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Understanding nuclear binding energy with nucleon mass difference via strong coupling constant and strong nuclear gravity

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Abstract: With reference to electromagnetic interaction and Abdus Salam's strong (nuclear) gravity, 1) Square root of 'reciprocal' of the strong coupling constant can be considered as the strength of nuclear elementary charge. 2) 'Reciprocal' of the strong coupling constant can be considered as the maximum strength of nuclear binding energy. 3) In deuteron, strength of nuclear binding energy is around unity and there exists no strong interaction in between neutron and proton. $G_s \cong 3.32688\times10^{28}\ \mathrm{m^3kg^{-1}sec^{-2}}$ being the nuclear gravitational constant, nuclear charge radius can be shown to be, $R_0 \cong \frac{2G_s m_p}{c^2} \cong 1.24$ fm. $e_s \cong \left(\frac{G_s m_p^2}{\hbar c}\right) e \cong 4.716785\times10^{-19}\,\mathrm{C}$ being the nuclear elementary charge, proton magnetic moment can be shown to be, $\mu_p \cong \frac{e_s\hbar}{2m_p} \cong \frac{eG_s m_p}{2c} \cong 1.48694\times10^{-26}\ \mathrm{J.T^{-1}}$. $\alpha_s \cong \left(\frac{\hbar c}{G_s m_p^2}\right)^2 \cong 0.1153795$ being the strong coupling constant, strong interaction range can be shown to be proportional to $\exp\left(\frac{1}{\alpha_s^2}\right)$. Interesting points to be noted are: An increase in the value of α_s helps in decreasing the interaction range indicating a more strongly bound nuclear system. A decrease in the value of α_s helps in increasing the interaction range indicating a more weakly bound nuclear system. From $Z \cong 30$ onwards, close to stable mass numbers, nuclear binding energy can be addressed with, $(B)_{A_s} \cong Z\times\left\{\left(\frac{1}{\alpha_s}+1\right)+\sqrt{\sqrt{30\times31}}\right\}(m_n-m_p)c^2 \approx Z\times19.66$ MeV. With further study, magnitude of the Newtonian gravitational constant can be estimated with nuclear elementary physical constants. One sample relation is, $\left(\frac{G_N}{G_s}\right) \cong \frac{1}{2}\left(\frac{m_e}{m_p}\right)^{10}\left[\sqrt{\frac{G_F}{\hbar c}}\bigg/\left(\frac{\hbar}{m_e c}\right)\right]$ where G_N represents the Newtonian gravitational constant and G_F represents the Fermi's weak coupling constant. Two interesting coincidences are, $(m_p/m_e)^{10} \cong \exp(1/\alpha_s^2)$ and $2G_s m_e/c^2 \cong \sqrt{G_F/\hbar c}$.

Keywords: strong (nuclear) gravity, nuclear elementary charge, strong coupling constant, nuclear charge radius, beta stability line, nuclear binding energy, nucleon mass difference, Fermi's weak coupling constant, Newtonian gravitational constant, deuteron, interaction range, super heavy elements.

1. Introduction

Low energy nuclear scientists assume 'strong interaction' as a strange nuclear interaction associated with binding of protons and neutrons. High-energy nuclear scientists consider nucleons as composite states of quarks and try to understand the nature and strength of strong interaction [1] at sub nuclear level. Very unfortunate thing is that, strong interaction is mostly hidden at low energy scales in the form of 'residual nuclear force'. At this juncture, one important question to be answered and reviewed at the basic level is: How to understand nuclear interactions in terms of sub nuclear interactions? Unfortunately, the famous nuclear models like, Liquid drop model and Fermi's gas model [2-5] are lagging in answering this question. To find a way, we would like to suggest that, by considering 'square root' of reciprocal of the strong coupling constant'

$(\alpha_s \cong 0.1186)$, as an index of strength of nuclear elementary charge, nuclear binding energy and nuclear stability can be understood. In this direction, we have developed interesting concepts and produced many semi empirical relations [6-12]. Even though it is in its budding stage, our model seems to be simple and realistic compared to the new integrated model proposed by N. Ghahramany et al [13,14]. It needs further study at a fundamental level.

2. About Strong (nuclear) gravity

Microscopic physics point of view, one very interesting concept is that- elementary particles can be considered as 'micro black holes'. 'Strong (nuclear) gravity' concept proposed by Abdus Salam, C. Sivaram, K.P. Sinha, K. Tennakone, Roberto Onofrio, O. F. Akinto and Farida Tahir [15-20], seems to be very attractive. The main object of unification is to understand the origin of elementary particles mass, (Dirac) magnetic moments and their forces. Right now and till today 'string theory' with 10 dimensions is not in a position to explain the unification of gravitational and non-gravitational forces. More clearly speaking it is not in a position to bring down the Planck scale to the nuclear size. The most desirable cases of any unified description are:

a) To implement gravity in microscopic physics and to estimate the magnitude of the Newtonian gravitational constant (G_N).
b) To develop a model of microscopic quantum gravity.
c) To simplify the complicated issues of known physics.
d) To predict new effects, arising from a combination of the fields inherent in the unified description.

3. About quantum chromo dynamics (QCD)

The modern theory of strong interaction is quantum chromo dynamics (QCD) [21]. It explores baryons and mesons in broad view with 6 quarks and 8 gluons. According to QCD, the four important properties of strong interaction are: 1) color charge; 2) confinement; 3) asymptotic freedom [22]; 4) short-range nature (<10^{-15} m). Color charge is assumed to be responsible for the strong force to act on quarks via the force carrying agent, gluon. Experimentally it is well established that, strength of strong force depends on the energy through the interaction or the distance between particles. At lower energies or longer distances: a) color charge strength increases; b) strong force becomes 'stronger'; c) nucleons can be considered as fundamental nuclear particles and quarks seem to be strongly bound within the nucleons leading to 'Quark confinement'. At high energies or short distances: a) color charge strength decreases; b) strong force gets 'weaker'; 3) colliding protons generate 'scattered free quarks leading to 'Quark Asymptotic freedom'. Based on these points, low energy nuclear scientists assume 'strong interaction' as a strange nuclear interaction associated with binding of nucleons. High-energy nuclear scientists consider nucleons as composite states of quarks and try to understand the nature and strength of strong interaction at sub nuclear level.

With reference to the picture of 'Strong (nuclear) gravity'[15-20], if $G_f \approx 10^{38} G_N$,

1) Schwarzschild radius of nucleon mass can be addressed with, $R_0 \approx \frac{2G_f m_p}{c^2} \approx 1.2$ fm .
2) Strong coupling constant can be expressed with $$\alpha_s \approx \left(\frac{\hbar c}{G_f m_p^2}\right)^2 \approx 0.115$$
3) Characteristic temperature associated with nucleon can be expressed with, $$T_{proton} \approx \frac{\hbar c^3}{8\pi k_B G_f m_p} \approx 10^{12}\ \text{K}$$

Note: Considering the relativistic mass of proton, it is possible to show that, $\alpha_s \propto \left(\frac{1}{m_p}\right)^4 \left[1-\frac{v}{c}\right]^2$ where v can be considered as the speed of proton. Qualitatively, at higher energies, strength of strong interaction seems to decrease with speed of proton.

4. About the semi empirical mass formula

Let A be the total number of nucleons, Z the number of protons and N the number of neutrons. According to the semi-empirical mass formula [2,3,4], nuclear binding energy:

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} \quad (1)$$

Here a_v = volume energy coefficient, a_s is the surface energy coefficient, a_c is the coulomb energy coefficient, a_a is the asymmetry energy coefficient and a_p is the pairing energy coefficient. If we consider the sum of the volume energy, surface energy, coulomb energy, asymmetry energy and pairing energy, then the picture of a nucleus as a drop of incompressible liquid roughly accounts for the observed variation of binding energy of the nucleus. By maximizing $B(A,Z)$ with respect to Z, one can find the number of protons Z of the stable nucleus of atomic weight A as,

$$Z \approx \frac{A}{2+(a_c/2a_a)A^{2/3}} \text{ and } A-2Z \approx \frac{0.4A^2}{A+200} \quad (2)$$

By substituting the above value of Z back into B one obtains the binding energy as a function of the atomic weight, $B(A)$. Maximizing $B(A)/A$ with respect to A gives the nucleus which is most strongly bound or most stable.

5. Three simple assumptions

With reference to our recent paper publications and conference proceedings [6-12], [23-33], we propose the following three assumptions.

1) Nuclear gravitational constant is very large in such a way that, $R_0 \cong \frac{2G_s m_p}{c^2} \approx 1.25$ fm
2) Strong coupling constant can be expressed with,
$$\alpha_s \cong \left(\frac{\hbar c}{G_s m_p^2}\right)^2 \approx 0.115$$
3) There exists a strong elementary charge in such a way that, $e_s \cong \left(e/\sqrt{\alpha_s}\right) \approx 4.7 \times 10^{-19}$ C

For detailed information, readers are encouraged to refer authors recent publications [23-33].

6. To fix the magnitudes of $(G_s, \alpha_s \text{ and } e_s)$

To fix the magnitudes of $(G_s, \alpha_s \text{ and } e_s)$, we assume that,

$$R_0 \cong \frac{2G_s m_p}{c^2} \cong 1.24 \text{ fm} \quad (3)$$

Based on this relation,

$$G_s \cong \left(\frac{R_0 c^2}{2m_p}\right) \cong 3.32688 \times 10^{28} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2} \quad (4)$$

$$\alpha_s \cong \left(\frac{\hbar c}{G_s m_p^2}\right)^2 \cong 0.1153795 \quad (5)$$

$$e_s \cong \frac{e}{\sqrt{\alpha_s}} \cong \left(\frac{G_s m_p^2}{\hbar c}\right) e \cong 4.716785 \times 10^{-19} \text{C} \quad (6)$$

7. New concepts and semi empirical relations

We would like to suggest that,

1) Fine structure ratio can be addressed with,
$$\alpha \cong \left(\frac{e_s^2}{4\pi\varepsilon_0 G_s m_p^2}\right)\left(\frac{\hbar c}{G_s m_p^2}\right) \cong 7.297348 \times 10^{-3}$$
2) Proton magnetic moment can be addressed with
$$\mu_p \cong \frac{e_s \hbar}{2m_p} \cong \frac{eG_s m_p}{2c} \cong 1.48694 \times 10^{-26} \text{ J.T}^{-1}$$
3) Neutron magnetic moment can be addressed with
$$\mu_n \cong \frac{(e_s - e)\hbar}{2m_n} \cong 9.805 \times 10^{-27} \text{ J.T}^{-1}.$$
4) Nuclear unit radius can be expressed as,
$$R_0 \cong \frac{2G_s m_p}{c^2} \cong \left(\frac{e_s}{e}\right)\left\{\frac{\hbar}{m_p c} + \frac{\hbar}{m_n c}\right\}$$
5) Root mean square nuclear charge radii [33] can be addressed with,
$$R_{(Z,A)} \cong \left\{1 - 0.349\left(\frac{N-Z}{N}\right)\right\} N^{1/3} \times 1.262 \text{ fm}$$
$$\cong \left\{Z^{1/3} + \left(\sqrt{Z(A-Z)}\right)^{1/3}\right\}\left(\frac{G_s m_p}{c^2}\right)$$
6) Nuclear potential energy can be understood with ,
$$\cong \frac{e_s^2}{4\pi\varepsilon_0 \left(G_s m_p/c^2\right)} \cong 20.129 \text{ MeV}$$
7) Close to stable mass numbers, nuclear binding energy can be understood with a single energy coefficient [30,31] ,
$$\frac{e^2 G_s m_p^3}{8\pi\varepsilon_0 \hbar^2} \cong \frac{e_s e}{8\pi\varepsilon_0 \left(\hbar/m_p c\right)} \cong \frac{e_s^2}{8\pi\varepsilon_0 \left(G_s m_p/c^2\right)}$$
$$\cong 10.0647 \text{ MeV}$$

8) With reference to $(\hbar/2)$, a useful quantum energy constant can be expressed with, $E_{(\hbar/2)} \cong \left(\frac{e^2 G_s m_p^3}{4\pi\varepsilon_0 (\hbar/2)^2}\right) \cong 80.517725 \text{ MeV}$
9) Close to magic and semi magic proton numbers [31], nuclear binding energy seems to approach $\left[2.531\left(n+\frac{1}{2}\right)\right]^2 10.06 \text{ MeV}$ where $n = 0,1,2,3,...$ and $(m_n - m_p/m_e) = 2.531$.
10) Characteristic melting temperature associated with proton can be expressed with, $T_{proton} \cong \frac{\hbar c^3}{8\pi k_B G_s m_p} \cong 0.15\times10^{12} \text{ K}$
11) Characteristic nuclear neutral mass unit [32] can be addressed with, $\sqrt{\frac{\hbar c}{G_s}} \cong 546.844 \text{ MeV}/c^2$.

8. To fit neutron-proton mass difference

Neutron-proton mass difference can be understood with:

$$\left(\frac{m_n c^2 - m_p c^2}{m_e c^2}\right) \cong \ln\sqrt{\frac{E_{(\hbar/2)}}{m_e c^2}} \cong \ln\sqrt{\frac{4e^2 G_s m_p^3}{4\pi\varepsilon_0 \hbar^2 m_e c^2}} \quad (4)$$

9. To fit neutron life time

Neutron life time t_n can be understood with the following relation:

$$t_n \cong \exp\left(\frac{E_{(\hbar/2)}}{(m_n - m_p)c^2}\right)\times\left(\frac{\hbar}{m_n c^2}\right) \cong 763.514 \text{ sec} \quad (5)$$

This value can be compared with recommended value of (878.5 ± 0.8) sec.

10. Understanding beta stability line with respect to proton and electron specific charge ratios

Nuclear beta stability line can be addressed with a relation of the form [4],

$$\begin{aligned} A_s &\cong 2Z + s(2Z)^2 \cong 2Z + (4s)Z^2 \\ &\cong 2Z + 0.0064Z^2 \cong Z(2+kZ) \end{aligned} \quad (7)$$

where,

$$s \cong \left\{\left(\frac{e_s}{m_p}\right) \div \left(\frac{e}{m_e}\right)\right\} \cong \left(\frac{G_s m_p m_e}{\hbar c}\right) \cong 0.00160335$$

Based on the relation (7),

Let, $4s = k = 0.0064134$

$$\left.\begin{aligned} &\text{A)}\ \frac{(A_s - 2Z)^2}{A_s} \cong k^2 A_s N_s \sqrt{Z} \\ &\text{B)}\ \frac{Z}{\sqrt{A_s - 2Z}} \cong \frac{1}{\sqrt{k}} \cong 4\pi \\ &\text{C)}\ \frac{N_s}{Z} \cong (1+kZ) \cong \sqrt{1+kA_s} \\ &\text{D)}\ A_s \cong \frac{(1+kZ)^2 - 1}{k} \end{aligned}\right\} \quad (8)$$

11. Nuclear binding energy at stable mass numbers

Interesting points to be noted are:

1. With reference to electromagnetic interaction, and based on proton number, $(1/\alpha_s) \cong 8.67$ can be considered as the maximum strength of nuclear binding energy.
2. $Z \approx 30$ seems to represent a characteristic reference number in understanding nuclear binding of light and heavy atomic nuclides.

Based on these points, at stable mass numbers of Z, nuclear binding energy can be expressed by the following simple empirical relation.

$$(B)_{A_s} \cong \gamma \times Z \times (m_n - m_p)c^2 \quad (9)$$

$$\left.\begin{aligned} &\text{If } (Z<30),\ \text{coefficient, } \gamma \cong \left[\left(\frac{1}{\alpha_s}+1\right)+\sqrt{Z}\right] \\ &\text{If } (Z\geq30),\ \gamma \cong \left[\left(\frac{1}{\alpha_s}+1\right)+\sqrt{\sqrt{30\times31}}\right] \cong 15.2 \\ &\text{where, } \alpha_s \cong 0.1153795 \\ &\text{and } 15.2\times1.2933 \text{ MeV} \cong 19.66 \text{ MeV} \end{aligned}\right\}$$

Thus, for, $(Z\geq30)$

$$(B)_{A_s} \cong Z\times19.66 \text{ MeV} \quad (10)$$

See table 1. Close to the stable mass numbers, binding energy is estimated with relations (7) and (9) and compared with Semi empirical mass formula (SEMF). It needs further study with respect to its surprising results against a single energy coefficient.

Above and below the stable mass number, binding energy can be approximately estimated with the following relation.

$$(B)_A \approx (B)_{A_s} - \left[\ln\left(\frac{A_s}{kZ}\right)\frac{(A_s - A)A_s}{A}\right](m_n - m_p)c^2 \quad (11)$$

It needs further study with reference to unstable nuclides . See table 2 for Z=50.

12. Very simple approach for understanding nuclear stability starting form Z=21 to 118

With this simple method, super heavy elements lower stable mass numbers can be estimated. With even-odd corrections, accuracy can be improved. For $(Z \geq 11)$,

$$A_s \cong \left[Z + \left(\frac{e_s}{e}\right)\right]^{1.2} \cong \left(Z + \sqrt{\frac{1}{\alpha_s}}\right)^{1.2} \cong (Z + 2.944)^{1.2} \quad (12)$$

$$\text{where, } \left(\sqrt{\frac{1}{\alpha_s}}\right)^{\frac{1}{6}} \cong \left(\frac{e_s}{e}\right)^{\frac{1}{6}} \cong 1.197 \cong 1.2$$

13. Understanding nuclear binding energy of Deuteron

If it is assumed that, there exists no strong interaction in between proton and neutron, nuclear binding of deuteron can be expressed as,

$$BE \text{ of } {}_1^2H \cong 2 \times (m_n - m_p)c^2 \cong 2.59 \text{ MeV} \quad (13)$$

$$\text{where, } \begin{cases} \left(\frac{1}{\alpha_s} + 1\right) \cong 1 \\ \rightarrow \left(\frac{1}{\alpha_s} \rightarrow \left(\frac{e_s}{e}\right)^2 \rightarrow 0\right) \Rightarrow e_s \rightarrow 0 \end{cases}$$

This can be compared with the experimental value of 2.225 MeV.

14. To fix the magnitude of Fermi's weak coupling constant

With trial-error we noticed that,

$$\left.\begin{array}{l} R_0 \cong \frac{2G_s m_p}{c^2} \cong \left(\frac{m_p}{m_e}\right)\sqrt{\frac{G_F}{\hbar c}} \\ \rightarrow \left(\frac{2G_s m_e}{c^2}\right) \cong \sqrt{\frac{G_F}{\hbar c}} \end{array}\right\} \quad (14)$$

where G_F is the Fermi's weak coupling constant [1]. Based on this relation,

$$\alpha_s G_F \cong \frac{4\hbar^3 m_e^2}{m_p^4 c} \quad (15)$$

$$\left.\begin{array}{l} G_F \cong \left(\frac{1}{\alpha_s}\right)\frac{4\hbar^3 m_e^2}{m_p^4 c} \cong \frac{4G_s^2 m_e^2 \hbar}{c^3} \\ \cong \hbar c\left(\frac{2G_s m_e}{c^2}\right)^2 \cong 1.43789 \times 10^{-62} \text{ J.m}^3 \end{array}\right\} \quad (16)$$

Recommended value of $G_F \cong 1.43586 \times 10^{-62}$ J.m^3. It may be noted that, relations (15) and (16) seem to play a key role in understanding the basics of final unification and needs further study.

15. To fix the magnitude of Newtonian Gravitational constant

With reference to Planck scale and considering the following two semi empirical relations, magnitude of the Newtonian gravitational constant (G_N) can be fitted [23, 34].

$$\left(\frac{m_p}{m_e}\right) \cong \left(\frac{G_s m_p^2}{\hbar c} \times \frac{G_s}{G_N}\right)^{\frac{1}{12}} \cong \left(\frac{e_s G_s}{e G_N}\right)^{\frac{1}{12}} \quad (17)$$

$$\left(\frac{m_p}{m_e}\right)^{10} \cong \exp\left(\frac{1}{\alpha_s^2}\right) \quad (18)$$

Based on relations (14) to (18),

$$\left(\frac{G_s}{G_N}\right) \cong \sqrt{\alpha_s}\left(\frac{m_p}{m_e}\right)^{12} \cong \sqrt{\frac{4\hbar^3 m_e^2}{m_p^4 c F_W}}\left(\frac{m_p}{m_e}\right)^{12} \quad (19)$$

$$\left(\frac{G_N}{G_s}\right) \cong \frac{1}{2}\left(\frac{m_e}{m_p}\right)^{10}\left[\sqrt{\frac{G_F}{\hbar c}} \Big/ \left(\frac{\hbar}{m_e c}\right)\right] \quad (20)$$

$\rightarrow G_N \cong 6.66439\times10^{-11}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$
where $G_F \cong 1.43586\times10^{-62}\ \text{J.m}^3$

$$\left(\frac{G_s}{G_N}\right) \cong \exp\left(\frac{1}{\alpha_s^2}\right)\sqrt{\frac{4\hbar^3}{m_e^2 c G_F}} \tag{21}$$

$\rightarrow G_N \cong 6.9121744\times10^{-11}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$
where $\alpha_s \cong 0.1153795$

$$G_N \cong \frac{1}{4}\left(\frac{m_e}{m_p}\right)^{11}\sqrt{\frac{c^5 G_F R_0^2}{\hbar^3}} \tag{22}$$

Accuracy of (G_N) seems to depend on $(G_s, R_0, \alpha_s, G_F)$.

16. To understand the range of strong interaction

From above relations, in terms of nuclear charge radius, $(R_0 \approx 1.24\ \text{fm})$,

$$R_0 \cong \left(\frac{m_p}{m_e}\right)^{11}\left(\frac{4G_N\hbar}{c^3}\right)\sqrt{\frac{\hbar c}{G_F}} \tag{23}$$

With reference to the known values,

$$R_0 \cong \left(\frac{m_p}{m_e}\right)^{11}\left(\frac{4G_N\hbar}{c^3}\right)\sqrt{\frac{\hbar c}{G_F}} \cong 1.24\ \text{fm} \tag{24}$$

$$\text{where}\begin{cases}\left(\frac{4G_N\hbar}{c^3}\right) \cong \left[\frac{2G_N M_{pl}}{c^2}\right]^2 \text{ where } M_{pl} \cong \sqrt{\frac{\hbar c}{G_N}} \\ \cong (\text{Schwarzschild radius of Planck mass})^2 \\ \text{and } \sqrt{\frac{G_F}{\hbar c}} \cong 6.7139189\times10^{-19}\,\text{m}\end{cases}$$

Based on relations (18) and (24), strong interaction range can be understood with the following relation.

$$R_0 \cong \exp\left(\frac{1}{\alpha_s^2}\right)\left\{\left(\frac{m_p}{m_e}\right)\left(\frac{4G_N\hbar}{c^3}\right)\sqrt{\frac{\hbar c}{G_F}}\right\} \tag{25}$$

It seems interesting to infer that,

a) $\left(\frac{1}{\alpha_s^2}\right)$ and $\exp\left(\frac{1}{\alpha_s^2}\right)$ play a crucial role in deciding the strong interaction range.

b) An increase in the value of α_s helps in decreasing the interaction range. This may be an indication of more strongly bound nuclear system.

c) A decrease in the value of α_s helps in increasing the interaction range. This may be an indication of more weakly bound nuclear system.

d) Poportionality constant being $\exp\left(\frac{1}{\alpha_s^2}\right)$,

$$\left.\begin{aligned} R_0 &\propto \left(\frac{m_p}{m_e}\right) \\ R_0 &\propto \left(\frac{4G_N h}{c^3}\right) \\ R_0 &\propto 1\Big/\sqrt{\frac{G_F}{hc}} \end{aligned}\right\}$$

17. Conclusion

Even though our approach to nuclear physics seems to be speculative, proposed assumptions show a wide range of applications embedded with in-depth physical meaning connected with low energy nuclear physics and high energy nuclear physics. With reference to the famous semi empirical mass formula having 5 different energy terms and 5 different energy coefficients, qualitatively and quantitatively, our proposed relations (7) and (9) are very simple to follow and a special study seems to be required for understanding the binding energy of isotopes above and below the stability line. With further research, current nuclear models and strong interaction concepts can be studied in a unified manner with respect to strong nuclear gravity. Finally, value of the Newtonian gravitational constant can be estimated with nuclear elementary physical constants.

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Table 1: Estimated nuclear binding close to stable mass numbers

Proton number	Est. Mass number close to stability	Neutron number	Value of γ	Est. BE (MeV)	SEMF BE (MeV)	Error (MeV)
2	4	2	11.08	28.7	22.0	-6.7
3	6	3	11.40	44.2	31.8	-12.4
4	8	4	11.67	60.4	52.9	-7.5
5	10	5	11.90	77.0	66.1	-10.9
6	12	6	12.12	94.0	87.4	-6.6
7	14	7	12.31	111.5	102.0	-9.5
8	16	8	12.50	129.3	123.2	-6.0
9	19	10	12.67	147.4	146.1	-1.3
10	21	11	12.83	165.9	167.5	1.6
11	23	12	12.98	184.7	183.6	-1.1
12	25	13	13.13	203.8	204.7	0.9
13	27	14	13.27	223.2	220.9	-2.2
14	29	15	13.41	242.8	241.6	-1.1
15	31	16	13.54	262.7	257.8	-4.8
16	34	18	13.67	282.8	290.8	8.0
17	36	19	13.79	303.2	307.1	3.9
18	38	20	13.91	323.8	327.2	3.4
19	40	21	14.03	344.7	343.4	-1.3
20	43	23	14.14	365.7	371.6	5.8
21	45	24	14.25	387.0	387.8	0.8
22	47	25	14.36	408.5	407.5	-1.0
23	49	26	14.46	430.2	423.5	-6.7
24	52	28	14.57	452.1	454.6	2.5
25	54	29	14.67	474.2	470.5	-3.7
26	56	30	14.77	496.5	489.6	-6.9
27	59	32	14.86	519.0	513.6	-5.4
28	61	33	14.96	541.7	532.5	-9.2
29	63	34	15.05	564.5	548.2	-16.4
30	66	36	15.20	589.7	577.9	-11.8
31	68	37	15.20	609.4	593.4	-16.0
32	71	39	15.20	629.1	619.8	-9.3
33	73	40	15.20	648.7	635.2	-13.5
34	75	41	15.20	668.4	653.3	-15.1
35	78	43	15.20	688.0	679.2	-8.8
36	80	44	15.20	707.7	697.0	-10.6
37	83	46	15.20	727.4	720.0	-7.4
38	85	47	15.20	747.0	737.6	-9.4

39	88	49	15.20	766.7	762.9	-3.8
40	90	50	15.20	786.3	780.2	-6.1
41	93	52	15.20	806.0	802.6	-3.4
42	95	53	15.20	825.6	819.7	-5.9
43	98	55	15.20	845.3	844.4	-0.9
44	100	56	15.20	865.0	861.2	-3.7
45	103	58	15.20	884.6	883.2	-1.4
46	106	60	15.20	904.3	909.6	5.3
47	108	61	15.20	923.9	923.9	-0.1
48	111	63	15.20	943.6	947.6	4.1
49	113	64	15.20	963.2	961.7	-1.5
50	116	66	15.20	982.9	987.5	4.6
51	119	68	15.20	1002.6	1008.6	6.0
52	121	69	15.20	1022.2	1024.6	2.4
53	124	71	15.20	1041.9	1047.6	5.7
54	127	73	15.20	1061.5	1070.4	8.9
55	129	74	15.20	1081.2	1084.0	2.8
56	132	76	15.20	1100.9	1108.7	7.9
57	135	78	15.20	1120.5	1129.0	8.5
58	138	80	15.20	1140.2	1153.3	13.1
59	140	81	15.20	1159.8	1166.6	6.7
60	143	83	15.20	1179.5	1188.5	9.0
61	146	85	15.20	1199.1	1210.3	11.2
62	149	87	15.20	1218.8	1231.9	13.1
63	151	88	15.20	1238.5	1244.9	6.4
64	154	90	15.20	1258.1	1268.2	10.1
65	157	92	15.20	1277.8	1287.5	9.7
66	160	94	15.20	1297.4	1310.4	13.0
67	163	96	15.20	1317.1	1329.4	12.3
68	166	98	15.20	1336.8	1352.0	15.3
69	169	100	15.20	1356.4	1370.8	14.4
70	171	101	15.20	1376.1	1385.1	9.0
71	174	103	15.20	1395.7	1405.4	9.7
72	177	105	15.20	1415.4	1425.7	10.3
73	180	107	15.20	1435.0	1445.7	10.7
74	183	109	15.20	1454.7	1465.7	11.0
75	186	111	15.20	1474.4	1485.4	11.1
76	189	113	15.20	1494.0	1505.1	11.1
77	192	115	15.20	1513.7	1524.6	10.9
78	195	117	15.20	1533.3	1544.0	10.6
79	198	119	15.20	1553.0	1563.2	10.2

80	201	121	15.20	1572.7	1582.3	9.7
81	204	123	15.20	1592.3	1601.3	9.0
82	207	125	15.20	1612.0	1620.2	8.2
83	210	127	15.20	1631.6	1638.9	7.3
84	213	129	15.20	1651.3	1657.5	6.2
85	216	131	15.20	1670.9	1676.0	5.0
86	219	133	15.20	1690.6	1694.3	3.7
87	223	136	15.20	1710.3	1717.8	7.5
88	226	138	15.20	1729.9	1737.5	7.6
89	229	140	15.20	1749.6	1753.8	4.3
90	232	142	15.20	1769.2	1773.2	4.0
91	235	144	15.20	1788.9	1789.4	0.5
92	238	146	15.20	1808.6	1808.5	0.0
93	241	148	15.20	1828.2	1824.5	-3.7
94	245	151	15.20	1847.9	1848.3	0.4
95	248	153	15.20	1867.5	1865.5	-2.0
96	251	155	15.20	1887.2	1882.6	-4.6
97	254	157	15.20	1906.8	1899.6	-7.2
98	258	160	15.20	1926.5	1922.7	-3.8
99	261	162	15.20	1946.2	1938.0	-8.2
100	264	164	15.20	1965.8	1956.1	-9.7

Table 2: Estimated approximate nuclear binding of isotopes of Z=50

Proton number	Mass number	Neutron number	Est. BE (MeV)	SEMF BE (MeV)	Error (MeV)
50	100	50	841.5	809.3	-32.2
50	101	51	851.7	822.3	-29.4
50	102	52	861.6	837.2	-24.4
50	103	53	871.4	849.2	-22.1
50	104	54	880.9	863.2	-17.7
50	105	55	890.3	874.5	-15.9
50	106	56	899.5	887.6	-11.9
50	107	57	908.6	898.1	-10.5
50	108	58	917.4	910.4	-7.0
50	109	59	926.2	920.1	-6.0
50	110	60	934.7	931.8	-2.9
50	111	61	943.1	940.7	-2.4
50	112	62	951.3	951.6	0.3
50	113	63	959.4	960.0	0.5
50	114	64	967.4	970.2	2.8

50	115	65	975.2	977.9	2.7
50	116	66	982.9	987.5	4.6
50	117	67	990.5	994.6	4.1
50	118	68	997.9	1003.5	5.7
50	119	69	1005.2	1010.1	4.9
50	120	70	1012.4	1018.5	6.1
50	121	71	1019.4	1024.4	5.0
50	122	72	1026.4	1032.3	5.9
50	123	73	1033.2	1037.7	4.5
50	124	74	1039.9	1045.1	5.2
50	125	75	1046.5	1050.1	3.5
50	126	76	1053.0	1056.9	3.9
50	127	77	1059.5	1061.4	1.9
50	128	78	1065.8	1067.8	2.0
50	129	79	1072.0	1071.8	-0.2
50	130	80	1078.1	1077.7	-0.3
50	131	81	1084.1	1081.4	-2.8
50	132	82	1090.0	1086.9	-3.2
50	133	83	1095.9	1090.1	-5.8
50	134	84	1101.6	1095.1	-6.5
50	135	85	1107.3	1098.0	-9.3
50	136	86	1112.9	1102.7	-10.2
50	137	87	1118.4	1105.1	-13.3
50	138	88	1123.8	1109.4	-14.4
50	139	89	1129.1	1111.5	-17.6
50	140	90	1134.4	1115.5	-18.9