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Article

Probability study on the thermal stress distribution in the thick HK40 stainless steel pipe using finite element method

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ABSTRACT: The present work deals with the development of finite element methodology for obtaining the stress distributions in thick cylindrical HK40 stainless steel pipe that carry high temperature fluids. The material properties and loading are assumed to be random variables. Thermal stresses that are generated along radial, axial and tangential directions are computed generally using analytical expressions which are very complex. To circumvent such an issue, the probability theory and mathematical statistics have been applied to many engineering problems which allows to determine the safety both quantitatively and objectively based on the concepts of reliability. Monte Carlo simulation methodology is used to study the probabilistic characteristics of thermal stresses which is used for estimating the probabilistic distributions of stresses against the variations arising due to material properties and load. A 2-D Probabilistic finite element code is developed in MATLAB and the deterministic solution is compared with ABAQUS solutions. The values of stresses that are obtained from the variation of elastic modulus are found to be low as compared to the case where the load alone is varying. The probability of failure of the pipe structure is predicted against the variations in internal pressure and thermal gradient. These finite element framework developments are useful for the life estimation of piping structures in high temperature applications and subsequently quantifying the uncertainties in loading and material properties.

Keywords: Probabilistic finite element method; HK40 stainless steel; axisymmetric finite elements; random variables; Material and load variability; Monte Carlo simulation

1. INTRODUCTION

The axisymmetric pressurized thick cylindrical pipes are widely used in chemical, petroleum, military industries, fluid transfer plants and power plants as well as in nuclear power plants due to ever-increasing industrial demand. These pipes are generally introduced to excessive pressures & temperatures which are to be steady or continuous. In general, exactly estimating the thermal stresses generating on structural components like thick pipe due to pressure and temperature change is very difficult. Therefore, the probability theory and mathematical statistics have been applied which allows us to determine the safety both quantitatively and objectively based on the concepts of reliability. The stress distribution in the nuclear power plant piping system remains a main concern and deterministic structural integrity assessment need to be combined with probabilistic approaches to consider uncertainties in material and load properties. The deterministic finite element method for a defined problem can be transformed to a probabilistic approach by considering some of the inputs

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to be random variables. The uncertainty associated with the strength prediction may be calculated by simulation techniques, such as Monte-Carlo simulation, which allow the values for basic stiffness variables to be generated based on their statistical distributions (probability density functions).Relevant strength variable for pipe is elastic modulus and load variables are internal pressure and temperature change. The objective herein is to compile statistical information and data based on literature review on both strength and loads random variables relevant to thick pipe structure for quantifying the probabilistic characteristics of these variables. Quantification of random variables of loads and material properties in terms of their means, standard deviations or coefficient of variation and probability of distributions can be achieved by data collection and data analysis. The initial step of the assignment is to gather as much as data for considering which is appropriate for the unarranged variables under study. The second is concerned with statistically analyzing the data to determine the probabilistic characteristics of these variables.

Zhou and Tu [1] carried out the work to estimate the service life of high temperature furnace which is very difficult due variability of creep data. To study the random nature of service life, a new stochastic creep damage model is proposed in his work. A comparison with results calculated by use of the Monte Carlo method verifies the creep damage model. The randomness of the creep damage is demonstrated with a calculation on HK-40 furnace tubes which provides an effective means to assess the reliability of the furnace tubes. In the present work the material parameters of the HK 40 are adapted from Zhou and Tu [1] .Chanylew Taye and Alem Bazezew [2] studied the creep analysis of boiler tubes by *finite element method*, in his work an analysis is developed for the determination of creep deformation of an axisymmetric boiler tube subjected to axisymmetric loads. Heat flux determination finds it application in the field of materials processing [3,4,5].Holm Altenbach et al. [6] presented the creep model is to reflect the basis features of creep in structures including the evaluation of inelastic deformations, relaxation and redistribution of stresses as well as the local reduction of material strength. The solutions are compared with the finite element solutions of ANSYS and ABAQUS finite element codes with user creep model subroutines. The geometric parameters and loading conditions for the present work are adopted from Holm Altenbach [6] and finally Oliver C. Ibe [7] presented the study of fundamentals of applied probability and random processes, the present work follows the probabilistic equations and finally a comprehensive review on the contour method has already been carried out by Prime and DeWald [8]. The node correction in a control volume mesh is not possible because the mesh needs to be regular. That weakness is not present in Finite Element Based Finite Volume Method (FEMFVM)

The stress induced by Thermal gradient through the wall is given by [9]

$$\sigma_r^T \text{(Radial stress)} = \frac{E\alpha(\Delta T)}{2(1-\mu)\ln a}\left[-2\ln\left(\frac{r_0}{r}\right) + \frac{\left(\frac{r_0}{r}\right)^2 - 1}{a^2-1}\ln a\right] \quad (1)$$

$$\sigma_\theta^T \text{(Circumferential stress)} = \frac{E\alpha(\Delta T)}{2(1-\mu)\ln a}\left[1 - \ln\left(\frac{r_0}{r}\right) - \frac{\left(\frac{r_0}{r}\right)^2 + 1}{a^2-1}\ln a\right] \quad (2)$$

$$\sigma_z^T \text{(Axial stress)} = \frac{E\alpha(\Delta T)}{2(1-\mu)\ln a}\left[1 - 2\ln\left(\frac{r_0}{r}\right) - \frac{2\ln a}{a^2-1}\right] \quad (3)$$

$$\sigma_r^T\text{(Radial stress)}=\frac{E\alpha(\Delta T)}{2(1-\mu)\ln a}\left[-2\ln\left(\frac{r_0}{r}\right)+\frac{\left(\frac{r_0}{r}\right)^2-1}{a^2-1}\ln a\right]\sigma_\theta^T(=\frac{E\alpha(\Delta T)}{2(1-\mu)\ln a}\left[1-\ln\left(\frac{r_0}{r}\right)-\right.$$

$$\left.\frac{\left(\frac{r_0}{r}\right)^2+1}{a^2-1}\ln a\right]\sigma_z^T\text{(Axial stress)}=\frac{E\alpha(\Delta T)}{2(1-\mu)\ln a}\left[1-2\ln\left(\frac{r_0}{r}\right)-\frac{2\ln a}{a^2-1}\right] \quad (4)$$

2. ANALYTICAL SOLUTION

2.1. The stresses for thick walled cylinder pipe under internal pressure (P) and thermal gradient (ΔT)

Radial stress is given by

$$\sigma_r = \sigma_r^P + \sigma_r^T \quad (5)$$

$$\sigma_r = \frac{P}{a^2-1}\left[1-\left(\frac{r_0}{r}\right)^2\right] + \frac{E\alpha(\Delta T)}{2(1-\mu)\ln a}\left[-2\ln\left(\frac{r_0}{r}\right) + \frac{\left(\frac{r_0}{r}\right)^2-1}{a^2-1}\ln a\right]$$

$$\sigma_r = \frac{P}{a^2-1}\left[1-\left(\frac{r_0}{r}\right)^2\right] + \frac{E\alpha(\Delta T)}{2(1-\mu)\ln a}\left[-2\ln\left(\frac{r_0}{r}\right) + \frac{\left(\frac{r_0}{r}\right)^2-1}{a^2-1}\ln a\right] \quad (6)$$

Circumferential stress is given by

$$\sigma_\theta = \sigma_\theta^P + \sigma_\theta^T \quad (7)$$

$$\sigma_\theta = \frac{P}{a^2-1}\left[1+\left(\frac{r_0}{r}\right)^2\right] + \frac{E\alpha(\Delta T)}{2(1-\mu)\ln a}\left[1-\ln\left(\frac{r_0}{r}\right) - \frac{\left(\frac{r_0}{r}\right)^2+1}{a^2-1}\ln a\right] \quad (8)$$

Axial stress is given by

$$\sigma_z = \sigma_z^P + \sigma_z^T \quad (9)$$

$$\sigma_z = \frac{P}{a^2-1}2\mu + \frac{E\alpha(\Delta T)}{2(1-\mu)\ln a}\left[1-2\ln\left(\frac{r_0}{r}\right) - \frac{2\ln a}{a^2-1}\right]$$

And finally Von-Mises stress is given by

$$\sigma_e = \frac{1}{\sqrt{2}}\sqrt{(\sigma_r-\sigma_\theta)^2 + (\sigma_\theta-\sigma_z)^2 + (\sigma_z-\sigma_\theta)^2} \quad (10)$$

where σ_θ^P is the hoop stress induced by pressure (MPa); σ_z^P,the axial stress induced by pressure (MPa); σ_r^P,the radial stress induced by pressure (MPa); P, the pressure in (MPa);r_0, the outer radius (mm); r_i,the inner radius (mm); a, the ratio of outer to inner radius; a = r_0/r_i; r, the radius at any position of tube wall(mm) and μ ,the poisson's ratio. σ_θ^T is the hoop stress induced by thermal stress (MPa); σ_z^T,the axial stress induced by thermal stress (MPa); σ_r^T,the radial stress induced by thermal stress (MPa); E, the elastic modulus of material (MPa);α,the thermal expansion coefficient of material ($1/{}^oC$); ΔT ,the Thermal gradient of outer wall and inner wall temperature is

$$\Delta T = T_i - T_0 \quad (11)$$

A thick-walled cylinder pipe carrying high temperature liquid is considered. The fluid inside the pipe is assumed to completely fill the pipe and exerts a constant pressure P. The analysis is carried out in the 2-D plane of the cross section of the pipe see Figure 1

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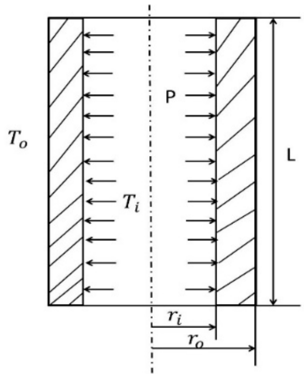

Figure.1. *Schematic diagram of the axisymmetric pipe section.*

The pipe is made up of material HK-40 Zhou and Tu [1]. It is stressed to a pressure of $= 40MPa$. The temperature of the fluid flowing inside the pipe is $T_i = 500$°C and the outside temperature is$T_0 = 420$°C. i.e the pipe is subjected to thermal gradient$\Delta T = 80^0 C$. [20]:The dimensions of the thick pipe section are taken as L $= 100mm$, $r_i = 25\ mm$ and $r_0 = 50\ mm$ respectively. The material properties of HK-40 are given as follows: Elastic modulus $E = 1.38 \times 10^5$ Pa, Poisson's ratio μ =0.313, Thermal expansion coefficient $\alpha = 1.5 \times 10^{-5}(1/^0C)$. The obtained values of the Radial stress, circumferential stress, axial stress and Von Mises stress are shown in the table 1. The graph of different stresses vs. radius are shown in figure 2 and figure 3.

Table 1. Analytical Results of the pipe.

Radius(mm)	Radial stress ($\boldsymbol{\sigma_r}$) MPa	Circumferential stress($\boldsymbol{\sigma_\theta}$) MPa	Axial stress($\boldsymbol{\sigma_z}$) MPa	Von Mises stress($\boldsymbol{\sigma_v}$) MPa
26.25	-35.04	61.70	8.34	83.93
28.75	-26.99	53.66	8.34	70.02
31.25	-20.80	47.46	8.34	59.33
33.75	-15.93	42.59	8.34	50.93
36.25	-12.03	38.70	8.34	44.21
38.75	-8.86	35.53	8.34	38.77
41.25	-6.25	32.92	8.34	34.29
43.75	-4.08	30.74	8.34	30.57
46.25	-2.24	28.91	8.34	27.44
48.75	-0.69	27.35	8.34	24.80

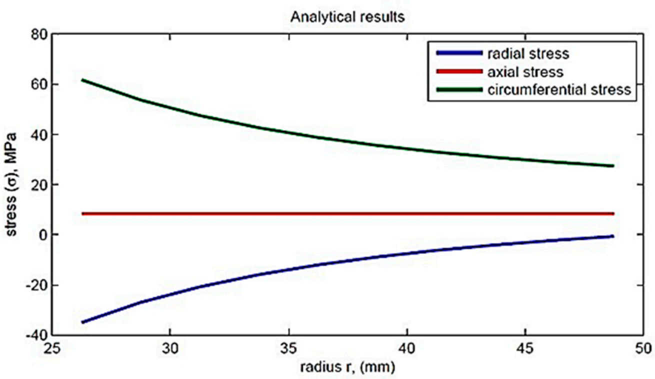

Figure.2. *Variation of Radial, Circumferential, Axial stresses.*

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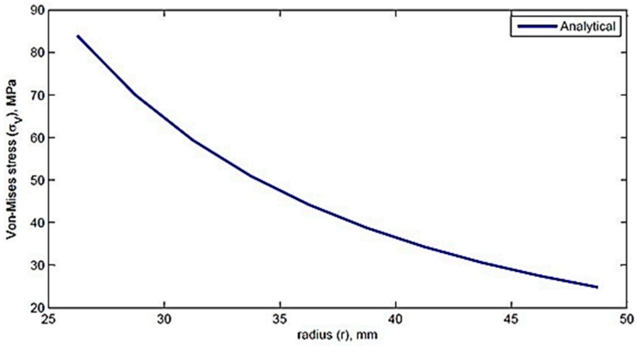

Figure.3. *Von-Mises stress verses Radius.*

3. Axisymmetric Finite Element Analysis Using Abaqus Software

The pipe is made up of material HK40 is considered with pipe length L = 100 mm, Inner radius r_i = 25 mm, outer radius r_0 = 50 mm. The material properties of the material are Elastic Modulus E = 1.38 X 10^5 MPa, Poisson's ratio υ = 0.313, Coefficient of thermal expansion α = 1.5 × 10^{-5}(1/⁰C). The model is meshed with element type CAX4R, A 4-noded bilinear quadrilateral element and the Mesh grid is 10 x10 elements and is fixed in Axial direction U2 =0 as shown in the figure 4. The loading conditions are Internal Pressure P = 40 MPa, Inside temperature T_i = 500 ℃, Outside temperature T_0 = 420℃ , Thermal gradient ΔT = 80 ℃. Figure 4 shows the model with meshing and applied boundary conditions in abaqus.

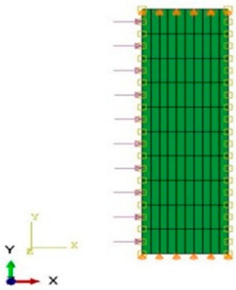

Figure.4. *Meshing, Boundary conditions, internal pressure and Thermal gradient of Axisymmetric Thick pipe*

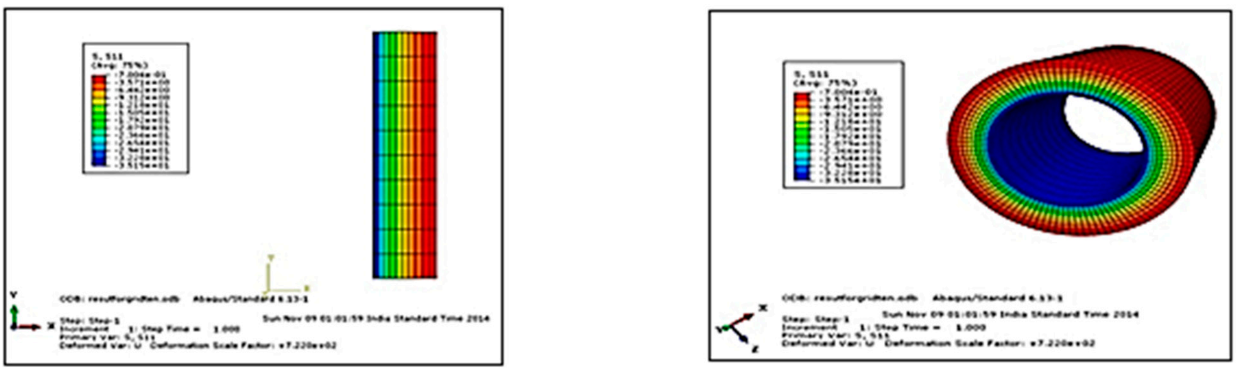

Figure.5. *Analysis on before sweep and after sweep of Radial stress elements.*

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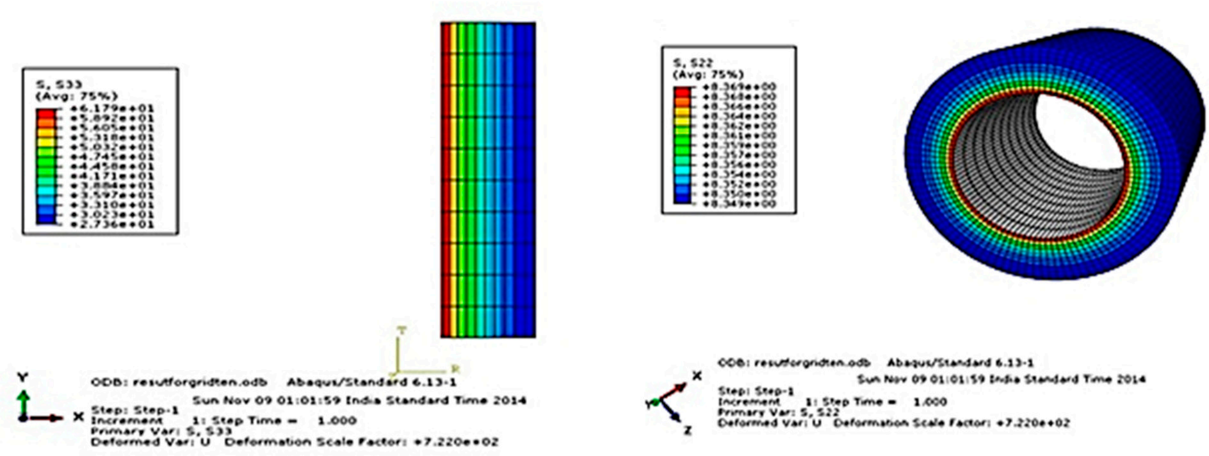

Figure.6. *Analysis on Before sweep and after sweep of axial stress elements* .

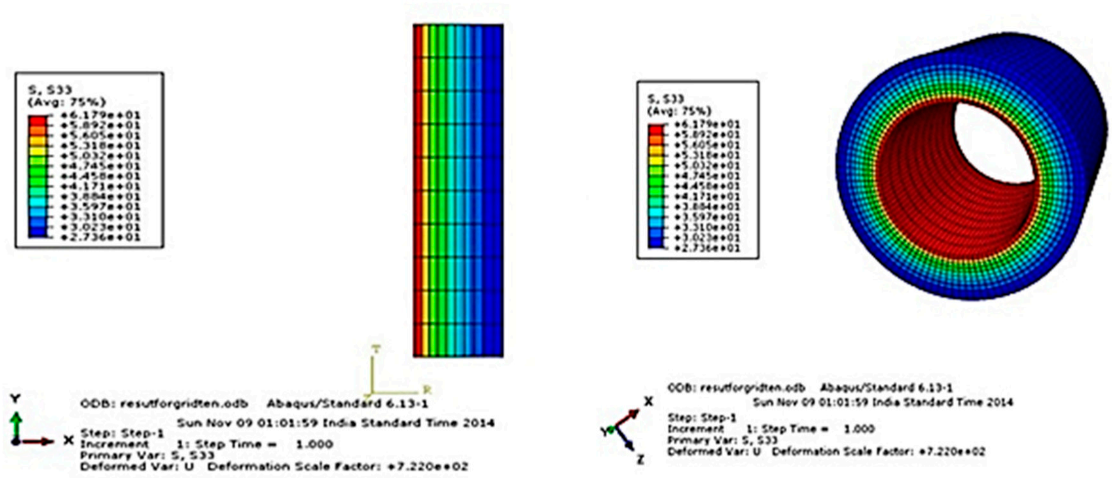

Figure.7.*Analysis on* before *sweep and after sweep of circumferential stress elements*

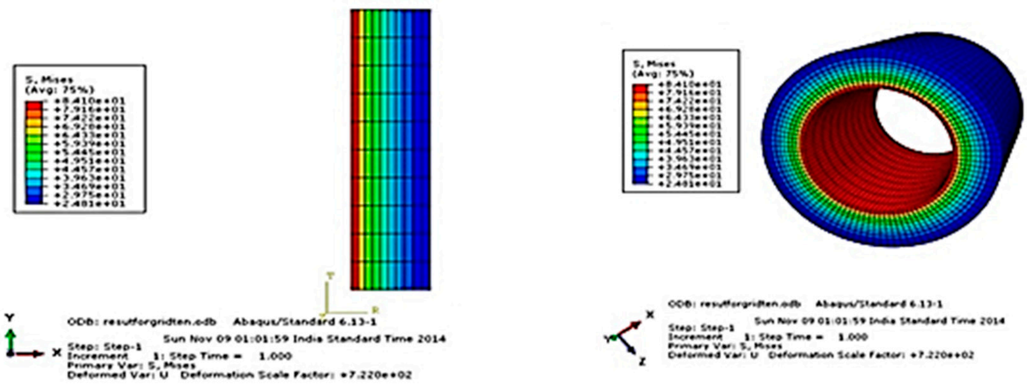

Figure.8. *Analysis on before sweep and after sweep of von-Mises stress elements*

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Table 2 . Comparison of Analytical and FEA using Abaqus Results by using Von mises stress

Radius(mm)	Analytical Results			FEA using Abaqus		
	σ_r(MPa)	σ_θ(MPa)	σ_V(MPa)	σ_r(MPa)	σ_θ(MPa)	σ_V(MPa)
26.2500	-35.0416	61.7082	83.9361	-35.1470	61.7910	84.0936
28.7500	-26.9943	53.6610	70.0273	-27.0669	53.7178	70.1337
31.2500	-20.8000	47.4667	59.3306	-20.8515	47.5068	59.4038
33.7500	-15.9305	42.5972	50.9312	-15.9679	42.6262	50.9821
36.2500	-12.0333	38.7000	44.2184	-12.0611	38.7214	44.2537
38.7500	-8.8658	35.5324	38.7720	-8.8868	35.5485	38.7961
41.2500	-6.2565	32.9232	34.2951	-6.2727	32.9354	34.3109
43.7500	-4.0816	30.7483	30.5730	-4.0942	30.7578	30.5825
46.2500	-2.2498	28.9165	27.4476	-2.2597	28.9239	27.4520
48.7500	-0.6925	27.3592	24.8000	-0.7004	27.3650	24.8004

a. Comparison of Analytical and FEA using Abaqus:

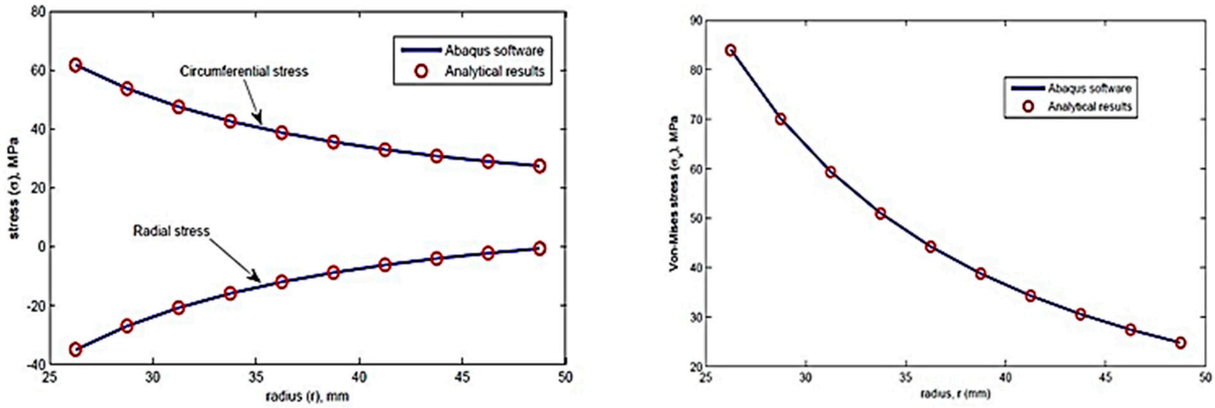

Figure.9. *Radial , circumferential stress and von-Mises stress*

Table 3. Comparison of Analytical and FEM using Matlab Results.

Radius(mm)	Analytical Results			FEM using Mat lab		
	σ_r(MPa)	σ_θ(MPa)	σ_V(MPa)	σ_r(MPa)	σ_θ(MPa)	σ_V(MPa)
26.250	-35.041	61.708	83.936	-34.950	61.883	81.636
28.750	-26.994	53.661	70.027	-26.926	53.768	68.244
31.250	-20.800	47.466	59.330	-20.748	47.532	57.917
33.750	-15.930	42.597	50.931	-15.890	42.635	49.789
36.250	-12.033	38.700	44.218	-12.002	38.720	43.280
38.750	-8.865	35.532	38.772	-8.841	35.540	37.990
41.250	-6.256	32.923	34.295	-6.237	32.923	33.636
43.750	-4.081	30.748	30.573	-4.067	30.742	30.010
46.250	-2.249	28.916	27.447	-2.238	28.906	26.963
48.750	-0.692	27.359	24.800	-0.684	27.346	24.378

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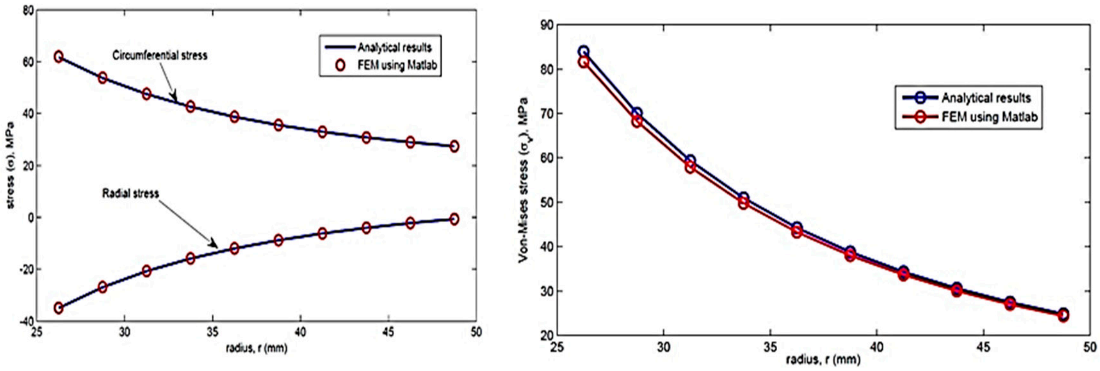

Figure.10. *Radial, circumferential and von-Mises stress.*

b. Comparison of Analytical, FEA using Abaqus and FEM using Mat lab Results

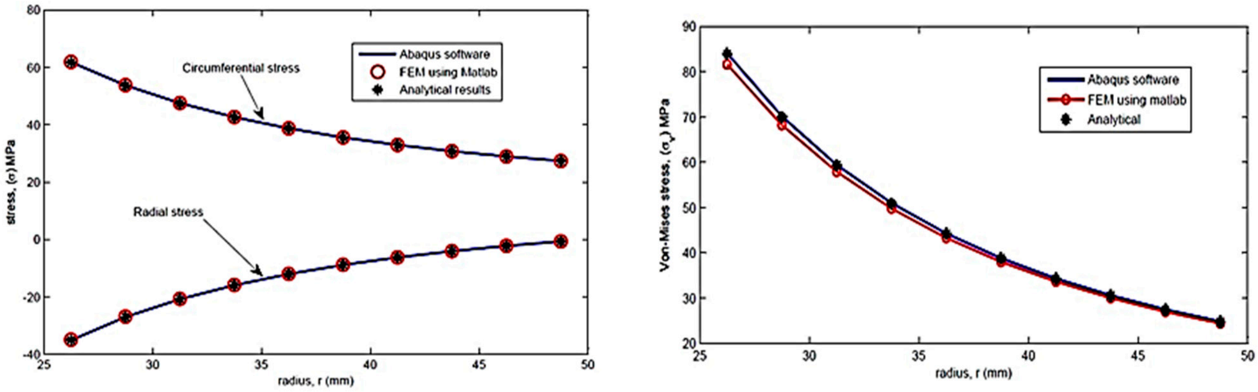

Figure.11. *Radial, circumferential and von-Mises stress.*

4. PROBABILISTIC STUDY

4.1. Monte-Carlo Simulation

The Monte-Carlo method involves randomly sampling the distributed input variables many times so as to build a statistical picture of the output quantities, The method has a wide range of applicability, engineering applications being only one. The method is particularly appropriate when there are a large number of independent variables which can influence the outcome. The Monte-Carlo method is being used increasingly in structural integrity applications. The probabilistic simulation uses a Monte-Carlo method with Latin Hypercube sampling [11]. This is an efficient technique which permits a large number of distributed variables to be addressed. Each variable take a finite number of values each of which represents a range of values (a 'bin'). All bins are of equal probability. The Latin Hypercube algorithm ensures that all bins of variables are sampled in the minimum number of trails (though not, of course, in all possible combinations). Moreover, because all bins are of equal probability it follows that all trails of equal probability, thus ensuring that all trails are of equal weight in the simulation [12].

4.2. Distributed structural parameters

The parameters which are required to calculate thermal stresses and which are taken as distributed in simulations are: elastic modulus, internal pressure and temperature change. Normal distributions are used for stresses and lognormal distribution is used for material properties.

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4.3. Lognormal distribution

A random variable X is consider to have a lognormal distribution if Y = ln(X) has a normal probability distribution. The density function of lognormal distribution is given by:

$$f_X(x) = \frac{1}{x\sigma\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right], \qquad 0 < x < \infty \qquad (12)$$

The notations $X \sim LN(\mu, \sigma^2)$ to provide an abbreviated description of a lognormal distribution. The notation states that X is log normally distributed with mean μ and variance σ^2

4.4. Input Distribution

4.4.1. Due to variability in material property i.e. Young's modulus (E)

Lognormal distribution for Young's modulus of elasticity (E):

$$f_X(E; \mu, \sigma) = \frac{1}{x\sigma_x\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{\ln x - \mu_x}{\sigma_x}\right)^2\right] \qquad (13)$$

Lognormal distribution for E=1.38×10^5 Pa

Monte-Carlo simulations (MCS) N = 1000 runs are carried out to estimate the stress distribution for number of elements in radial direction

Where Mean is E, Variance is σ^2

Coefficient of variance = 0.2 = $\frac{\sigma}{\mu} = \frac{\sigma}{E}$

Mean = $(\mu_x) = \log\frac{\mu^2}{\sqrt{\sigma^2+\mu^2}}$

Standard deviation = $(\sigma_x) = \sqrt{\log\frac{\sigma^2}{\mu^2+1}}$

Elastic modulus is lognormal distributed with mean and standard deviation as 1.38 x 10^5 Pa respectively.

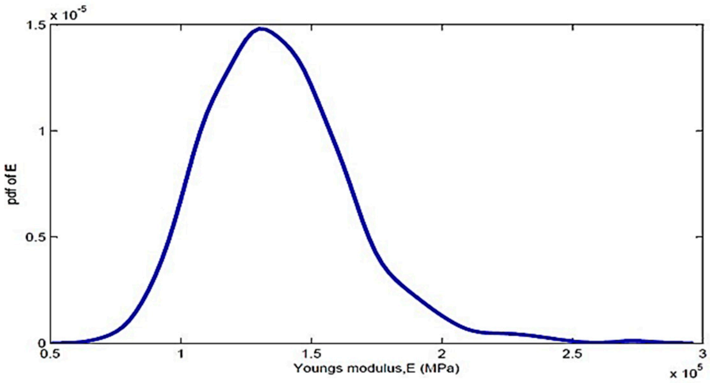

Figure.12. *Young's modulus due to the variation of the material property.*

4.4.2. Due to load variability

Normal distribution:

This distribution is basis for many statistical methods. The normal density function for a random variable X is given by:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right], \qquad -\infty < x < \infty. \qquad (14)$$

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It is common to use the notation $X \sim N(\mu, \sigma^2)$ to provide an abbreviated description of a normal distribution. The notation states that X is normally distributed with mean μ and variance σ^2.

In this study, P and ΔT are random variables in r-radial and z-axial direction. Normal distribution of the form

$$f_X(x;\mu,\sigma) = f_X(P,\Delta T;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right], \quad where \quad -\infty < x < \infty. \qquad (15)$$

Normal distribution for pressure (P):

Mean = P, Variance = σ^2, Coefficient of variance = 0.1 = $\frac{\sigma}{\mu} = \frac{\sigma}{P}$, Standard deviation = (σ_x) = $\sqrt{V}$,

Pressure $(P) = x * \sigma_x + \mu$

Pressure is normal distributed with mean and standard deviation of 40 and 4 respectively

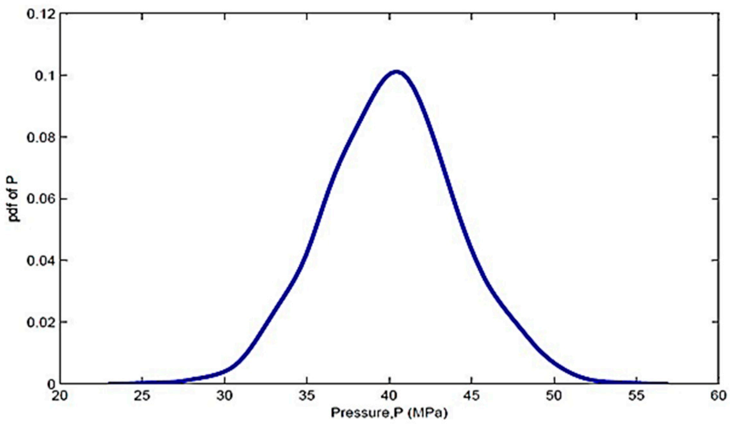

Figure.13. *Nominal distribution of Pressure.*

Normal distribution for Thermal gradientΔT:

Mean = $\Delta T = T_{i(x)} - T_0$ (16)

Variance = σ^2, Coefficient of variance = 0.05 = $\frac{\sigma}{\mu} = \frac{\sigma}{\Delta T = (T_{i(x)} - T_0)}$, Standard deviation = (σ_x) = $\sqrt{V}$

Thermal gradient $(\Delta T) = x * \sigma_x + \mu$

Inside temperature is varying and outside temperature is kept constant

Thermal gradient is normal distributed with mean and standard deviation as 80 and 4 respectively

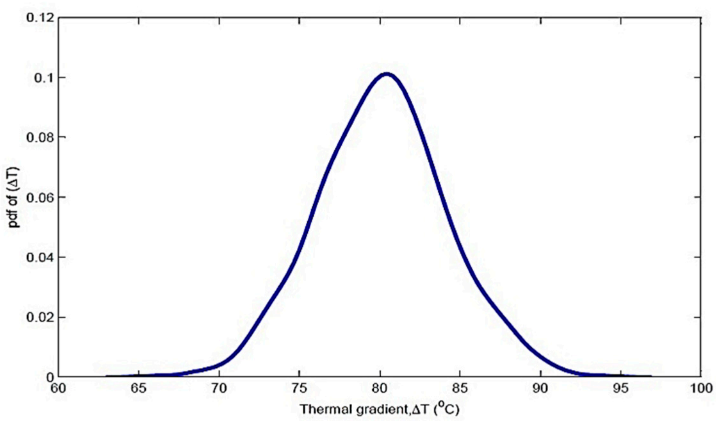

Figure.14. *Nominal distribution of Thermal gradient.*

5. Probabilistic finite element formulation:

Development of probabilistic finite element formulation for an axisymmetric pipe section is based on available procedure for determining finite element analysis of axis symmetric pipe section. The axisymmetric section of the pipe used for finite element analysis is discretized into ten finite elements in the radial direction and 10 elements in the axial direction and the geometry of the FE Mesh if the shown in the figure below.

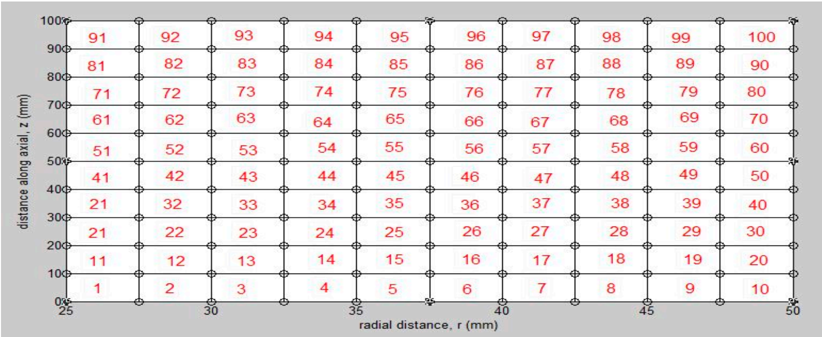

Figure.15. *finite element mesh for the geometry of axisymmetric pipe section.*

the constitutive equations for the axisymmetric pipe is given by

$$\sigma = C(\epsilon - \epsilon_0) \tag{17}$$

here, σ is the resultant stress vector induced due to combined effect of pressure and temperature gradient. the components of stress and strain vector are represented as

$$\sigma = (\sigma_r \; \sigma_z \; \sigma_\theta \; \tau_{rz})^T; \epsilon = (\epsilon_r \; \epsilon_z \; \epsilon_\theta \; \gamma_{rz})^T, \; \epsilon_0 = (\alpha\Delta T(r,z) \;\; \alpha\Delta T(r,z) \;\; \alpha\Delta T(r,z) \;\; 0) \tag{18}$$

ϵ_0 is the initial strain vector due to temperature change, here α is the coefficient of linear expansion and superscript , t is the transpose operator. c is the constitutive matrix which is a function of e, the young's modulus of elasticity of the isotropic material and μ is the poisson's ratio. e is considered to be a random variable and is of the form

$$E = E(x) = LN(\mu, \sigma^2) \tag{19}$$

therefore material matrix can be expressed as

$$C(r,z) = \frac{E(r,z)}{(1-2\vartheta)(1+\vartheta)} \begin{pmatrix} 1-\vartheta & \vartheta & \vartheta & 0 \\ \vartheta & 1-\vartheta & \vartheta & 0 \\ \vartheta & \vartheta & 1-\vartheta & 0 \\ 0 & 0 & 0 & \frac{1-2\vartheta}{2} \end{pmatrix} \tag{20}$$

6. Derivation of stiffness matrix:

In the next process the 4-noded finite element quadrilateral element for axisymmetric probabilistic finite element analysis with two degrees per node, denoted by $\delta = [u, w]$, where $u(r,z)$ is the vector of radial displacement and $w(r,z)$ is the vector of axial displacement. Since it is the axisymmetric case, displacement in θ direction is zero. The internal strain energy, U can be written as

$$U = \frac{1}{2}\delta_e^T K_e \delta_e \tag{21}$$

Where δ_e is the element displacement vector and K_e is the element stiffness matrix of the pipe section expressed as

$$K_e(r,z) = \int_{r1}^{r_2}\int_{z_1}^{z_2} [BC(r,z)B^T] 2\pi r \, dr \, dz \tag{22}$$

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B is the strain-displacement transfer matrices (derivatives of FE shape functions) and independent of material properties. The limits r_1 and r_2 are the inner and outer radii of the cylindrical pipe and L is the length of pipe from $z_1$toz_2. When C is considered to be random variable.

The element load matrix is given by

$$F_e(r,z) = \int_{r1}^{r_2}\int_{z_1}^{z_2}[BC(r,z)\in_o]2\pi r\,dr\,dz \qquad (23)$$

A Gauss quadrature integration scheme is used to evaluate the above integrals. The global stiffness matrix K is obtained by assembling all the element stiffness matricesK_e. Subsequently, the nodal displacements are estimated by solving the finite element governing equation

The global load vector F and the displacement vector obtained from this governing equation used for calculating strains for each element at centroid location.

$$[K(\omega)]\,[\delta(\omega)] = F(\omega) \qquad (24)$$

Where ω is the random variable. So, Monte Carlo simulations are used to simulate the stresses for each element. Finally, the stress contours for each element in radial direction is obtained.

6.1. Material Specifications

The pipe is made up of material HK40. It is stressed to a pressure of $P = 40MPa$ and subjected to thermal gradient $\Delta T = 80^0C$. The dimension of Thick pipe section are taken as$L = 100\ mm, r_i = 25\ mm$ and $r_0 = 50\ mm$ respectively. The material properties of HK40 are given as follows: Elastic modulus $E = 1.38 \times 10^5\ Pa$, Poisson's ratio $\mu = 0.313$, Thermal expansion coefficient $\alpha = 1.5X10^{-5}(1/{}^0C)$. For the numerical calculations, the random variable for Young's modulus of elasticity is E

7. PROBABILISTIC STUDY: OUTPUT DISTRIBUTION

The output distribution is a complete and systematic framework for probabilistic modeling of material variability and load fluctuations for the fatigue design expected.

7.1. Due to material variability

In order to characterize the material variability on the cyclic stress-strain and strain-life responses of the various elements under multi axial fatigue, the various random variables have been calculated

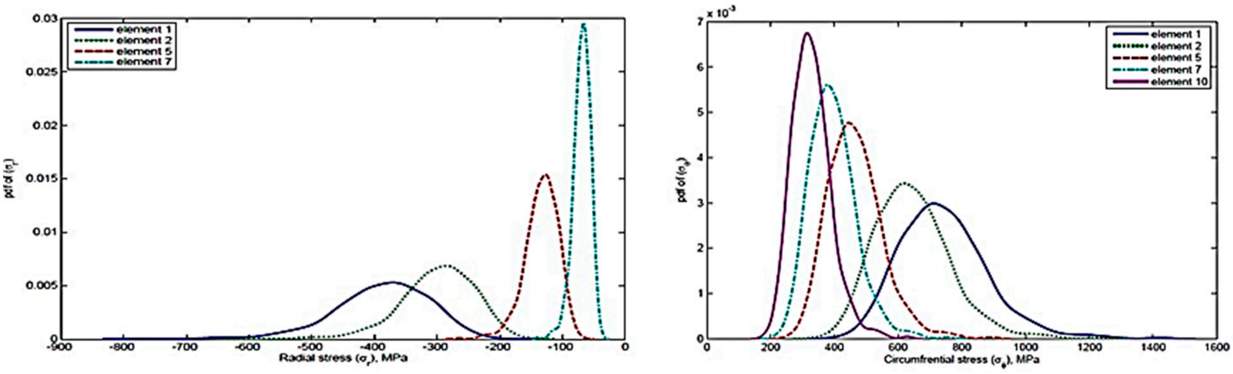

Figure.16. *comparison of radial stress and circumferential stress for different elements when* E *is a random variable.*

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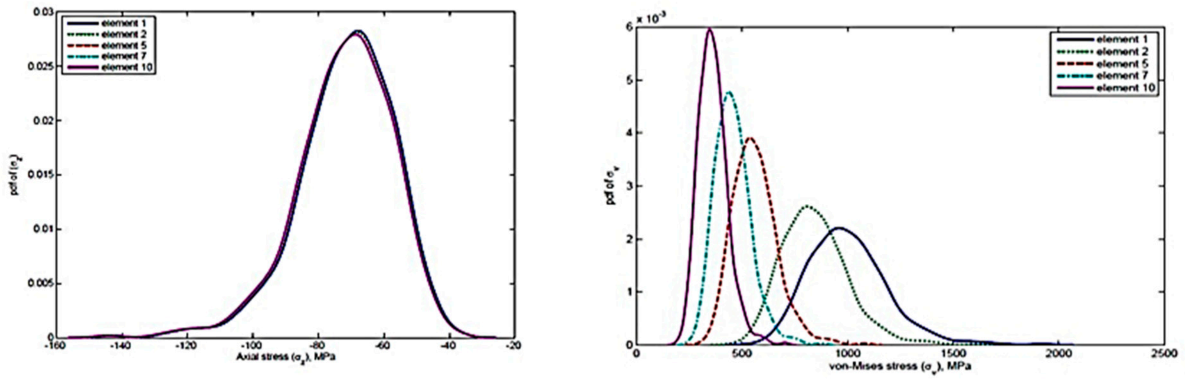

Figure.17. Comparison of axial stress and Von-mises stress for different elements when E is a random variable.

7.2. Due to both material and load variability

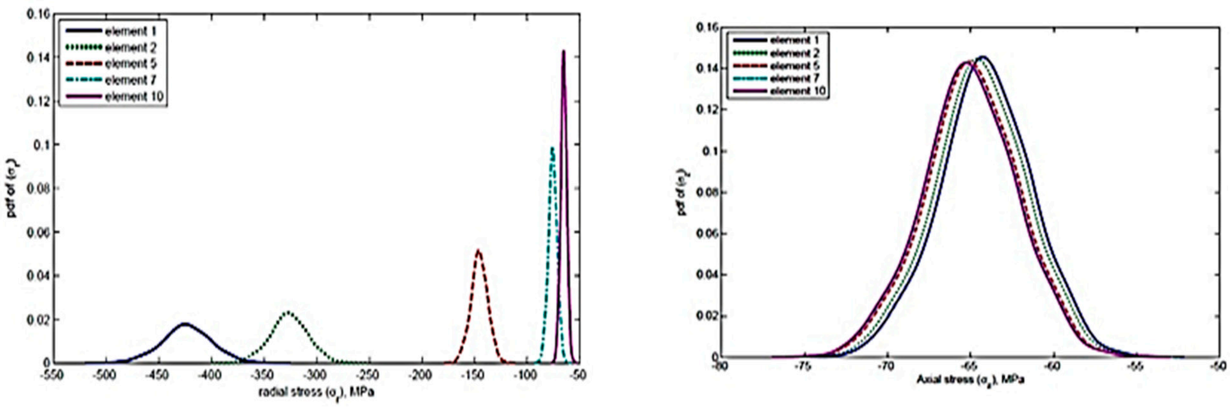

Figure.18. Comparison of radial stress and axial stress for different elements when E, P and ΔT are random variables.

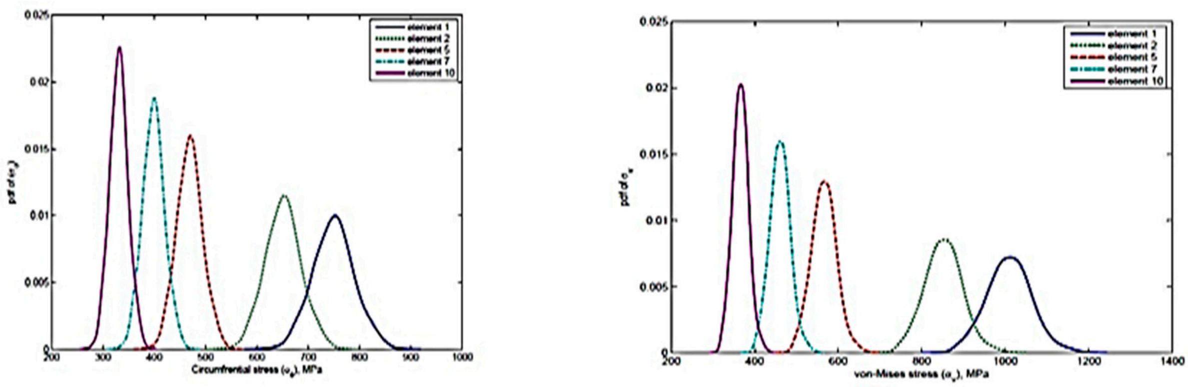

Figure.19. *Comparison of circumferential stress and Von mises stress for different elements when* E, P *and* ΔT *are random variables*

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8. Probability of failure of Von-Mises stress with respect to Yield strength:

The difference in the von-Mises stress lies in the selection of the shell face either positive or negative commonly known as SPOS and SNEG in ABAQUS [13]

Probability of failure:

$$P_F = P[\sigma_V > S_Y] \quad (25)$$
$$= 1 - P[\sigma_V \leq S_Y]$$

Yield strength (σ_Y)= 241 MPa for HK40 (Austenitic heat resistant stainless steel) material.

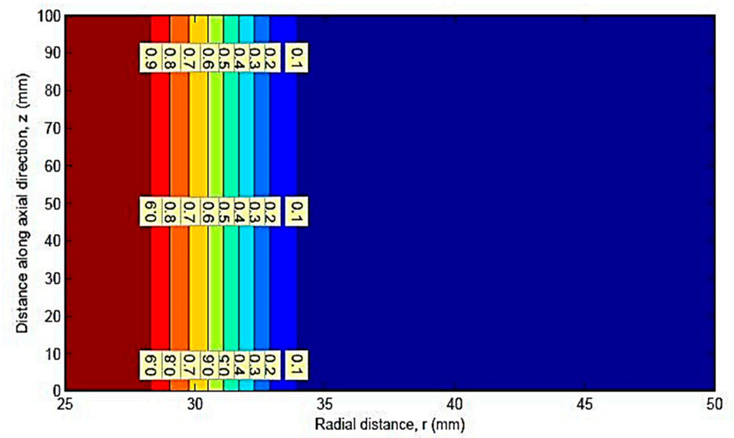

Figure.20. Probability of failure of von-Mises stress.

9. STRESS CONTOURS

The contour method is used to measure the stress in the component which is normal to the section surface for both left and right sides as shown in figures 21-24. Generally in the contour method we can study the numerical data in order to verify that it could accurately measure various types of stresses. In our study we have measured various stresses such as radial, circumferential, Axial, Von mises of HK40 Material

9.1. Mean stress contours for HK40 material

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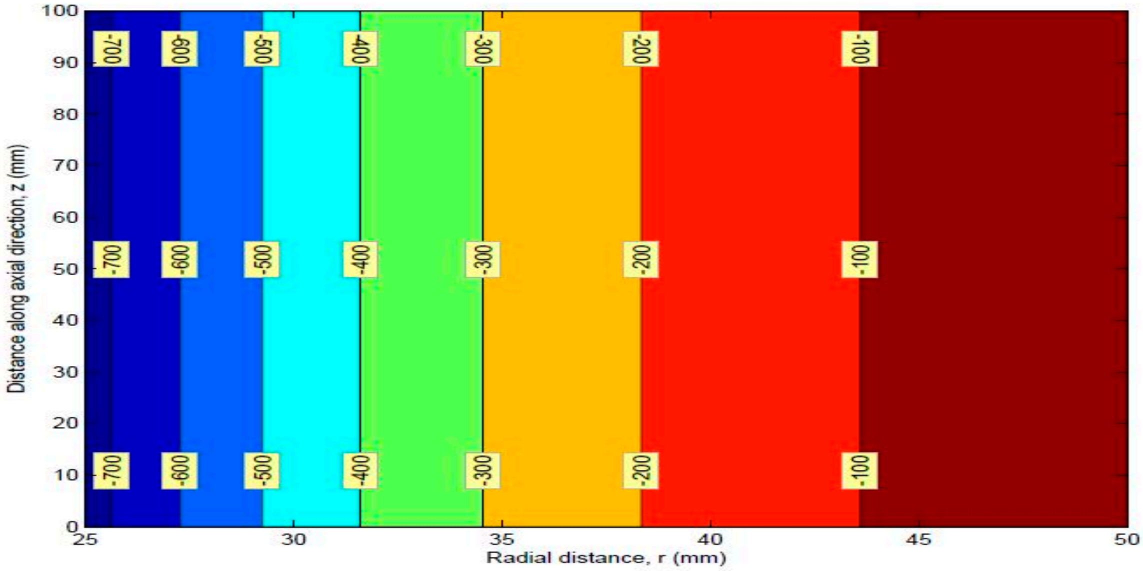
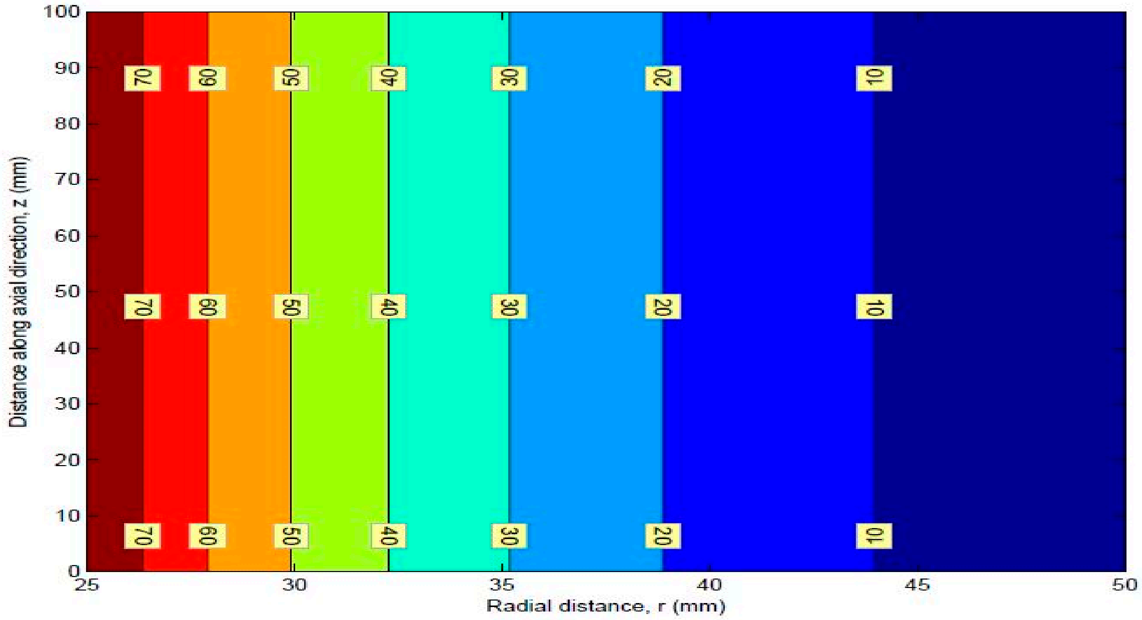
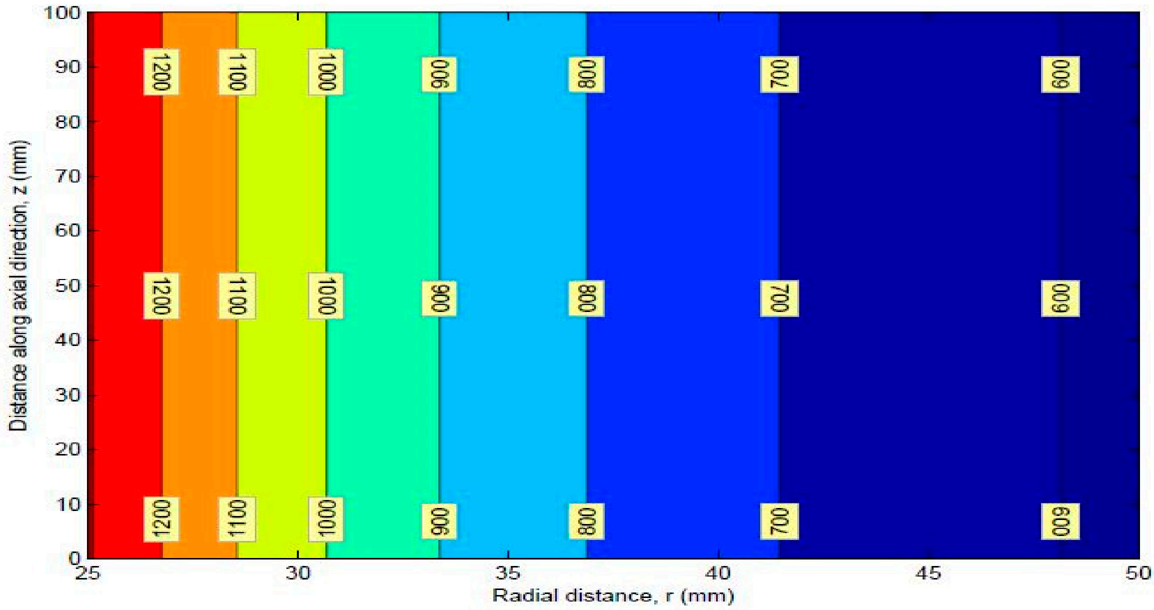


(a)



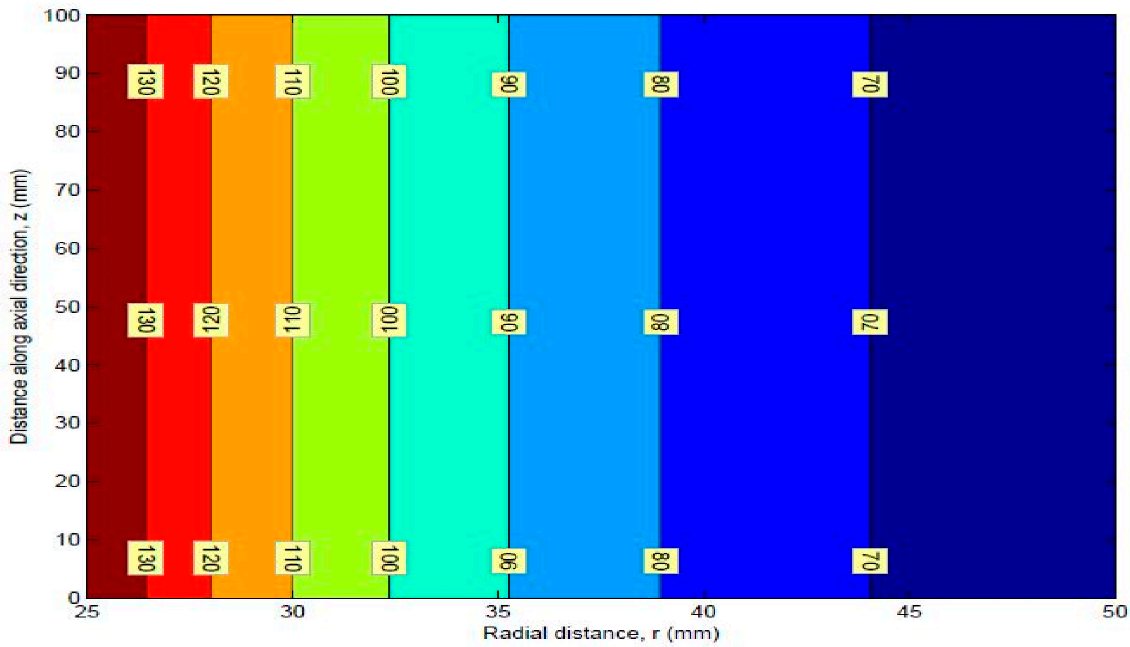
(b)

Figure.21.(a) *Mean (b) Standard deviation of radial stress*



325

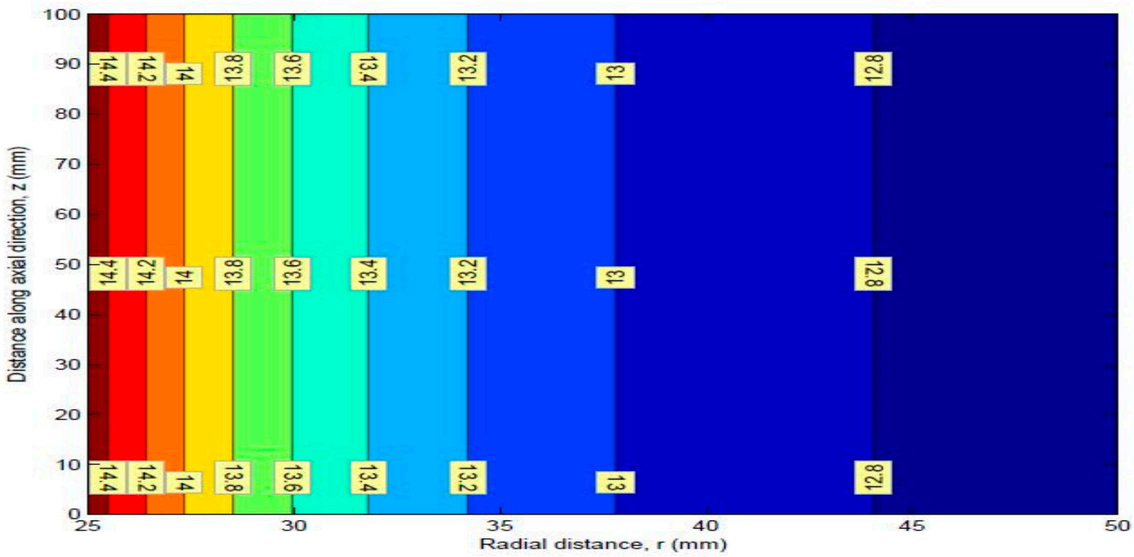
326 (a)



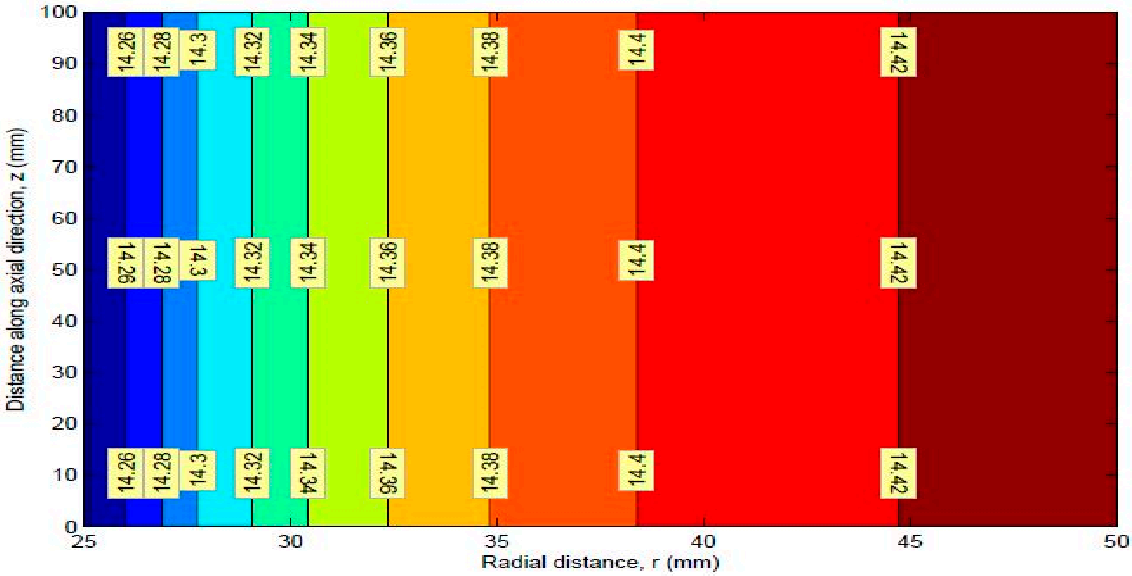
327

328 (b)

329 **Figure.22.** (a) *Mean (b) Standard deviation of circumferential stress.*



(a)



(b)

Figure.23. (a) *Mean (b) Standard deviation of axial stress.*

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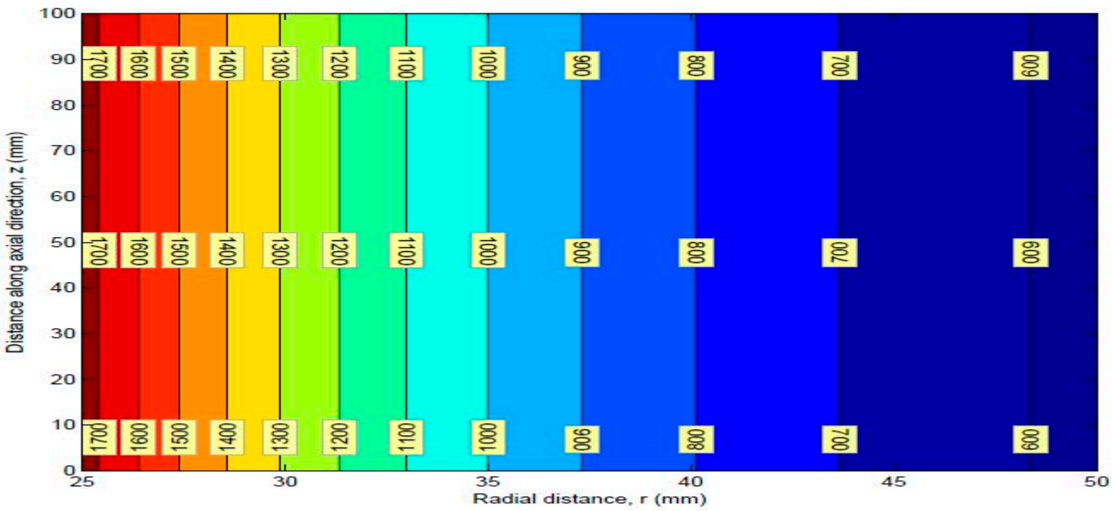
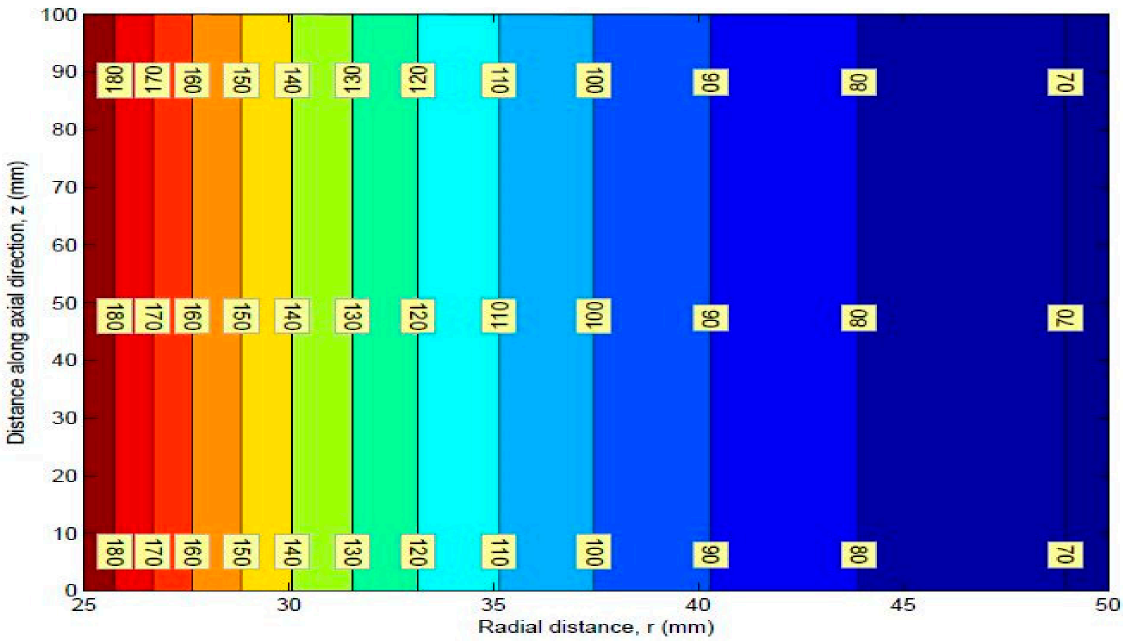


(a)



(b)

Figure.23. (a) *Mean (b) Standard deviation of Von-Mises stress.*

10. CONCLUSION

The results are predicated and proved with the help of ABAQUS software and FEM using MATLAB and a very good match is obtained among all of these findings. Finite element frame work is enhanced to find the effects of uncertainties in pipe structure due to material properties and loading. Random variable models are used to model the variability's in material properties and load by using Monte Carlo simulations. Monte Carlo simulations are used to study the probabilistic characteristics of stress distribution of pipe structure. The values of thermal stresses that are obtained from the variation in material properties like modulus of elasticity are found to be low as compared to the case where the load alone is varying.

The present methodology is used for estimating the probabilistic distributions of thermal stresses against the variations arising due to material properties and as well as variations due to thermal

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loading. The probability of failure of the pipe structure is predicted against the variations in internal pressure and thermal gradient and finally the results in the contour method indicated that it could be very similar with the result obtained using the analytical formula, when asymmetrical cut was made by averaging the stress components of both sides of the cut. The developed methodology can be helpful for the life assessment of piping structures that can be used for high temperature practices against creep, fatigue failures for further studies.

NOMENCLATURE

σ_r^T Radial stress

ΔT Thermal gradient

σ_z^T Axial stress

σ_θ^T Circumferential stress

σ_θ^P hoop stress induced by pressure

σ_z^P axial stress induced by pressure

σ_r^P radial stress induced by pressure

r_0 outer radius (mm)

r_i inner radius (mm)

a ratio of outer to inner radius

μ poisson's ratio

r radius at any position of tube wall(mm)

E elastic modulus of material (MPa)

α thermal expansion coefficient of material

CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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