest regeneration harvesting of any stand. We apply the empirical model of Bollandsås et al. (31, 32), and parametrize it with reference to a recently introduced stationary state (33, 30).

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We define any regeneration harvesting with respect to a natural stationary state (33, 30). In particular, we introduce a diameter-limit cutting to breast height diameter 25 cm, and adjust the number of remaining trees within any diameter class below 25 cm to the number of trees in the natural stationary state (33, 30). Then, we allow any stand to develop according to the empirical model by Bollandsås *et al*. [31, 32], until a particular rotation age, where another regeneration harvesting is assumed to occur. We parametrize the model with site fertility index 20, which corresponds to dominant height in meters for an even-aged stand of 40 years of age [31, 32].

The number of trees per hectare in different diameter classes, according to the model described above, is shown in Fig. 1. The initial setup naturally contains only trees of diameter less than 250 mm. The number of trees in any diameter class, except the smallest, is smaller than in developed stages. A consequence is that the regeneration harvesting not only contains diameter-limit cutting, but also some thinning of smaller trees.

On the basis of the number of trees per hectare, we clarify the volumetric amount of two assortments, pulpwood and sawlogs, according to an appendix given by of Rämö and Tahvonen [34, 35]. The monetary value of the assortment volumes is clarified according to stumpage prices given by Rämö and Tahvonen [34]. In the initial setup, the number of trees (breast-height diameter at least 50 mm) per hectare is 154. The basal area is 2.72 m^2/ha, and the combined volume of the assortments 18.5 m^3/ha. The stumpage value of the trees in the initial setup is 670 Eur/ha.

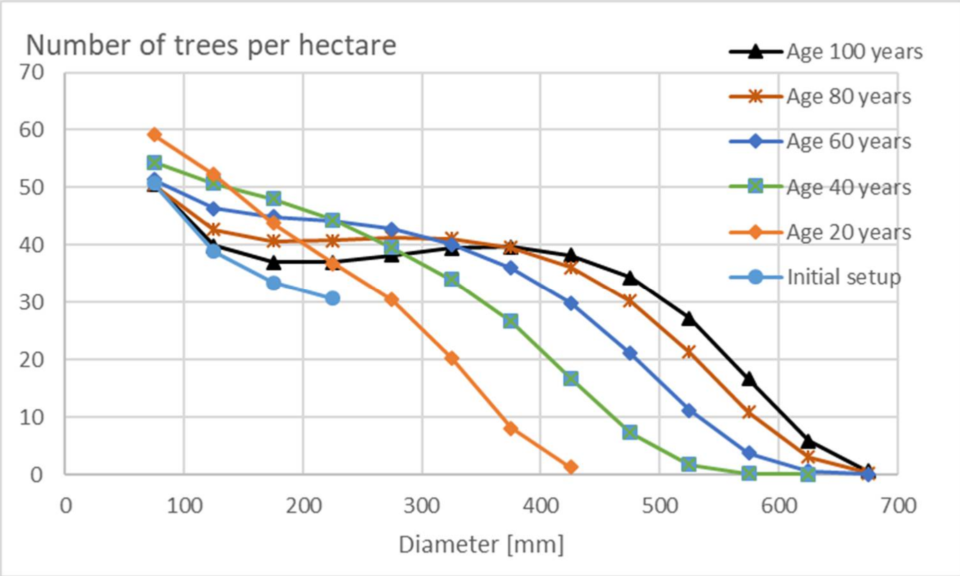

Fig. 1. Number of trees in 50 mm diameter classes in the initial setup, as well as later stages of stand development, according to the spruce growth model (31, 32, 33, 30). Stand age refers to lime elapsed after the latest regeneration harvesting.

24. Analytical procedures

For any of the three growth models (Eq. (11), and (11) combined with (12), and (31, 32, 33, 30), we optimize the rotation age according to any of the three different objective functions given in Eqs. (9), (10) and (8). Maximization of the net present value of proceeds according to Eq. (9) requires an external discount interest rate. We adopt three different rates, 2%, 3% and 4%. Maximization of the internal rate of return according to Eq. (10), as well as the expected value of capital return according to Eq. (8) require a bare land value, as an opportunity cost in Eq. (10) and as a part of the applied capitalization in Eq. (8). We apply three different bare land values: zero, a moderate, and a high bare land value. In the case of pine stand models with a regeneration expense, the moderate value equals half of the regeneration expense, and the high value two times the regeneration expense. In the case of the spruce model with natural regeneration, the moderate value is 70% of the value of trees in the initial setup, and the high value 300% of the value of trees in the initial setup. Since we are discussing stationary forestry, we do not consider any appreciation of the real estate prices.

After determining the optimal rotation age for three different growth models, using the three objective functions, we find out what kind of expected value of relative wealth increment rate

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those rotation ages yield according to Eq. (8). Again, this is done using the three different bare land values as explained above. Eq. (8) of course can be substituted to Eq. (7) or to Eq. (2).

In order to apply Eq. (8), we need an amortizations schedule for any eventual initial investment. We choose to make a one-time amortization at the end of any rotation, simultaneously as the final proceeds are gained. A consequence is that there is no amortization of initial investment affecting the denominator of Eq. (8); the investment expense contributes to the capitalization during the entire rotation. On the other hand, the change of capitalization during the rotation, appearing in the numerator of Eq. (8), is net of regeneration expense.

3. Results

31. Pine volumetric growth model

Let us first plot the momentary capitalization $\kappa(a)$, as well as the increment rate of momentary capitalization $\frac{d\kappa}{da}$ as a function of stand age a in Fig. 2. The result is an outcome of the growth model only and does not depend on the objective function applied. The increment rate of capitalization does not depend on the bare land value, whereas the capitalization itself does. We find that the average increment rate of capitalization reaches the momentary increment rate at stand age 95 years, which corresponds to the rotation age of maximum sustainable volumetric yield.

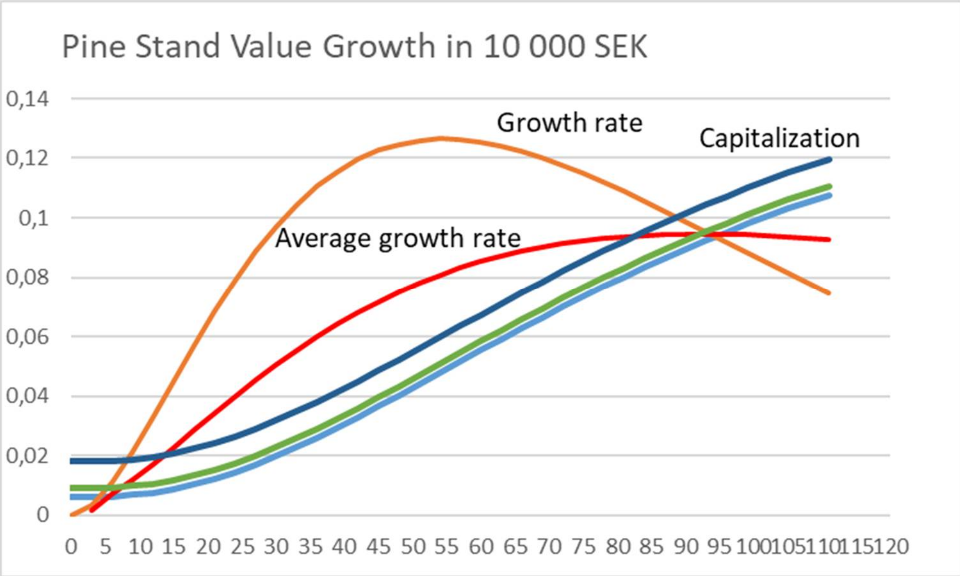

Fig. 2. Pine stand value growth as a function of stand age, according to a North-Swedish growth function (11), [9]. Value growth is given in units of 10 000 SEK, whereas the capitalization in millions per hectare. Three curves of capitalization correspond to bare land values 0, 3000 SEK/ha and 12 000 SEK/ha.

Let us then plot the momentary capital return rate $r(a)=\frac{d\kappa(a)}{\kappa(a)dt}$ as a function of stand age *a*, and the expected value of the wealth appreciation rate $\langle r\rangle$ (Eq. (8)) as a function of rotation age τ. These both are drawn in Fig. 3. We find that for any of the three bare land values, the expected value of the wealth appreciation rate reaches the momentary value at the maximum value of the former. The regeneration expense, as well as the bare land value taken as constants, the actual wealth appreciation rate depends on the rotation age according to Eq. (8). The optimal rotation ages for bare land values 0, 3000 SEK/ha and 12 000 SEK/ha are 45, 48 and 56 years, corresponding to wealth appreciation rates to 3.396%, 2.877% and 2.047% per annum.

It is worth noting that in Fig. 3, the momentary capital return rate is given as a function of stand age *a*, without any reference to any particular rotation age τ. We chose to make a one-time amortization at the end of any rotation, the momentary capital return rate does not contain any amortization. On the other hand, the wealth appreciation rate $\langle r\rangle$ is given as a function of rotation age, and correspondingly the amortization contributes to it. That is why $\langle r\rangle$ first becomes nonnegative at rotation age of 21 years.

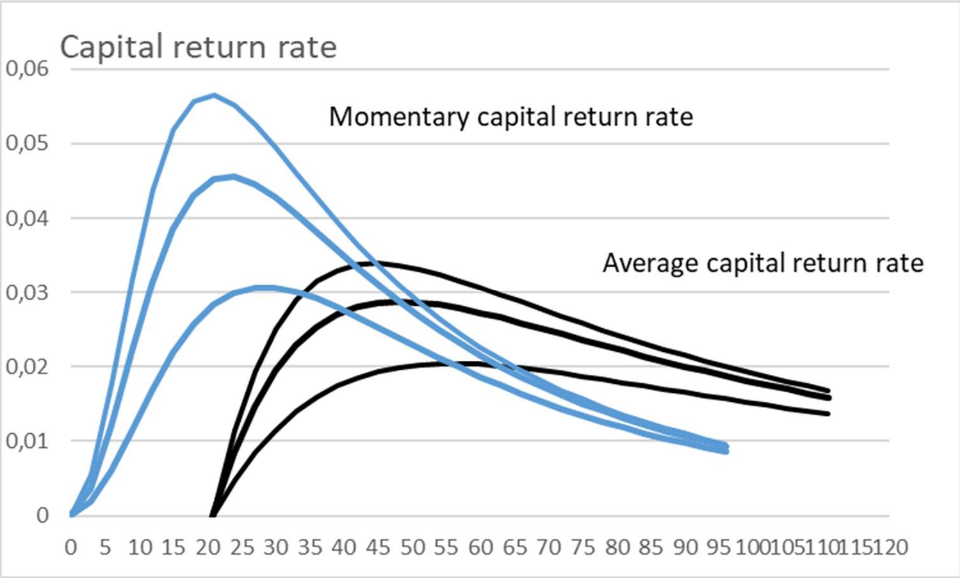

Fig. 3. Momentary capital return rate $r(a) = \dfrac{d\kappa(a)}{\kappa(a)dt}$, as well as expected wealth appreciation rate $\langle r \rangle$ (Eq. (8)), for three bare land values 0, 3000 SEK/ha and 12 000 SEK/ha.

Let us then consider the objective functions (9) and (10). According to Eq. (9), the optimal rotation ages are 60, 52 and 45 years with discounting interest rates 2%, 3%, and 4%, respectively. The corresponding net present values are 12 700, 2 830 and -1 150 SEK/ha. The negativity of the net present values would disqualify the largest discounting interest rate in most applications. The moderate bare land value of SEK 3000 is gained with 2,97% discounting interest, at rotation age 52 years. The high bare land value of SEK 12 000 is gained with 2,043% discounting interest, at rotation age 60 years. Zero bare land value is gained with 3,624% discounting interest, at rotation age 48 years.

According to Eq. 10, the internal rate of return reaches its maximum value at rotation age 52 with the moderate bare land value and 60 with the high bare land value. The corresponding internal return rates are 2.972% and 2,043%. In the absence of any bare land value, the internal rate of return would be 3,624% at rotation age 48 years.

Obviously, the three different objective functions (Eqs. (8), (9) and (10)) give different optimal rotation ages. With zero bare land value, the rotation ages are 45, 48 and 48 years. The corresponding annual wealth accumulation rates, computed from Eq. (8), are 3.396%, 3.373% and 3.373%. These returns possibly should be contrasted to the outcome of Eq. (9) with 2%

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and 3% discounting interests, which would give optimal rotation ages 60 and 52 years. These rotation ages, according to Eq. (8), give capital appreciation rates 3,075% and 3,300%, respectively.

With the moderate bare land value of 3000 SEK/ha, the rotation ages are 48, 52 and 52 years. The corresponding annual wealth accumulation rates, computed from Eq. (8), are 2.877%, 2.857% and 2.857%. These returns possibly should be contrasted to the outcome of Eq. (9) with 2% and 3% discounting interests, which again give optimal rotation ages 60 and 52 years. These rotation ages, according to Eq. (8) but now with the bare land value 3000 SEK/ha, give capital appreciation rates 2,726% and 2.857%, respectively.

With the high bare land value of 12 000 SEK/ha, the rotation ages are 56, 60 and 60 years. The corresponding annual wealth accumulation rates, computed from Eq. (8), are 2.047%, 2.035% and 2.035%. These returns again should be contrasted to the outcome of Eq. (9) with 2% and 3% discounting interests, which again give optimal rotation ages 60 and 52 years. These rotation ages, according to Eq. (8) but now with the bare land value 12 000 SEK/ha, give capital appreciation rates 2,035% and 2.037%, respectively.

We find that with this growth model, Eq. (10) gives rotation ages and wealth appreciation rates rather close to the optimum. Since the outcome of Eq. (9) depends on the discounting interest rate, the results vary. In the case of zero and moderate bare land values, the outcome with 2% discounting interest differs from the optimal. In the of the high bare land value, the outcome of Eq. (9) differs only slightly from the optimum, with any of the two discounting interest rates.

Rather interestingly, the results from Eqs. (9) and (10), regarding rotation time and wealth appreciation rate, are identical provided the discounting interest rate in Eq. (9) is adjusted to result in any known bare land value as a net present value of future proceeds. The adjusted discounting interest rate in Eq. (9) is the same as the internal rate of return in Eq. (10). This leads us to suspect that the two objective functions possibly are identical.

32. Pine value growth model

Let us plot the momentary capitalization $\kappa(a)$, as well as the increment rate of momentary capitalization $\frac{d\kappa}{da}$ for the value growth model given in Eqs. (11) and (12), as a function of stand age a in Fig. 4. Again, the result is an outcome of the growth model only and does not depend on the objective function applied. The increment rate of capitalization does not depend on the bare land value, whereas the capitalization itself does. The average increment rate of capitalization reaches the momentary increment rate at stand age 130 years, which corresponds to the rotation age of maximum sustainable value yield.

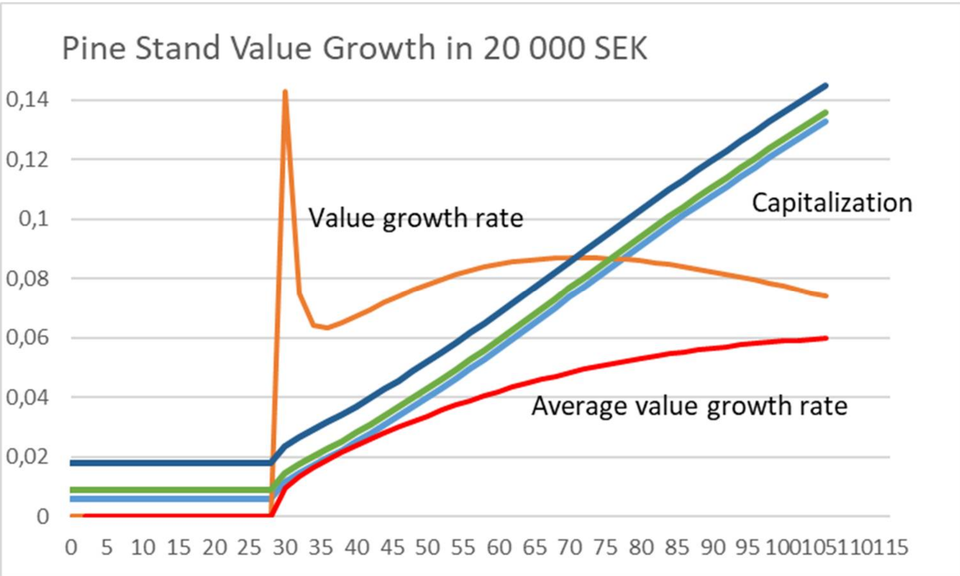

Fig. 4. Pine stand value growth as a function of stand age, according to a North-Swedish growth functions (11) and (12), [9]. Value growth is given in units of 20 000 SEK, whereas the capitalization in millions per hectare. Three curves of capitalization correspond to bare land values 0, 3000 SEK/ha and 12 000 SEK/ha.

Let us then plot the momentary capital return rate $r(a) = \frac{d\kappa(a)}{\kappa(a)dt}$ as a function of stand age a, and the expected value of the wealth appreciation rate $\langle r \rangle$ (Eq. (8)) as a function of rotation age τ. These both are drawn in Fig. 5. We again find that for any of the three bare land values, the expected value of the wealth appreciation rate reaches the momentary value at the maximum value of the former. The regeneration expense, as well as the bare land value taken as constant, the actual wealth appreciation rate depends on the rotation age according to Eq. (8). The optimal rotation ages for bare land values 0, 3000 SEK/ha and 12 000 SEK/ha are 49, 54 and 63 years, corresponding to wealth appreciation rates to 4.034%, 3.347% and 2.354% per annum.

It is worth noting that in Fig. 5, the momentary capital return rate is given as a function of stand age a, without any reference to any particular rotation age τ . We chose to make a one-time amortization at the end of any rotation, so the momentary capital return rate does not contain any amortization. On the other hand, the wealth appreciation rate $\langle r \rangle$ is given as a function of rotation age, and correspondingly the amortization contributes to it. That is why $\langle r \rangle$ first becomes nonnegative at rotation age of 31 years.

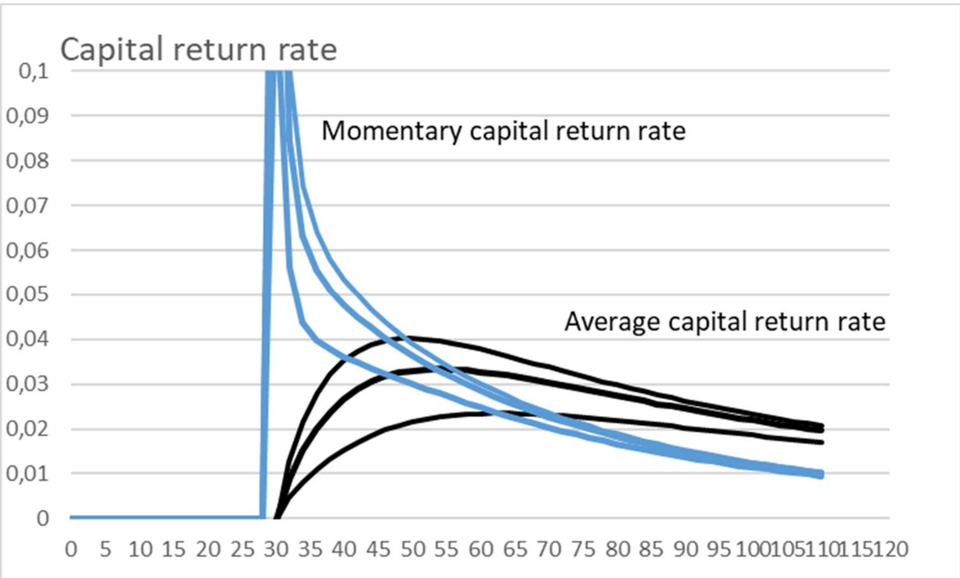

Fig. 5. Momentary capital return rate $r(a) = \frac{d\kappa(a)}{\kappa(a)dt}$, as well as expected wealth appreciation rate $\langle r \rangle$ (Eq. (8)), for three bare land values 0, 3000 SEK/ha and 12 000 SEK/ha.

Let us then consider the objective functions (9) and (10). According to Eq. (9), the optimal rotation ages are 73, 62 and 56 years with discounting interest rates 2%, 3%, and 4%, respectively. The corresponding net present values are 14 200, 2 820 and -1 120 SEK/ha. The negativity of the net present values would disqualify the largest discounting interest rate in most applications. The moderate bare land value of SEK 3000 is gained with 2.972% discounting interest, at rotation age 63 years. The high bare land value of SEK 12 000 is gained with 2.126% discounting interest, at rotation age 71 years. Zero bare land value is gained with 3.549% discounting interest, at rotation age 58 years.

According to Eq. 10, the internal rate of return reaches its maximum value at rotation age 63 with the moderate bare land value and 71 with the high bare land value. The corresponding internal return rates are 2.972% and 2.126%, respectively. In the absence of any bare land value, the internal rate of return would be 3.549% at rotation age 58 years.

Obviously, the three different objective functions (Eqs. (8), (9) and (10)) shall give different optimal rotation ages. With zero bare land value, the rotation ages are 49, 58 and 58 years. The corresponding annual wealth accumulation rates, computed from Eq. (8), are 4.034%, 3.859% and 3.859%. With the moderate bare land value of 3000 SEK/ha, the rotation ages are 54, 63 and 63 years. The corresponding annual wealth accumulation rates, computed from Eq. (8), are 3.347%, 3.222% and 3.222%. With the high bare land value of 12 000 SEK/ha, the rotation ages are 63, 71 and 71 years. The corresponding annual wealth accumulation rates, computed from Eq. (8), are 2.354%, 2.308% and 2.308%.

Interestingly, the results based on Eqs. (9) and (10) are again identical. This is the case provided the discounting interest used in Eq. (9) is calibrated to gain the same bare land value as is used as input variable in Eq. (10). The situation is different if arbitrary discounting interests are used in Eq. (9). With 2% and 3% discounting interests, the optimal rotation ages would be 73 and 62 years, respectively. According to Eq. (8), these rotation ages would yield capital appreciation rates 3.263% and 3.713% for zero bare land value, 2.948% and 3.244% for moderate bare land value, and 2.286% and 2.353% for the high bare land value. In the case of zero and moderate bare land values, the outcome with 2% discounting interest again differs from the optimal, this time more severely than in the case of the volumetric growth model discussed above.

33. Spruce value growth model

Let us plot the momentary capitalization $\kappa(a)$, as well as the increment rate of momentary capitalization $\frac{d\kappa}{da}$ for the spruce value growth model (31, 32, 33, 30), as a function of stand age a in Fig. 6. Again, the result is an outcome of the growth model only and does not depend on the objective function applied. The increment rate of capitalization does not depend on the bare land value, whereas the capitalization itself does. The average increment rate of capitalization reaches the momentary increment rate at stand age 70 years, which corresponds to the rotation age of maximum sustainable value yield.

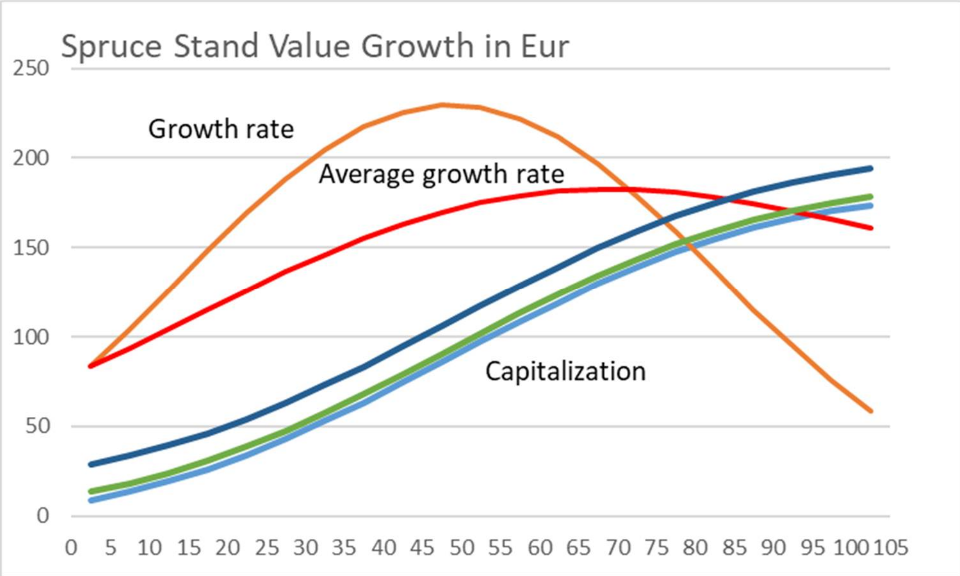

Fig. 6. Spruce stand value growth as a function of stand age, according to Bollandsås et al. (Fig. 1, 31, 32, 33, 30). The capitalization is given in hundreds of Euros per hectare. Three curves of capitalization correspond to bare land values 0, 469 Eur/ha and 2009 Eur/ha.

Let us then plot the momentary capital return rate $r(a) = \frac{d\kappa(a)}{\kappa(a)dt}$ as a function of stand age a, and the expected value of the wealth appreciation rate $\langle r \rangle$ (Eq. (8)) as a function of rotation age τ. These both are drawn in Fig. 7. With bare land values 0 and 469 Eur/ha the capital return rate reaches its maximum during the first five-year growth period, being 9.48% and 6.18%. With bare land value 2 009 Eur/ha the wealth appreciation rate reaches its maximum during the fifth five-year growth period, being 3.12%.

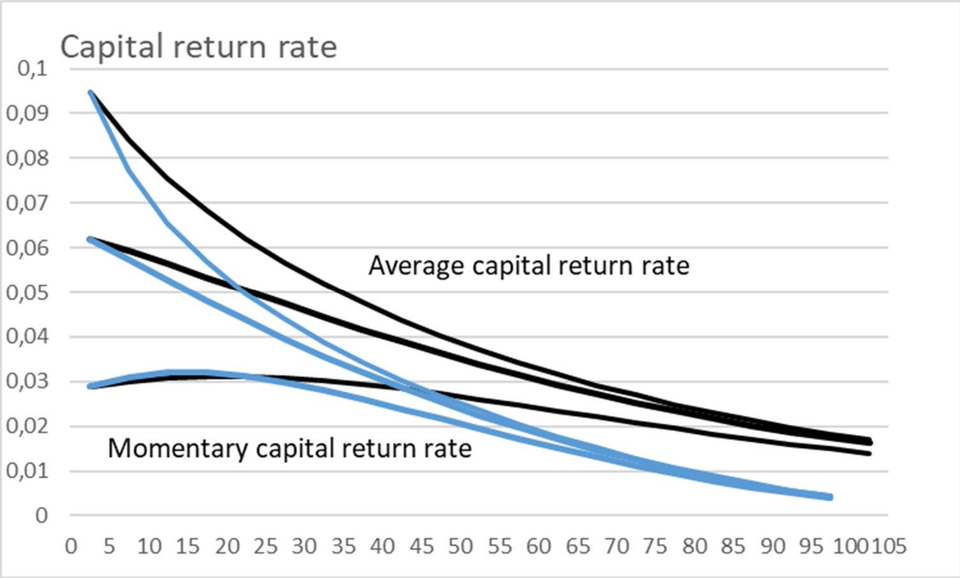

Fig. 7. Momentary capital return rate $r(a)=\dfrac{d\kappa(a)}{\kappa(a)dt}$, as well as expected wealth appreciation rate $\langle r\rangle$ (Eq. (8)), for three bare land values 0, 469 Eur/ha and 2 009 Eur/ha.

Let us then consider the objective functions (9) and (10). According to Eq. (9), the optimal rotation ages are 40, 25 and 10 years with discounting interest rates 2%, 3%, and 4%, respectively. The corresponding net present values of the initial setup (bare land and trees standing after the regeneration harvesting) are 5062, 2821 and 1905 Eur/ha. The moderate initial setup value of Eur 1139 is gained with 6.24% discounting interest, at rotation age 5 years. The high initial setup value of Eur 2679 is gained with 3.11% discounting interest, at rotation age 25 years. Initial setup value consisting only of trees remaining after the regeneration cutting is gained with 9.68% discounting interest, at rotation age 5 years.

According to Eq. 10, the internal rate of return reaches its maximum value at rotation age 5 with the moderate bare land value and 25 with the high bare land value. The corresponding internal return rates are 6.24% and 3.11%, respectively. In the absence of any bare land value, the internal rate of return would be 9.68% at rotation age 5 years.

Again, the results based on Eqs. (9) and (10) are identical, provided the discounting interest in Eq. (9) is calibrated to produce a known net present value for the initial setup. In the case of this spruce growth model, also direct optimization according to Eq. (8) gives the same rotation times. Internal rates of return resulting from Eq. (10) slightly differ from wealth accumulation rates coming from Eq. (8), the former Equation neglecting the shape of the yield curve, whereas Eq. (8) does not. The discounting interest rates needed in Eq. (9) to produce known initial setup values as net present values are the same as the internal return rates in Eq. (8).

The economic views diverge if arbitrary discounting interests are used in Eq. (9). With 2%, 3% and 4% discounting interests, the optimal rotation ages would be 40, 25 and 10 years, respectively. According to Eq. (8), these rotation ages would yield capital appreciation rates 4.76%, 6.21% and 8.41% for zero bare land value, 4.16%, 5.04% and 5.92% for moderate bare land value, and 2.94%, 3.12% and 3.00% for the high bare land value.

4. Discussion

Two of the objective functions discussed above have yielded financially satisfactory results, whereas the third is clearly inferior. However, the inferior method of maximization of net present value appear to yield numerical results identical to those of internal rate of return, provided that the discounting interest in Eq. (9) is calibrated to yield an appropriate bare land value. This raises a question whether the two objective functions are identical with the mentioned boundary condition. We can readily rewrite Eq. (10) for the special case where there are nonzero net proceeds a two time instants: at the beginning of each growth cycle, and at the end of the growth cycle. In the beginning, there is an initial investment I and a purchase expense for the bare land B. At the end of the cycle, there is harvesting revenue H, and sales proceeds for the bare land B. Then, Eq. (10) can be rewritten as

$$(R+B)e^{-o\tau}-(I+B)=0 \qquad (13).$$

Resolving the bare land value yields

$$B=\left(Re^{-o\tau}-I\right)\frac{1}{1-e^{-o\tau}} \qquad (14).$$

Eq. (12) however is the same as Eq. (9), with the same boundary conditions. One can readily show that the equality applies to any schedule of proceeds where a resource is occupied at the beginning of a period and released at the end.

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Eqs. (13) and (14) demonstrate that two of the above-discussed objective functions are the same, provided the discounting interest rate in Eq. (9) is adjusted to yield an appropriate bare land value. Then, one must ask how can one clarify what is an appropriate bare land value? In the mind of the Author, this is not too difficult. There is bare forest land available in the real estate market. There also are young plantations on the market, and the bare land value is achievable by deducting their regeneration expense. On the other hand, there is no reliable way of determining a valid discounting interest rate, apart from the calibration to yield a valid bare land value. No market interest rate can be selected for the discounting interest; at the time of writing, mortgage interest rates within the Eurozone vary 1%...2%, but an opportunity cost for neglected alternative investments easily becomes 6%...9%. The range is far too wide.

In a few cases, the wealth appreciation rate is not very sensitive to the objective function selected. However, there are rather sensitive cases. Fig. 8 shows wealth accumulation within the value growth model described in Eqs. (11) and (12), for a period of 70 years, in the absence of any bare land value. We find that there are rather significant differences between the different objective functions. Direct maximization of Eq. (8) results as a rotation age of 49 years. Maximization of internal rate of return according to Eq. (10) results as rotation age 58 years. Net present value maximization according to Eq. (9) results as rotation ages 73 and 62 years, with 2% and 3% discounting interest, respectively. In Fig. 8, wealth accumulation is only slightly suboptimal if rotation age is determined either with Eq. (10) or with Eq. (9) with discounting interest rate 3%. However, with discounting interest 2%, Eq. (9) yields clearly inferior results.

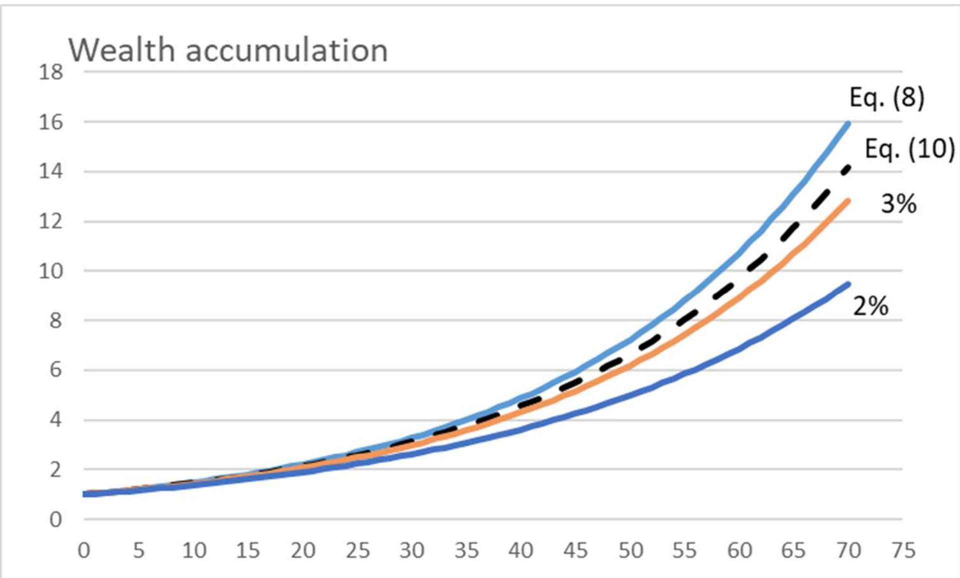

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Fig. 8. Wealth accumulation as a function of time (in years) for the value growth model introduced in Eqs. (11) and (12), with bare land value 0 SEK/ha.

Fig. 9 shows wealth accumulation within the value growth model described in Eqs. (11) and (12), for a period of 70 years, for the moderate bare land value 3000 SEK/ha. We find that there again are rather significant differences between the different objective functions, even if somewhat less than in Fig. 8. Direct maximization of Eq. (8) results as a rotation age of 54 years. Maximization of internal rate of return according to Eq. (10) results as rotation age 63 years. Net present value maximization according to Eq. (9) results as rotation ages 73 and 62 years, with 2% and 3% discounting interest, respectively. We find from Fig. 9 that Eq. (9) with 3% discounting interest yields a slightly greater wealth accumulation rate than Eq. (10). This is naturally accidental. The 3% discounting rate in Eq. (9) being selected arbitrarily, it happens to result in one year younger rotation age in the case of Fig. 9. Again, Eq. (9) with discounting interest 2%, yields clearly inferior results.

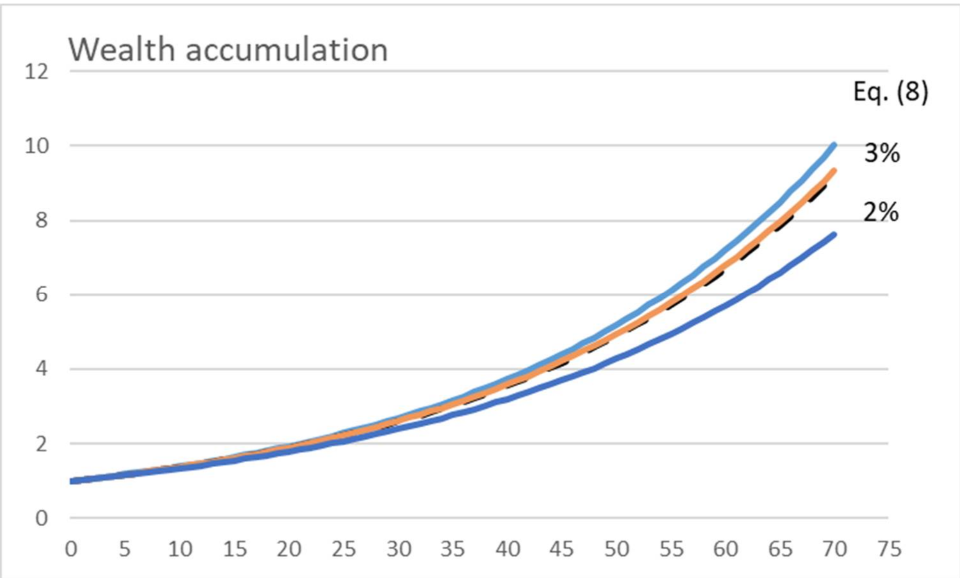

Fig. 9. Wealth accumulation as a function of time (in years) for the value growth model introduced in Eqs. (11) and (12), with moderate bare land value 3000 SEK/ha. The dotted line corresponds to the internal rate of return optimization according to Eq. (10).

Fig. 10 shows wealth accumulation within the spruce value growth model (31, 32, 33, 30), in the absence of any bare land value. We find that there are rather significant differences between the different objective functions. Direct maximization of Eq. (8) results as a rotation age of five years. Maximization of internal rate of return according to Eq. (10) results as the same rotation

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age. Net present value maximization according to Eq. (9) results as rotation ages 40, 25 and 10 years, with 2%, 3%, and 4% discounting interest, respectively. We find that all these three discounting interest rates in Eq. (9) result as financially inferior procedures. The lower the discounting interest is, the more devastating are the consequences.

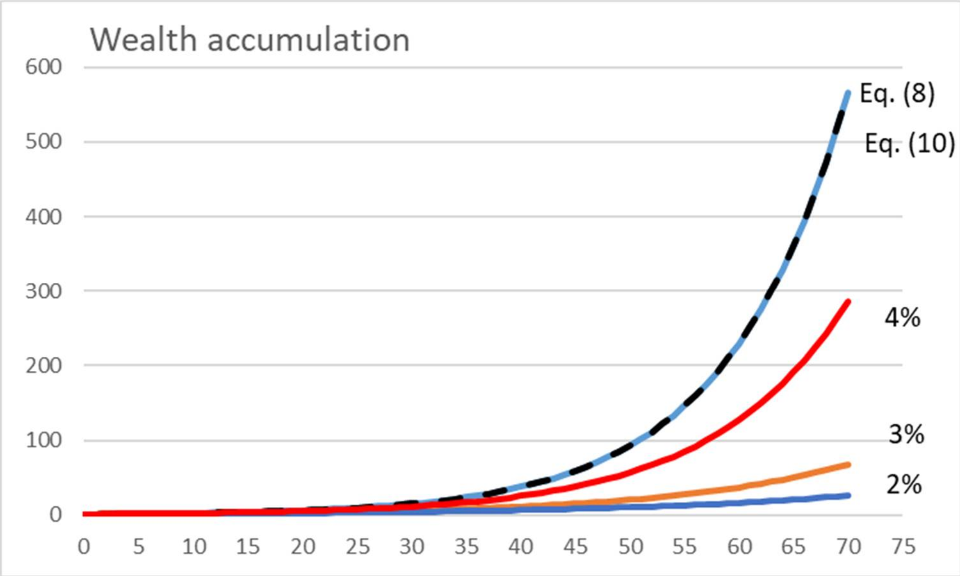

Fig. 10. Wealth accumulation as a function of time (in years) for according to the spruce value growth model (31, 32, 33, 30), with bare land value 0 Eur/ha.

Fig. 11 shows wealth accumulation within the spruce value growth model (31, 32, 33, 30), with the moderate any bare land value 469 Eur/ha. We find that there again are rather significant differences between the different objective functions. Direct maximization of Eq. (8) results as a rotation age of five years. Maximization of internal rate of return according to Eq. (10) results as the same rotation age. Net present value maximization according to Eq. (9) results as rotation ages 40, 25 and 10 years, with 2%, 3%, and 4% discounting interest, respectively. We again find that the lower the discounting interest is, the more devastating are the consequences. It is, however, worth noting that discounting interest rates exceeding the internal rate of return 6,23% would not produce any net present value matching the value of the initial setup.

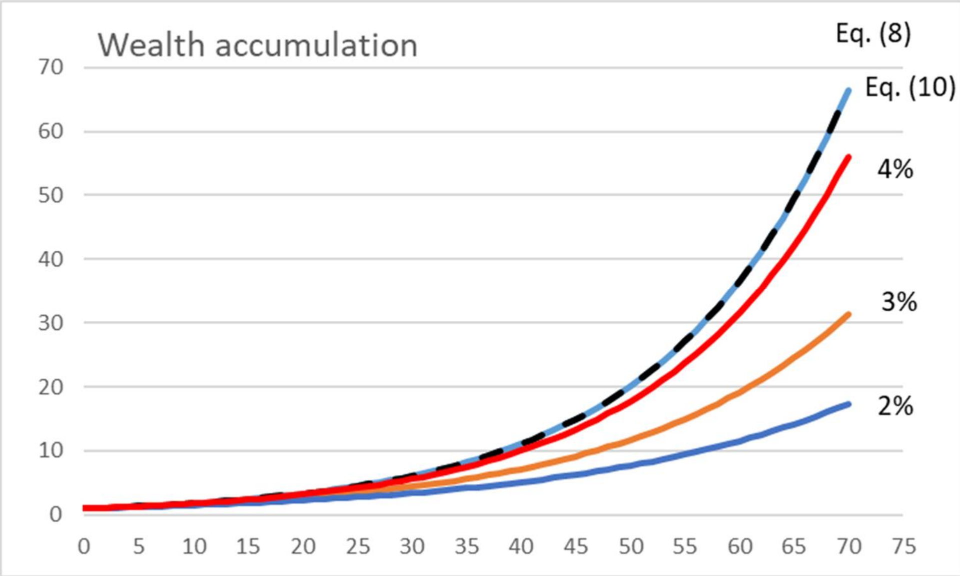

Fig. 11. Wealth accumulation as a function of time (in years) according to the spruce value growth model (31, 32, 33, 30), with moderate bare land value 469 Eur/ha.

It is of interest to consider why the maximization of the internal rate of return according to Eq. (10) yields wealth appreciation rates lower than direct maximization of Eq. (8). The reason naturally is that the IRR is determined on the basis of cash proceeds and neglects the details of the yield curve. The details of the yield curve however do contribute to the wealth accumulation rate. Regardless of this, maximizing the internal rate of return, in general, does not lead to any large deviation from financially optimal management procedures (Figs. 8, 9, 10 and 11).

Unlike the internal rate of return, maximization of net present value with external discounting interest rate does induce large deviations from financially optimal management (Figs. 8, 9, 10 and 11). However, different discounting interest rates differ. There always is a discounting interest rate where maximizing the net present value becomes equal to maximizing the internal rate of return. However, arbitrarily adopting an external discounting interest leads to highly variable and at worst devastating financial consequences (Figs. 8, 9, 10 and 11). This also applies to discounting interest rates commonly utilized in net present value computations in forestry [36, 34, 37, 38, 39, 40].

It is worth noting that a 3% discounting rate in Eq. (9), relatively commonly used in forestry (37, 38, Sinha et al., 2017), performs relatively well in the pine growth models discussed above (Figs. 8 and 9), but produces clearly inferior results in the case of the spruce value growth model (Figs. 10 and 11).

In this paper, we have been discussing stationary rotation forestry. In other words, stand ages have been assumed to be evenly distributed. Prices and expenses have been treated as constants in real terms. It has been recently shown that an eventually appreciating real estate price possibly dictates financially sustainable management practices (30). It would be of interest to investigate such an effect on the performance of the different objective functions.

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