Bayesian VAR Modeling and Forecasting of the Dynamic Interrelationship between Economic Growth and Revenue from Oil and Non-Oil Sectors in Nigeria[+]

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Abstract

The present reality about the Nigerian economy calls for investment and development in the non-oil sector. This becomes necessary as a result of fall in the oil price in the global market. This paper examined the Bayesian Vector Autoregression (BVAR) modeling and forecasting of the dynamic interrelationship between Economic growth and revenue from the oil and non-oil sectors in Nigeria. To achieve this, annual data on Gross Domestic Product (GDP), revenue from oil and non-oil sectors were collected from Central Bank of Nigeria (CBN) bulletin, the sample from 1981 to 2008 was used for analysis, while sample from 2009 to 2014 was used for model validation. Six (6) versions of Sims-Zha BVAR models were compared for out-of-sample forecast, the result revealed the superiority of the BVAR6 model over the other BVAR models. Lastly, evidence from the decomposition forecast errors revealed that revenue of oil sector contributed 7.69% to GDP while revenue from non-oil sector contributed 0.12% to GDP in Nigeria. This paper therefore recommended that the present government should encourage investment that is geared toward development in the non-oil sector, of which it has the capacity to improve the Economic growth of the Nigerian economy.

Keywords: Bayesian Vector Autoregression (BVAR), Modeling, Forecasting, Gross Domestic Product (GDP), Economic Growth, Revenue, Oil sector, Non-oil Sector

Introduction

Revenue is defined as all amounts of money received by a government from external sources for example those originating from "outside the government" net of refunds and other correcting transaction, proceeds from issuance debt, sale of investments, agency or private trust transactions, and intra-governmental transfers (Ahmed, 2010; Okwori & Sule, 2016). For instance in Nigeria, provision of economic infrastructures, social services, debt services, providing the Army, the Police, the court system and any other operation of the government, are provided by the Nigerian government through oil revenue, non-oil revenue and federal government independent revenue (Omachi, 2011). Unfortunately, the Nigerian economy has suffers decline in revenue during this period of recession, leading to high inflation rate and leading to shortfalls in government capital and recurrent expenditures.

The following are related literature review: Ude and Agodi (2012) used cointegration and error correction model to investigate the impact of non-oil revenue on economic growth in Nigeria. They used annual data from 1980 to 2013 and the study revealed that agricultural revenue, manufacturing revenue and interest rate have significant impact on economic growth in Nigeria. Offia (2012) examined the impact of non-oil exports on economic growth in Nigeria from 1986-2010. Results from multiple regression revealed that non-oil export is statistically significant to Nigeria economic growth. In a similar work by Abogan et al (2014) investigated the impact of non-oil export on economic growth in Nigeria between 1980 and 2010. Evidence from ordinary least squares method involving error correction mechanism revealed that the impact of non-oil export on economic growth was moderate. Akwe (2014) investigated the impact of non-oil tax revenue on economic growth in Nigeria. Results from the empirical study revealed a statistical significance of economic growth effects of non-oil tax revenue. Oziengbe (2015) investigated the effects of important penetration and FDI inflows on the performance of Nigera's non-oil exports in the period from 1981-2012 using the methodology of ARDL (Bounds test) approach to cointegration and error correction analysis. The study indicated that import penetration impacted positively on the performance of Nigeria's non-oil export in the short run, though its long run impact was negative. The short run and long run impact of FDI on non-oil export performance were not statistically significant. Furthermore, the currency depreciation positively impact the performance of Nigeria's non-oil export in the long run, but its short run impact was not significant. Okwori & Sule (2016) fitted a long run relationship between GDP and Oil revenue, non-oil revenue, domestic debt and external debt in Nigeria. Evidence from their study revealed that oil revenue, non-oil revenue and external debt are positively related to GDP while domestic debt was negatively related to GDP. Okezie and Azubike (2016) evaluated the contribution of non-oil revenue to government revenue and economic growth in Nigeria from 1980 to 2014. Results from ordinary least squares regression revealed a positive and significant contribution of non-oil revenue to economic growth and positive insignificant contribution to government revenue.

This study therefore examined the contribution of oil and non-oil revenues to Gross Domestic Product (GDP) growth rate in Nigeria using six (6) versions of Sims-Zha Bayesian Vector Autoregressive (BVAR) models for out-of-sample forecast comparison.

Model Specification and Description
Bayesian Vector Autoregression with Sims-Zha Prior

One of the advantages of Bayesian VAR over the Classical VAR, is its suitability in small sample time series. In time series literature, many versions of BVAR are available. These include Litterman prior, Jeffery prior and Sims-Zha prior etc. In recent times, the BVAR model of Sims and Zha (1998) has gained popularity both in economic time series and political analysis. One major advantage of the Sims-Zha BVAR is that it allows for a more general specification and can produce a tractable multivariate normal posterior distribution. Again, the

Sims-Zha BVAR estimates the parameters for the full system in a multivariate regression (Brandt and Freeman, 2006).

Given the reduced form model

$$y_t = c + y_{t-1}B_1 + \ldots + y_{t-p}B_p + u_t$$

where $c = dA_0^{-1}$, $B_l = -A_lA_0^{-1}$, $l = 1,2,\ldots p$, $u_t = \varepsilon_t A_0^{-1}$ and $\Sigma = A_0^{-1'}A_0^{-1}$

The matrix representation of the reduced form is given as

$$\underset{T\times m}{Y} = \underset{T\times(mp+1)}{X}\underset{(mp+1)\times m}{\beta} + \underset{T\times m}{U}, U \sim MVN(0,\Sigma)$$

We can then construct a reduced form Bayesian SUR with the Sims-Zha prior as follows. The prior means for the reduced form coefficients are that $B_1=I$ and $B_2, \ldots B_p=0$. We assume that the prior has a conditional structure that is multivariate Normal-inverse Wishart distribution for the parameters in the model. To estimate the coefficients for the system of the reduced form model with the following estimators

$$\hat{\beta} = (\Psi^{-1} + X'X)^{-1}(\Psi^{-1}\overline{\beta} + X'Y)$$

$$\hat{\Sigma} = T^{-1}(Y'Y - \hat{\beta}'(X'X + \Psi^{-1})\hat{\beta} + \overline{\beta}'\Psi^{-1}\overline{\beta} + \overline{S})$$

where the Normal - inverse Wishart prior for the coefficients is

$$\beta/\Sigma \sim N(\overline{\beta},\Psi) \text{ and } \Sigma \sim IW(\overline{S},v)$$

This representation translates the prior proposed by Sims and Zha form from the structural model to the reduced form (Brandt and Freeman, (2006, 2009), and Sims and Zha, (1998, 1999). The summary of the Sims-Zha prior is given in Table 1.0.

Table 1.0: Hyperparameters of Sims-Zha reference prior

Parameter	Range	Interpretation
λ_0	[0,1]	Overall scale of the error covariance matrix
λ_1	>0	Standard deviation around A_1 (persistence)
λ_2	=1	Weight of own lag versus other lags
λ_3	>0	Lag decay
λ_4	≥0	Scale of standard deviation of intercept
λ_5	≥0	Scale of standard deviation of exogenous variable coefficients
μ_5	≥0	Sum of coefficients/Cointegration (long-term trends)
μ_6	≥0	Initial observations/dummy observation (impacts of initial conditions)
v	>0	Prior degrees of freedom

Source: Brandt and Freeman, (2006)

This study considered six (6) versions of Sims-Zha Bayesian VAR model given below

$$BVAR1 = (\lambda_0 = 0.6, \lambda_1 = 0.1, \lambda_3 = 1, \lambda_4 = 0.1, \lambda_5 = 0.07, \mu_5 = \mu_6 = 5)$$
$$BVAR2 = (\lambda_0 = 0.8, \lambda_1 = 0.1, \lambda_3 = 1, \lambda_4 = 0.1, \lambda_5 = 0.07, \mu_5 = \mu_6 = 5)$$
$$BVAR3 = (\lambda_0 = 0.6, \lambda_1 = 0.15, \lambda_3 = 1, \lambda_4 = 0.15, \lambda_5 = 0.07, \mu_5 = \mu_6 = 2)$$
$$BVAR4 = (\lambda_0 = 0.8, \lambda_1 = 0.15, \lambda_3 = 1, \lambda_4 = 0.15, \lambda_5 = 0.07, \mu_5 = \mu_6 = 2)$$
$$BVAR5 = (\lambda_0 = 0.9, \lambda_1 = 0.1, \lambda_3 = 1, \lambda_4 = 0.1, \lambda_5 = 0.07, \mu_5 = \mu_6 = 2)$$
$$BVAR6 = (\lambda_0 = 0.9, \lambda_1 = 0.15, \lambda_3 = 1, \lambda_4 = 0.15, \lambda_5 = 0.07, \mu_5 = \mu_6 = 5)$$

where nµ is prior degrees of freedom given as m+1 where m is the number of variables in the multiple time series data. In this work nµ is 4 (that is three (3) time series variables plus 1(one)). The following are the criteria for Forecast assessments used:

1. Mean Absolute Error (MAE) has a formular $MAE_j = \dfrac{\sum_{i=1}^{n}|e_i|}{n}$. This criterion measures deviation from the series in absolute terms, and measures how much the forecast is biased. This measure is one of the most common ones used for analyzing the quality of different forecasts.

2. The Root Mean Square Error (RMSE) is given as $RMSE_j = \sqrt{\dfrac{\sum_{i}^{n}(y_i - y^f)^2}{n}}$ where y_i is the time series data and y^f is the forecast value of y (Caraiani, 2010).

For the two measures above, the smaller the value, the better the fit of the model (Cooray, 2008)

Materials and Methods

The data used in this paper was sourced from CBN 2014 Statistical Bulletin. The data on annual GDP, Oil revenue and non-oil revenue in Nigeria spanned from 1981 to 2014. The unit of measurement for GDP, Oil revenue and non-Oil revenue are all in Billion naira (₦). The sample from 1981 to 2008 was used for analysis, while sample from 2009 to 2014 was used for model validation.

Results and Discussion

To carry out a BVAR analysis, it is required to obtain the optimal lag for the model. Result from the VAR lag specification revealed lag 6 as the optimal (see table 2 below for detail)

Table 2: VAR lag Specification

```
$ldets
     Lags   Log-Det     Chi^2       p-value
[1,]    6   8.090737 44.163130 1.317016e-06
[2,]    5  22.811780 21.768731 9.641553e-03
[3,]    4  26.439902 38.292377 1.544683e-05
[4,]    3  30.694610 39.015096 1.144858e-05
[5,]    2  33.945868  5.607955 7.784235e-01
[6,]    1  34.319732  0.000000 0.000000e+00

$results
```

```
      Lags       AIC       BIC        HQ
[1,]    1 35.41064 36.00576 35.55083
[2,]    2 35.85496 36.89641 36.10029
[3,]    3 33.42188 34.90967 33.77236
[4,]    4 29.98536 31.91948 30.44098
[5,]    5 27.17542 29.55587 27.73618
[6,]    6 13.27255 16.09935 13.93846


attr(,"class")
[1] "var.lag.specification"
```

The analysis of the variables (GDP, Oil revenue and non-Oil revenue) using BVAR in R, and the forecast were compared with the actual values of the variable using the forecast assessment criteria and the result is presented in table 3 below.

Table 3: Performance Ratings of the six (6) version of Sims-Zha Bayesian VAR

Models	RMSE	MAE
BVAR1	12532.12	8453.85
BVAR2	12294.701	8360.998
BVAR3	12206.281	8329.391
BVAR4	12020.970	8270.493
BVAR5	12206.281	8329.391
BVAR6	**11924.605**	**8237.353**

From the table 3 above the minimum RMSE and MAE is associated with BVAR6. Hence BVAR6 is the best model among others. Details of BVAR6 are presented at the appendix.

Table 4A: Actual Series from 2009 to 2014

Actual Series			
Year	GDP	Oil Revenue	Non-Oil Revenue
2009	24794.24	3191.94	1652.65
2010	54612.26	5396.09	1907.58
2011	62980.4	8878.97	2237.88
2012	71713.94	8025.97	2628.78
2013	80092.56	6809.23	2950.56
2014	89043.62	6793.72	3275.12

Table 4B: BVAR6 Forecast Series from 2009 to 2014

Forecast from BVAR6 Model			
Year	GDP	Oil Revenue	Non-Oil Revenue
2009	29037.08	8001.757	1612.811
2010	34639.98	9464.051	1932.150
2011	41239.29	11119.764	2293.535

2012	49022.26	13167.680	2724.771
2013	58297.99	15700.070	3239.744
2014	69353.94	18699.247	3853.369

Model validation used the actual series in Table 4A to compare the forecast from BVAR6 in Table 4B. The Impulse response function for the model is presented at the appendix.

Table 5: Decomposition of Forecast Errors for a Shock to GDP

```
-------------------------------------------------------------
     Std. Error      gdp   oilrev nonoilrev
[1,]   768.5614 84.28936 15.249626 0.4610175
[2,]  1143.1991 88.12734 11.659314 0.2133470
[3,]  1474.9375 90.73917  9.121355 0.1394709
[4,]  1819.1347 92.18394  7.693131 0.1229263
-------------------------------------------------------------
```

In table 5 above on decomposition of forecast errors for a shock to GDP at step 4 revealed that Oil revenue contributed 7.69% to GDP while Non-Oil revenue contributed only 0.12% to GDP in Nigeria. This result revealed that oil revenue still dominates the non-oil revenue in terms of their contribution to GDP in Nigeria which agrees with Omachi (2011) and contradict the findings of Okezie and Azubike (2016). The implication of this finding show some level of overdependence on oil sector and a level of neglect on the non-oil sector.

Conclusion

This paper examined the Bayesian Vector Autoregression (BVAR) modeling and forecasting of the dynamic interrelationship between Economic growth and revenue from the oil and non-oil sectors in Nigeria. To achieve this, annual data on Gross Domestic Product (GDP), revenue from oil and non-oil sectors were collected from Central Bank of Nigeria (CBN) bulletin from 1981 to 2014. The sample from 1981 to 2008 was used for analysis, while sample from 2009 to 2014 was used for model validation. Six (6) versions of Sims-Zha BVAR models were compared for out-of-sample forecast, the result revealed the superiority of the BVAR6 model over the other BVAR models. Lastly, evidence from the decomposition forecast errors revealed that revenue of oil sector contributed 7.69% to GDP while revenue from non-oil sector contributed 0.12% to GDP in Nigeria. This paper therefore recommended that the present government should encourage investment that is geared toward development in the non-oil sector (for instance agriculture, mining etc), of which it has the capacity to improve the Economic growth of the Nigerian economy.

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Appendix A

```
-------------------------------------------
Sims-Zha Prior reduced form Bayesian VAR
-------------------------------------------
Prior form :  Normal-inverse Wishart
Prior hyperparameters :
lambda0 = 0.9
lambda1 = 0.15
lambda3 = 1
lambda4 = 0.15
lambda5 = 0.07
mu5     = 5
mu6     = 5
nu      = 4
-------------------------------------------
Number of observations :  26
Degrees of freedom per equation :  7
-------------------------------------------
Posterior Regression Coefficients :
-------------------------------------------
Autoregressive matrices:
B(1)
[,1]      [,2]      [,3]
[1,] 1.065863 0.053400 0.026101
[2,] 0.234580 0.733754 0.035377
[3,] 0.265913 0.159131 0.710972

B(2)
[,1]       [,2]       [,3]
[1,]  0.020815  0.031796 -0.004895
[2,] -0.070519 -0.106017 -0.017032
[3,]  0.096299  0.020217 -0.000877

B(3)
[,1]      [,2]       [,3]
[1,]  0.029628 0.024648 -0.001369
[2,] -0.005843 0.053943 -0.005075
[3,]  0.019319 0.078149 -0.008379

B(4)
[,1]      [,2]      [,3]
[1,] 0.009202 0.011388 0.001951
[2,] 0.015156 0.067673 0.004021
[3,] 0.049522 0.047226 0.001880

B(5)
[,1]      [,2]       [,3]
[1,] 0.006134 0.007031  0.000549
[2,] 0.033225 0.033936 -0.003672
[3,] 0.028314 0.006616 -0.006481

B(6)
[,1]       [,2]        [,3]
```

```
[1,]   0.002496   0.000892  -0.000704
[2,]   0.001570  -0.009863  -0.000472
[3,]  -0.017998  -0.018567   0.004818


-------------------------------------------
Constants
22.09845 -7.788908 0.06743
-------------------------------------------
-------------------------------------------
Posterior error covariance
[,1]        [,2]       [,3]
[1,] 497885.96 211774.23 -36821.56
[2,] 211774.23 254821.12 -11341.83
[3,] -36821.56 -11341.83  29473.70

-------------------------------------------
```
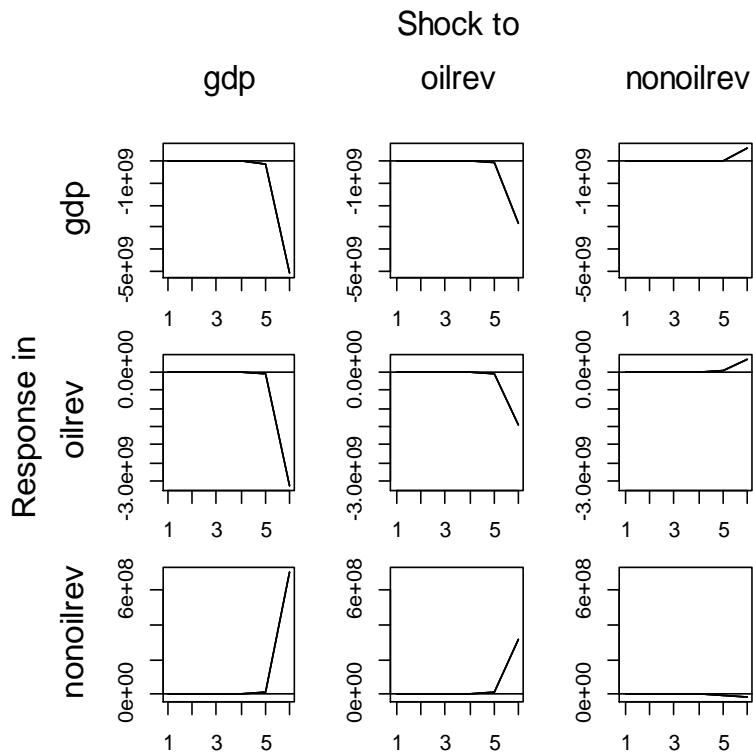
Appendix B

Shock to

Response in

Appendix C

```
Decomposition of Forecast Errors for a Shock to gdp
-------------------------------------------------------------
     Std. Error       gdp    oilrev nonoilrev
[1,]   768.5614 84.28936 15.249626 0.4610175
```

```
[2,]   1143.1991 88.12734 11.659314 0.2133470
[3,]   1474.9375 90.73917  9.121355 0.1394709
[4,]   1819.1347 92.18394  7.693131 0.1229263
---------------------------------------------------------------
Decomposition of Forecast Errors for a Shock to oilrev
---------------------------------------------------------------
     Std. Error       gdp   oilrev  nonoilrev
[1,]    406.0257  0.000000 99.93128 0.06871849
[2,]    514.4502  3.632023 96.14352 0.22445996
[3,]    568.3475 10.222174 89.44650 0.33132222
[4,]    617.2525 18.473032 81.15791 0.36905860
---------------------------------------------------------------
Decomposition of Forecast Errors for a Shock to nonoilrev
---------------------------------------------------------------
     Std. Error       gdp   oilrev nonoilrev
[1,]    163.2092  0.000000 0.000000 100.00000
[2,]    206.5427  4.415172 1.581163  94.00367
[3,]    247.8532 18.995612 4.127369  76.87702
[4,]    303.8179 37.475378 7.197465  55.32716
---------------------------------------------------------------
>
```