Theoretical studies on the creation of artificial magnetic monopoles

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Abstract

In this study, we discuss the theoretical studies on the creation of artificial magnetic monopole, and new electromagnetic equations. Employing Lorentz transformation, radial electrostatic fields, and a stationary wave derived from a superconducting loop, we demonstrate the existence of a magnetic monopole whereby the divergence of the magnetic field is not zero. We develop a device wherein a condenser provides electrostatic fields along the radial direction to the superconducting loop and discuss the nodes of the resulting stationary wave along the superconducting loop. We employ the Lorentz transformation with respect to the vector and electrostatic potentials. Then, because the nodes have no three-dimensional vector potential and have zero magnetic field rotation, the conserved energy is converted into new form that is associated with the magnetic field potential to yield the Lorentz transformation. As a result, we derived the relationship between the electric and the magnetic fields. This dependent relationship involves the exchange of the distribution characteristics of the static electric and static magnetic fields, and new electromagnetic equations of both electric and magnetic fields are obtained. We also analyzed the magnetic field from the magnetic monopole whose result assists the theory.

Keywords: Lorentz transformation, new electromagnetic equations Cooper pairs

1. Introduction

In this study, we discuss the artificial creation of a magnetic monopole. The first description of a monopole was presented by Dirac [1]. Based on his suggestions, some experiments were conducted [2,3] The magnetic monopole in spin ice[4,5] and other systems [6-8] have also been discussed. Recently, by employing the concepts of spin and Bose–Einstein condensation, a potential artificial magnetic monopole was created [9,10]. Moreover, the emergent magnetic monopoles was reported [14].

In this paper, we propose another method for creating a magnetic monopole and present its theoretical basis, which includes new equations of the relationship between static electric and static magnetic fields.

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Dirac's paper described the monopole using the gauge. However, as long as the spatially dependent vector potential ***A*** is not zero, the monopole cannot be defined because a nonzero ***A*** results in the existing Maxwell's second equation, whereby a nonzero ***A*** eventually leads to a loop magnetic flux. By employing Lorentz transformation, radial electrostatic fields, and a stationary wave derived from a superconducting loop, we demonstrate in this study that Maxwell's second equation can be modified, such that the divergence of the magnetic field is not zero.

In the cylindrical coordinates, the condenser generates electrostatic fields along the radial direction to the superconducting loop. Given these conditions, we discuss the nodes in the stationary wave along the superconducting loop and consider the Lorentz transformation with respect to the vector and electrostatic potentials. Because the nodes have no three-dimensional vector potential ***A*** and have zero magnetic field rotation, the conserved energy is converted into new form that is associated with the magnetic field potential to yield the Lorentz transformation. As a result, we obtain the relationship between the electric and magnetic fields. Because this is a dependent relationship, Maxwell's first equation becomes a modified Maxwell's second equation. As the result of consideration, the magnetic monopole is found to be the combination between Cooper pair and Cooper pair.

We then derived new electromagnetism equations, which describe the exchange of the distribution characteristics of the electric and magnetic fields. Finally, we numerically analyze the magnetic field distribution from the monopole. The results demonstrate the divergence of the distribution of the magnetic fields. The significance of the paper is that it obtained new equations of both electric and magnetic fields theoretically. Moreover, this paper presents easier method to gain a magnetic monopole.

2. Method

Fig. 1 shows the schematic of the superconducting loop and cylindrical condenser setup. In this study, the poles of the cylindrical condenser are charged. Using the abovementioned method, we can apply the radial static electric fields to the superconducting loop. Because of these electric fields (i.e., centripetal forces), Cooper pairs move along the loop, and eventually form a stationary wave.

As will be discussed later, creation of a monopole requires nodes of the superconducting stationary wave, and Cooper pairs in this node constitutes the monopole. The detail will be described later.

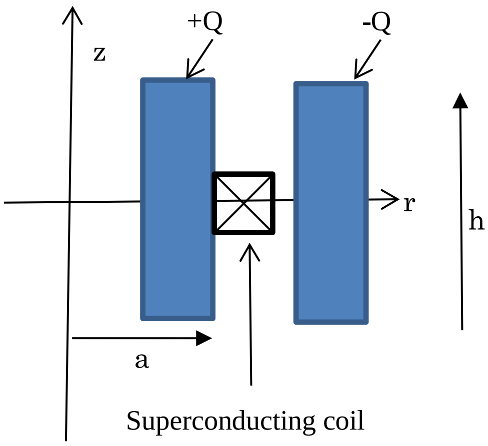

Fig. 1 cross section of the device to create magnetic monopole

3. Theory

3.1 Creation of a magnetic monopole

The superconducting state is generally a wave state. In the macroscopic scale, the state of the superconductivity can be approximated as plane waves:

$$\psi = |\psi| \exp(j\theta) + |\psi| \exp(-j\theta) = 2|\psi| cos\theta, \qquad (1)$$

However, when applying a constant electrostatic field, i.e., the constant potential is distributed along the length *l* of a loop, the wave function is changed as

$$\psi_n = \sqrt{\frac{2}{l}} sin \frac{n\pi}{l} x \quad (2)$$

where *l*, *n* and *x* denote the length of a loop, integer and axis along the loop.
Due to the period characteristics, each point has the coordinate *l* from arbitrary origin.

$$\psi_n = \sqrt{\frac{2}{l}} \sin(n\pi), \ (3)$$

Thus, in each node, the superconductivity state is broken. As discussed later, however, Copper pairs in each node remain. The monopole will be constituted by the combination of two Cooper pairs as discussed later.

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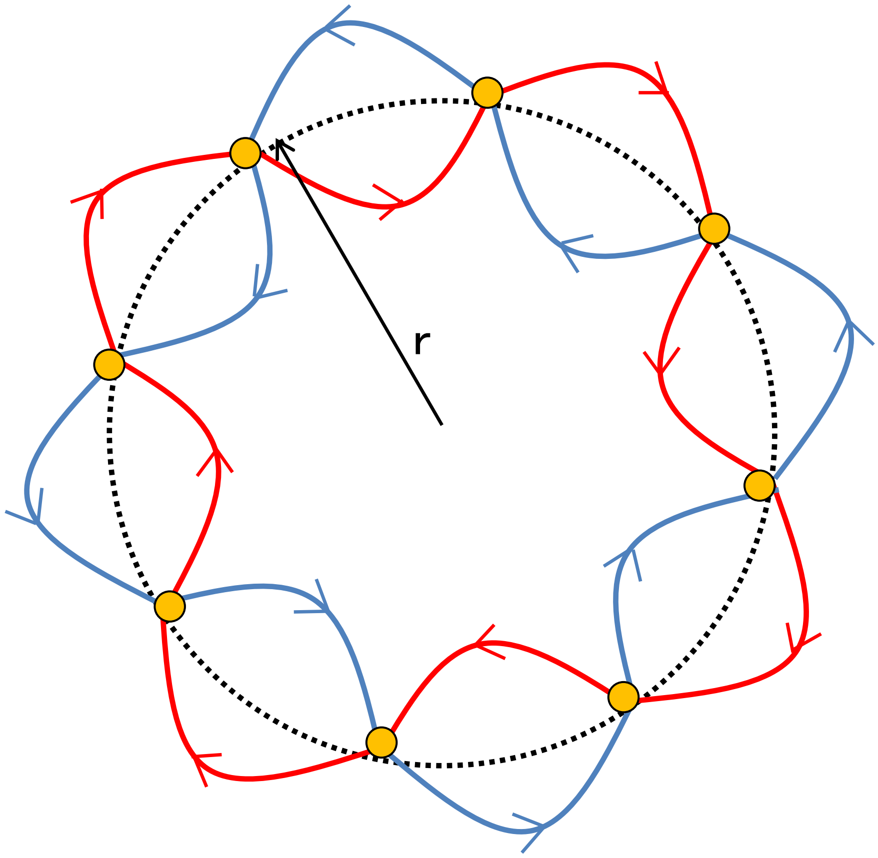

Fig. 2 schematic of the stationary wave formed along the superconducting loop

As shown in Fig.2, we derive nodes that have no oscillation. From the existing Maxwell's equation, we derive Eq. (5) from Eq. (4), considering that the macroscopic current is approximately zero at each node because local existence of a few Cooper pairs in each node is not able to produce the macroscopic current:

$$\mathrm{rot}\vec{H} = \vec{J} \qquad (4)$$

$$\mathrm{rot}\vec{H} = 0 \qquad (5)$$

Eq. (5) implies that the magnetic field ***H*** becomes a conservative force. That is, at each node, we can consider both the electrostatic and magnetic potentials. We further examine this process in the following discussion.

Let us review the relative theory here :

From the unchangeable light velocity principle, in 4 dimensional vector (x, y, z, ct), the Lorentz transformation is obtained. From this, the unchangeable relation is gained:

$$x^2 + y^2 + z^2 - (ct)^2 = constant. \qquad (6)$$

The same unchangeable is formed in another 4 dimensional vector, such as (A_x, A_y, A_z, φ_E/c).

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In this paper, we consider a four-dimensional vector potential:

$$\vec{A}_4 = \left(\vec{A}_3, \frac{\varphi_E}{c}\right), \qquad (7)$$

where φ_E and c denote the electrostatic potential and velocity of light, respectively.
Considering the Lorentz transformation, the following equation should be conserved:

$$|\overrightarrow{A_3}|^2 - \left(\frac{\varphi_E}{c}\right)^2. \qquad (8)$$

In the above equation, the conservation values are given as follows:

$$|\overrightarrow{A_3}|^2 - \left(\frac{\varphi_E}{c}\right)^2 = const. \qquad (9\text{-}1)$$

At each node, the current density is zero. In the vicinity of the zero point, the vector potential $\boldsymbol{A}_3$ becomes zero [11,12].

$$-\left(\frac{\varphi_E}{c}\right)^2 = \left(\frac{1}{ec}\frac{1}{2}\hbar\omega\right)^2, \qquad (9\text{-}2)$$

where e denotes the electric charge of the electron. The right hand side is constituted by the consideration of the dimension. However, in this equation, because the left-hand side of Eq. (9-1) is related to the electromagnetism, the zero-point energy in eq. (9-2) is that of a photon, not phonon. The electrostatic potential φ_E stems from the applied electric field.

Eq. (9-2) is not generally formed mathematically. That is, this equation does not meet the requirements for a Lorentz transformation. Thus, an energy conservation must occur to form the Lorentz-transformation equation. In addition, at each node, the following equation holds:

$$\mathrm{rot}\vec{H} = 0. \qquad (10)$$

Therefore, the right-hand side in Eq. (9-2) is consumed and a new term appears that is associated with the magnetic potential φ_B:

$$(\mu_0\varphi_B)^2 - \left(\frac{\varphi_E}{c}\right)^2 = 0\,. \qquad (11)$$

Thus, we obtain the following equation:

$$\varphi_E = c\mu_0\varphi_B. \qquad (12)$$

Therefore, at each node, we derive the following important relation:

$$\vec{E} = c\vec{B}. \qquad (13)$$

This equation is substituted with the existing Gauss equation.

$$\mathrm{div}\vec{B} = \frac{\rho_v}{\varepsilon_0 c} \to \rho_B \neq 0 \quad , \qquad (14\text{-}1)$$

where ρ_v is the electric charge density. When this electric charge combines with ε_0 and c, we derive the "magnetic charge density ρ_B." That is, the right-hand side implies the magnetic charge density.

The above equation does not imply an existing Gauss equation relationship between the electric fields; hence, the distribution of the electric fields must be approached in another way. Because Eq. (14-1) is not consistent with the existing Maxwell's second equation, the following equation must be derived from Eq. (13):

$$\mathrm{div}\vec{E} = 0. \quad (14\text{-}2)$$

Eqs. (14-1) and (14-2) imply that the distribution characteristics of the electric and magnetic fields have been exchanged. We know that the looped magnetic flux is generally distributed when a current is supplied along a coil. In this case, however, we can infer that the looped electric flux is distributed by the following method.

After the creation of monopoles, we consider the rotation of the entire coil loop mechanically. By this conduction, the created monopoles gain rotational kinetic energy. Thus, combining the μ_0 and *c*, a magnetic current can be considered in this case. Because the rotation of the electric field is not zero, we can consider a modified Ampere's law:

$$\mathrm{rot}\vec{E} = \mu_0 c\vec{\imath_s} \quad \to \vec{\imath}_B. \quad (15)$$

The right-hand side is the magnetic current density $\boldsymbol{i}_B$.

Note that the following equation holds.

$$\vec{\imath}_B = \rho_B \vec{v}, \quad (16)$$

where ***v*** denotes the velocity of rotational magnetic monopole motion.

Table 1 Summary of the differences between the existing equations and new equations of this study

Existing Maxwell's equation	Proposed Maxwell's equation
$\mathrm{div}\vec{E} = \frac{\rho_v}{\varepsilon_0}$	$\mathrm{div}\vec{E} = 0$
$\mathrm{div}\vec{B} = 0$	$\mathrm{div}\vec{B} = 4q\frac{\delta(r_0 - r)}{2\pi r}\delta(z)\frac{1}{\varepsilon_0 c}$
$\mathrm{rot}\vec{E} = 0$	$\mathrm{rot}\vec{E} = \frac{4q}{\varepsilon_0 c}\frac{\delta(r_0 - r)}{2\pi r}\delta(z)\vec{v}$
$\mathrm{rot}\vec{H} = \vec{\imath}$	$\mathrm{rot}\vec{H} = 0$

Table 1 provides a summary of the existing and our proposed equations. Note that we do not consider the time-dependent fields in this study. This table is provided by the fact that charge density can be generally represented by the delta function and that the coordinate is cylindrical. Considering the above description, the electric charge density in Eq. (14-1) may be represented as

$$\rho_v = 4q\delta(r_0 - r)\delta(z)/2\pi r \quad (17)$$

where *q*, *z*, *r*, and r_0 denote the electric charge, the variables of the cylindrical coordinates, and the radius of the stationary wave, respectively.

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3.2 Structure of a monopole

As discussed in the previous section, to create a monopole, the photon $\hbar\omega$ (=$2mc^2$), as defined by the Dirac equation, must be consumed. Considering the stationary wave derived from both the incident wave and the reflected wave, 2 Cooper pairs (i.e., 4 electrons) gather at each node. At each node, as indicated in Fig. 3, a Cooper pair shifts from the energy level of the vacuum, while maintaining the pair, to the ground state. At this point, we can derive the following equation:

$$2mc^2 = \hbar\omega, \quad (18)$$

where m and ω denote the electron mass and the angular frequency, respectively.

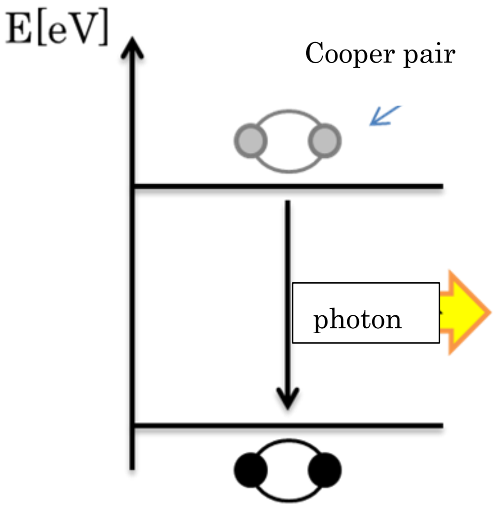

Fig. 3 Transition of a Cooper pair between the energy gap of vacuum

As shown in Fig. 4, when Cooper pair 1 shifts to the grand state, a photon is emitted. This photon works into Cooper pair 2, which is located at the same node as Cooper pair 1, and which is at the ground state and the energy level of this Cooper pair increases to the energy level of the vacuum. In turn, Cooper pair 2 shifts from the energy level of the vacuum to the ground state, and a photon is again emitted. Similarly this photon works into Cooper pair 1 and the energy of this pair increases to the energy level of the vacuum. In this sense, a monopole is a multiple particle with photons acting as an interaction force. We estimate the interaction as follows:

$$\hbar\omega = 2mc^2 = 1.64 \times 10^{-13}[J]. \quad (19)$$

The energy gap ΔE and time period Δt have an uncertain relation. This fact provides some basis for the extremely small radius of the monopole. That is, the pair is not broken. As a result of the transition to the ground state of the Cooper pair, this Cooper pair decreases its energy. This implies that the coherence of the Cooper pair becomes substantially shorter.

In short, the magnetic monopole is the combination between Cooper pair and Cooper pair.

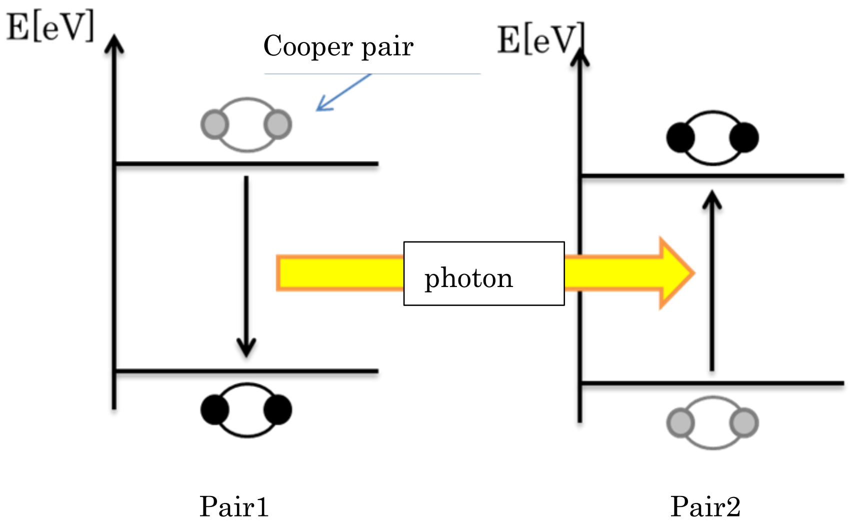

Fig. 4 schematic of photon work to a Cooper pair in the vicinity of another Cooper

3.3 Numerical calculation

To calculate magnetic fields from the monopoles, we employ the relationship between the electric and magnetic fields, as indicated in Eq. (13). Because the monopole has the Dirac function, we consider that the magnetic fields are within an order of about 10^{-14} [m]. Figs. 5 and 6 indicate the B_r and B_z components of the magnetic fields, respectively. Because the analysis is conducted within very small range, it seems that the magnetic field at the origin is extremely large. However, at 10^{-12}[m] range, the magnetic field becomes 7.5 [G].

Figs. 5 and 6 imply that the distribution of the magnetic fields represent the Dirac function, which was discussed in the previous section in this paper. Moreover, in Fig. 6, both plus magnetic component and minus component appear. Considering these facts, we find that divergent magnetic fields are obtained.

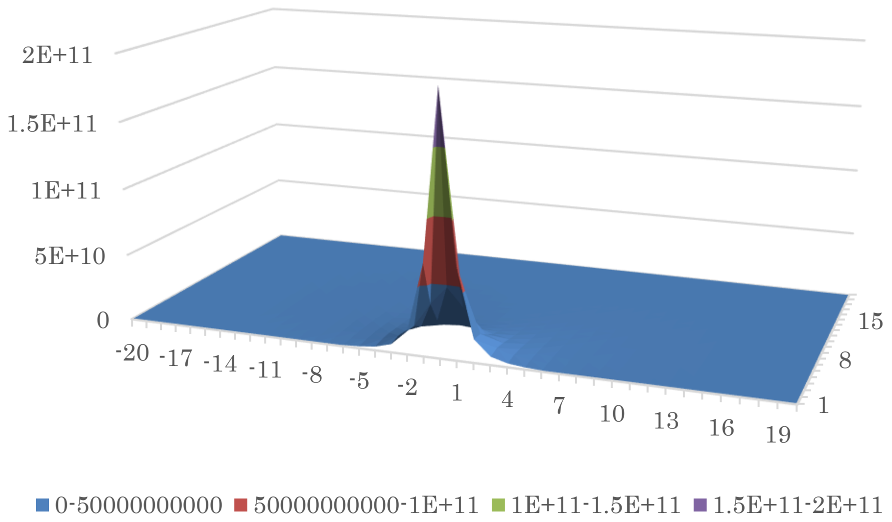

Fig. 5 B_r component distribution from the monopole. At the origin, a monopole exists. The unit of magnetic field is [T] and z-and r-axis is 10^{-14} [m] Note that we distinguish the microscopic origin for the origin indicated in Fig. 1

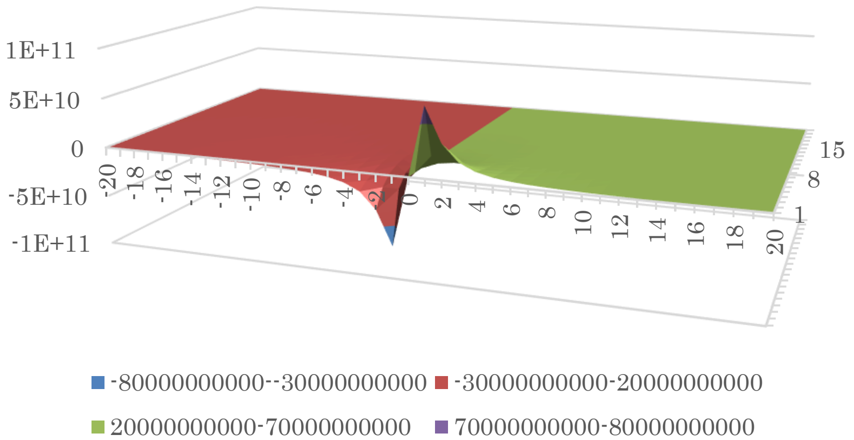

Fig. 6 B_z component distribution from the monopole. The unite of magnetic field is [T] and z-and r-axis is 10^{-14} [m] At the origin, a monopole exists.

4. Discussion

The purpose of this paper is two points. That is,

1. We propose the new method of creating magnetic monopole.
2. By the above, the new Maxwell equations are presented.

The method device is constituted by a superconducting loop and cylindrical condenser. This condenser provides the static electric fields to the loop. From this application of the electric field, the Lorentz transformation can be considered, and the existence of the stationary wave in the loop provides nodes, at which magnetic monopoles are created to support the Lorentz transformation. By employing the Lorentz transformation, radial electrostatic fields, and a stationary wave obtained from the superconducting loop, we demonstrated that the existing Maxwell's second equation can be modified, such that the divergence of a magnetic field is not zero. (Note that unless the spatially dependent vector potential ***A*** is zero, a monopole cannot be provided, since a nonzero ***A*** results in the existing Maxwell's second equation. In other words, a nonzero ***A*** eventually leads to a loop magnetic flux.)

Moreover, as a result of theoretical calculation, we can obtain the new electromagnetic equations, which present the exchange of the role of the static magnetic and electric fields. Moreover, we confirmed the existence of the monopole by numerical calculation. In the numerical analysis, magnetic fields sharp distribution, and the magnetic fields are divergent, which demonstrates the existence of the new electromagnetic fields.

From eq. (19), we derive the wavelength of a monopole:

$$\omega = \frac{2mc^2}{\hbar}\ , \quad \omega = \frac{2\pi c}{\lambda} \quad (20)$$

$$\lambda = \frac{\pi\hbar}{mc} = 1.2\times 10^{-12}[\,m] \quad (21)$$

Moreover, from the condition of forming stationary waves,

$$n = \frac{2\pi}{\lambda} r = \frac{2\pi r}{1.2\times 10^{-12}} \quad (22)$$

The above equation implies that quantum number, i.e., the number of monopoles increases with the arbitrary radius r. Therefore, in a loop of superconductor, plural monopoles are created, and macroscopic magnetic fields are derived. In adhering the microscopic approach, however, there are many deferential points at which the magnetic monopoles are located, and it is not good way to see the conversion by the microscopic model [13]. This can be analogized by the difficulty that microscopic many body problem jumps to the macroscopic Newton model. However, it can be solved by considering the fact that the conversion by the microscopic approach must be agreed with the macroscopic approach. In Fig.1, the macroscopic electric field is calculated as

$$E = \frac{Q}{2\pi a\varepsilon_0 h}, \quad (23)$$

where a and h denote the device parameters defined in Fig. 1

From eq. (13), the magnetic field is

$$B = \frac{Q}{2\pi a \varepsilon_0 hc} \quad . \qquad (24)$$

For example, if Q=10^{-7} [C], a=10 [cm], h=20 [cm], the magnetic field is calculated as
$B = 1.2 \times 10^{-4}\ [T]$
According to the above method, we are able to confirm the existence of the divergent magnetic fields by detecting the magnetic fields whose directions are divergent. We would like to put this as the future work.

5. Conclusion

In this paper, we described the potential existence of an artificial magnetic monopole and proposed new electromagnetism equations. That is, we proposed the new method of creating magnetic monopoles, which is easier than the conventional methods. Considering the stationery wave and the Lorentz transformation, we obtained the existence of the magnetic monopole. In the process of the theory, the new Maxwell equations were obtained. Moreover, the numerical calculations assisted them. As a result, we could obtain the divergent distribution of the magnetic fields.

Let us review the result: In the cylindrical coordinates, the cylindrical condenser provides electrostatic fields along the radial direction to the superconducting loop. Considering this device, we described the nodes of the stationary wave along the superconducting loop and discussed the Lorentz transformation with respect to the vector and electrostatic potentials. Since these nodes have no three-dimensional vector potential and have zero magnetic field rotation, the conserved energy is converted into new form associated with the magnetic field potential to yield the Lorentz transformation. As a result, we derived the dependent relationship between the electric and the magnetic fields. Because this relationship is dependent, Maxwell's first equation (i.e., the existing Gauss equation with respect to the electric field) becomes the modified Maxwell's second equation. By employing this derived relationship between the electric and magnetic fields, we further introduced novel electromagnetic equations, which identify the exchange of the distribution characteristics of the electric and magnetic fields. Based on this relationship, we then numerically analyzed the magnetic-field distribution from the monopole, which was divergent distribution. As a follow-up study, we will examine the existence of the magnetic monopoles experimentally.

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Additional information

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