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system under study. With the current innovations in big data sciences and Artificial Intelligence (AI), these signatures should be used as a new family of informations.

Consequently, since the structure of complex biological, social and professional organizations is known to exhibit memory effects, conditioning, local and global synchronization, hysteresis, threshold effects, amplitude dependence, and saturation at various states, an extension of this mesoscopic signal processing could be introduced in order to quantify the new proportion of stochasticity in the (nonlinear) response of any complex system. Since stochastics signal are actually produced by deterministic mesoscopic systems that are capable of nonlinear stochastic responses, their behavior should be associated to invariant properties, such as symmetries like time invariance and stationarity. For example, the stability of such mesoscopic system is also conditioned by a complex skeleton of elementary rules or elements which is necessary for synchronized behaviour. Under external excitations, nonlinear systems (or complex organizations) can produce non deterministic responses which increases the stochastic part with a non intuitive proportion that needs to be considered in biology, modern engineering organizations, and social monitoring, in order to understand the breaking of synchronized states.

The huge variety of information extracted from this small stochastic part of the response coming from a complex system induces an increase of uncertainty associate to the linear part. This linear part, with its underlying hypothesis of stationarity and determinism, should be consequently associated to a greater uncertainty if the system under study presents intrinsically a complex structure with mesoscopic properties, memory effects, conditioning, and ageing. Of course, these properties are breaking now the stationarity hypothesis implicitly assumed in any linear signal processing, since linear systems theory dominates the field of engineering.

The systematic analysis proposed in this paper where the memristor technology is associated to a classical ultrasonic imaging technique can be seen as a world-view where standard linear approaches are supplemented by the framework of multiscale and nonlinear analysis. The final motivation of these results is to promote complex system engineering encouraging the application of the findings in the field of Non Destructive Testing integrity engineering; in engineering departments, medical center and institutions and other relevant structures throughout the world.

In 1971, Leon Chua [1] predicted presence of Memristor, which is a fundamental circuit element that connects between charge $q(t)$ and flux $\phi(t)$. Memristor is a 2 terminal circuit element characterized by a relation between

$$q(t) = \int_{-\infty}^{t} i(\tau) d\tau$$

and

$$\phi(t) = \int_{-\infty}^{t} v(\tau) d\tau.$$

He defined charge controlled memristor by the relation [2, 3]

$$\begin{aligned} v(t) &= M(q(t))i(t) \\ M(q) &= d\phi(q)/dq, \end{aligned} \tag{1}$$

and flux controlled memristor by the relation

$$\begin{aligned} i(t) &= W(\phi(t))v(t) \\ W(\phi) &= dq(\phi)/d\phi. \end{aligned} \tag{2}$$

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The relation $v(t) = M(q(t))i(t)$ of memristive system sometimes shows in $v - i$ plane, strange attractors and chaotic behaviors. In the study of generalized time-dependent Ginzburg-Landau equation [4], spatiotemporal transitions of the solution to Chaos was observed and transition between spacial chaos and temporal chaos was discussed. Grebogi, Ott, Perikan and York [5] gave definition of strange attractors and chaotic attractors, and showed that there are strange attractors that are not chaotic.

In 1976, Chua and Kang [6] proposed a generic equation to describe memristive devices and systems

$$\begin{aligned} Y &= g(X,U,t)U \\ dX/dt &= f(X,U,t) \end{aligned} \tag{3}$$

where X is the state variable, U is the input and Y is the output variable.

Muthuswamy and Chua [7] studied 3 dimensional memristive system $X = (x, y, z)$ defined by

$$\begin{aligned} dx/dt &= \frac{y}{C} \\ dy/dt &= -\frac{1}{L}[x + \beta(z^2 - 1)y] \\ dz/dt &= -y - \alpha z + yz \end{aligned} \tag{4}$$

where C, L, α and β are constant parameters, and $Y = v_M$, $U = i_M$, $g(X,U,t) = M(q)$ and $i_M = dq/dt$. They found chaotic strange attractors, and [8] studied routes to chaos of the system via bifurcation of attractors. Chaos in surface acoustic waves (SAWs) on $YZ - LiNbO_3$ material was studied in [14].

For simple input function

$$i(t) = A \sin \omega t$$

the period $t = \pi/\omega$ goes to 0 as ω goes to ∞. Thus $v(t) = R(q(t))i(t)$ goes to $R(0)i(t)$ and for $i(t) = A \sin \omega t$ form Lissajous figure always have $(v, i) = (0, 0)$ point and hysteresis loop become pinched when ω can be chosen arbitrary. In acoustic wave crack detections, output voltage depends on ω and instability and chaotic behavior appear [14].

Hysteresis can be seen also in time dependent $B - H$ plane diagram of ferromagnetic materials [10], and in $Pressure - Strain$ plane diagram of nonlinear elastic materials [11, 12, 13].

Under certain condition, eigenvectors of time reversed acoustic or electromagnetic waves in nonlinear elastic materials can be decomposed and focused. The technique of decomposition of time reversal operator (DORT) was applied in [15, 16, 17, 18, 19, 20, 21], and to reduce the effect of noise, focused transmission of waves and study of correlation of acoustic or electromagnetic nonlinear waves and its time reversed wave using FDORT technique was proposed [22, 23, 24, 25].

In these work, vibration signal from a transducer is emitted into non-linear elastic material and received by transducers placed on the boundary of a sphere surrounding the emitter, and time reversed signal was emitted from each transducers. Correlation of original signal and time reversed signal were measured and tomographic figures were produced.

If one uses transducers which emit and receive signals, the time reversed signal can be emitted before receiving the signal. Vejvodka, Prevorovsky and Dos Santos [26] proposed nonlinear time reversal tomography using emitting and receiving transducers array in a cubic domain. When amplitude inverted pulses are emitted in non-linear medium, the received signals do not cancel

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in certain time domain, and property of nonlinear medium can be detected. Since time reversal (TR) is performed before non linear elastic wave spectroscopy (NEWS) is performed, the system is called TR-NEWS [27]. The system was applied to non destructive testing (NDT) of complex materials as carbon fibre reinforced polymer (CFRP) and biological materials[28, 29, 38].

In the similar system: NEWS-TR, signals are emitted from transducers placed on an $x-y$ planes in the cube of $z = 2n\Delta z$ and received on transducers placed on another $x-y$ plane of $z = (2n+1)\Delta z$ and time reversed signal is emitted to transducers on $z = 2n\Delta z$ or $z = (2n+2)\Delta z$ [33].

In the measurement of waves travelling in materials which contain hysteresis, it is necessary to take into account time reversal symmetry of the waves which are expressed by using Green's function. McCall and Guyer[11, 12, 13] presented equation of state and wave propagation in hysteretic nonlinear elastic materials, in which the Preisach-Mayergoyz (PM) space model of Hysteresis [34] was adopted. In the PM space model, a material like transducer or biological skin is assumed to be composed of a large number of small elastic units: hysteretic elementary unit (HEU), and one can take a HEU as a memristor[2, 9] whose operation depends on time.

On the time reversal and complexity of images, Fink[23] considered two-dimensional wave for finding its evolution in the 3rd dimension $p(\mathbf{r}_i, t)$ and its time reversed wave $p(\mathbf{r}_i, T-t)$, where T comes from the boundary condition. Performing the Fourier transform, the initial wave is written as $A_i \cos(\omega t + \phi_i)$ and its time reversed wave is $A_i \cos(\omega t - \phi_i)$. In the study of duality between space and time variables in wave physics[24, 25], differential equation

$$[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c(\mathbf{r})^2}\frac{\partial^2}{\partial t^2}]\phi(\mathbf{r}, t) = 0 \tag{5}$$

in a 4 dimensional hypervolume containing 3 dimensional hypervolume V with boundary S.

1) Cauchy spatial boundary condition prescribes

$$\{\phi(\mathbf{r}, t), \partial_n \phi(\mathbf{r}, t)\}$$

for $\mathbf{r} \in S$, for all t, in which 2 spatial and 1 temporal dimension are taken.

2) Cauchy initial conditions prescribe

$$\{\phi(\mathbf{r}, t = t_i), \partial_t \phi(\mathbf{r}, t = t_i)\}$$

for all $\mathbf{r} \in V$, in which 3 spatial dimensions are taken.

In non dissipative heterogeneous medium with a punctual source, the Green's function satisfies

$$[\Delta - \frac{1}{c^2(\mathbf{r})}\frac{\partial^2}{\partial t^2}]G(\mathbf{r}, \mathbf{r}_0; t) = -\delta(\mathbf{r} - \mathbf{r}_0)\delta(t) \tag{6}$$

To obtain the wave function at t_f, he showed the Loschmidt approach and the Time Reversal Mirror approach as follows.

1) The Loschmidt approach: on the whole volume V, the final conditions at time t_f is

$$\{\phi(\mathbf{r}', t_f); \partial_t \phi(\mathbf{r}', t_f)\}$$

and new initial conditions

$$\phi(\mathbf{r}', t_i) = \phi(\mathbf{r}', t_f) \text{ and } \partial_t \phi(\mathbf{r}', t_i) = -\partial_t \phi(\mathbf{r}', t_f)$$

But one may wonder meanings of choosing $\partial_t\phi(\mathbf{r}', t_f)$ to $-\partial_t\phi(\mathbf{r}', t_f)$ on the boundary.

2) The Time Reversal Mirror (TRM) approach: One defines record on the boundary, only $\phi(\mathbf{r}', t)$, and transmit from the boundary $\phi(\mathbf{r}', T-t)$ in the far field of the source.

Fink adopted the TRM approach [22, 44], in the medical ultrasound micro calcification detection. In this approach, technique of decomposition of the time reversal operator (Decomposition d'Operateur de Renversement de Temps: DORT) (décomposition d'opérateur de retournement temporel) and that of focused transmission scheme (FDORT) were used.

Signal processing concept for focusing the ultrasonic energy into a specific part of the test object FDORT originates from diffraction of waves from a source to a scatterer and a scatterer to a receiver and the two waves are different directional [23].

Time reversal acoustics (TRA) developed by Fink was applied by Sutin et al. [40, 41] in different set-ups. They developed TRA focusing reverberation media with an external resonator and between two media two electronic units, 1) for receiver channel and 2) for transmitter channel. The propagation of waves are same directional [32, 36, 37].

Born and Wolf [39] pointed out that in the analysis of interference of diffraction of waves, cross correlation and autocorrelation are useful tools for analysis. It can avoid the problem of boundary condition in time space, and take into account nonlinear property of waves.

In the TR-NEWS analysis, the autocorrelation of nonlinear direct solitonic sound wave and the time reversed solitonic wave are considered and two waves are one directional [61]. Presence of time reversed wave in addition to the direct wave is similar to the Gribov ambiguity in Coulomb gauge field equations [84].

In the section of propagation of Solitary waves in matters with hysteresis, we describe the propagation of nonlinear wave which is treated as a soliton in matter described by Preisach-Mayergoyz's model.

In the section of TR-NEWS technology, we explain application to memoducer and non destructive testing (NDT), and in the section of discussion and conclusion, genera,l overview of memristor and remaining problems of TR-NEWS will be summarized.

2 Propagation of Solitary waves in matters with hysteresis

Born and Wolf [39] considered light disturbance at a point P in the volume $\mathbf{v}$ bounded by a surface S by first taking the monochromatic scalar wave

$$V(x, y, z, t) = U(x, y, z)e^{-i\omega t}$$

where space dependent part $U(x, y, z)$ satisfies

$$(\nabla^2 + k^2)U = 0$$

where $k = \omega/c$. If $U'(x, y, z)$ is any other function which satisfies the same continuity requirements as U, The Green's theorem says that

$$\int\int\int_{\mathbf{v}}(U\nabla^2U' - U'\nabla^2U)d\mathbf{v} = -\int\int_S(U\frac{\partial U'}{\partial n} - U'\frac{\partial U}{\partial n})dS \tag{7}$$

where $\partial/\partial n$ denotes differentiation along the inward normal to S. By choosing a small sphere S' of radius ϵ and the wave function $U'(x,y,z) = e^{iks}/s$, where s is the distance from the point P and (x,y,z), and taking the limit $\epsilon \to 0$ one can derive

$$U(P) = \frac{1}{4\pi}\int\int_S [U\frac{\partial}{\partial n}(\frac{e^{iks}}{s}) - \frac{e^{iks}}{s}\frac{\partial U}{\partial n}]dS. \tag{8}$$

Assuming that $V(x,y,z,t)$ can be expressed by its Fourier transform, one can show that

$$V(P,t) = \frac{1}{4\pi}\int\int_S \{[V]\frac{\partial}{\partial n}(\frac{1}{s}) - \frac{1}{cs}\frac{\partial s}{\partial n}[\frac{\partial V}{\partial t}] - \frac{1}{s}[\frac{\partial V}{\partial n}]\}dS \tag{9}$$

where the square brackets denote retarded values, i.e. values of the function taken at the time $t - s/c$.

In order to formulate description of propagation of solitary waves in matter, we consider a one parameter transformation group generated by a vector X:

$$\Phi(\mathbf{x}_0; t+s) = \Phi(\Phi(\mathbf{x}_0;t);s) \tag{10}$$

and the evolution equation

$$u_t = H(u, u_1, \cdots, u_m) \tag{11}$$

Fink et al., [15, 16, 17] adopted the Kirchhoff's diffraction theory in the propagation of sound waves.

Trajectories of orbits have time reversal symmetry, and Loschmidt considered the probability of observing only one trajectory, but we need to consider forward and reversal trajectory sets [45]. Lints et al. considered forward and reversal trajectories, but time evolutions are one directional.

2.1 Two directional focusing of direct and time-reversed ultrasonic energy

Experimental results showing the reversibility of transient acoustic waves through high order multiple scattering by means of an acoustic time-reversal mirror were reported in 1995-97 by Fink et al [15, 16, 17]. Superposition of an acoustic pulse and time reversed pulse produced by piezoelectric transducers were measured. Detection and focusing technique of acoustic pulse, Focused decomposition of the time-reversal operator (DORT), was explained in [22].

In [17], one-channel time reversal on circular with a corner cut-off silicon wafer cavity. It was shown that time reversal in wave systems with *chaotic ray dynamics* is feasible. It was proven that, using single time-reversal channel, a good reversal quality can be obtained, but residual temporal sidelobes persisted even for time-reversal windows of infinite size [16].

Importance of nonlinearity in TRM in a wave *chaotic* system was discussed in [44]. Transmission of a four-color (two bits per pixel) image using the time-reversed linear sona, which is reconstructed only at the linear port, and transmission of a different four colour image using the time-reversed nonlinear sona, which is reconstructed only at the nonlinear port were presented.

Fink et al. [16, 20] extended the time reversal acoustic (TRA) focusing technique from 2D silicon wafer to elastic solids. A new algorithm of focused decomposition of the time reversal operator (FDORT) was presented in [22]. Dependence of signals on location of speckle was reported.

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In the [22], signal emitters $\mathbf{E}(\omega)$, signal receivers $\mathbf{R}(\omega)$ and two scatterers at positions $\mathbf{P}$ and $\mathbf{Q}$ were considered.

For

$$\mathbf{E}(\omega) = [E_1(\omega), E_2(\omega), \cdots, E_M(\omega)]^T$$
$$\mathbf{R}(\omega) = \mathbf{K}(\omega)\mathbf{E}(\omega) = [R_1(\omega), R_2(\omega), \cdots, R_N(\omega)]^T$$

The scattering amplitude producing interference of direct and time-reversed wave was described by each scatterer reflectivity $D(\mathbf{P})$ and $D(\mathbf{Q})$ as

$$\mathbf{K}_{foc} = (\mathbf{H}_{Rx})^T \mathbf{D} \mathbf{H}_{Tx}$$

where

$$\mathbf{H}_{Rx} = \begin{pmatrix} H_{Rx}(P)_1 & H_{Rx}(P)_2 & \cdots & H_{Rx}(P)_N \\ H_{Rx}(Q)_1 & H_{Rx}(Q)_2 & \cdots & H_{Rx}(Q)_N \end{pmatrix}$$

and

$$\mathbf{D} = \begin{pmatrix} D(P) & 0 \\ 0 & D(Q) \end{pmatrix}.$$

i.e. there is no Jost function term. Since spatio temporal transition to chaos occurs in Nonlinear Schrödinger (NS) equation [4], it is necessary to study stability of superposition of waves that propagate in nonlinear media[48]. It was shown that self-focused light beams that are uniform along the propagation direction are unstable in a nonlinear transparent medium.

We think the evolution equation of time should be chosen one directional, in order to suppress chaotic spectra, which appears near the boundary of classical spectra.

2.2 Solitary waves in Preisach-Mayergoyz (PM) space and one directional focusing of TRM

Wave equation of a pulse in solid bodies regarded as continuous media is explained in the text book of Landau and Lifshitz [46]. When equations of motion are non-linear, un-harmonic vibrations occur, and when one considers un-harmonic effects of third order, arising from terms in the elastic energy which are cubic in the strains, quadratic terms in the stress tensor appears, and also in the equations of motion. The strain tensor needs to be taken the complete expression

$$u_{ik} = \frac{1}{2}\left(\frac{\partial x_i}{\partial x_k} + \frac{\partial x_k}{\partial x_i} + \frac{\partial u_l}{\partial x_i}\frac{\partial u_l}{\partial x_k}\right)$$

where $u_i = x_i' - x_i$ is the displacement vector, $dl = \sqrt{{dx_1}^2 + {dx_2}^2 + {dx_3}^2}$ and $dl' = \sqrt{{dx_1'}^2 + {dx_2'}^2 + {dx_3'}^2}$.

Effect of cracks on compression of rocks, which is represented in stress-strain hysteretic curve was discussed in [42] and [43]. McCall[11] studied the wave equation for an isotropic homogeneous elastic solids with cubic un-harmonicity in the moduli, accounting for attenuation by introducing complex linear and nonlinear moduli, using a Green function technique.

Guyer, McCall and Boitnott[13] described nonlinear wave propagation in rocks characterized by density in Preisach-Mayergoyz space (PM space) using the Green function technique.

First, the rock is taken to be in the elastic state E_0, and when there is a pressure disturbance δP at (x,t)

$$M(x,t;E_0) = M_0\{1 + \tilde{\kappa}[\delta P(x,t)]\}$$

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with a set of switching instants

$$A_t = \{t \in [0,T] : v(t) = \alpha \text{ or } \beta\}$$

and

$$u(t) = \begin{cases} u(0) & \text{when} \quad A_t = 0 \\ 1 & \text{when} \quad A_t \neq 0 \quad \text{and} \quad v(max(A_t = \alpha)) \\ -1 & \text{when} \quad A_t \neq 0 \quad \text{and} \quad v(max(A_t = \beta)) \end{cases}$$

With a hysteretic relay $R_{\beta,\alpha}$ and output value at time 0: $u_{\beta,\alpha}(0)$,

$$u(t) = \int\int_{\alpha\geq\beta} \mu(\beta,\alpha)\cdot R_{\beta,\alpha}(v(t), u_{\beta,\alpha}(0))d\beta d\alpha$$

was calculated. The time evolution was one directional, but different boundary conditions were chosen.

2.3 Evolution equation of nonlinear waves in Hamiltonian formalism

Nonlinearity of acoustic waves propagating in hysteretic materials, which causes emergence and propagation of solitary wave in materials like carbon fibre reinforced polymer can be investigated in Hamiltonian formalism.

In the standard study of propagation of solitary waves [78], the evolution equation for unknown function $u(\mathbf{x},t)$ of the form

$$u_t = H(u, u_1, \cdots, u_m) = \mathcal{H}\frac{\partial h}{\partial u}$$

where $u_t = \frac{\partial u}{\partial t}$ and $u_k = \frac{\partial^k u}{\partial x^k}$ is postulated.

We consider $\mathbf{x}$ is two dimensional (x,y) and a function $F(x,y)$ is invariant under one parameter transformation $\Phi(\mathbf{x},t)$ which means that

$$\bar{\mathbf{x}} = \Phi(\mathbf{x};t) = (\bar{x}(t), \bar{y}(t)) = (\phi^1(x,y;t), \phi^2(x,y;t))$$

also satisfy $F(\bar{x}(t), \bar{y}(t)) = 0$

When the vector field X is expressed as

$$X = \xi(x,y)\frac{\partial}{\partial x} + \eta(x,y)\frac{\partial}{\partial y}$$

since $\frac{d\bar{x}}{dt} = \xi(\bar{x},\bar{y})$, $\frac{d\bar{y}}{dt} = \eta(\bar{x},\bar{y})$,

$$\frac{dF}{dt}(\bar{x},\bar{y}) = \frac{\partial F}{\partial x}\xi(\bar{x},\bar{y}) + \frac{\partial F}{\partial y}\eta(\bar{x},\bar{y}) = XF(\bar{x},\bar{y}) = 0 \tag{14}$$

If one chooses $t = 0$, $F(x,y) = 0$ gives

$$XF(x,y) = \xi(x,y)\frac{\partial F}{\partial x} + \eta(x,y)\frac{\partial F}{\partial y} = 0.$$

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We express $\mathbf{x} = (x, y) = (x^1, x^2)$ and defne J_n as the space of nth order partial derivative space of u:

$$J^{(0)} = \{(x, y, u)\}, J_1 = \{(u_x, u_y)\}, J_2 = \{(u_{xx}, u_{xy}, u_{yy})\}, \cdots$$

We define nth order space $J^{(n)} = J^{(0)} \times J_1 \times \cdots \times J_n$ and consider

$$F(\mathbf{x}, \mathbf{u}^{(n)}) = 0.$$

On $J^{(0)}$

$$X = \xi^1(\mathbf{x}, u)\frac{\partial}{\partial x^1} + \xi^2(\mathbf{x}, u)\frac{\partial}{\partial x^2} + \eta(\mathbf{x}, u)\frac{\partial}{\partial u} \tag{15}$$

and define one parameter transformation group

$$\Phi(\mathbf{x}, u; t) = (\bar{x}^1(\mathbf{x}, u; t), \bar{x}^2(\mathbf{x}, u; t), \bar{u}(\mathbf{x}, u; t)),$$

and prolong the operation to $J^{(1)}$ as

$$(x^1, x^2, u, u_{(1,0)}, u_{(0,1)}) \to (\bar{x}^1, \bar{x}^2, \bar{u}, \bar{u}_{(1,0)}, \bar{u}_{(0,1)})$$

and one can define $\Phi^{(1)}(\mathbf{x}, u^{(1)}; t)$.

The Lie symmetry of the differential equation $F(\mathbf{x}, \mathbf{u}^{(n)}) = 0$ is the 1 parameter group $\Phi(\cdot; t)$.

The prolongation of the vector field X to nth order $X^{(n)}$ is

$$\begin{aligned} X^{(n)} &= X + \sum_{0<|l|\le n} \eta_l \frac{\partial}{\partial u_l} \\ &= \sum_{j=1}^{2} \xi^j (\frac{\partial}{\partial x^j} + \sum_{|l|\le n} u_{l+e_j}\frac{\partial}{\partial u_l} + \sum_{|l|\le n} D_l(\eta - \sum_{j=1}^{2} u_{e_j}\xi^j)\frac{\partial}{\partial u_l} \end{aligned} \tag{16}$$

where

$$D(\eta) = [D_j\bar{\eta}^i] = \begin{bmatrix} D_1\eta^1 & D_2\eta^1 \\ D_1\eta^2 & D_2\eta^2 \end{bmatrix}$$

When $\mathbf{R}[u]$ is defined as a ring of u and its derivative with respect to x and its higher orders : $\{u_0, u_1, \cdots, u_k, \cdots\}$.

On $\mathbf{R}[u]$, derivatives $D_{u_i} = u_{i+1}$ are defined.

$$u_t = H \to \partial_\eta(u_t - H) = 0$$

is the condition for ∂_η becomes the symmetry of the evolution equation.

When the new symmetry of the evolution equation contains non-trivial symmetry whose time evolution is different from the original time evolution is not known [78].

2.4 Hamiltonian of Faddeev and Takhtajan, and matrix solutions of solitonic wave equation

Propagation of solitonic waves and their orbits described by Hamiltonian is well formulated in the book of Faddeev and Takhtajan [65]. They considered solutions of a modified Nonlinear Schrödinger equation

$$i\frac{\partial\psi}{\partial t} = -\frac{\partial^2\psi}{\partial x^2} + 2\kappa(|\psi|^2 - \rho^2)\psi \tag{17}$$

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with the following three cases of boundary condition.

A) Rappidly decreasing boundary condition [66]: As $|x| \to \infty$ $\psi(x.t) \to 0$ and $\rho = 0$.

B) Finite density boundary condition: As $x \to \pm\infty$, $\psi(x,t) \to \rho e^{i\phi_\pm(t)}$.

C) Quasi periodic boundary condition: Smooth function ψ that satisfy $\psi(x+2L,t) = e^{i\theta}\psi(x,t)$, $0 \leq \theta < 2\pi$ is independent of (x,t).

The Riemann problem of matrix value function $G_\pm(x,\lambda)$, consists of factorizing the function

$$G_+(x,\lambda)G_-(x,\lambda) = G(x,\lambda)$$

where matrices $G_-(x,\lambda) = S_-(s,\lambda)E^{-1}(x,\lambda)$, $E(x,\lambda) = exp\{\frac{\lambda}{2i}x\sigma_3\}$ and $G_+(x,\lambda) = a(\lambda)E(x,\lambda){S_+}^{-1}(x,\lambda)$, where $detS_+(\lambda) = a(\lambda)$.

The matrices $S_\pm(x,\lambda)$ are defined by Jost solutions of the scattering problem in the space of Schwartz's rapidly decreasing function $T_\pm(x,\lambda)$

$$T_-(x,\lambda) = E(x,\lambda) + \int_{-\infty}^{x} E(x-z,\lambda)U_0(z)T_-(z,\lambda)dz \quad (18)$$

$$T_+(x,\lambda) = E(x,\lambda) - \int_{x}^{\infty} E(x-z,\lambda)U_0(z)T_+(z,\lambda)dz \quad (19)$$

as

$$S_+(x,\lambda) = T_+(x,\lambda)\begin{pmatrix} a(\lambda) & 0 \\ b(\lambda) & 1 \end{pmatrix} = T_-(x,\lambda)\begin{pmatrix} 1 & -\epsilon\bar{b}(\lambda) \\ 0 & a(\lambda) \end{pmatrix}, \quad (20)$$

$$S_-(x,\lambda) = T_+(x,\lambda)\begin{pmatrix} 1 & \epsilon\bar{b}(\lambda) \\ 0 & \bar{a}(\lambda) \end{pmatrix} = T_-(x,\lambda)\begin{pmatrix} \bar{a}(\lambda) & 0 \\ -b(\lambda) & 1 \end{pmatrix}, \quad (21)$$

where $\epsilon = sign\ \kappa$ which is introduced to make the pair relation

$$\bar{U}(x,\lambda) = \sigma U(x,\bar{\lambda})\sigma.$$

When $\kappa > 0$, $\sigma = \sigma_1$ and when $\kappa < 0$, $\sigma = \sigma_2$.

Reduced monodromy matrix $T(\lambda)$ is a $x \to \infty, y \to \infty$ limit of the adjoin matrix of $T(x,y,\lambda)$

$$T(\lambda) = \lim_{x\to\infty, y\to-\infty} E(-x,\lambda)T(x,y,\lambda)E(y,\lambda) \quad (22)$$

where

$$\begin{aligned} T(x,y,\lambda) &= T_+(x,\lambda)T_+^{-1}(y,\lambda) \\ &= T_-(x,\lambda)T_-^{-1}(y,\lambda) \end{aligned} \quad (23)$$

$$\begin{aligned} \frac{\partial F}{\partial x} &= U(x,t,\lambda)F, \\ \frac{\partial F}{\partial t} &= V(x,t,\lambda)F. \end{aligned} \quad (24)$$

Here

$$F = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \quad (25)$$

and

$$\begin{aligned} U &= U_0 + U_1 \\ &= \sqrt{\kappa}(\bar{\psi}\sigma_+ + \psi\sigma_-) + \frac{\lambda}{2i}\sigma_3, \end{aligned} \tag{26}$$

$$\begin{aligned} V &= V_0 + \lambda V_1 + \lambda^2 V_2 \\ &= i\kappa|\psi|^2\sigma_3 - i\sqrt{\kappa}(\frac{\partial\bar{\psi}}{\partial x}\sigma_+ - \frac{\partial\psi}{\partial x}\sigma_-) - \lambda U_0 - \lambda^2 U_1 \end{aligned} \tag{27}$$

where

$$\sigma_+ = \frac{\sigma_1 + i\sigma_2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \ \sigma_- = \frac{\sigma_1 - i\sigma_2}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \tag{28}$$

In the case of finite density boundary condition $V_\rho = V - i\kappa\rho^2\sigma_3$ should be adopted.

The matrices U and V satisfy zero-curvature condition

$$\frac{\partial U}{\partial t} - \frac{\partial V}{\partial x} + [U, V] = 0$$

Corresponding to the transformation due to local choice of the bases expressed by $G(x, t, \lambda)$

$$F(x, y, \lambda) \to \tilde{F}(x, t, \lambda) = G(x, t, \lambda)F(x, t, \lambda) \tag{29}$$

the connection coefficients $U(x, t, \lambda)$ and $V(x, t, \lambda)$ are gauge transformed as

$$\begin{aligned} U \to \tilde{U} &= \frac{\partial G}{\partial x}G^{-1} + GUG^{-1} \\ V \to \tilde{V} &= \frac{\partial G}{\partial t}G^{-1} + GVG^{-1}. \end{aligned} \tag{30}$$

Next one solves the inverse scattering problem [69, 70]

2.5 The Khokhlov-Zabolatskaya-Kuznetsov (KZK) equation

Propagation of sound beams in nonlinear media was described by KZK equation [76]

$$\begin{aligned} \frac{\partial^2 p}{\partial z \partial \tau} &= \frac{c_0}{2}\nabla^2 p \\ &+ \frac{\theta}{2{c_0}^3}\frac{\partial^2 p^2}{\partial \tau^2}(p_t - pp_x - \beta p_{xx})_x - \gamma\triangle_y p = 0 \end{aligned} \tag{31}$$

where the sound propagation along x and y axes are assumed to be order $\sqrt{\zeta}$ smaller than that along z axis, and expressed by coordinates

$$z' = \zeta z, x' = \sqrt{\zeta}x, y' = \sqrt{\zeta}y, \tau = t - \frac{z'}{c_0} \tag{32}$$

as

$$p_z = \zeta p_z(\tau, x', y', z'), \tag{33}$$

$$p_{x,y} = \zeta\sqrt{\zeta}p_{x,y}(\tau, x', y', z') \tag{34}$$

Similar equation was derived by Westervelt[77] and studied in the framework of nonlinear partial differential equation, as Burger's equation [78, 80].

When $\theta = 0$, the non-linear acoustic equations becomes KZ equation

$$\frac{\partial^2 p}{\partial z \partial \tau} = \frac{c_0}{2}\nabla_\perp^2 p + \frac{\beta}{2\rho_0 {c_0}^3}\frac{\partial^2 p^2}{\partial \tau^2} \tag{35}$$

where $\tau = t - \frac{z}{c_0}$, $z > 0$, and by choosing transverse coordinate (r, θ), the equation of longitudinal pressure $u = p/u_0$ reduces to

$$KZ(r, \theta, z) = N u_{\theta\theta}^2 - u_{\theta z} + u_{rr} + \frac{1}{r}u_r = 0 \tag{36}$$

Here the pressure is normalized by $u_0 = \frac{p_0}{\rho_0 c_0}$, and the density is expressed as $\rho = \rho_0 + \rho'$ with fluctuations ρ'.

Applying an a priori excitation condition as

$$u(r, \theta, z) = f(z)h(r, \theta, z), \tag{37}$$

$$u(r, \theta, z) = g(\theta)k(r, \theta, z) \tag{38}$$

In order to incorporate hysteretic behavior, Dos Santos et al. [51] defined normalized particle velocity $V = v/v_0$, where v_0 is the initial amplitude $v_0 = v_{max}(z = 0)$, θ is the retarded time normalized to the initial characteristic duration τ_0, $\theta = \tau/\tau_0$, and the characteristic length

$$z_{NL} = 2c_0^2\tau_0/hv_0$$

where h is the nondimensional parameter of hysteretic quadratic nonlinearity, and $\xi = z/z_{NL}$,

$$\frac{\partial V}{\partial \xi} + V\frac{\partial V}{\partial \theta} = 0, \quad \text{if} \quad \frac{\partial V}{\partial \theta} > 0 \tag{39}$$

$$\frac{\partial V}{\partial \xi} + (V_M - V)\frac{\partial V}{\partial \theta} = 0, \quad \text{if} \quad \frac{\partial V}{\partial \theta} < 0 \tag{40}$$

The solution of hysteretic equation $V(\xi, \theta)$ was assumed to have the form

$$V(\xi, \theta) = V_M(\xi)g(\xi, \theta)$$

and by studying associate invariant

$$\eta(\xi, V(\xi, \theta)) = (\beta + \alpha\xi)^{-\frac{\gamma}{\alpha}}V$$

where α, β and γ are defined from Lie reduction to the hysteretic equation.

The time evolution of waves characterized by $\pm\frac{\partial V}{\partial \xi}$ is one directional, since two signs of $\frac{\partial V}{\partial \theta}$ is due to directions of the wave propagation and $\xi = z/z_{NL}$ is independent of t. Lie group properties of Hysteretic equation can be applied as in the analysis of generalized Burgers equation [60], although they considered cases of $t > 0$ and different boundary conditions. The Jost solution contain both $t \geq 0$ and $t < 0$ cases.

3 Setup of transducers of excitation symmetry analysis (ESAM) in TR-NEWS

Concepts of excitation symmetry analysis method for calculation of higher order nonlinearity (ESAM) was proposed in [30], and the simulation and experiments were performed by Vejvodova, Prevorovsky and Dos Santos[26] by using nonlinear time reversal tomography of structural defects. In this experiment, five transducers $T_1, T_2, \cdots, T_5$, on a line separated by 1mm, two receivers R_1, R_2 on a line parallel to the line of transducers separated by 1mm, and two scatterers S_1, S_2 are placed inside a 10mm $\times$ 4mm rectangular plane.

The nonlinear response $y(t)$ is written as

$$y(t) = N_1 x(t) + N_2 x^2(t) + N_3 x^3(t) \tag{41}$$

Excitations x_E, x_ϵ and x_{ϵ^*} and corresponding response y_E, y_ϵ and y_{ϵ^*} are given by

$$x_E = x(t) \;\to\; y_E,\; x_\epsilon = x(t)e^{\frac{2i\pi}{3}} \to y_\epsilon,\; x_{\epsilon^*} = x(t)e^{-\frac{2i\pi}{3}} \to y_{\epsilon^*}.$$

$$N_3 x^3(t) = \frac{y_E + y_\epsilon + y_{\epsilon^*}}{3}. \tag{42}$$

Experimentally

$$x_E(t) = x(t) \to y_E,\; x_A(t) = -\frac{1}{2}x(t) \to y_A$$

$$x_{B1}(t) = \frac{\sqrt{3}}{2}x(t) \to y_{B1},\; x_{B2}(t) = -\frac{\sqrt{3}}{2}x(t) \to y_{B2}$$

were defined and N_1, N_2 and N_3 were calculated from responses as

$$s_3(t) = N_3 x^3(t) = \frac{4}{3}[y_E(t) + 2y_A(t) - y_{B1}(t) - y_{B2}(t)]$$

$$s_2(t) = N_2 x^2(t) = \frac{2}{3}[y_{B1}(t) + y_{B2}(t)]$$

$$s_1(t) = N_1 x(t) = y_E(t) - s_2(t) - s_3(t).$$

We define energies related to excitations

$$E_{01} = \int_{-\infty}^{\infty} |x(t)|^2 dt,\; E_{02} = \int_{-\infty}^{\infty} |x^2(t)|^2 dt,\; E_{03} = \int_{-\infty}^{\infty} |x^3(t)|^2 dt$$

$$E_1 = \int_{-\infty}^{\infty} |s_1(t)|^2 dt = N_1^2 E_{01},$$

$$E_2 = \int_{-\infty}^{\infty} |s_2(t)|^2 dt = N_2^2 E_{02},$$

$$E_3 = \int_{-\infty}^{\infty} |s_3(t)|^2 dt = N_3^2 E_{03}$$

Hence $N_1 = \sqrt{E_1/E_{01}}, N_2 = \sqrt{E_2/E_{02}}$ and $N_3 = \sqrt{E_3/E_{03}}$.

4 Multistep simulation of hysteresis using Adams-Bashforth method

Memristor is the fourth relation between circuit variables, current i, voltage v, charge q and flux ϕ, Resistor connects between v and i, Capacitor connects between v and q, Inductor connects between i and ϕ and Memristor connects between q and ϕ [2, 3, 47].

Main difference of Memristor from other three connections are nonlinearity and hysteresis. From Muthuswamy-Chua system, one can observe chaotic behavior [8].

Starting from non-linear Schrödinger equation, which is the generalized time-dependent Ginzburg-Landau equation [4]

$$i\psi_t + p_{xx} + q|\psi|^2\psi = -\gamma\psi$$

where $p = p_r + ip_i$ and $q = q_r + iq_i$ and $|p_r| = |q_r|$ are assumed to be 1, and $p_i < 0, q_i > 0$ and $\gamma > 0$, which provides unstable solitonic wave function, it may be possible to incorporate hysteretic effects of materials.

They considered three different states: 1) motionless state ($\psi = 0$), 2) periodic patterns described by finite-amplitude plane wave solution and 3) chaotic states.

The transition from a quiescent state to a periodic pattern was described by a shock solution:

$$\psi = \psi_0 e^{i(Kx-\Omega t)}\bar{\psi}(x - vt)$$

where $\Omega = p_r K^2 - q_r|\psi_0|^2$ and $|\psi_0|^2 = (\gamma + p_i K^2)/q_i$. K and v were chosen so that $|\bar{\psi}|$ vanishes at one infinity and approaches 1 at the other infinity.

Numerical integration was performed by means of the Adams-Bashforth method. In general form y_n is written as

$$\begin{aligned} y_n &= \alpha_1 y_{n-1} + \alpha_2 y_{n-2} + \cdots + \alpha_k y_{n-k} \\ &+ h(\beta_0 f(x_n, y_n) + \beta_1 f(x_{n-1}, y_{n-1}) + \cdots + \beta_k f(x_{n-k}, y_{n-k})). \end{aligned} \tag{44}$$

Approximate solution at x_n is defined as

$$y_n = y_{n-1} + h\beta_0 f(x_n, y_n) + \beta_1 f(x_{n-1}, y_{n-1}) + \cdots + \beta_k f(x_{n-k}, y_{n-k}) \tag{45}$$

and

$$f(x_n, y_n), f(x_{n-1}, y_{n-1}), \cdots, f(x_{n-k}, y_{n-k})$$

are approximated by

$$y'(x_n), y'(x_{n-1}), \cdots, y'(x_{n-k}).$$

Taylor series expansion about x_n becomes in the form

$$C_1 hy'(x_n) + C_2 h^2 y"(x_n) + \cdots + C_p h^p y^{(p)}(x_n) + O(h^{p+1}) \tag{46}$$

In the fourth order Adams-Bashforth method,

$$\begin{aligned} C_1 &= 1 - \beta_0 - \beta_1 - \beta_2 - \beta_3 - \beta_4 \\ C_2 &= -\frac{1}{2} + \beta_1 + 2\beta_2 + 3\beta_3 + 4\beta_4 \\ C_3 &= \frac{1}{6} - \frac{1}{2}(\beta_1 + 4\beta_2 + 9\beta_3 + 16\beta_4) \\ C_4 &= -\frac{1}{24} + \frac{1}{6}(\beta_1 + 8\beta_2 + 27\beta_3 + 64\beta_4) \end{aligned}$$

and conditions $C_1 = C_2 = C_3 = C_4 = 0$ gives $\beta_0 = 0, \beta_1 = \frac{55}{24}, \beta_2 = \frac{59}{24}, \beta_3 = \frac{37}{24}$, and $\beta_4 = -\frac{3}{8}$. In the 2nd order Adams-Bashforth, $\beta_1 = 3/2$ and $\beta_2 = -1/2$.

Test problem for getting stability region of differential equation $y' = qy$ is obyained by defining $z = hq$ and solving

$$(1 - z\beta_0)y_n - (\alpha_1 + z\beta_1)y_{n-1} - \cdots - (\alpha_k + z\beta_k)y_{n-k} = 0 \tag{47}$$

The auxiliary polynomial

$$\begin{aligned}\Phi(w,z) &= (1 - z\beta_0)w^k - (\alpha_1 + z\beta_1)w^{k-1} - \cdots - (\alpha_k + z\beta_k) \\ &= \rho(w) - z\sigma(w) \\ &= w^k(\alpha(w^{-1}) - z\beta(w^{-1}))\end{aligned} \tag{48}$$

We consider stable region in complex plane defined by $\Phi(w,z) = 0$ on the circle $|w| = 1$. $\alpha(w^{-1}) - z\beta(w^{-1}) = 0$ is equivalent to $\alpha(w) - z\beta(w) = 0$. As pointed out in [81], if $f(z)$ is meromorphic in a complex region Ω with the zeros a_j and poles b_k, then

$$\frac{1}{2\pi i}\int_\gamma \frac{f'(z)}{f(z)}dz = \sum_j n(\gamma, a_j) - \sum_k n(\gamma, b_k)$$

for every circle γ which is homologous to zero in Ω and does not pass through any of the zeros or poles [82]. Hence if $|\alpha(w) - z\beta(w)| < |\alpha(w)|$ on γ

$$|\frac{z\beta(w)}{\alpha(w)} - 1| < 1 \tag{49}$$

and the values of $F(w) = z\beta(w)/\alpha(w)$ on γ are contained in the open disk of center 1 and radius 1in the complex w plane, and for image cycle of γ denoted by Γ, $n(\gamma, 0) = 0$.

In the case of Adams-Bashforth method, β_0 is chosen to be 0, and $\beta(w) = \beta_1 w + \beta_2 w^2 + \cdots \beta_k w^k$, which becomes 0 at $w = 0$. Stability of solutions in linear multistep methods needs care. In the 4th order Adams-Bashforth method, "doubly unstable regions" in $Re(z) > 0$ symmetric in $Im(z) > 0$ and $Im(z) < 0$ appear in addition to stable and unstable regions as shown in Fig, 2 whose nature of the solution is not clear. In the 3rd order Adams-Bashforth method, stable regions appear close to the imaginary axis, $Re(z) > 0$ symmetric in $Im(z) > 0$ and $Im(w) < 0$. In the 2nd order Adams-Bashford method, only a stable region in $Re(z) \leq 0$ appear[81].

The coefficient β_0 of Adams-Moulton method is not zero, and stability region of the 3rd order Adams-Moulton method is close to that of the 3rd order Adams-Bashforth method. In the "doubly unstable" region observed in the 4th order Adams-bashforth method, transition from periodic pattern to chaos may occur spatially and temporally[4].

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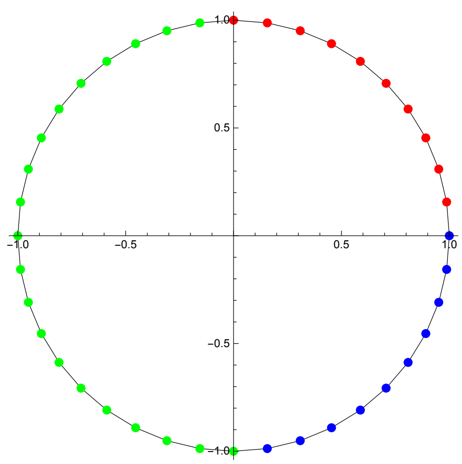

Figure 1: Points used in the Adams-Bashforth method in the complex plane of w.

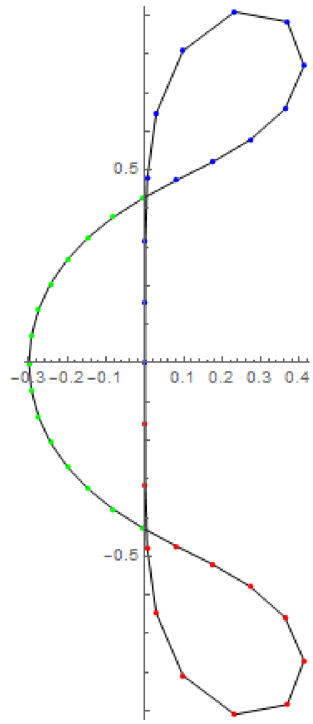

Figure 2: Stability regions in z plane of the 4th order Adams-Bashforth method. The region surrounded by the line with red points and the line with blue points are called doubly unstable.

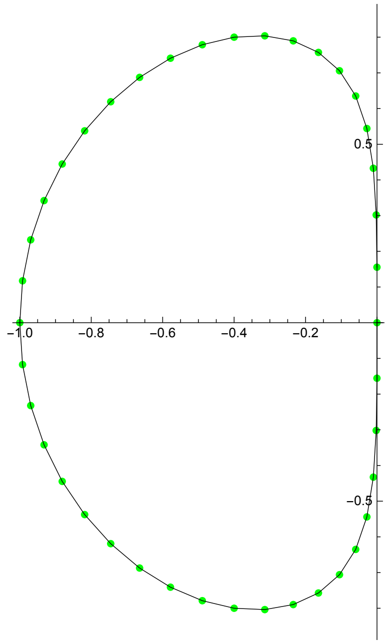

Figure 3: Stability region in z plane of the 2nd order Adams-Bashforth method.

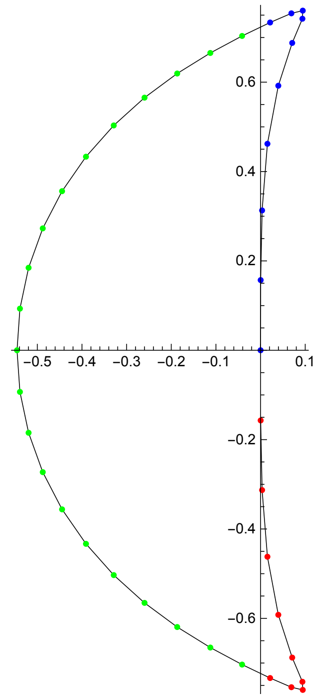

Figure 4: Stability regions in z plane of the 3rd order Adams-Bashforth method.

5 Simulation of ultrasound propagation in elastic media

In solid mechanics, strain vectors

$$\{\epsilon\}^T = [\epsilon_x, \epsilon_y, \epsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}]$$

and stress vectors

$$\{\sigma\}^T = [\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}]$$

are related by the generalized Hook's law.

In the case of axysymmetry about the z axis, elastic coefficients tensor in Voigt notation ($xx \to 1$, $yy \to 2$, $zz \to 3$, $yz, zy \to 4$, $xz, zx \to 5$ and $xy, yx \to 6$) has the general form[52, 53]

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \tag{50}$$

$$\{\sigma\} = [C]\{\epsilon\}$$

In numerical simulation of TR-NEWS, spacial evolution of the variable $y(x,t)$ and its partial derivative $\frac{\partial y(x,t)}{\partial x}$ was calculated by using Fast Fourier Transform (FFT) [52, 56, 59]

$$\frac{\partial y(x,t)}{\partial x} = FFT^{-1}[(ik_x e^{\pm ik_x}\frac{\Delta x}{2}.FFT[y(x,t)])], \tag{51}$$

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where k_x is the wave number of the wave along the x direction, and Δx is the spacial step of the numerical calculation. In the temporal domain $y(g(t),t)$ is defined as $\partial g/\partial t$, and $g(t)$ was numerically derived from

$$g(t+\Delta t) = g(t) + \frac{\Delta t}{24}[26g(t+\frac{\Delta t}{2}) - 5g(t-\frac{\Delta t}{2}) + 4g(t-\frac{3\Delta t}{2}) - g(t-\frac{5\Delta t}{2})]. \tag{52}$$

By definitions

$$\begin{aligned} g(t+\Delta t) &= y_n, \quad g(t) = y_{n-2}, \\ g(t+\frac{\Delta t}{2}) &= y_{n-1}, \quad g(t-\frac{\Delta t}{2}) = y_{n-3}, \\ g(t-\frac{3\Delta t}{2}) &= y_{n-5}, \quad g(t-\frac{5\Delta t}{2}) = y_{n-7}, \end{aligned} \tag{53}$$

the equation becomes

$$\begin{aligned} y_n - y_{n-2} &= \Delta t[\frac{26}{24}(y_{n-1} - y_{n-3}) + \frac{21}{24}(y_{n-3} - y_{n-5}) \\ &+ \frac{25}{24}(y_{n-5} - y_{n-7}) + y_{n-7}] = 0. \end{aligned} \tag{54}$$

Goursolle [33] solved numerically the sound waves equation

$$\begin{aligned} y_n - y_{n-2} &= \Delta t[\frac{26}{24}(y_{n-1} - y_{n-3}) + \frac{21}{24}(y_{n-3} - y_{n-5}) \\ &+ \frac{25}{24}(y_{n-5} - y_{n-7}) + y_{n-7}] = 0 \end{aligned} \tag{55}$$

which can be written as

$$v_n = \frac{1}{2}(\frac{26}{24}v_{n-1} + \frac{21}{24}v_{n-3} + \frac{25}{24}v_{n-5}) + \frac{1}{2\Delta t}y_{n-7} \tag{56}$$

where $v_n = \frac{y_n - y_{n-2}}{2\Delta t}$. The coordinate y can be derived by integrating v and choosing the arbitrary coordinate y_{n-7} to be 0.

The 4th order Adams-Bashforth method was applied in the multistep ultrasound simulation in active solid media[33, 52].

6 Discussion and Conclusion

The main difference of TR-NEWS approach and FDORT approach is direction of time evolution, and the boundary condition of the sound wave. Solutions of TR-NEWS correspond to solitons, in which time evolution is essential. Solutions of FDORT correspond to that of Chaos, in which phase correlation is not essential.

Zakharov [48] showed that self-focused light beams that are uniform along the propagation direction in a nonlinear transparent medium becomes unstable, or become Chaotic, but the Soliton solution of the Korteweg-de Vries (KdV) equation

$$u_t + 6uu_x + u_{xxx} = 0$$

is stable [49]. In the case of nonlinear Schrödinger equation [69], exact Soliton solution was obtained using the Hamiltonian formalism.

The KdV equation can be also solved exactly by direct method or Hirota method [72, 78].

On the stability of nonlinear partial differential equation, symmetry of Hmiltonians of solitons, i.e. Painlevé properties are well known [78, 79, 80]. Studies of integrable modified KdV soliton wave function on lattice [73, 74], and a study of singularity confinement in discrete systems was done by Hietarinta and Viallet [75]. Soliton solution of modified or discretized KdV equation [71] were studied by Singularity Confinement and Painlevé test [73, 74]. Propagation of localized finite-amplitude didturbance of solitary wave on slightly dissipation-perturbed KdV is numerically calculated in [68].

In Einstein's theory of special relativity, time distance between two events that occur at ξ^μ and $\xi^\mu + d\xi^\mu$ on one space-time S is [90]

$$d\tau = \frac{ds}{c} = \frac{1}{c}\sqrt{\eta_{\mu\nu}d\xi^\mu d\xi^\nu} = \sqrt{c^2 + \eta_{mn}\frac{d\xi^m}{dt}\frac{d\xi^n}{dt}}\frac{dt}{c} = \sqrt{1 - \frac{v^2}{c^2}}dt \tag{57}$$

where $v^m = d\xi^m/dt$.

In general relativity, on a general space-time $S|x$, where x^0 is time coordinate and x^1, x^2, x^3 are space coordinate

$$d\tau = \frac{ds}{c} = \frac{1}{c}\sqrt{g_{\mu\nu}dx^\mu dx^\nu} = \frac{1}{c}\sqrt{g_{mu\nu}\dot{x}^\mu(t)\dot{x}^\nu(t)}dt \tag{58}$$

The Lie group, that is used for propagation of sound in solids has time-reversal symmetry, and consequently the direct and time-reversed wave function are allowed. One could imagine the Minkowski space as three dimensional space with two directional time evolution $\mathbf{R}^3 \times \mathbf{R}^{1,1}$, however it is more natural to consider $\mathbf{R}^4 \times S^3$ for direct and $\mathbf{R}^4 \times S^{3\prime}$ for time-reversed solution [85, 86, 87].

In FDORT approach, boundary condition is taken to avoid Loschmidt paradox. The paradox comes from ignorance of ghost degrees of freedom in a local coordinate system. We do not think the Kirchhoff's diffraction is not applicable to low sound wave propagation, and we need to take into account the reduced monodromy term $T(\lambda)$.

The ghost degrees of freedom is important also in infrared Yang Mills equation that has the Becchi-Rouet-Stora-Tyntin (BRST) symmetry. The Coulomb gauge Yang Mills equation satisfying the BRST symmetry was studied by Zwanziger [83]. Kugo and Ojima [88] argued that the gluons and ghost do not appear in the physical spectrum if the BRST symmetry is preserved. Furui and Nakajima [84] performed lattice simulation of infra-red gluon propagator and ghost propagator, and observed Kugo-Ojima color confinement parameter c obtained by the simulation becomes close to the theoretical value 1 when unquenched lattice configurations are adopted.

In Blakiston's Gould Medical Dictionary [89], Hysteresis is defined as:

• In medicine, a delayed reaction in the formation of gels, as the reaction of a blood clot after coagulation

• In physics, the retention of a magnetic state of iron in a changing magnetic field

• In chemistry, the lag of a chemical system in attaining equilibrium.

The present work shows that the definition in medicine and in physics become closer.

Freeman Dyson, who contributed very much in the formulation of Quantum Field Theory as Feynman, Schwinger and Tomonaga, mentioned about collaboration with Vietnamese physicist Nguyen Huu Xuong, and wrote why they gave up particle physics, and he switched attention to astrophysics and Xuong switched attention to biology[91].

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