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ON CONFORMABLE DOUBLE LAPLACE TRANSFORM AND ONE DIMENSIONAL FRACTIONAL COUPLED BURGERS' EQUATION

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ABSTRACT. This article deals with the conformable double Laplace transforms and their some properties with examples and also the existence Condition for the conformable double Laplace transform is studied. Finally, in order to obtain the solution of nonlinear fractional problems, we present a modified conformable double Laplace that we call conformable double Laplace decomposition methods (CDLDM). Then, we apply it to solve, Regular and singular conformable fractional coupled burgers equation illustrate the effectiveness of our method some examples are given.

1. INTRODUCTION

The fractional partial differential equations play a crucial role in fields of physics, chemistry and engineering. Cheng and Yao, in [8] studied the solution of some time-fractional partial differential equations by simplest equation method. In this work, we deal with burgers equation, this equations appear in the area of applied sciences such as fluid mechanics, mathematic model. The Burgers Equation was first proposed by Bateman [1] who found its steady solutions, descriptive of certain viscous flows and modified by J. M. Burgers (1895-1981) then it is widely named as Burgers' Equation [2]. Many researchers are Concentrated to studying the exact and numerical solutions of this equation. The conformable double Laplace transform method was introduced by Özkan and Kurt [3] in the study of fractional partial differential equations. Çenesiz et al. in [11] applied the first integral method to establish the exact solutions for time-fractional Burgers' equation. Jincun and Guolin in [12] is applied the generalized two-dimensional differential transform method (DTM) to obtain the solution of the coupled Burgers equations with space- and time-fractional derivatives. Recently M.S. Hashemi [13] applied conformable fractional Laplace transform to solve the coupled system of conformable fractional differential equation. The aim of this article is to propose an analytic solution of the one dimensional Regular and singular conformable fractional coupled burgers equation by using a conformable double Laplace decomposition method (CDLDM). However, conformable double Laplace decomposition method can be used to approximate the solutions of the nonlinear differential equations with the linearization

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of non-linear terms by using Adomian polynomials. A new type of Burger's equation was proposed in further work named as time-space fractional order coupled Burger's equations [7], which has the form

$$\begin{aligned} \frac{\partial^{\beta} u}{\partial t^{\beta}} - \frac{\partial^{2\alpha} u}{\partial x^{2\alpha}} + \eta u \frac{\partial^{\alpha}}{\partial x^{\alpha}} u + \zeta \frac{\partial^{\alpha}}{\partial x^{\alpha}} (uv) &= f(\frac{x^{\alpha}}{\alpha}, \frac{t^{\beta}}{\beta}) \\ \frac{\partial^{\beta} v}{\partial t^{\beta}} - \frac{\partial^{2\alpha} v}{\partial x^{2\alpha}} + \eta v \frac{\partial^{\alpha}}{\partial x^{\alpha}} v + \mu \frac{\partial^{\alpha}}{\partial x^{\alpha}} (uv) &= g\left(\frac{x^{\alpha}}{\alpha}, \frac{t^{\beta}}{\beta}\right). \end{aligned} \tag{1.1}$$

Conformable fractional derivatives were studied by Khalil et al. [4] and extended by Abdeljawad [5]. First of all, we start to recall the definition of the conformable fractional derivatives, which are used in this article

Definition 1. *Given a function $f : (0, \infty) \to R$, then the conformable fractional derivative of f of order β is defined by*

$$\frac{d^{\beta}}{dt^{\beta}} f(t) = \lim_{\epsilon \to 0} \frac{f\left(\frac{t^{\beta}}{\beta} + \epsilon t^{1-\beta}\right) - f(t)}{\epsilon}, \quad t > 0, \; 0 < \beta \leq 1,$$

see [4, 9, 10].

Conformable Partial Derivatives:

Definition 2. *Given a function $f(\frac{x^{\alpha}}{\alpha}, \frac{t^{\beta}}{\beta}) : R \times (0, \infty) \to R$. Then, the conformable space fractional partial derivative of order α a function $f(\frac{x^{\alpha}}{\alpha}, \frac{t^{\beta}}{\beta})$ is defined as:*

$$\frac{\partial^{\alpha}}{\partial x^{\alpha}} f(\frac{x^{\alpha}}{\alpha}, \frac{t^{\beta}}{\beta}) = \lim_{\epsilon \to 0} \frac{f\left(\frac{x^{\alpha}}{\alpha} + \epsilon x^{1-\alpha}, t\right) - f\left(\frac{x^{\alpha}}{\alpha}, \frac{t^{\beta}}{\beta}\right)}{\epsilon}, \quad x, t > 0, 0 < \alpha, \beta \leq 1,$$

see [6].

Definition 3. *Given a function $f(\frac{x^{\alpha}}{\alpha}, \frac{t^{\beta}}{\beta}) : R \times (0, \infty) \to R$. Then, the conformable time fractional partial derivative of order β a function $f(\frac{x^{\alpha}}{\alpha}, \frac{t^{\beta}}{\beta})$ is defined as:*

$$\frac{\partial^{\beta}}{\partial t^{\beta}} f(\frac{x^{\alpha}}{\alpha}, \frac{t^{\beta}}{\beta}) = \lim_{\sigma \to 0} \frac{f\left(\frac{x^{\alpha}}{\alpha}, \frac{t^{\beta}}{\beta} + \sigma t^{1-\beta}\right) - f\left(\frac{x^{\alpha}}{\alpha}, \frac{t^{\beta}}{\beta}\right)}{\sigma}, \quad x\ , t > 0, \; 0 < \alpha, \beta \leq 1,$$

see [6].

Conformable fractional derivative of certain functions:

Example 1.

$$\begin{aligned} \frac{\partial^{\alpha}}{\partial x^{\alpha}} (\frac{x^{\alpha}}{\alpha})(\frac{t^{\beta}}{\beta}) &= (\frac{t^{\beta}}{\beta}), \qquad \frac{\partial^{\alpha}}{\partial x^{\alpha}} (\frac{x^{\alpha}}{\alpha})^{n}(\frac{t^{\beta}}{\beta}) = n(\frac{x^{\alpha}}{\alpha})^{n-1}(\frac{t^{\beta}}{\beta}) \\ \frac{\partial^{\beta}}{\partial t^{\beta}} (\frac{x^{\alpha}}{\alpha})(\frac{t^{\beta}}{\beta}) &= (\frac{x^{\alpha}}{\alpha}), \qquad \frac{\partial^{\beta}}{\partial t^{\beta}} (\frac{x^{\alpha}}{\alpha})^{n}(\frac{t^{\beta}}{\beta})^{m} = m(\frac{x^{\alpha}}{\alpha})^{n}(\frac{t^{\beta}}{\beta})^{m-1} \\ \frac{\partial^{\beta}}{\partial t^{\beta}} \left(\sin(\frac{x^{\alpha}}{\alpha}) \sin(\frac{t^{\beta}}{\beta}) \right) &= \sin(\frac{x^{\alpha}}{\alpha}) \cos(\frac{t^{\beta}}{\beta}), \\ \frac{\partial^{\alpha}}{\partial x^{\alpha}} \left(\sin a(\frac{x^{\alpha}}{\alpha}) \sin(\frac{t^{\beta}}{\beta}) \right) &= a \cos(\frac{x^{\alpha}}{\alpha}) \sin(\frac{t^{\beta}}{\beta}) \\ \frac{\partial^{\alpha}}{\partial x^{\alpha}} \left(e^{\lambda \frac{x^{\alpha}}{\alpha} + \frac{\tau t^{\beta}}{\beta}} \right) &= \lambda e^{\lambda \frac{x^{\alpha}}{\alpha} + \frac{\tau t^{\beta}}{\beta}}, \quad \frac{\partial^{\beta}}{\partial t^{\beta}} \left(e^{\lambda \frac{x^{\alpha}}{\alpha} + \frac{\tau t^{\beta}}{\beta}} \right) = \tau e^{\lambda \frac{x^{\alpha}}{\alpha} + \frac{\tau t^{\beta}}{\beta}}. \end{aligned}$$

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Conformable Laplace transform:

Definition 4. *Let $f : [0,\infty) \to \mathbb{R}$ be a real valued function. The conformable Laplace transform of f is defined by*

$$L_t^\beta\left(f(\frac{t^\beta}{\beta})\right) = \int_0^\infty e^{-s\frac{t^\beta}{\beta}} f\left(\frac{t^\beta}{\beta}\right) t^{\beta-1}dt$$

for all values of s, the integral is correct, see [?].

Definition 5. *see*[3]*: Let $u(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta})$ be a piecewise continuous function on the interval $[0,\infty)\times[0,\infty)$ of exponential order. Consider for some $a,b\in\mathbb{R}$ $\sup_{\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}}>0$, $\frac{\left|u\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)\right|}{e^{\frac{ax^\alpha}{\alpha}+\frac{bt^\beta}{\beta}}}$. Under these conditions conformable double Laplace transform is defined by*

$$L_x^\alpha L_t^\beta\left(u\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)\right) = U(p,s) = \int_0^\infty\int_0^\infty e^{-p\frac{x\alpha}{\alpha}-s\frac{t^\beta}{\beta}} u\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right) t^{\beta-1}x^{\alpha-1}dtdx$$

where $p,s\in\mathbb{C}$, $0<\alpha,\beta\le 1$ and the integrals are by means of conformable fractional integral with respect to $\frac{x^\alpha}{\alpha}$ and $\frac{t^\beta}{\beta}$ respectively.

Example 2. *this example we calculate the double fractional Laplace for certain functions*

1. $L_x^\alpha L_t^\beta\left[(\frac{x^\alpha}{\alpha})^n(\frac{t^\beta}{\beta})^m\right] = L_xL_t\left[(x)^n(t)^m\right] = \frac{n!m!}{p^{n+1}s^{m+1}}$.

2. $L_x^\alpha L_t^\beta\left[(e^{\lambda\frac{x^\alpha}{\alpha}+\frac{\tau t^\beta}{\beta}}\right] = L_xL_t\left[(e^{\lambda x+\tau t}\right] = \frac{1}{(p-\lambda)(s-\tau)}$.

3. $L_x^\alpha L_t^\beta\left[(\sin(\lambda\frac{x^\alpha}{\alpha})\sin(\tau\frac{t^\beta}{\beta})\right] = L_xL_t\left[(\sin(x)\sin(t)\right] = \frac{1}{p^2+\lambda^2}\frac{1}{s^2+\tau^2}$.

4. If $a(>-1)$ and $b(>-1)$ are real numbers, then double fractional Laplace of the function $f(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}) = (\frac{x^\alpha}{\alpha})^a(\frac{t^\beta}{\beta})^b$ is given by

$$L_x^\alpha L_t^\beta\left[(\frac{x^\alpha}{\alpha})^a(\frac{t^\beta}{\beta})^b\right] = \frac{\Gamma(a+1)\Gamma(b+1)}{p^{a+1}s^{b+1}}.$$

Theorem 1. *Let $0<\alpha,\beta\le 1$ and $m,n\in\mathbb{N}$ such that $u\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)\in C^l(\mathbb{R}^+\times\mathbb{R}^+)$, $l=\max(m,n)$. Also let the conformable Laplace transforms of the functions $u\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)$, $\frac{\partial^{m\alpha}u}{\partial x^{m\alpha}}$ and $\frac{\partial^{n\beta}u}{\partial t^{n\beta}}$, $i=1;...,m$, $j=1;...,n$ exist. Then*

$$L_x^\alpha L_t^\beta\left(\frac{\partial^{m\alpha}u}{\partial x^{m\alpha}}\right) = p^mU(p,s) - p^{m-1}U(0,s) - \sum_{i=1}^{m-1}p^{m-1-i}L_t^\beta\left(\frac{\partial^{i\alpha}}{\partial x^{i\alpha}}u\left(0,\frac{t^\beta}{\beta}\right)\right)$$

$$L_x^\alpha L_t^\beta\left(\frac{\partial^{n\beta}}{\partial t^{n\beta}}u\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)\right) = s^nU(p,s) - s^{n-1}U(p,0) - \sum_{j=1}^{n-1}s^{n-1-j}L_x^\alpha\left(\frac{\partial^{j\beta}}{\partial t^{j\beta}}u\left(\frac{x^\alpha}{\alpha},0\right)\right)$$

where $\frac{\partial^{m\alpha}u}{\partial x^{m\alpha}}$ and $\frac{\partial^{n\beta}v}{\partial t^{n\beta}}$ denotes m,n times conformable fractional derivatives of function $u(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta})$ with order b and a respectively. for more details see [3]

In the following theorem, we study double Laplace transform of the function $\left(\frac{x^\alpha}{\alpha}\right)^n \frac{\partial^\beta}{\partial t^\beta}f(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta})$ as follows:

Theorem 2. *If conformable double Laplace transform of the partial derivatives* $\frac{\partial^\beta}{\partial t^\beta} f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})$ *is given by Eqs.(2.21), then double Laplace transform of* $\left(\frac{x^\alpha}{\alpha}\right)^n \frac{\partial^\beta}{\partial t^\beta} f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})$ *and* $\frac{x^\alpha}{\alpha} g\left(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta}\right)$ *are given by*

$$(-1)^n \frac{d}{dp}\left(L_x^\alpha L_t^\beta \left[\frac{\partial^\beta}{\partial t^\beta} f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})\right]\right) = L_x^\alpha L_t^\beta \left[\left(\frac{x^\alpha}{\alpha}\right)^n \frac{\partial^\beta}{\partial t^\beta} f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})\right] \tag{1.2}$$

and

$$(-1)\frac{d}{dp}\left(L_x^\alpha L_t^\beta \left[g\left(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta}\right)\right]\right) = L_x^\alpha L_t^\beta \left[\frac{x^\alpha}{\alpha} g\left(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta}\right)\right], \tag{1.3}$$

where $n = 1, 2, 3, \ldots$

Proof. Using the definition of double Laplace transform of the fractional partial derivatives one gets

$$L_x^\alpha L_t^\beta \left[\frac{\partial^\beta}{\partial t^\beta} f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})\right] = \int_0^\infty \int_0^\infty e^{-p\frac{x^\alpha}{\alpha} - s\frac{t^\beta}{\beta}} \left(\frac{\partial^\beta}{\partial t^\beta} f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})\right) t^{\beta-1} x^{\alpha-1} dt\, dx, \tag{1.4}$$

by taking the ^{n}th derivative with respect to p for both sides of Eq.(**??**), we have

$$\begin{aligned}
\frac{d^n}{dp^n}\left(L_x^\alpha L_t^\beta \left[\frac{\partial^\beta}{\partial t^\beta} f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})\right]\right) &= \int_0^\infty \int_0^\infty \frac{d^n}{dp^n}\left(e^{-p\frac{x^\alpha}{\alpha} - s\frac{t^\beta}{\beta}} \frac{\partial^\beta}{\partial t^\beta} f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})\right) t^{\beta-1} x^{\alpha-1} dt dx \\
&= (-1)^n \int_0^\infty \int_0^\infty \left(\frac{x^\alpha}{\alpha}\right)^n e^{-p\frac{x^\alpha}{\alpha} - s\frac{t^\beta}{\beta}} t^{\beta-1} x^{\alpha-1} \frac{\partial^\beta}{\partial t^\beta} f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta}) dt\, dx \\
&= (-1)^n L_x^\alpha L_t^\beta \left[\left(\frac{x^\alpha}{\alpha}\right)^n \frac{\partial^\beta}{\partial t^\beta} f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})\right],
\end{aligned}$$

we obtain

$$(-1)^n \frac{d}{dp}\left(L_x^\alpha L_t^\beta \left[\frac{\partial^\beta}{\partial t^\beta} f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})\right]\right) = L_x^\alpha L_t^\beta \left[\left(\frac{x^\alpha}{\alpha}\right)^n \frac{\partial^\beta}{\partial t^\beta} f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})\right].$$

Similarly, we can prove Eq.(1.3). ∎

Existence Condition for the conformable double Laplace transform:

If $f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})$ is an exponential order a and b as $\frac{x^\alpha}{\alpha} \to \infty$, $\frac{t^\beta}{\beta} \to \infty$, if there exists a positive constant K such that for all $x > X$ and $t > T$

$$\left| f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta}) \right| \le K e^{a\frac{x^\alpha}{\alpha} + b\frac{t^\beta}{\beta}}, \tag{1.5}$$

it is easy to get,

$$f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta}) = O(e^{a\frac{x^\alpha}{\alpha} + b\frac{t^\beta}{\beta}}) \quad \text{as} \quad \frac{x^\alpha}{\alpha} \to \infty, \frac{t^\beta}{\beta} \to \infty.$$

Or, equivalently,

$$\lim_{\substack{\frac{x^\alpha}{\alpha} \to \infty \\ \frac{t^\beta}{\beta} \to \infty}} e^{-\mu\frac{x^\alpha}{\alpha} - \eta\frac{t^\beta}{\beta}} \left| f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta}) \right| = K \lim_{\substack{\frac{x^\alpha}{\alpha} \to \infty \\ \frac{t^\beta}{\beta} \to \infty}} e^{-(\mu - a)\frac{x^\alpha}{\alpha} - (\eta - b)\frac{t^\beta}{\beta}} = 0,$$

where $\mu > a$ and $\eta > a$. The function $f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})$ is called an exponential order as $\frac{x^\alpha}{\alpha} \to \infty$, $\frac{t^\beta}{\beta} \to \infty$, and clearly, it does not grow faster than $Ke^{a\frac{x^\alpha}{\alpha}+b\frac{t^\beta}{\beta}}$ as $\frac{x^\alpha}{\alpha} \to \infty$, $\frac{t^\beta}{\beta} \to \infty$.

Theorem 3. *If a function $f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})$ is a continuous function in every finite intervals $(0, X)$ and $(0, T)$ and of exponential order $e^{a\frac{x^\alpha}{\alpha}+b\frac{t^\beta}{\beta}}$, then the conformable double Laplace transform of $f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})$ exists for all* $\operatorname{Re} p > \mu, \operatorname{Re} s > \eta$.

Proof. From the definition of the conformable double Laplace transform of $f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})$, we have

$$\begin{aligned} |U(p,s)| &= \left| \int_0^\infty \int_0^\infty e^{-p\frac{x^\alpha}{\alpha} - s\frac{t^\beta}{\beta}} f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta}) t^{\beta-1} x^{\alpha-1} dt\, dx \right| \\ &\leq K \left| \int_0^\infty \int_0^\infty e^{-(p-a)\frac{x^\alpha}{\alpha} - (s-b)\frac{t^\beta}{\beta}} t^{\beta-1} x^{\alpha-1} dt\, dx \right| \\ &= \frac{1}{(p-a)(s-b)}. \end{aligned} \tag{1.6}$$

For $\operatorname{Re} p > \mu, \operatorname{Re} s > \eta$, from Eq.(1.6), we have

$$\lim_{\substack{p\to\infty \\ s\to\infty}} |U(p,s)| = 0 \text{or} \lim_{\substack{p\to\infty \\ s\to\infty}} U(p,s) = 0.$$

∎

2. One dimensional fractional coupled Burgers' equation

In this Section we discuss the solution of regular One dimensional conformable fractional coupled burgers equation and singular one dimensional conformable fractional coupled burgers equation by using conformable double Laplace decomposition methods (CDLDM).

The first problem: One dimensional conformable fractional coupled burgers equation is given by

$$\begin{aligned} \frac{\partial^\beta u}{\partial t^\beta} - \frac{\partial^{2\alpha} u}{\partial x^{2\alpha}} + \eta u \frac{\partial^\alpha}{\partial x^\alpha} u + \zeta \frac{\partial^\alpha}{\partial x^\alpha}(uv) &= f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta}) \\ \frac{\partial^\beta v}{\partial t^\beta} - \frac{\partial^{2\alpha} v}{\partial x^{2\alpha}} + \eta v \frac{\partial^\alpha}{\partial x^\alpha} v + \mu \frac{\partial^\alpha}{\partial x^\alpha}(uv) &= g\left(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta}\right), \end{aligned} \tag{2.1}$$

subject to

$$u\left(\frac{x^\alpha}{\alpha}, 0\right) = f_1\left(\frac{x^\alpha}{\alpha}\right), \quad v\left(\frac{x^\alpha}{\alpha}, 0\right) = g_1\left(\frac{x^\alpha}{\alpha}\right). \tag{2.2}$$

for $t > 0$. Here, $f(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})$, $g(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta})$, $f_1(x)$ and $g_1(x)$ are given functions, η, ζ and μ are arbitrary constants depending on the system parameters such as Peclet number, Stokes velocity of particles due to gravity and Brownian diffusivity [14]. By taking conformable double Laplace transform for both sides of Eq(2.1) and conformable single Laplace transform for Eq(2.2), we have

$$U(p,s) = \frac{F_1(p)}{s} + \frac{F(p,s)}{s} + \frac{1}{s} L_x^\alpha L_t^\beta \left[\frac{\partial^{2\alpha} u}{\partial x^{2\alpha}} - \eta u \frac{\partial^\alpha}{\partial x^\alpha} u - \zeta \frac{\partial^\alpha}{\partial x^\alpha} (uv)_x \right], \tag{2.3}$$

and

$$V(p,s)=\frac{G_1(p)}{s}+\frac{G(p,s)}{s}+\frac{1}{s}L_x^{\alpha}L_t^{\beta}\left[\frac{\partial^{2\alpha}v}{\partial x^{2\alpha}}-\eta v\ \frac{\partial^{\alpha}}{\partial x^{\alpha}}v-\mu\ \frac{\partial^{\alpha}}{\partial x^{\alpha}}(uv)\right]. \quad (2.4)$$

The conformable double Laplace decomposition methods (CDLDM) defines the solution of one dimensional conformable fractional coupled burgers equation as $u\left(\frac{x^{\alpha}}{\alpha},\frac{t^{\beta}}{\beta}\right)$ and $v\left(\frac{x^{\alpha}}{\alpha},\frac{t^{\beta}}{\beta}\right)$ by the infinite series

$$u\left(\frac{x^{\alpha}}{\alpha},\frac{t^{\beta}}{\beta}\right)=\sum_{n=0}^{\infty}u_n\left(\frac{x^{\alpha}}{\alpha},\frac{t^{\beta}}{\beta}\right),\quad v\left(\frac{x^{\alpha}}{\alpha},\frac{t^{\beta}}{\beta}\right)=\sum_{n=0}^{\infty}v_n\left(\frac{x^{\alpha}}{\alpha},\frac{t^{\beta}}{\beta}\right). \quad (2.5)$$

We can give Adomian's polynomials A_n, B_n and C_n respectively as follows

$$A_n=\sum_{n=0}^{\infty}u_nu_{xn},\ B_n=\sum_{n=0}^{\infty}\ v_nv_{xn},\ \ C_n=\sum_{n=0}^{\infty}u_nv_n. \quad (2.6)$$

The Adomian polynomials for the nonlinear terms uu_x , vv_x and uv are given by

$$\begin{aligned}
A_0&=u_0u_{0x}\\
A_1&=u_0u_{1x}+u_1u_{0x}\\
A_2&=u_0u_{2x}+u_1u_{1x}+u_2u_{0x},\\
A_3&=u_0u_{3x}+u_1u_{2x}+u_2u_{1x}+u_3u_{0x},\\
A_4&=u_0u_{4x}+u_1u_{3x}+u_2u_{2x}+u_3u_{1x}+u_4u_{0x},
\end{aligned} \quad (2.7)$$

$$\begin{aligned}
B_0&=v_0v_{0x}\\
B_1&=v_0v_{1x}+v_1v_{0x},\\
B_2&=v_0v_{2x}+v_1v_{1x}+v_2v_{0x},\\
B_3&=v_0v_{3x}+v_1v_{2x}+v_2v_{1x}+v_3v_{0x},\\
B_4&=v_0v_{4x}+v_1v_{3x}+v_2v_{2x}+v_3v_{1x}+v_4v_{0x}.
\end{aligned} \quad (2.8)$$

and

$$\begin{aligned}
C_0&=u_0v_0\\
C_1&=u_0v_1+u_1v_0\\
C_2&=u_0v_2+u_1v_1+u_2v_0.\\
C_3&=u_0v_3+u_1v_2+u_2v_1+u_3v_0,\\
C_3&=u_0v_4+u_1v_3+u_2v_2+u_3v_1+u_4v_0.
\end{aligned} \quad (2.9)$$

By applying inverse conformable double Laplace transform on both sides of Eq.(2.3) and Eq.(2.4), making use of Eq.(2.6), we have

$$\begin{aligned}
\sum_{n=0}^{\infty}u_n\left(\frac{x^{\alpha}}{\alpha},\frac{t^{\beta}}{\beta}\right)&=f_1(x)+L_p^{-1}L_s^{-1}\left[\frac{F(p,s)}{s}\right]\\
&+L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^{\alpha}L_t^{\beta}\left[\frac{\partial^{2\alpha}u}{\partial x^{2\alpha}}_n\right]\right]\\
&-L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^{\alpha}L_t^{\beta}\left[\eta A_n\right]\right]\\
&-L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^{\alpha}L_t^{\beta}\left[\zeta\left(C_n\right)_x\right]\right],
\end{aligned} \quad (2.10)$$

and

$$\begin{aligned}\sum_{n=0}^{\infty} v_n\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right) &= g_1(x) + L_p^{-1}L_s^{-1}\left[\frac{G(p,s)}{s}\right] \\ &+L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\frac{\partial^{2\alpha} v}{\partial x^{2\alpha}}_n\right]\right] \\ &-L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\eta B_n\right]\right] \\ &-L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\mu\left(C_n\right)_x\right]\right]. \qquad (2.11)\end{aligned}$$

On comparing both sides of the Eq.(2.10) and Eq.(2.11) we have

$$\begin{aligned}u_0 &= f_1(x) + L_p^{-1}L_s^{-1}\left[\frac{F(p,s)}{s}\right], \\ v_0 &= g_1(x) + L_p^{-1}L_s^{-1}\left[\frac{G(p,s)}{s}\right]. \qquad (2.12)\end{aligned}$$

In general, the recursive relation is given by

$$\begin{aligned}u_{n+1} = L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\frac{\partial^{2\alpha} u}{\partial x^{2\alpha}}_n\right]\right] \quad &-L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\eta A_n\right]\right] \\ &-L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\zeta\left(C_n\right)_x\right]\right], (2.13)\end{aligned}$$

and

$$\begin{aligned}v_{n+1} = L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\frac{\partial^{2\alpha} v}{\partial x^{2\alpha}} v_n\right]\right] \quad &-L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\eta B_n\right]\right] \\ &-L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\mu\left(C_n\right)_x\right]\right]. (2.14)\end{aligned}$$

Here, we provide double inverse laplace transform with respect to p and s exist for each terms in the right hand side of above equations. To illustrate this method for one dimensional conformable fractional coupled burgers equation we take the following example:

Example 3. *Consider the following homogeneous form of a one dimensional conformable fractional coupled burgers equation*

$$\begin{aligned}\frac{\partial^\beta u}{\partial t^\beta} - \frac{\partial^{2\alpha} u}{\partial x^{2\alpha}} - 2u\,\frac{\partial^\alpha}{\partial x^\alpha}u + \frac{\partial^\alpha}{\partial x^\alpha}(uv) &= 0 \\ \frac{\partial^\beta v}{\partial t^\beta} - \frac{\partial^{2\alpha} v}{\partial x^{2\alpha}} - 2v\,\frac{\partial^\alpha}{\partial x^\alpha}v + \frac{\partial^\alpha}{\partial x^\alpha}(uv) &= 0, \qquad (2.15)\end{aligned}$$

with initial condition

$$u\left(\frac{x^\alpha}{\alpha},0\right) = \sin\frac{x^\alpha}{\alpha}, \quad v\left(\frac{x^\alpha}{\alpha},0\right) = \sin\frac{x^\alpha}{\alpha}. \qquad (2.16)$$

By using Eqs. (2.12), Eq. (2.13) and Eq. (2.14) we have

$$
\begin{aligned}
u_0 &= \sin\frac{x^\alpha}{\alpha},\ v_0=\sin\frac{x^\alpha}{\alpha}\\
u_1 &= L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\frac{\partial^{2\alpha}u}{\partial x^{2\alpha}}_0+2u_{0x}D_\alpha^{(1)}u_0-\frac{\partial^\alpha}{\partial x^\alpha}(u_0v_0)\right]\right]\\
&= L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[-\sin\frac{x^\alpha}{\alpha}\right]\right]=L_p^{-1}L_s^{-1}\left[\frac{1}{s^2(p^2+1)}\right]=-\frac{t^\beta}{\beta}\sin\frac{x^\alpha}{\alpha},\\
v_1 &= L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\frac{\partial^{2\alpha}v}{\partial x^{2\alpha}}_0+2v_{0x}D_\alpha^{(1)}v_0-\frac{\partial^\alpha}{\partial x^\alpha}(u_0v_0)\right]\right]\\
&= L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[-\sin\frac{x^\alpha}{\alpha}\right]\right]=L_p^{-1}L_s^{-1}\left[\frac{1}{s^2(p^2+1)}\right]=-\frac{t^\beta}{\beta}\sin\frac{x^\alpha}{\alpha}
\end{aligned}
$$

$$
\begin{aligned}
u_2 &= L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\frac{\partial^{2\alpha}u}{\partial x^{2\alpha}}_1+2\left(u_0\frac{\partial^\alpha}{\partial x^\alpha}u_1+u_{1x}D_\alpha^{(1)}u_0\right)-\frac{\partial^\alpha}{\partial x^\alpha}(u_0v_1+u_1v_0)\right]\right]\\
&= L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\frac{t^\beta}{\beta}\sin\frac{x^\alpha}{\alpha}\right]\right]=L_p^{-1}L_s^{-1}\left[\frac{1}{s^3(p^2+1)}\right]=\frac{\left(\frac{t^\beta}{\beta}\right)^2}{2}\sin\frac{x^\alpha}{\alpha},
\end{aligned}
$$

$$
\begin{aligned}
v_2 &= L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\frac{\partial^{2\alpha}v}{\partial x^{2\alpha}}_1+2\left(v_0\frac{\partial^\alpha}{\partial x^\alpha}v_1+v_{1x}D_\alpha^{(1)}v_0\right)-\frac{\partial^\alpha}{\partial x^\alpha}(u_0v_1+u_1v_0)\right]\right]\\
&= L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\frac{t^\beta}{\beta}\sin\frac{x^\alpha}{\alpha}\right]\right]=L_p^{-1}L_s^{-1}\left[\frac{1}{s^3(p^2+1)}\right]=\frac{\left(\frac{t^\beta}{\beta}\right)^2}{2}\sin\frac{x^\alpha}{\alpha},
\end{aligned}
$$

and

$$
\begin{aligned}
u_3 &= L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\frac{\partial^{2\alpha}u}{\partial x^{2\alpha}}_2+2\left(u_{0x}D_\alpha^{(1)}u_2+u_{1x}D_\alpha^{(1)}u_1+u_{2x}D_\alpha^{(1)}u_0\right)\right]\right]\\
&\quad -L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\frac{\partial^\alpha}{\partial x^\alpha}(u_0v_2+u_1v_1+u_2v_0)\right]\right]\\
&= -\frac{\left(\frac{t^\beta}{\beta}\right)^3}{6}\sin\frac{x^\alpha}{\alpha},\\
v_3 &= L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\frac{\partial^{2\alpha}v}{\partial x^{2\alpha}}_2+2\left(v_{0x}D_\alpha^{(1)}v_2+v_{1x}D_\alpha^{(1)}v_1+v_{2x}D_\alpha^{(1)}v_0\right)\right]\right]\\
&\quad -L_p^{-1}L_s^{-1}\left[\frac{1}{s}L_x^\alpha L_t^\beta\left[\frac{\partial^\alpha}{\partial x^\alpha}(u_0v_2+u_1v_1+u_2v_0)\right]\right]\\
&= -\frac{\left(\frac{t^\beta}{\beta}\right)^3}{6}\sin\frac{x^\alpha}{\alpha},
\end{aligned}
$$

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and so on for other components. Using Eq.(2.5), the series solutions are therefore given by

$$u\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right) = u_0+u_2+u_3+... = \left(1-\left(\frac{t^\beta}{\beta}\right)+\frac{\left(\frac{t^\beta}{\beta}\right)^2}{2!}-\frac{\left(\frac{t^\beta}{\beta}\right)^3}{3!}+...\right)\sin\frac{x^\alpha}{\alpha}$$

$$v\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right) = v_0+v_2+v_3+... = \left(1-\left(\frac{t^\beta}{\beta}\right)+\frac{\left(\frac{t^\beta}{\beta}\right)^2}{2!}-\frac{\left(\frac{t^\beta}{\beta}\right)^3}{3!}+...\right)\sin\frac{x^\alpha}{\alpha}$$

and hence the exact solutions become

$$u\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)=e^{-\frac{t^\beta}{\beta}}\sin\frac{x^\alpha}{\alpha},\qquad v\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)=e^{-\frac{t^\beta}{\beta}}\sin\frac{x^\alpha}{\alpha}.$$

By taking $\alpha=1$ and $\beta=1$, the fractional solution become

$$u\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)=e^{-t}\sin x,\qquad v\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)=e^{-t}\sin x.$$

The second problem: Singular one dimensional conformable fractional coupled burgers equation with Bessel operator are given by

$$\begin{aligned}\frac{\partial^\beta u}{\partial t^\beta}-\frac{\alpha}{x^\alpha}\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}u\right)+\eta u\frac{\partial^\alpha}{\partial x^\alpha}u+\zeta\frac{\partial^\alpha}{\partial x^\alpha}(uv) &= f\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)\\ \frac{\partial^\beta v}{\partial t^\beta}-\frac{\alpha}{x^\alpha}\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}v\right)+\eta u\frac{\partial^\alpha}{\partial x^\alpha}v+\mu\frac{\partial^\alpha}{\partial x^\alpha}(uv) &= g\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right),\end{aligned} \tag{2.17}$$

with initial conditions

$$u\left(\frac{x^\alpha}{\alpha},0\right)=f_1\left(\frac{x^\alpha}{\alpha}\right),\qquad v\left(\frac{x^\alpha}{\alpha},0\right)=g_1\left(\frac{x^\alpha}{\alpha}\right), \tag{2.18}$$

where the linear terms $\frac{\alpha}{x^\alpha}\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}\right)$ is the called conformable Bessel operator where ζ, μ and η are real constants. In order to obtain the solution of Eq.(2.17), First: multiply both sides of Eq.(2.17) by $\frac{x^\alpha}{\alpha}$ we have

$$\begin{aligned}\frac{x^\alpha}{\alpha}\frac{\partial^\beta u}{\partial t^\beta}-\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}u\right)+\eta\frac{x^\alpha}{\alpha}u\frac{\partial^\alpha}{\partial x^\alpha}u+\zeta\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}(uv) &= \frac{x^\alpha}{\alpha}f\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)\\ \frac{x^\alpha}{\alpha}\frac{\partial^\beta v}{\partial t^\beta}-\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}v\right)+\eta\frac{x^\alpha}{\alpha}v\frac{\partial^\alpha}{\partial x^\alpha}v+\mu\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}(uv) &= \frac{x^\alpha}{\alpha}g\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right).\end{aligned} \tag{2.19}$$

Second: we apply conformable double Laplace transform on both sides of Eq.(2.19) and single conformable Laplace transform for initial condition, we get

$$\begin{aligned}L_x^\alpha L_t^\beta\left[\frac{x^\alpha}{\alpha}\frac{\partial^\beta u}{\partial t^\beta}\right] &= L_x^\alpha L_t^\beta\left[\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}u\right)-\eta\frac{x^\alpha}{\alpha}u\frac{\partial^\alpha}{\partial x^\alpha}u-\zeta\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}(uv)+\frac{x^\alpha}{\alpha}f\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)\right],\\ L_x^\alpha L_t^\beta\left[\frac{x^\alpha}{\alpha}\frac{\partial^\beta v}{\partial t^\beta}\right] &= L_x^\alpha L_t^\beta\left[\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}v\right)-\eta\frac{x^\alpha}{\alpha}v\frac{\partial^\alpha}{\partial x^\alpha}v-\mu\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}(uv)+\frac{x^\alpha}{\alpha}g\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)\right]\end{aligned} \tag{2.20}$$

by applying theorem 1 and theorem 2, we have

$$\begin{aligned}
-s\frac{d}{dp}U(p,s)+\frac{d}{dp}L_x^\alpha\left[f_1(x)\right] &= L_x^\alpha L_t^\beta\left[\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}u\right)-\eta\frac{x^\alpha}{\alpha}u\frac{\partial^\alpha}{\partial x^\alpha}u-\zeta\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}(uv)\right] \\
&\quad -\frac{d}{dp}\left(L_x^\alpha L_t^\beta\left[f\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)\right]\right), \\
-s\frac{d}{dp}V(p,s)+\frac{d}{dp}L_x^\alpha\left[g_1(x)\right] &= L_x^\alpha L_t^\beta\left[\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}v\right)-\eta\frac{x^\alpha}{\alpha}v\frac{\partial^\alpha}{\partial x^\alpha}v-\mu\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}(uv)\right] \\
&\quad -\frac{d}{dp}\left(L_x^\alpha L_t^\beta\left[g\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)\right]\right), \qquad (2.21)
\end{aligned}$$

simplifing Eq.(2.21), we obtain

$$\begin{aligned}
\frac{d}{dp}U(p,s) &= \frac{1}{s}\frac{d}{dp}L_x^\alpha\left[f_1(x)\right]-\frac{1}{s}L_x^\alpha L_t^\beta\left[\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}u\right)-\eta\frac{x^\alpha}{\alpha}u\frac{\partial^\alpha}{\partial x^\alpha}u-\zeta\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}(uv)\right] \\
&\quad +\frac{1}{s}\frac{d}{dp}\left(L_x^\alpha L_t^\beta\left[f\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)\right]\right). \\
\frac{d}{dp}V(p,s) &= \frac{1}{s}\frac{d}{dp}L_x^\alpha\left[g_1(x)\right]-\frac{1}{s}L_x^\alpha L_t^\beta\left[\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}v\right)-\eta\frac{x^\alpha}{\alpha}v\frac{\partial^\alpha}{\partial x^\alpha}v-\mu\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}(uv)\right] \\
&\quad +\frac{1}{s}\frac{d}{dp}\left(L_x^\alpha L_t^\beta\left[g\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)\right]\right). \qquad (2.22)
\end{aligned}$$

Third: by integrating both sides of Eq.(2.22) from 0 to p with respect to p, we have

$$\begin{aligned}
U(p,s) &= \frac{1}{s}\int_0^p\left(\frac{d}{dp}L_x^\alpha\left[f_1(x)\right]\right)dp-\frac{1}{s}\int_0^p L_x^\alpha L_t^\beta\left[\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}u\right)-\eta\frac{x^\alpha}{\alpha}N_1-\zeta\frac{x^\alpha}{\alpha}N_2\right]dp \\
&\quad +\frac{1}{s}\int_0^p\left(\frac{d}{dp}\left(L_x^\alpha L_t^\beta\left[f\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)\right]\right)\right)dp, \\
V(p,s) &= \frac{1}{s}\int_0^p\left(\frac{d}{dp}L_x^\alpha\left[g_1(x)\right]\right)dp-\frac{1}{s}\int_0^p L_x^\alpha L_t^\beta\left[\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}v\right)-\eta\frac{x^\alpha}{\alpha}N_3-\mu\frac{x^\alpha}{\alpha}N_2\right]dp \\
&\quad +\frac{1}{s}\int_0^p\left(\frac{d}{dp}\left(L_x^\alpha L_t^\beta\left[g\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)\right]\right)\right)dp. \qquad (2.23)
\end{aligned}$$

Using conformable double Laplace decomposition method to defines the solution of the system as $u\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)$ and $v(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta})$ by the infinite series

$$u\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)=\sum_{n=0}^{\infty}u_n\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right),\quad v\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right)=\sum_{n=0}^{\infty}v_n\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right). \qquad (2.24)$$

The nonlinear operators can be defined as

$$N_1=\sum_{n=0}^{\infty}A_n,\quad N_2=\sum_{n=0}^{\infty}C_n\ ,N_3=\sum_{n=0}^{\infty}B_n \qquad (2.25)$$

$$\begin{aligned}\sum_{n=0}^{\infty} u_n\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right) &= f_1(x) + L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p dF(p,s)\right] \\ &\quad -L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p \left(L_x^\alpha L_t^\beta\left[\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}\left(\sum_{n=0}^{\infty} u_n\right)\right)\right]\right)dp\right] \\ &\quad +L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p \left(L_x^\alpha L_t^\beta\left[\eta\frac{x^\alpha}{\alpha}\sum_{n=0}^{\infty} A_n\right]\right)dp\right] \\ &\quad +L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p \left(L_x^\alpha L_t^\beta\left[\zeta\frac{x^\alpha}{\alpha}\sum_{n=0}^{\infty} C_n\right]\right)dp\right], \qquad (2.26)\end{aligned}$$

and

$$\begin{aligned}\sum_{n=0}^{\infty} v_n\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right) &= g_1(x) + L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p dG(p,s)\right] \\ &\quad -L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p \left(L_x^\alpha L_t^\beta\left[\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}\left(\sum_{n=0}^{\infty} v_n\right)\right)\right]\right)dp\right] \\ &\quad +L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p \left(L_x^\alpha L_t^\beta\left[\eta\frac{x^\alpha}{\alpha}\sum_{n=0}^{\infty} B_n\right]\right)dp\right] \\ &\quad +L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p \left(L_x^\alpha L_t^\beta\left[\mu\frac{x^\alpha}{\alpha}\sum_{n=0}^{\infty} C_n\right]\right)dp\right]. \qquad (2.27)\end{aligned}$$

The first few components can be written as

$$\begin{aligned}u_0 &= f_1(x) + L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p dF(p,s)\right], \\ v_0 &= g_1(x) + L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p dG(p,s)\right], \qquad (2.28)\end{aligned}$$

and

$$\begin{aligned}u_{n+1}\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right) &= -L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p \left(L_x^\alpha L_t^\beta\left[\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}\left(\sum_{n=0}^{\infty} u_n\right)\right)\right]\right)dp\right] \\ &\quad +L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p \left(L_x^\alpha L_t^\beta\left[\eta\frac{x^\alpha}{\alpha}\sum_{n=0}^{\infty} A_n\right]\right)dp\right] \\ &\quad +L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p \left(L_x^\alpha L_t^\beta\left[\zeta\frac{x^\alpha}{\alpha}\sum_{n=0}^{\infty} C_n\right]\right)dp\right], \qquad (2.29)\end{aligned}$$

and

$$\begin{aligned}v_{n+1}\left(\frac{x^\alpha}{\alpha},\frac{t^\beta}{\beta}\right) &= -L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p \left(L_x^\alpha L_t^\beta\left[\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}\left(\sum_{n=0}^{\infty} v_n\right)\right)\right]\right)dp\right] \\ &\quad +L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p \left(L_x^\alpha L_t^\beta\left[\eta\frac{x^\alpha}{\alpha}\sum_{n=0}^{\infty} B_n\right]\right)dp\right] \\ &\quad +L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p \left(L_x^\alpha L_t^\beta\left[\zeta\frac{x^\alpha}{\alpha}\sum_{n=0}^{\infty} C_n\right]\right)dp\right]. \qquad (2.30)\end{aligned}$$

Here we provide double inverse Laplace transform with respect to p and s exist for each terms in the right hand side of of Eqs. (2.28), (2.29) and (2.30).

Example 4. *Singular one dimensional conformable fractional coupled burgers equation*

$$\begin{aligned} \frac{\partial^{\beta}u}{\partial t^{\beta}}-\frac{\alpha}{x^{\alpha}}\frac{\partial^{\alpha}}{\partial x^{\alpha}}\left(\frac{x^{\alpha}}{\alpha}\frac{\partial^{\alpha}}{\partial x^{\alpha}}u\right)-2u\frac{\partial^{\alpha}}{\partial x^{\alpha}}u+\frac{\partial^{\alpha}}{\partial x^{\alpha}}(uv) &= \left(\frac{x^{\alpha}}{\alpha}\right)^{2}e^{\frac{t^{\beta}}{\beta}}-4e^{\frac{t^{\beta}}{\beta}} \\ \frac{\partial^{\beta}v}{\partial t^{\beta}}-\frac{\alpha}{x^{\alpha}}\frac{\partial^{\alpha}}{\partial x^{\alpha}}\left(\frac{x^{\alpha}}{\alpha}\frac{\partial^{\alpha}}{\partial x^{\alpha}}v\right)-2v\frac{\partial^{\alpha}}{\partial x^{\alpha}}v+\frac{\partial^{\alpha}}{\partial x^{\alpha}}(uv) &= \left(\frac{x^{\alpha}}{\alpha}\right)^{2}e^{\frac{t^{\beta}}{\beta}}-4e^{\frac{t^{\beta}}{\beta}}, \end{aligned} \tag{2.31}$$

subject to

$$u(x,0)=\left(\frac{x^{\alpha}}{\alpha}\right)^{2}, \qquad v(x,0)=\left(\frac{x^{\alpha}}{\alpha}\right)^{2}. \tag{2.32}$$

By applying the above steps, we obtain

$$\begin{aligned} \sum_{n=0}^{\infty}u_{n}\left(\frac{x^{\alpha}}{\alpha},\frac{t^{\beta}}{\beta}\right) &= \left(\frac{x^{\alpha}}{\alpha}\right)^{2}e^{\frac{t^{\beta}}{\beta}}-4e^{\frac{t^{\beta}}{\beta}}+4 \\ &\quad -L_{p}^{-1}L_{s}^{-1}\left[\frac{1}{s}\int_{0}^{p}\left(L_{x}^{\alpha}L_{t}^{\beta}\left[\frac{\partial^{\alpha}}{\partial x^{\alpha}}\left(\frac{x^{\alpha}}{\alpha}\frac{\partial^{\alpha}}{\partial x^{\alpha}}\left(\sum_{n=0}^{\infty}v_{n}\right)\right)\right]\right)dp\right] \\ &\quad -L_{p}^{-1}L_{s}^{-1}\left[\frac{1}{s}\int_{0}^{p}\left(L_{x}^{\alpha}L_{t}^{\beta}\left[2\frac{x^{\alpha}}{\alpha}\sum_{n=0}^{\infty}A_{n}\right]\right)dp\right] \\ &\quad +L_{p}^{-1}L_{s}^{-1}\left[\frac{1}{s}\int_{0}^{p}\left(L_{x}^{\alpha}L_{t}^{\beta}\left[\frac{x^{\alpha}}{\alpha}\frac{\partial^{\alpha}}{\partial x^{\alpha}}\left(\sum_{n=0}^{\infty}C_{n}\right)\right]\right)dp\right], \end{aligned} \tag{2.33}$$

and

$$\begin{aligned} \sum_{n=0}^{\infty}v_{n}\left(\frac{x^{\alpha}}{\alpha},\frac{t^{\beta}}{\beta}\right) &= \left(\frac{x^{\alpha}}{\alpha}\right)^{2}e^{\frac{t^{\beta}}{\beta}}-4e^{\frac{t^{\beta}}{\beta}}+4 \\ &\quad -L_{p}^{-1}L_{s}^{-1}\left[\frac{1}{s}\int_{0}^{p}\left(L_{x}^{\alpha}L_{t}^{\beta}\left[\frac{\partial^{\alpha}}{\partial x^{\alpha}}\left(\frac{x^{\alpha}}{\alpha}\frac{\partial^{\alpha}}{\partial x^{\alpha}}\left(\sum_{n=0}^{\infty}v_{n}\right)\right)\right]\right)dp\right] \\ &\quad -L_{p}^{-1}L_{s}^{-1}\left[\frac{1}{s}\int_{0}^{p}\left(L_{x}^{\alpha}L_{t}^{\beta}\left[2\frac{x^{\alpha}}{\alpha}\sum_{n=0}^{\infty}B_{n}\right]\right)dp\right] \\ &\quad +L_{p}^{-1}L_{s}^{-1}\left[\frac{1}{s}\int_{0}^{p}\left(L_{x}^{\alpha}L_{t}^{\beta}\left[\frac{x^{\alpha}}{\alpha}\sum_{n=0}^{\infty}C_{n}\right]\right)dp\right] \end{aligned} \tag{2.34}$$

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where A_n, B_n and C_n are defined in Eqs.(2.7), (2.8)and (2.9) respectively. On using Eqs. (2.28), (2.29) and (2.30) the components are given by

$$\begin{aligned}
u_0 &= \left(\frac{x^\alpha}{\alpha}\right)^2 e^{\frac{t^\beta}{\beta}} - 4e^{\frac{t^\beta}{\beta}} + 4,\ v_0 = \left(\frac{x^\alpha}{\alpha}\right)^2 e^{\frac{t^\beta}{\beta}} - 4e^{\frac{t^\beta}{\beta}} + 4, \\
u_1 &= -L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p L_x^\alpha L_t^\beta \left[\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}u_0\right) + 2\frac{x^\alpha}{\alpha}u_0\frac{\partial^\alpha}{\partial x^\alpha}u_0 - \frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}(u_0v_0)\right]dp\right] \\
u_1 &= -L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p L_x^\alpha L_t^\beta \left[\left(4\frac{x^\alpha}{\alpha}e^{\frac{t^\beta}{\beta}}\right)\right]dp\right] = 4e^{\frac{t^\beta}{\beta}} - 4, \\
v_1 &= -L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p L_x^\alpha L_t^\beta \left[\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}v_0\right) + 2\frac{x^\alpha}{\alpha}v_0\frac{\partial^\alpha}{\partial x^\alpha}v_0 - \frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}(u_0v_0)\right]dp\right] \\
v_1 &= -L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p L_x^\alpha L_t^\beta \left[\left(4\frac{x^\alpha}{\alpha}e^{\frac{t^\beta}{\beta}}\right)\right]dp\right] = 4e^{\frac{t^\beta}{\beta}} - 4,,
\end{aligned}$$

In the same manner, we obtain that

$$\begin{aligned}
u_2 &= -L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p L_x^\alpha L_t^\beta \left[\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}u_0\right)\right]dp\right] \\
&\quad -L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p L_x^\alpha L_t^\beta \left[2\frac{x^\alpha}{\alpha}\left(u_0\frac{\partial^\alpha}{\partial x^\alpha}u_1 + u_1\frac{\partial^\alpha}{\partial x^\alpha}u_0\right)\right]dp\right] \\
&\quad +L_p^{-1}L_s^{-1}\left[\frac{1}{s}\int_0^p L_x^\alpha L_t^\beta \left[\frac{x^\alpha}{\alpha}\frac{\partial^\alpha}{\partial x^\alpha}(u_0v_1 + u_1v_0)\right]dp\right] \\
u_2 &= 0 \\
v_2 &= 0,
\end{aligned}$$

It is obvious that the self-cancelling some terms appear between various components and the connected by coming terms, then we have,

$$u\left(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta}\right) = u_0 + u_1 + u_2 + ..., v\left(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta}\right) = v_0 + v_1 + v_2 + ...$$

Therefore, the exact solution is given by

$$u\left(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta}\right) = \left(\frac{x^\alpha}{\alpha}\right)^2 e^{\frac{t^\beta}{\beta}} \text{ and } v\left(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta}\right) = \left(\frac{x^\alpha}{\alpha}\right)^2 e^{\frac{t^\beta}{\beta}}$$

By taking $\alpha = 1$ and $\beta = 1$, the fractional solution becomes

$$\begin{aligned}
u\left(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta}\right) &= x^2e^t \\
v\left(\frac{x^\alpha}{\alpha}, \frac{t^\beta}{\beta}\right) &= x^2e^t
\end{aligned}$$

Conclusion 1. *In this work, we give the solution of the one dimensional Regular and singular conformable fractional coupled burgers equation by using conformable double Laplace decomposition method. Moreover, two examples were given to validate our method. This method can be apply to solve some nonlinear time-fractional differential equations with conformable derivative.*

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