Understanding nuclear binding energy with nucleon mass difference via strong coupling constant and strong nuclear gravity

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Abstract: With reference to electromagnetic interaction and Abdus Salam's strong (nuclear) gravity, 1) Square root of 'reciprocal' of the strong coupling constant can be considered as the strength of nuclear elementary charge. 2) 'Reciprocal' of the strong coupling constant can be considered as the maximum strength of nuclear binding energy. 3) In deuteron, strength of nuclear binding energy is around unity and there exists no strong interaction in between neutron and proton. $G_s \cong 3.3293665\times10^{28}$ m^{3}kg^{-1}sec^{-2} being the nuclear gravitational constant, nuclear charge radius can be shown to be, $R_0 \cong 2G_s m_p/c^2 \cong 1.2392185$ fm. $e_s \cong \left(G_s m_p^2/\hbar c\right)e \cong 4.7203105\times10^{-19}$ C being the nuclear elementary charge, proton magnetic moment can be shown to be, $\mu_p \cong e_s\hbar/2m_p \cong eG_s m_p/2c \cong 1.488055\times10^{-26}$ J.T^{-1}. $\alpha_s \cong \left(\hbar c/G_s m_p^2\right)^2 \cong 0.1152072$ being the strong coupling constant, strong interaction range can be shown to be proportional to $\exp\left(1/\alpha_s^2\right)$. Interesting points to be noted are: An increase in the value of α_s helps in decreasing the interaction range indicating a more strongly bound nuclear system. A decrease in the value of α_s helps in increasing the interaction range indicating a more weakly bound nuclear system. One interesting approximation is $\left(m_p/m_e\right)^{10} \approx \exp\left(1/\alpha_s^2\right)$. From $Z \cong 30$ onwards, close to stable mass numbers, nuclear binding energy can be addressed with, $(B)_{A_s} \cong Z\times\left\{\left((1/\alpha_s)+1\right)+\sqrt{30}\right\}\left(m_n-m_p\right)c^2 \approx Z\times19.6$ MeV. To improve the accuracy, we tried to understand nuclear binding with two simple terms having a single energy coefficient of $\left[e_s^2/8\pi\varepsilon_0\left(G_s m_p/c^2\right)\right] \approx 10.06$ MeV. With further study, magnitude of the Newtonian gravitational constant can be estimated with nuclear elementary physical constants. One sample relation is, $\left(G_N/G_s\right) \cong \left(m_e/m_p\right)^{10}\left[G_s m_e^2/\hbar c\right]$ where G_N represents the Newtonian gravitational constant. Electroweak gravitational constant can be expressed as, $G_{ew} \cong \left(m_e/m_p\right)^{10} G_N$. G_F being the Fermi's weak coupling constant, we noticed that, $2G_s m_e/c^2 \cong \sqrt{G_F/\hbar c}$ and $G_F \cong 4G_{ew}\hbar^2/c^2$. Based on the estimated and recommended values of G_F, estimated average value of $G_N \cong 6.674224\times10^{-11}$ m^{3}kg^{-1}sec^{-2}. Finally it is possible to show that, $R_0 \cong \left(m_p/m_e\right)\sqrt{4G_{ew}\hbar/c^3}$ and $G_s \cong \sqrt{\left(G_{ew}\right)\left(\hbar c/m_e^2\right)}$.

Keywords: strong (nuclear) gravity, nuclear elementary charge, strong coupling constant, nuclear charge radius, beta stability line, nuclear binding energy, nucleon mass difference, Fermi's weak coupling constant, Newtonian gravitational constant, deuteron, interaction range, super heavy elements.

1. Introduction

Low energy nuclear scientists assume 'strong interaction' as a strange nuclear interaction associated with binding of protons and neutrons. High-energy nuclear scientists consider nucleons as composite states of quarks and try to understand the nature and strength of strong interaction [1] at sub nuclear level. Very unfortunate thing is that, strong interaction is mostly hidden at low energy scales in the form of 'residual nuclear force'. At this juncture, one important question to be answered and reviewed at the basic level is: How to understand nuclear

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interactions in terms of sub nuclear interactions? Unfortunately, the famous nuclear models like, Liquid drop model and Fermi's gas model [2-5] are lagging in answering this question. To find a way, we would like to suggest that, by considering 'square root' of reciprocal of the strong coupling constant' $(\alpha_s \cong 0.1186)$, as an index of strength of nuclear elementary charge, nuclear binding energy and nuclear stability can be understood. In this direction, we have developed interesting concepts and produced many semi empirical relations [6-12]. Even though it is in its budding stage, our model seems to be simple and realistic compared to the new integrated model proposed by N. Ghahramany et al [13,14]. It needs further study at a fundamental level.

2. About Strong (nuclear) gravity

Microscopic physics point of view, one very interesting concept is that- elementary particles can be considered as 'micro black holes'. 'Strong (nuclear) gravity' concept proposed by Abdus Salam, C. Sivaram, K.P. Sinha, K. Tennakone, Roberto Onofrio, O. F. Akinto and Farida Tahir [15-20], seems to be very attractive. The main object of unification is to understand the origin of elementary particles mass, (Dirac) magnetic moments and their forces. Right now and till today 'string theory' with 10 dimensions is not in a position to explain the unification of gravitational and non-gravitational forces. More clearly speaking it is not in a position to bring down the Planck scale to the nuclear size. The most desirable cases of any unified description are:

a) To implement gravity in microscopic physics and to estimate the magnitude of the Newtonian gravitational constant (G_N).
b) To develop a model of microscopic quantum gravity.
c) To simplify the complicated issues of known physics.
d) To predict new effects, arising from a combination of the fields inherent in the unified description.

3. About quantum chromo dynamics (QCD)

The modern theory of strong interaction is quantum chromo dynamics (QCD) [21]. It explores baryons and mesons in broad view with 6 quarks and 8 gluons. According to QCD, the four important properties of strong interaction are: 1) color charge; 2) confinement; 3) asymptotic freedom [22]; 4) short-range nature (<10^{-15} m). Color charge is assumed to be responsible for the strong force to act on quarks via the force carrying agent, gluon. Experimentally it is well established that, strength of strong force depends on the energy through the interaction or the distance between particles. At lower energies or longer distances: a) color charge strength increases; b) strong force becomes 'stronger'; c) nucleons can be considered as fundamental nuclear particles and quarks seem to be strongly bound within the nucleons leading to 'Quark confinement'. At high energies or short distances: a) color charge strength decreases; b) strong force gets 'weaker'; 3) colliding protons generate 'scattered free quarks leading to 'Quark Asymptotic freedom'. Based on these points, low energy nuclear scientists assume 'strong interaction' as a strange nuclear interaction associated with binding of nucleons. High-energy nuclear scientists consider nucleons as composite states of quarks and try to understand the nature and strength of strong interaction at sub nuclear level.

4. About the semi empirical mass formula

Let A be the total number of nucleons, Z the number of protons and N the number of neutrons. According to the semi-empirical mass formula [2,3,4], nuclear binding energy:

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} \quad (1)$$

Here $a_v \cong 15.78$ MeV = volume energy coefficient, $a_s \cong 18.34$ MeV = surface energy coefficient, $a_c \cong 0.71$ MeV = coulomb energy coefficient, $a_a \cong 23.21$ MeV = asymmetry energy coefficient and $a_p \cong 12.0$ MeV = pairing energy coefficient. If we consider the sum of the volume energy, surface energy, coulomb energy, asymmetry energy and pairing energy, then the picture of a nucleus as a drop of incompressible liquid roughly accounts for the observed variation of binding energy of the nucleus. By maximizing $B(A,Z)$ with respect to Z, one can find the number of protons Z of the stable nucleus of atomic weight A as,

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$$Z \approx \frac{A}{2+\left(a_c/2a_a\right)A^{2/3}} \text{ and } A-2Z \approx \frac{0.4A^2}{A+200} \quad (2)$$

By substituting the above value of Z back into B one obtains the binding energy as a function of the atomic weight, $B(A)$. Maximizing $B(A)/A$ with respect to A gives the nucleus which is most strongly bound or most stable.

5. Three simple assumptions

With reference to our recent paper publications and conference proceedings [6-12], [23-33], we propose the following three assumptions.

1) Nuclear gravitational constant is very large in such a way that,

$$R_0 \cong \frac{2G_s m_p}{c^2} \quad (3)$$

2) Strong coupling constant can be expressed with,

$$\alpha_s \cong \left(\frac{\hbar c}{G_s m_p^2}\right)^2 \quad (4)$$

3) There exists a strong elementary charge in such a way that,

$$e_s \cong \left(\frac{G_s m_p^2}{\hbar c}\right)e \cong \frac{e}{\sqrt{\alpha_s}} \quad (5)$$

Note: Considering the relativistic mass of proton, it is possible to show that, $\alpha_s \propto \left(\frac{1}{m_p}\right)^4 \left[1-\frac{v}{c}\right]^2$ where v can be considered as the speed of proton. Qualitatively, at higher energies, strength of strong interaction seems to decrease with speed of proton.

6. To fix the magnitudes of $(G_s, \alpha_s \text{ and } e_s)$

Considering neutron, proton and electron rest masses, and based on relation (6), proposed nuclear gravitational constant can be estimated. Based on that, other values can be estimated.

$$\begin{Bmatrix} G_s \cong 3.3293665\times10^{28} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2} \\ R_0 \cong \frac{2G_s m_p}{c^2} \cong 1.2392185 \text{ fm} \\ \alpha_s \cong 0.1152072 \\ e_s \cong 4.7203105\times10^{-19} \text{ C} \end{Bmatrix}$$

7. New concepts and semi empirical relations

We would like to suggest that,

1) Fine structure ratio can be addressed with,

$$\alpha \cong \left(\frac{e_s^2}{4\pi\varepsilon_0 G_s m_p^2}\right)\left(\frac{\hbar c}{G_s m_p^2}\right) \cong 7.297352533\times10^{-3}$$

2) Proton magnetic moment can be addressed with

$$\mu_p \cong \frac{e_s\hbar}{2m_p} \cong \frac{eG_s m_p}{2c} \cong 1.488055\times10^{-26} \text{ J.T}^{-1}$$

3) Neutron magnetic moment can be addressed with

$$\mu_n \cong \frac{(e_s-e)\hbar}{2m_n} \cong 9.816235\times10^{-27} \text{ J.T}^{-1}.$$

4) Nuclear unit radius can be expressed as,

$$R_0 \cong \frac{2G_s m_p}{c^2} \cong \left(\frac{e_s}{e}\right)\left\{\frac{\hbar}{m_p c}+\frac{\hbar}{m_n c}\right\}$$

5) Root mean square nuclear charge radii [33] can be addressed with,

$$R_{(Z,A)} \cong \left\{1-0.349\left(\frac{N-Z}{N}\right)\right\}N^{1/3}\times1.262 \text{ fm}$$
$$\cong \left\{Z^{1/3}+\left(\sqrt{Z(A-Z)}\right)^{1/3}\right\}\left(\frac{G_s m_p}{c^2}\right)$$

6) Nuclear potential energy can be understood with ,

$$\cong \frac{e_s^2}{4\pi\varepsilon_0\left(G_s m_p/c^2\right)} \cong 20.17225 \text{ MeV}$$

7) Close to stable mass numbers, nuclear binding energy can be understood with a single energy coefficient [30,31] ,

$$\frac{e^2 G_s m_p^3}{8\pi\varepsilon_0\hbar^2} \cong \frac{e_s e}{8\pi\varepsilon_0\left(\hbar/m_p c\right)} \cong \frac{e_s^2}{8\pi\varepsilon_0\left(G_s m_p/c^2\right)}$$
$$\cong 10.086124 \text{ MeV}$$

8) With reference to $(\hbar/2)$, a useful quantum energy constant can be expressed with,

$$E_{(\hbar/2)} \cong \left(\frac{e^2 G_s m_p^3}{4\pi\varepsilon_0(\hbar/2)^2}\right) \cong 80.6889925 \text{ MeV}$$

9) Close to magic and semi magic proton numbers [31], nuclear binding energy seems to approach

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$\left[2.531\left(n+\frac{1}{2}\right)\right]^2 10.09$ MeV where n = 0,1,2,3,... and $(m_n - m_p/m_e) = 2.531$.

10) Characteristic melting temperature associated with proton can be expressed with, $T_{proton} \cong \frac{\hbar c^3}{8\pi k_B G_s m_p} \cong 0.15\times 10^{12}$ K

11) Characteristic nuclear neutral mass unit [32] can be addressed with, $\sqrt{\frac{\hbar c}{G_s}} \cong 546.6365\ \text{MeV}/c^2$.

8. To fit neutron-proton mass difference

Neutron-proton mass difference can be understood with:

$$\left(\frac{m_n c^2 - m_p c^2}{m_e c^2}\right) \cong \ln\sqrt{\frac{E_{(\hbar/2)}}{m_e c^2}} \cong \ln\sqrt{\frac{4e^2 G_s m_p^3}{4\pi\varepsilon_0 \hbar^2 m_e c^2}} \quad (6)$$

9. To fit neutron life time

Neutron life time t_n can be understood with the following relation:

$$t_n \cong \exp\left(\frac{E_{(\hbar/2)}}{(m_n - m_p)c^2}\right)\times\left(\frac{\hbar}{m_n c^2}\right) \cong 871.62 \text{ sec} \quad (7)$$

This can be compared with recommended value [1] of the neutron life time, (880.2 ± 1.0) sec

10. Understanding beta stability line with respect to proton and electron specific charge ratios

Nuclear beta stability line can be addressed with a relation of the form [4],

$$\begin{aligned} A_s &\cong 2Z + s(2Z)^2 \cong 2Z + (4s)Z^2 \\ &\cong 2Z + 0.0064Z^2 \cong Z(2+kZ) \end{aligned} \quad (8)$$

where,

$$s \cong \left\{\left(\frac{e_s}{m_p}\right)\div\left(\frac{e}{m_e}\right)\right\} \cong \left(\frac{G_s m_p m_e}{\hbar c}\right) \cong 0.00160454$$

Based on relation (8), let, $4s = k = 0.0064182$

$$\left.\begin{aligned} &\text{A)}\ \frac{(A_s - 2Z)^2}{A_s} \cong k^2 A_s N_s \sqrt{Z} \\ &\text{B)}\ \frac{Z}{\sqrt{A_s - 2Z}} \cong \frac{1}{\sqrt{k}} \cong 4\pi \\ &\text{C)}\ \frac{N_s}{Z} \cong (1+kZ) \cong \sqrt{1+kA_s} \\ &\text{D)}\ A_s \cong \frac{(1+kZ)^2 - 1}{k} \end{aligned}\right\} \quad (9)$$

11. Nuclear binding energy at stable mass numbers

Interesting points to be noted are:

1. With reference to electromagnetic interaction, and based on proton number, $(1/\alpha_s) \cong 8.68$ can be considered as the maximum strength of nuclear binding energy.
2. $Z \approx 30$ seems to represent a characteristic reference number in understanding nuclear binding of light and heavy atomic nuclides.

Based on these points, at stable mass numbers of Z, nuclear binding energy can be expressed by the following simple empirical relation.

$$(B)_{A_s} \cong \gamma \times Z \times (m_n - m_p)c^2 \quad (10)$$

$$\left.\begin{aligned} &\text{If } (Z<30),\ \text{coefficient},\ \gamma \cong \left[\left(\frac{1}{\alpha_s}+1\right)+\sqrt{Z}\right] \\ &\text{If } (Z \geq 30),\ \gamma \cong \left[\left(\frac{1}{\alpha_s}+1\right)+\sqrt{30}\right] \cong 15.157 \\ &\text{and } 15.157\times 1.29333 \text{ MeV} \cong 19.6033 \text{ MeV} \end{aligned}\right\}$$

Thus, for, $(Z \geq 30)$

$$(B)_{A_s} \cong Z \times 19.6033 \text{ MeV} \quad (11)$$

See table 1. Close to the stable mass numbers, binding energy is estimated with relations (8) and (10) and compared with Semi empirical mass formula (SEMF). It needs further study with respect to its surprising results against a single energy coefficient. In this context, we tried to understand nuclear binding with two simple terms. See section -14.

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Above and below the stable mass number, binding energy can be approximately estimated with the following relation.

$$(B)_A \approx (B)_{A_s} - \left[\ln\left(\frac{A_s}{kZ}\right)\frac{(A_s - A)A_s}{A}\right](m_n - m_p)c^2 \quad (12)$$

It needs further study with reference to unstable nuclides . See table 2 for Z=50.

12. Very simple approach for understanding nuclear stability starting form Z=21 to 118

With this simple method, super heavy elements lower stable mass numbers can be estimated. With even-odd corrections, accuracy can be improved. For $(Z \geq 11)$,

$$A_s \cong \left[Z + \left(\frac{e_s}{e}\right)\right]^{1.2} \cong \left(Z + \sqrt{\frac{1}{\alpha_s}}\right)^{1.2} \cong (Z + 2.9462)^{1.2} \quad (13)$$

$$\text{where, } \left(\sqrt{\frac{1}{\alpha_s}}\right)^{\frac{1}{6}} \cong \left(\frac{e_s}{e}\right)^{\frac{1}{6}} \cong 1.19732 \cong 1.2$$

13. Understanding nuclear binding energy of Deuteron

If it is assumed that, there exists no strong interaction in between proton and neutron, nuclear binding of deuteron can be expressed as,

$$BE \text{ of } {}_1^2H \cong 2\times(m_n - m_p)c^2 \cong 2.59 \text{ MeV} \quad (14)$$

$$\text{where, } \begin{cases} \left(\frac{1}{\alpha_s} + 1\right) \cong 1 \\ \rightarrow \left(\frac{1}{\alpha_s} \rightarrow \left(\frac{e_s}{e}\right)^2 \rightarrow 0\right) \Rightarrow e_s \rightarrow 0 \end{cases}$$

This can be compared with the experimental value of 2.225 MeV.

14. Understanding nuclear binding energy with two terms (close to stable mass numbers)

Based on the new integrated model proposed by N. Ghahramany et al [13,14],

$$B(Z,N) = \left\{A - \left(\frac{(N^2 - Z^2) + \delta(N-Z)}{3Z} + 3\right)\right\}\frac{m_n c^2}{\gamma} \quad (15)$$

where, $\gamma =$ Adjusting coefficient $\approx$ (90 to 100).
if $N \neq Z$, $\delta(N-Z) = 0$ and if $N = Z$, $\delta(N-Z) = 1$.

Readers are encouraged to see references there in [13,14] for derivation part. Point to be noted is that, close to the beta stability line, $\left[\frac{N^2 - Z^2}{3Z}\right]$ takes care of the combined effects of coulombic and asymmetric effects. In this context, we would like suggest that,

$$\left.\begin{aligned} \frac{m_n c^2}{\gamma} &\cong \frac{m_n c^2}{(90 \text{ to } 100)} \cong \text{Constant} \\ &\cong \frac{e_s^2}{8\pi\varepsilon_0 \left(G_s m_p / c^2\right)} \cong 10.09\text{MeV} \end{aligned}\right\} \quad (16)$$

Proceeding further, with reference to relation (8), it is also possible to show that, for $Z \cong (40 \text{ to } 83)$, close to the beta stability line,

$$\left[\frac{N_s^2 - Z^2}{Z}\right] \cong kA_s Z \quad (17)$$

$$\left[\frac{N_s^2 - Z^2}{3Z}\right] \cong \frac{kA_s Z}{3} \quad (18)$$

Based on the above relations and close to the stable mass numbers of $(Z \approx 5 \text{ to } 118)$, with a common energy coefficient of 10.06 MeV, we would like to suggest two terms for fitting and understanding nuclear binding energy.

First term helps in **increasing** the binding energy and can be considered as,

$$\text{Term\_1} = A_s \times 10.06 \text{ MeV} \quad (19)$$

Second term helps in **decreasing** the binding energy and can be considered as,

$$\text{Term\_2} = \left(\frac{kA_s Z}{2.531} + 3.531\right) \times 10.06 \text{ MeV} \quad (20)$$

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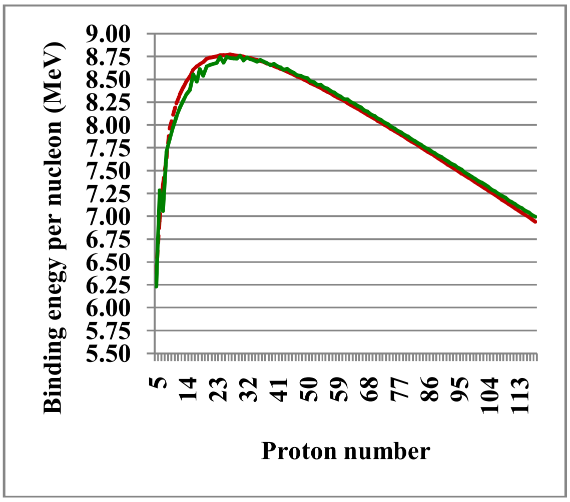
$$\text{where}\begin{cases}\left(\dfrac{(m_n-m_p)c^2}{m_e c^2}\right)\cong \ln\left(\dfrac{1}{\sqrt{k}}\right)\cong 2.531.\\ 3.531\cong 1+2.531\cong 1+\ln\left(\dfrac{1}{\sqrt{k}}\right)\end{cases}$$

Thus, binding energy can be fitted with,

$$B_{A_s}\cong\left\{A_s-\left(\frac{kA_sZ}{2.531}+3.531\right)\right\}\times 10.06\text{ MeV} \qquad (21)$$

See the following figure 1. Dotted red curve plotted with relations (8) and (21) can be compared with the green curve plotted with the standard semi empirical mass formula (SEMF).

Figure 1: Binding energy per nucleon close to stable mass numbers of Z = 5 to 118



15. To fix the magnitude of Fermi's weak coupling constant

With trial-error we noticed that,

$$\left.\begin{array}{l}R_0\cong\dfrac{2G_s m_p}{c^2}\cong\left(\dfrac{m_p}{m_e}\right)\sqrt{\dfrac{G_F}{\hbar c}}\\ \to\left(\dfrac{2G_s m_e}{c^2}\right)\cong\sqrt{\dfrac{G_F}{\hbar c}}\end{array}\right\} \qquad (22)$$

where G_F is the Fermi's weak coupling constant [1,19] and $\sqrt{\dfrac{G_F}{\hbar c}}\cong$ Characteristic electroweak length.

Based on this relation,

$$\alpha_s G_F\cong\frac{4\hbar^3 m_e^2}{m_p^4 c} \qquad (23)$$

$$\left.\begin{array}{l}G_F\cong\left(\dfrac{1}{\alpha_s}\right)\dfrac{4\hbar^3 m_e^2}{m_p^4 c}\cong\dfrac{4G_s^2 m_e^2\hbar}{c^3}\\ \cong\hbar c\left(\dfrac{2G_s m_e}{c^2}\right)^2\cong 1.4400414\times10^{-62}\text{ J.m}^3\end{array}\right\} \qquad (24)$$

Recommended value of $G_F\cong 1.43586\times10^{-62}$ J.m^3. It may be noted that, relations (23) and (24) seem to play a key role in understanding the basics of final unification and needs further study.

16. To fix the magnitude of Newtonian Gravitational constant

With reference to Planck scale and considering the following semi empirical relation, magnitude of the Newtonian gravitational constant (G_N) can be fitted [23, 34].

$$\left(\frac{m_p}{m_e}\right)\cong\left(\frac{G_s m_p^2}{\hbar c}\times\frac{G_s}{G_N}\right)^{\frac{1}{12}}\cong\left(\frac{e_s G_s}{eG_N}\right)^{\frac{1}{12}} \qquad (25)$$

Based on relations (22) to (25),

$$\left(\frac{G_s}{G_N}\right)\cong\sqrt{\alpha_s}\left(\frac{m_p}{m_e}\right)^{12}\cong\sqrt{\frac{4\hbar^3 m_e^2}{m_p^4 c F_W}}\left(\frac{m_p}{m_e}\right)^{12} \qquad (26)$$

$$\left.\begin{array}{l}\left(\dfrac{G_N}{G_s}\right)\cong\dfrac{1}{2}\left(\dfrac{m_e}{m_p}\right)^{10}\left[\sqrt{\dfrac{G_F}{\hbar c}}\Big/\left(\dfrac{\hbar}{m_e c}\right)\right]\\ \to\begin{cases}G_N\cong\left(\dfrac{m_e}{m_p}\right)^{10}\left[\dfrac{G_s m_e^2}{\hbar c}\right]G_s\\ G_s\cong\left(\dfrac{m_p}{m_e}\right)^{5}\sqrt{\dfrac{G_N\hbar c}{m_e^2}}\end{cases}\end{array}\right\} \qquad (27)$$

where $\dfrac{\hbar}{m_e c}\cong$ Compton wavelength of electron. Based on the recommended and estimated values of G_F,

$G_N\cong 6.66937197\times10^{-11}$ m^{3}kg^{-1}sec^{-2} where $G_F\cong 1.43586\times10^{-62}$ J.m^3
$G_N\cong 6.679076\times10^{-11}$ m^{3}kg^{-1}sec^{-2} where $G_F\cong 1.440414\times10^{-62}$ J.m^3

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Average value of $G_N \cong 6.674224\times10^{-11}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$.

In terms of nuclear charge radius,

$$G_N \cong \frac{1}{4}\left(\frac{m_e}{m_p}\right)^{11}\sqrt{\frac{c^5 G_F R_0^2}{\hbar^3}} \quad (28)$$

Accuracy of (G_N) seems to depend on $(G_s, R_0, \alpha_s, G_F)$.

17. To fix the magnitude of electroweak gravitational constant

According to Roberto Onofrio [19], electro weak scale gravitational constant is roughly 10^{33} times the Newtonian gravitational constant. In this context, we would like suggest that,

$$\frac{G_{ew}}{G_N} \cong \left(\frac{m_p}{m_e}\right)^{10} \quad (29)$$

where $G_{ew} \cong 2.90723\times10^{22}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$ can be considered as the electroweak gravitational constant. Based on this idea,

$$R_0 \cong \left(\frac{m_p}{m_e}\right)\sqrt{\frac{4G_{ew}\hbar}{c^3}} \cong \left(\frac{m_p}{m_e}\right)^{11}\sqrt{\frac{4G_N\hbar}{c^3}} \quad (30)$$

where $\sqrt{\frac{4G_{ew}\hbar}{c^3}}$ can be called as the electroweak Planck length.

$$G_F \cong \hbar c\left(\frac{4G_{ew}\hbar}{c^3}\right) \cong \frac{4G_{ew}\hbar^2}{c^2} \quad (31)$$

Based on relation (27),

$$G_s \cong \left(\frac{m_p}{m_e}\right)^5\sqrt{\frac{G_N\hbar c}{m_e^2}} \cong \sqrt{(G_{ew})\left(\frac{\hbar c}{m_e^2}\right)} \quad (32)$$

Characteristic electroweak mass and its Schwarzschild radius can be expressed as,

$$M_{ew} \cong \sqrt{\frac{\hbar c}{G_{ew}}} \cong 584.983\ \text{GeV}/c^2 \quad (33)$$

$$\frac{2G_{ew}M_{ew}}{c^2} \cong \sqrt{\frac{4G_{ew}\hbar}{c^3}} \cong 6.74642\times10^{-19}\ \text{m} \quad (34)$$

$$\frac{M_{ew}}{m_p} \cong \sqrt{\frac{\hbar c}{G_{ew}m_p^2}} \cong \frac{\hbar c}{G_s m_p m_e} \quad (35)$$

$$\frac{M_{ew}}{m_e} \cong \frac{G_s}{G_{ew}} \quad (36)$$

18. To understand the range of strong interaction

One strange approximation is,

$$\left(\frac{m_p}{m_e}\right)^{10} \approx \exp\left(\frac{1}{\alpha_s^2}\right) \quad (37)$$

$$4.356\times10^{32} \approx 5.259\times10^{32}$$

Based on above relations, strong interaction range can be understood with the following relation.

$$R_0 \cong \exp\left(\frac{1}{\alpha_s^2}\right)\left\{\left(\frac{m_p}{m_e}\right)\left(\frac{4G_N\hbar}{c^3}\right)\sqrt{\frac{\hbar c}{G_F}}\right\} \quad (38)$$

It seems interesting to infer that,

a) $\left(\frac{1}{\alpha_s^2}\right)$ and $\exp\left(\frac{1}{\alpha_s^2}\right)$ play a crucial role in deciding the strong interaction range.

b) An increase in the value of α_s helps in decreasing the interaction range. This may be an indication of more strongly bound nuclear system.

c) A decrease in the value of α_s helps in increasing the interaction range. This may be an indication of more weakly bound nuclear system.

d) Poportionality constant being $\exp\left(\frac{1}{\alpha_s^2}\right)$,

$$\left.\begin{aligned} R_0 &\propto \left(\frac{m_p}{m_e}\right) \\ R_0 &\propto \left(\frac{4G_N\hbar}{c^3}\right) \\ R_0 &\propto 1\Big/\sqrt{\frac{G_F}{\hbar c}} \end{aligned}\right\}$$

19. Conclusion

Even though our approach to nuclear physics seems to be speculative, proposed assumptions show a wide range of applications embedded with in-depth physical meaning connected with low energy nuclear physics and high energy nuclear physics. With reference to the famous semi empirical mass formula having 5 different energy terms and 5 different energy coefficients, qualitatively and quantitatively, our proposed relations (8), (10) and (21) are very simple to follow and a special study seems to be required for understanding the binding energy of isotopes above and below the stability line. We are working in this direction.

With further research, current nuclear models and strong interaction concepts can be studied in a unified manner with respect to strong nuclear gravity. Finally, value of the Newtonian gravitational constant can be estimated with nuclear elementary physical constants.

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Table 1: Estimated nuclear binding close to stable mass numbers

Proton number	Est. Mass number close to stability	Neutron number	Value of γ	Est. BE (MeV)	SEMF BE (MeV)	Error (MeV)
2	4	2	11.09	28.7	22.0	-6.7
3	6	3	11.41	44.3	26.9	-17.4
4	8	4	11.68	60.4	52.9	-7.6
5	10	5	11.92	77.1	62.3	-14.8
6	12	6	12.13	94.1	87.4	-6.7
7	14	7	12.33	111.6	98.8	-12.8
8	16	8	12.51	129.4	123.2	-6.2
9	19	10	12.68	147.6	148.9	1.3
10	21	11	12.84	166.1	167.5	1.4
11	23	12	13.00	184.9	186.1	1.2
12	25	13	13.14	204.0	204.7	0.7
13	27	14	13.29	223.4	223.2	-0.2
14	29	15	13.42	243.0	241.6	-1.4
15	31	16	13.55	262.9	260.0	-2.9
16	34	18	13.68	283.1	290.8	7.7
17	36	19	13.80	303.5	305.1	1.6
18	38	20	13.92	324.1	327.2	3.1
19	40	21	14.04	345.0	341.5	-3.5
20	43	23	14.15	366.1	371.6	5.5

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21	45	24	14.26	387.4	389.6	2.2
22	47	25	14.37	408.9	407.5	-1.4
23	49	26	14.48	430.6	425.2	-5.4
24	52	28	14.58	452.5	454.6	2.0
25	54	29	14.68	474.7	468.9	-5.8
26	56	30	14.78	497.0	489.6	-7.4
27	59	32	14.88	519.5	515.2	-4.3
28	61	33	14.97	542.2	532.5	-9.7
29	63	34	15.07	565.0	549.7	-15.4
30	66	36	15.16	588.1	577.9	-10.2
31	68	37	15.16	607.7	592.0	-15.7
32	71	39	15.16	627.3	619.8	-7.5
33	73	40	15.16	646.9	636.6	-10.3
34	75	41	15.16	666.5	653.3	-13.2
35	78	43	15.16	686.1	677.9	-8.2
36	80	44	15.16	705.7	697.0	-8.7
37	83	46	15.16	725.3	721.3	-4.0
38	85	47	15.16	744.9	737.6	-7.3
39	88	49	15.16	764.5	761.6	-2.9
40	90	50	15.16	784.1	780.2	-3.9
41	93	52	15.16	803.7	803.9	0.2
42	95	53	15.16	823.3	819.7	-3.6
43	98	55	15.16	842.9	843.2	0.2
44	100	56	15.16	862.5	861.2	-1.3
45	103	58	15.16	882.1	884.4	2.2
46	106	60	15.16	901.7	909.6	7.9
47	108	61	15.16	921.3	922.7	1.4
48	111	63	15.16	940.9	947.6	6.7
49	113	64	15.16	960.5	962.8	2.3
50	116	66	15.16	980.2	987.5	7.3
51	119	68	15.16	999.8	1009.7	9.9
52	121	69	15.16	1019.4	1024.6	5.2
53	124	71	15.16	1039.0	1046.5	7.6
54	127	73	15.16	1058.6	1070.4	11.9
55	129	74	15.16	1078.2	1085.1	6.9
56	132	76	15.16	1097.8	1108.7	11.0
57	135	78	15.16	1117.4	1130.1	12.7
58	138	80	15.16	1137.0	1153.3	16.3
59	140	81	15.16	1156.6	1165.6	9.0
60	143	83	15.16	1176.2	1188.5	12.3
61	146	85	15.16	1195.8	1209.3	13.5
62	149	87	15.16	1215.4	1231.9	16.5
63	151	88	15.16	1235.0	1245.9	10.9
64	154	90	15.16	1254.6	1268.2	13.6
65	157	92	15.16	1274.2	1288.4	14.2
66	160	94	15.16	1293.8	1310.4	16.6
67	163	96	15.16	1313.4	1330.4	17.0
68	166	98	15.16	1333.0	1352.0	19.0
69	169	100	15.16	1352.6	1371.7	19.1
70	171	101	15.16	1372.2	1385.1	12.9
71	174	103	15.16	1391.8	1404.5	12.7
72	177	105	15.16	1411.4	1425.7	14.2
73	180	107	15.16	1431.0	1444.8	13.8
74	183	109	15.16	1450.6	1465.7	15.0

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75	186	111	15.16	1470.2	1484.6	14.3
76	189	113	15.16	1489.8	1505.1	15.3
77	192	115	15.16	1509.4	1523.7	14.3
78	195	117	15.16	1529.0	1544.0	14.9
79	198	119	15.16	1548.6	1562.4	13.7
80	201	121	15.16	1568.2	1582.3	14.1
81	204	123	15.16	1587.8	1600.5	12.6
82	207	125	15.16	1607.4	1620.2	12.7
83	210	127	15.16	1627.0	1638.1	11.0
84	213	129	15.16	1646.7	1657.5	10.8
85	216	131	15.16	1666.3	1675.2	8.9
86	219	133	15.16	1685.9	1694.3	8.5
87	223	136	15.16	1705.5	1718.6	13.1
88	226	138	15.16	1725.1	1737.5	12.4
89	229	140	15.16	1744.7	1754.6	10.0
90	232	142	15.16	1764.3	1773.2	9.0
91	235	144	15.16	1783.9	1790.2	6.3
92	238	146	15.16	1803.5	1808.5	5.1
93	241	148	15.16	1823.1	1830.2	7.1
94	245	151	15.16	1842.7	1848.3	5.6
95	248	153	15.16	1862.3	1864.8	2.5
96	251	155	15.16	1881.9	1882.6	0.7
97	254	157	15.16	1901.5	1898.9	-2.6
98	258	160	15.16	1921.1	1922.7	1.6
99	261	162	15.16	1940.7	1938.7	-2.0
100	264	164	15.16	1960.3	1956.1	-4.2

Table 2: Estimated approximate nuclear binding of isotopes of Z=50

Proton number	Mass number	Neutron number	Est. BE (MeV)	SEMF BE (MeV)	Error (MeV)
50	100	50	838.8	809.3	-29.5
50	101	51	848.9	822.3	-26.7
50	102	52	858.9	837.2	-21.7
50	103	53	868.6	849.2	-19.4
50	104	54	878.2	863.2	-15.0
50	105	55	887.6	874.5	-13.1
50	106	56	896.8	887.6	-9.2
50	107	57	905.8	898.1	-7.8
50	108	58	914.7	910.4	-4.3
50	109	59	923.4	920.1	-3.3
50	110	60	932.0	931.8	-0.2
50	111	61	940.4	940.7	0.4
50	112	62	948.6	951.6	3.0
50	113	63	956.7	960.0	3.3
50	114	64	964.7	970.2	5.5
50	115	65	972.5	977.9	5.4
50	116	66	980.2	987.5	7.3
50	117	67	987.7	994.6	6.8
50	118	68	995.1	1003.5	8.4
50	119	69	1002.4	1010.1	7.6
50	120	70	1009.6	1018.5	8.9

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50	121	71	1016.7	1024.4	7.8
50	122	72	1023.6	1032.3	8.7
50	123	73	1030.5	1037.7	7.3
50	124	74	1037.2	1045.1	7.9
50	125	75	1043.8	1050.1	6.3
50	126	76	1050.3	1056.9	6.6
50	127	77	1056.7	1061.4	4.7
50	128	78	1063.0	1067.8	4.8
50	129	79	1069.2	1071.8	2.6
50	130	80	1075.3	1077.7	2.4
50	131	81	1081.3	1081.4	0.0
50	132	82	1087.3	1086.9	-0.4
50	133	83	1093.1	1090.1	-3.1
50	134	84	1098.9	1095.1	-3.7
50	135	85	1104.5	1098.0	-6.6
50	136	86	1110.1	1102.7	-7.5
50	137	87	1115.6	1105.1	-10.5
50	138	88	1121.0	1109.4	-11.6
50	139	89	1126.4	1111.5	-14.9
50	140	90	1131.6	1115.5	-16.2