 doi:10.20944/preprints201812.0341.v1

Regarding rational exponential and hyperbolic functions solutions of conformable time-fractional Phi-four equation

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Abstract

In this study, we actually want to explore the time-fractional Phi-four equation via two methods, the exp_{a} function method and the hyperbolic function method. We transform a fractional order differential equation into an ordinary differential equation using a wave transformation and the fractional derivative in conformable form. Then, the resulting equation has successfully been explored for new explicit exact solutions. The procured solutions are simply showed the effectiveness and plainness of the projected methods.

Keywords: Time-fractional Phi-four equation; Conformable derivatives; exp_{a} function approach; Hyperbolic function approach; Exact solutions.

1 Introduction

A large number of physical prototypes has been expressed via nonlinear evolution equations in physical and natural sciences. These prototypes are discussed through different types of solutions of corresponding nonlinear PDEs. The exact solutions have always been a particular importance among the researchers in many fields of nonlinear sciences. To avoid the cumbersome calculations, a number of symbolic computation packages are available that facilitate to inaugurate new exact solutions to nonlinear evolution equations. Various methods have been presented and implemented to search exact solutions in different research articles such as: the ansatz [1–5], modified simple equation [6, 7], the extended

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trial equation [8,9], sine-cosine approach [10], the first integral [11,12], $(\frac{G'}{G})$-expansion [13], sine-Gordon expansion [14,15], the new extended direct algebraic method [16] and generalized projective Riccati equation method [17]. Furthermore, some other excellent works like various forms of the Kudryashov method [18–20], a modified form of Kudryashov and functional variable methods [21–23] have been done by different researchers. In [24–28], the auxiliary equation, the extended $tanh$-function, the improved $tan(\frac{\phi(\eta)}{2})$-expansion methods and the exp function approach have been explored for discrete and fractional order PDEs as well. In particular, Ali and Hassan, Hosseini et al., Zayed and Al-Nowehy all have utilized the $exp_{\ a}$ function method in [29], [30], [31] and [32] respectively. The hyperbolic function method is one of the most useful approach to procure exact solutions of nonlinear partial differential equations, for example one can study, Xie et al. [33], Bai [34] and Hosseini et al. [35]. The study of fractional calculus has been given in [36–39].
The Klein–Gordon equation have an important role in mathematical physics, particle and nuclear physics, particulary in condensed matter physics [40–42]. The Phi-four equation is a special form of the Klein-Gordon equation that prototypes the interaction of kink and anti-kink solitary waves in particle physics [43]. This paper aims to scrutinize the new explicit exact solutions for the time-fractional Phi-four equation [44,45] that may read as

$$D_t^{2\gamma} - u_{xx} + m^2 u + \delta u^3 = 0,\ \delta > 0, 0 < \gamma \leq 1, \tag{1}$$

here m and δ are reals. We employ the exp_a function approach [29,30,32] and the hyperbolic function approach [33–35] via traveling wave transformation with the conformable derivative.
The scheme of this paper is as follows: Section 2 presents a brief description of the exp_a and the hyperbolic functions methods. Section 3 illustrate how to utilize these methods for producing new exact solutions of time-fractional Phi-four equation. The latter part presents the results of the current study graphically.

We recall the conformable derivative with some of its properties [38].
Defination 1 Suppose $h : \mathbb{R}_{>0} \to \mathbb{R}$ be a function. Then, for all $t\ > 0$,

$$D_t^{\alpha}(p(t)) = \lim_{\varepsilon \to 0} \frac{p(t + \varepsilon t^{1-\alpha}) - p(t)}{\varepsilon}$$

is known as $\alpha,\ \ 0 < \alpha \leq 1$ order conformable fractional derivative of p. The followings are some useful properties:
$D_t^{\alpha}(a\ p + b\ g) = aD_t^{\alpha}(p) + bD_t^{\alpha}(g)$, for all $a,\ b \in \mathbb{R}$
$D_t^{\alpha}(p\ g) = p\ D_t^{\alpha}(g) + g\ D_t^{\alpha}(p)$
Let $p : \mathbb{R}_{>0} \to \mathbb{R}$ be an α-differentiable function, g be a differentiable function defined in the range of p.

$$D_t^{\alpha}(p \circ g(t)) = t^{1-\alpha}\ g'(t)\ p'(g(t)).$$

On the top of that, the following rules hold.
$D_t^{\alpha}(t^h) = h\ t^{h-\alpha}$, for all $h \in \mathbb{R}$

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$D_t^\alpha(\delta) = 0$, where δ is constant.
$D_t^\alpha(p/g) = \frac{gD_t^\alpha(p) - pD_t^\alpha(g)}{g^2}$.
Conjointly, if p is differentiable, then $D_t^\alpha(p(t)) = t^{1-\alpha}\frac{dp(t)}{dt}$.

2 Description of Methods

The present subsection provides a brief explanation for two reliable techniques in engendering new exact solutions to nonlinear conformable time-fractional Phi-four equation. For this purpose, suppose that we have a nonlinear conformable time FDE that can be presented in the form

$$F(u, D_t^\gamma u, D_x u, D_{tt}^{2\gamma} u, D_{xx} u, ...) = 0 \quad (2)$$

The FDE (2) can be changed into the following nonlinear ODE of integer order

$$\wp(U, U^{'}, U^{''}, ...,) = 0 \quad (3)$$

with the use of following wave transformation

$$u(x,t) = U(\xi), \xi = kx - l\frac{t^\gamma}{\gamma}, \quad (4)$$

where $\wp$ is a polynomial in $U(\xi)$ and its total derivatives with $' = d/d\xi$ while k and l are nonzero arbitrary constants.

2.1 The exp_a function approach

Let us try to search a non-trivial solution for (3) in the following form [29–32]

$$U(\xi) = \frac{A_0 + A_1 a^\xi + ... + A_N a^{N\xi}}{B_0 + B_1 a^\xi + ... + B_N a^{N\xi}} \quad (5)$$

where A_i and B_i, for $(0 \le i \le N)$, are found later and N is a free positive constant. Replacing the Eq.(5) in the nonlinear Eq.(3), yields

$$\wp(a^\xi) = q_0 + q_1 a^\xi + ... + q_\tau a^{\tau\xi} = 0 \quad (6)$$

Setting $l_i(0 \le i \le \tau)$ in (5) to be zero, results give a set of nonlinear equations as follows:

$$q_i = 0, \quad i = 0, ..., \tau \quad (7)$$

by solving the generated set (7), we acquire non-trivial solutions of the nonlinear PDE (2).

2.2 The hyperbolic function approach

Let us try to search a non-trivial solution to the Eq. (3) in the following form [33–35, 46]

$$U(\eta) = A_0 + \sum_{i=1}^{N} \sinh^{i-1}(\rho)[B_i \sinh(\rho) + A_i \cosh(\rho)] \tag{8}$$

where ρ is some specific functions. By calculating the positive integer N, setting the Eq. (8) in Eq. (3), and comparing coefficients, we will find a set of nonlinear equations whose solution, finally provides explicit exact solutions of the Eq.(2). It is worth mentioning that using the separation of variables techniques on $\frac{d\rho}{d\eta} = \sinh(\rho)$, we find $\sinh(\rho) = \pm\text{csch}(\eta)$, $\cosh(\rho) = -\coth(\eta)$ and $\sinh(\rho) = \pm\imath\text{sech}(\eta)$, $\cosh(\rho) = -\tanh(\eta)$. Accordingly, the solution (7) can be rewritten as

$$U(\eta) = A_0 + \sum_{i=1}^{N}(\pm\text{csch})^{i-1}(\eta))[\pm B_i csch(\eta) - A_i \coth(\eta)],$$

and

$$U(\eta) = A_0 + \sum_{i=1}^{N}(\pm\imath\text{sech})^{i-1}(\eta)[\pm\imath B_i\text{sech}(\eta) - A_i \tanh(\eta)].$$

Likewise, it is obvious that from $\frac{dw}{d\eta} = \cosh(\rho)$, we find $\sinh(\rho) = -\cot(\eta)$, $\cosh(\rho) = \pm\csc(\eta)$ and $\sinh(\rho) = \tan(\eta)$, $\cosh(\rho) = \pm\sec(\eta)$. Accordingly, the solution (7) can be rewritten as

$$U(\eta) = A_0 + \sum_{i=1}^{N}(-\cot)^{i-1}(\eta)[-B_i \cot(\eta) \pm A_i \csc(\eta)],$$

and

$$U(\eta) = A_0 + \sum_{i=1}^{N}(\tan^{i-1})(\eta)[B_i \tan(\eta) \pm A_i \sec(\eta)].$$

3 Application to time-fractional phi-four equation

In this section, the following form of time-fractional phi-four equation [44, 45] is going to be considered for solution via exp_a method. Using the transformation (4) in (1), we get

$$(l^2 - k^2)U'' + m^2U + \delta U^3 = 0; \delta > 0. \tag{9}$$

Through balancing the terms U'' and U^3, we select $N = 1$, the nontrivial solution (5) reduces to:

$$U(\xi) = \frac{a_1 a^{\xi} + a_0}{b_1 a^{\xi} + b_0}, \quad a \neq 1 \tag{10}$$

By setting the above solution in (9) and equating factors of each power of a^{ξ} in the resulting equation, we reach a set of nonlinear algebraic equations. which its solution yields the following new explicit exact solutions to (1).

$$Set-1: a_0 = -\frac{ib_0 m}{\sqrt{\delta}}, a_1 = \frac{ib_1 m}{\sqrt{\delta}}, b_0 = b_0, b_1 = b_1, k = k, l = \mp\frac{\sqrt{k^2\log^2(a)+2m^2}}{\log(a)}$$

$$u_1(x,t) = -\frac{im\left(b_0 - b_1 a^{kx-l\frac{t^\gamma}{\gamma}}\right)}{\sqrt{\delta}\left(b_1 a^{kx-l\frac{t^\gamma}{\gamma}} + b_0\right)} \tag{11}$$

$$Set-2: a_0 = \frac{ib_0 m}{\sqrt{\delta}}, a_1 = -\frac{ib_1 m}{\sqrt{\delta}}, b_0 = b_0, b_= b_1, k = k, l = \mp\frac{\sqrt{k^2\log^2(a)+2m^2}}{\log(a)}$$

$$u_2(x,t) = \frac{im\left(b_0 - b_1 a^{kx-l\frac{t^\gamma}{\gamma}}\right)}{\sqrt{\delta}\left(b_1 a^{kx-l\frac{t^\gamma}{\gamma}} + b_0\right)} \tag{12}$$

In particular if we put $a = e$ in u_1 and u_2 appear in (11), then we can obtain the following hyperbolic function solutions.

$$u_1^T(\xi) = -\frac{im\left(-b_1\sinh(kx - l\frac{t^\gamma}{\gamma}) - b_1\cosh(\xi) + b_0\right)}{\sqrt{\gamma}\left(b_1\sinh(kx - l\frac{t^\gamma}{\gamma}) + b_1\cosh(kx - l\frac{t^\gamma}{\gamma}) + b_0\right)}$$

$$u_2^T(\xi) = \frac{im\left(-b_1\sinh(\xi) - b_1\cosh(kx - l\frac{t^\gamma}{\gamma}) + b_0\right)}{\sqrt{\gamma}\left(b_1\sinh(kx - l\frac{t^\gamma}{\gamma}) + b_1\cosh(\xi) + b_0\right)}$$

We now again consider the Eq. (9) to solve by utilizing the hyperbolic function approach. **Case-1:** $\frac{d\rho}{d\xi} = \sinh(\rho)$ Through homogenous balancing principle, the terms U'' and U^3 gives $N = 1$ and the non-trivial solution (8) becomes

$$u(\xi) = B_1\sinh(\rho) + A_1\cosh(\rho) + A_0 \tag{13}$$

By setting the above non-trivial solution in (9) and equating the coefficients to zero in the resultant equation, we reach a set of nonlinear algebraic equations. which its solution yields the following new explicit exact solutions of (1).

$$Set-1: A_0=0, A_1=-\frac{im}{\sqrt{\delta}}, B_1=\frac{im}{\sqrt{\delta}}, k=k, l=\mp\sqrt{k^2+2m^2}$$
$$Set-2: A_0=0, A_1=\frac{im}{\sqrt{\delta}}, B_1=-\frac{im}{\sqrt{\delta}}, k=k, l=\mp\sqrt{k^2+2m^2}$$
$$u(\xi)=\mp\frac{im}{\sqrt{\delta}}\tanh\left(\frac{kx-l\frac{t^\gamma}{\gamma}}{2}\right) \tag{14}$$

$$Set-3: A_0=0, A_1=-\frac{im}{\sqrt{\delta}}, B_1=-\frac{im}{\sqrt{\delta}}, k=k, l=\mp\sqrt{k^2+2m^2}$$
$$Set-4: A_0=0, A_1=\frac{im}{\sqrt{\delta}}, B_1=\frac{im}{\sqrt{\delta}}, k=k, l=\mp\sqrt{k^2+2m^2}$$
$$u(\xi)=\mp\frac{im}{\sqrt{\delta}}\coth\left(\frac{kx-l\frac{t^\gamma}{\gamma}}{2}\right). \tag{15}$$

Some more hyperbolic function solutions of Equ.(1) can be written as

$$Set-5: A_0=0, A_1=0, B_1=\pm m\sqrt{\frac{2}{\delta}}, k=k, l=\pm\sqrt{k^2-m^2}$$
$$u(\xi)=\pm m\sqrt{\frac{2}{\delta}}\mathrm{csch}(kx-l\frac{t^\gamma}{\gamma}) \tag{16}$$
$$Set-6: A_0=0, A_1=\pm\frac{im}{\sqrt{\delta}}, B_1=0, k=k, l=\pm\frac{\sqrt{2k^2+m^2}}{\sqrt{2}}$$
$$u(\xi)=\pm\frac{im}{\sqrt{\delta}}\coth(kx-l\frac{t^\gamma}{\gamma}) \tag{17}$$

Case-2: $\frac{d\rho}{d\xi}=\cosh(\rho)$ and for $N=1$, we procured the following new exact trigonometric wave solutions

$$Set-1: A_0=0, A_1=-\frac{m}{\sqrt{\delta}}, B_1=-\frac{m}{\sqrt{\delta}}, k=k, l=\mp\sqrt{k^2-2m^2}$$
$$Set-2: A_0=0, A_1=\frac{m}{\sqrt{\delta}}, B_1=\frac{m}{\sqrt{\delta}}, k=k, l=\mp\sqrt{k^2-2m^2}$$
$$u_{1,2}(\xi)=\pm\frac{m}{\sqrt{\delta}}\tan\left(\frac{kx-l\frac{t^\gamma}{\gamma}}{2}\right) \tag{18}$$

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$$Set-3: A_0 = 0, A_1 = -\frac{m}{\sqrt{\delta}}, B_1 = \frac{m}{\sqrt{\delta}}, k = k, l = \mp\sqrt{k^2 - 2m^2}$$
$$Set-4: A_0 = 0, A_1 = \frac{m}{\sqrt{\delta}}, B_1 = -\frac{m}{\sqrt{\delta}}, k = k, l = \mp\sqrt{k^2 - 2m^2}$$
$$u_{3,4}(\xi) = \pm\frac{m}{\sqrt{\delta}}\cot\left(\frac{kx - l\frac{t^\gamma}{\gamma}}{2}\right) \tag{19}$$

The following are some more trigonometric function solutions for Equ. (1).

$$Set-5: A_0 = 0, A_1 = 0, B_1 = \pm\frac{m}{\sqrt{\delta}}, k = k, l = \pm\frac{\sqrt{2k^2 - m^2}}{\sqrt{2}}$$
$$u_{5,6}(\xi) = \pm\frac{m}{\sqrt{\delta}}\cot(kx - l\frac{t^\gamma}{\gamma}) \tag{20}$$
$$Set-6: A_0 = 0, A_1 = \pm im\sqrt{\frac{2}{\delta}}, B_1 = 0, k = k, l = \pm\sqrt{k^2 + m^2}$$
$$u_{7,8}(\xi) = \pm im\sqrt{\frac{2}{\delta}}\csc(kx - l\frac{t^\gamma}{\gamma}) \tag{21}$$

3.1 Numerical simulation for solutions of time-fractional Phi-four equation

The obtained solutions of Eq.(1) are graphed here for different γ-values corresponding to $m = \frac{1}{2}, \delta = 2$, and $k = 1$.

Case-I: The figures 1(a)-1(d) and 2(a)-2(d) reveal the two solutions given in (14) and (15) for $\gamma = 0.5, 0.75, 1$ and $t = 0$ respectively.

Case-II: The figures 3(a)-3(d) and 4(a)-4(d) present two solutions given in (18) and (19) for $\gamma = 0.5, 0.75, 1$ and $t = 0$ respectively.

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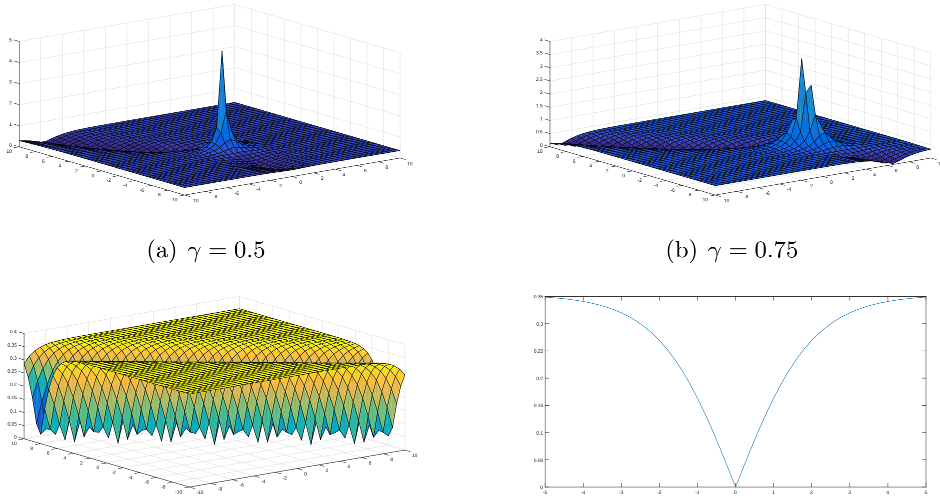


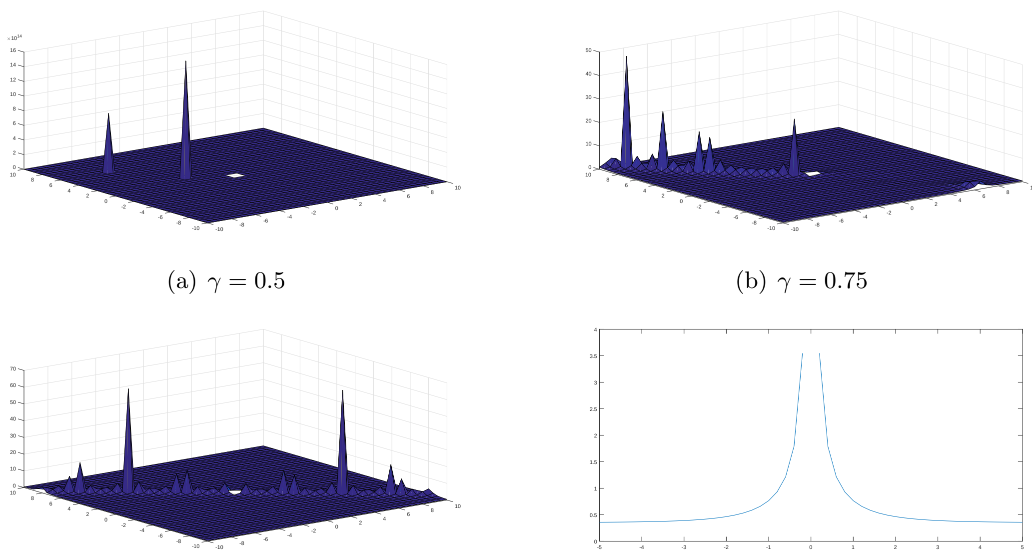
(a) $\gamma = 0.5$

(b) $\gamma = 0.75$

(c) $\gamma = 1$

(d) $t = 0$

Figure 1: Solitary wave profile of $u_{1,2}$ appears in Eq.(14)



(a) $\gamma = 0.5$

(b) $\gamma = 0.75$

(c) $\gamma = 1$

(d) $t = 0$

Figure 2: Solitary wave profile of $u_{3,4}$ appears in Eq.(15)

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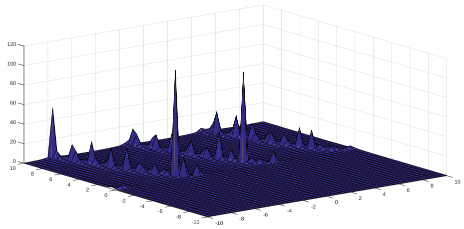
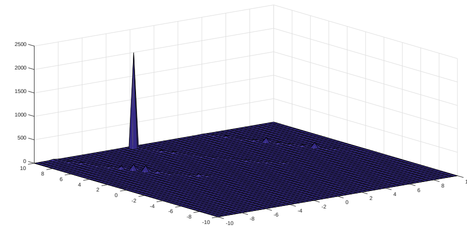
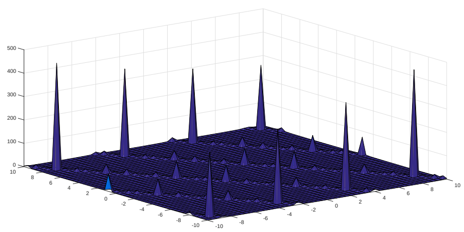


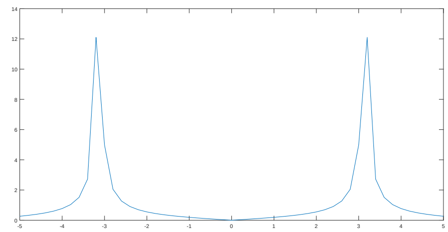
(a) $\gamma = 0.5$



(b) $\gamma = 0.75$

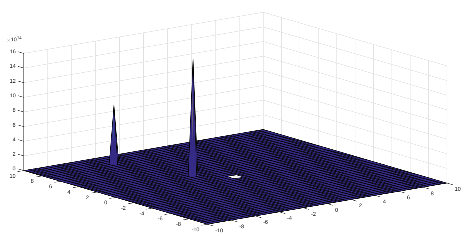


(c) $\gamma = 1$

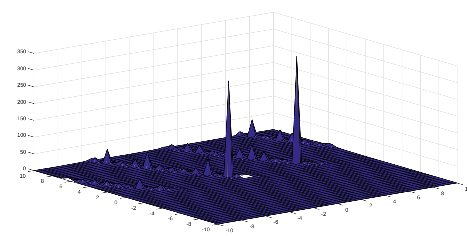


(d) $t = 0$

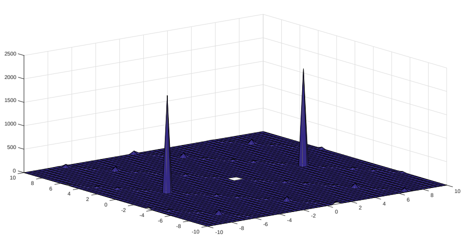
Figure 3: Solitary wave profile of $u_{1,2}$ appears in Eq.18



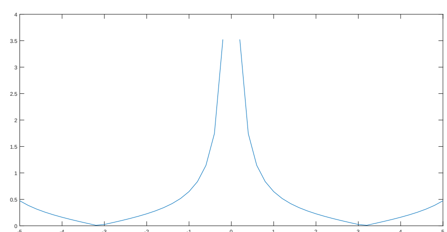
(a) $\gamma = 0.5$



(b) $\gamma = 0.75$



(c) $\gamma = 1$



(d) $t = 0$

Figure 4: Solitary wave profile of $u_{3,4}$ appears in Eq.19

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4 Conclusion

We have succeeded to explore two novel analytical approaches in finding many new exact solutions of time-fractional Phi-four equation. In other words, we utilized the $exp\ _a$ function method and the hyperbolic function method to construct the new explicit exact solutions of time-fractional Phi-four equation with the aid of MATHEMATICA. After comparing our findings with those appear in existing literature, we are confident that our solutions are new and the proposed methods are more efficient. Furthermore, the graphical representations of some solutions for particular parameters have been left for the reader.

References

[1] Q. Zhou, M. Mirzazadeh, E. Zerrad, A. Biswas, and B. Milivoj. Bright, dark, and singular solitons in optical fibers withspatio-temporal dispersion and spatially dependent coefficients. *Jornal of Modern Optics.*, 63(10):427–430, 2016.

[2] Kamyar Hosseini, Peyman Mayeli, and Reza Ansari. Bright and singular soliton solutions of the conformable time-fractional klein–gordon equations with different nonlinearities. *Waves in Random and Complex Media*, 28(3):426–434, 2018.

[3] Alper Korkmaz. Exact solutions of space-time fractional ew and modified ew equations. *Chaos, Solitons and Fractals*, 96:132–138, 2017.

[4] Alper Korkmaz. Explicit exact solutions to some one dimensional conformable time fractional equations. *Waves in Random and Complex Media*, 29(1):124–137, 2019.

[5] M. Mirzazadeh M. Eslami H. Rezazadeh Q. Zhou M.S. Osman, Alper Korkmaz. The unified method for conformable time fractional schrodinger equation with perturbation terms. *Chinese Journal of Physics*, 56(5):2500–2506, 2018.

[6] Muhammad Younis and Asim Zafar. The modified simple equation method for solving nonlinear phi-four equation. *International Journal of Innovation and Applied Studies*, 2(4):661–664, 2013.

[7] A. Biswas, M. O. Al-Amr, H. Rezazadeh, M. Mirzazadeh, M.Eslami, Q. Zhou, S.P. Moshokoa, and M. Belic. Resonant optical solitons with dual-power law nonlinearity and fractional temporal evolution. *Optik*, 165:233–239, 2018.

[8] Shuker Mahmood and Shuker Mahmood. Mathematical analysis of the generalized benjamin and burger-kdv equations via the extended trial equation method. *Journal of the Association of Arab Universities for Basic and Applied Sciences*, 16(1):91–100, 2014.

[9] Khaled A. Gepreel, Taher A. Nofal, and Nehal S. Al-Sayali. Exact solutions to the generalized hirota-satsuma kdv equations using the extended trial equation method. *Engineering Letters*, 24(3):274–283, 2016.

[10] Ahmet Bekir. New solitons and periodic wave solutions for some nonlinear physical models by using the sine–cosine method. *Physica Scripta*, 77(4):1–10, 2008.

[11] Mostafa Eslami and Hadi Rezazadeh. The first integral method for wu–zhang system with conformable time-fractional derivative. *Calcolo*, 53(3):475–485, 2016.

[12] M. Eslami. Trial solution technique to chiral nonlinear schrodinger's equation in (1+2)-dimensions. *Nonlinear Dynam*, 85(2):813–816, 2016.

[13] M. Younis and Asim Zafar. Exact solutions to nonlinear differential equations of fractional order via $(\frac{G'}{G})$-expansion method. *Appl. Math*, 5:1–6, 2014.

[14] Alper Korkmaz. Complex wave solutions to mathematical biology models i: Newell-whitehead-segel and zeldovich equations. *Journal of Computational and Nonlinear Dynamics*, 13(8):1–7, 2018.

[15] S. M. Mirhosseini-Alizamini A. Neirameh M. Eslami Q. Zhou H. Rezazadeh, M. Mirzazadeh. Optical solitons of lakshmanan–porsezian–daniel model with a couple of nonlinearities. *Optik*, 164:414–423, 2018.

[16] H. Rezazadeh. New solitons solutions of the complex ginzburg-landau equation with kerr law nonlinearity. *Optik*, 167:218–227, 2018.

[17] H. Rezazadeh, A. Korkmaz, M. Eslami, J. Vahidi, and R. Asghari. Traveling wave solution of conformable fractional generalized reaction duffing model by generalized projective riccati equation method. *Optical and Quantum Electronics*, 50(3):150, 2018.

[18] Alper Korkmaz. Exact solutions to (3+ 1) conformable time fractional jimbo-miwa, zakharov-kuznetsov and modified zakharov-kuznetsov equations. *Communications in Theoretical Physics*, 67(5):479, 2017.

[19] Alper Korkmaz and Kamyar Hosseini. Exact solutions of a nonlinear conformable time-fractional parabolic equation with exponential nonlinearity using reliable methods. *Optical and Quantum Electronics*, 49(8):278, 2017.

[20] Nikolai A. Kudryashov and Nadejda B. Loguinova. Extended simplest equation method for nonlinear differential equations. *Applied Mathematics and Computation*, 205(1):396–402, 2008.

[21] K. Hosseini, E. Yazdani Bejarbaneh, A. Bekir, and M. Kaplan. New exact solutions of some nonlinear evolution equations of pseudoparabolic type. *Optical and Quantum Electronics*, 49(7):241, 2017.

[22] Z. Ayati, K. Hosseini, and M. Mirzazadeh. Application of kudryashov and functional variable methods to the strain wave equation in microstructured solids. *Nonlinear Engineering*, 6:25–29, 2017.

[23] K. Hosseini, P. Mayeli, and D. Kumar. New exact solutions of the coupled sine-gordon equations in nonlinear optics using the modified kudryashov method. *Journal of Modern Optics*, 65(3):361–364, 2018.

[24] A. Bekir. Application of the extended tanh method for coupled nonlinear evolution equation. *Commun. Nonlinear Sci.*, 13:1742–1751, 2008.

[25] E. Fan. Extended tanh-function method and its application to nonlinear equation. *Phys. Lett. A.*, 277:212–218, 2000.

[26] AM. Wazwaz. The extended tanh method for the zakharov-kuznestsov(zk) equation, the modified zk equation, and its generalized forms. *Commun. Nonlinear Sci.*, 13:1039–1047, 2008.

[27] Kamyar Hosseini, Jalil Manafian, Farzan Samadani, Mohammadreza Foroutan, Mohammad Mirzazadeh, and Qin Zhou. Resonant optical solitons with perturbation terms and fractional temporal evolution using improved $tan(\phi(\eta)/2)$-expansion method and exp function approach. *Optik*, 158:933 – 939, 2018.

[28] S. Zhang. Application of exp-function to a kdv equation with variable-coefficients. *Phys. Lett. A*, 365:448–453, 2007.

[29] A. T. Ali and E. R. Hassan. General exp_a function method for nonlinear evolution equations. *Appl. Math. Comput.*, 217:451–459, 2010.

[30] K. Hosseini, Z. Ayati, and R. Ansari. New exact solution of the tzitzéica type equations in nonlinear optics using the exp_a function method. *J. Mod. Opt.*, 65(7):847–851, 2018.

[31] Asim Zafar. Rational exponential solutions of conformable space-time fractional equal-width equations. *Nonlinear Engineering*, 2018.

[32] E. M. E. Zayed and A. G. Al-Nowehy. Generalized kudryashov method and general $exp\ _a$ function method for solving a high order nonlinear schrödinger equation. *J. Space Explor.*, 6:1–26, 2017.

[33] Fuding Xie, Zhenya Yan, and Hongqing Zhang. Explicit and exact traveling wave solutions of whitham–broer–kaup shallow water equations. *Physics Letters A*, 285(1):76 – 80, 2001.

[34] Chenglin Bai. Exact solutions for nonlinear partial differential equation: a new approach. *Physics Letters A*, 288(3):191 – 195, 2001.

[35] K. Hosseini, A. Zabihi, F. Samadani, and R. Ansari. New explicit exact solutions of the unstable nonlinear schrödinger's equation using the exp_a and hyperbolic function methods. *Optical and Quantum Electronics*, 50(2):82, 2018.

[36] G. Samko, A.A. Kilbas, and Marichev. *Fractional Integrals and Derivatives: Theory and Applications*. Gordon and Breach, Yverdon, 1993.

[37] A. Kilbas, M. H. Srivastava, and J. J. Trujillo. Theory and application of fractional differential equations. *North Holland Mathematics Studies*, 204, 2006.

[38] R. Khalil, M. Al Horani, A. Yousef, and M. Sababheh. A new defination of fractional derivative. *J.Comput.Appl.Math.*, 264:65–70, 2014.

[39] W.S. Chung. Fractional newton mechanics with conformable fractional derivative. *J. Comput. Appl. Math.*, 209:150–158, 2015.

[40] Philip G Drazin and Robin S Johnson. *Solitons: an introduction*, volume 2. Cambridge university press, 1989.

[41] Sirendaoreji. Auxiliary equation method and new solutions of klein-gordon equations. *Choas Solitons Fractals*, 31:943–950, 2007.

[42] A. Ebaid. Exact solutions for the generalized klein–gordon equation via a transformation and exp-function method and comparison with adomian's method. *Journal of Computational and Applied Mathematics*, 223(1):278 − 290, 2009.

[43] Roger F Dashen, Brosl Hasslacher, and Andre Neveu. Particle spectrum in model field theories from semiclassical functional integral techniques. *Physical Review D*, 11(12):3424, 1975.

[44] Hira Tariq and Ghazala Akram. New approach for exact solutions of time fractional cahn–allen equation and time fractional phi-4 equation. *Physica A: Statistical Mechanics and its Applications*, 473:352–362, 2017.

[45] Ghazala Akram, Fiza Batool, and Ayesha Riaz. Two reliable techniques for the analytical study of conformable time-fractional phi-4 equation. *Optical and Quantum Electronics*, 50(1):22, 2018.

[46] Aly R. Seadawy, D. Kumar, K. Hosseini, and F. Samadani. The system of equations for the ion sound and langmuir waves and its new exact solutions. *Results in Physics*, 9:1631 − 1634, 2018.