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Special Relativity Leads to a Trans-Planckian Crisis that Is Solved by Haug's Maximum Velocity for Matter*

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Abstract

In gravity theory, there is a well-known trans-Planckian problem, which is that general relativity theory leads to a shorter than Planck length and shorter than Planck time in relation to so-called black holes. However, there has been little focus on the fact that special relativity also leads to a trans-Planckian problem, something we will demonstrate here. According to special relativity, an object with mass must move slower than light, but special relativity has no limits on how close to the speed of light something with mass can move. This leads to a scenario where objects can undergo so much length contraction that they will become shorter than the Planck length as measured from another frame, and we can also have shorter time intervals than the Planck time.

The trans-Planckian problem is easily solved by a small modification that assumes Haug's maximum velocity for matter is the ultimate speed limit for something with mass. This speed limit depends on the Planck length, which can be measured without any knowledge of Newton's gravitational constant or the Planck constant. After a long period of slow progress in theoretical physics, we are now in a Klondike "gold rush" period where many of the essential pieces are falling in place.

Key words: Special relativity theory, length contraction, Planck length, Planck time, trans-Planck.

1 Introduction: Is There A Quantum and Minimum Length?

One of the open questions in physics is whether there is a minimum length or not, and also how to interpret such a thing precisely. The Planck length is considered by many physicists to be the minimum length. According to the National Institute of Standards in the US (NIST CODATA 2014), it is only about 1.616229×10^{-35} meters. This is incredibly small. Looking to the history behind this unit, in 1899, Max Planck first suggested the Planck length as a component of what he called the natural units [1, 2]. He assumed that there were three essential universal constants, namely the speed of light c, Newton's gravitational constant G, and the Planck constant $\hbar$. Using only these three constants and dimensional analysis, he calculated what he thought were the fundamental length, time, mass, and temperature for matter. Today these are known as the Planck length, the Planck time, the Planck mass, and the Planck temperature. The Planck length is given as

$$l_p = \sqrt{\frac{G\hbar}{c^3}} \tag{1}$$

In 1883, George Johnstone Stoney [3] suggested a set of natural units that were not too different from those later given by Planck. The Stoney length was given as about 1.38×10^{-34} meters. However, the natural units of Planck are generally considered essential today, even though there are some disagreements on their importance. Some physicists would claim they are just mathematical artifacts with no implications for physics whatsoever, while others think there could be a unit smaller than the Planck length [4–6], and still others maintain that there should be no minimum length at all – that zero is the minimum. Nevertheless, the majority of physicists seem to agree that there is a minimum length and that it likely is the Planck length [7–11]. Later in this paper, we will point out recent progress in physics strongly indicating that the Planck length is indeed truly essential, and something that we can observe without relying on the Planck length formula. In other words, the Planck length is actually more than just a derived constant. However, first we will turn to special relativity and the notion of the speed limit and how it leads to a trans-Planckian problem.

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2 Special Relativity Speed Limit Leads to a Trans-Planckian Problem

Einstein's relativistic energy mass formula [12, 13] is given by

$$\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{2}$$

Further, Einstein commented on his own formula,

> *This expression approaches infinity as the velocity v approaches the velocity of light c. The velocity must therefore always remain less than c, however great may be the energies used to produce the acceleration.*[1]

Assume a planet with the same diameter as the Earth, about 12,742,000 meters. This planet is traveling at a velocity relative to Earth of[2]

$$v_1 = c \times 0.999999999999999999999999999999999999999999999999999999999999999999999999999999999999195$$

If we take the diameter of this planet and use standard length contraction on it, with this velocity we get

$$L = 12742000 \times \sqrt{1-\frac{v_1^2}{c^2}} \approx 1.616229 \times 10^{-35} \text{ m} \tag{3}$$

That is, this large planet has contracted to the Planck length as observed from the Earth. For a velocity higher than this, for example, a velocity of

$$v_2 = c \times 0.99999999999999999999999999999999999999999999999999999999999999999999999999999999999999195$$

then the planet would have a length contracted to only $\frac{1}{10}$ of the Planck length. And if Lorentz symmetry holds, then there is no preferred reference frame. So, we could even have an electron moving at this velocity relative to Earth. The velocity between two observers is the same as observed from each observer and if we had an electron that traveled past Earth, the velocity of the Earth relative to the electron would be the same as the velocity of the electron relative to the Earth. But our Earth is then shorter than the Planck length, due to length contraction as observed from the moving electron. Of course, it is unlikely that we would be able to build a measurement apparatus so small that it could fit inside an electron, so the idea remains theoretical, but that is not the point here. The main argument is that special relativity leads to a shorter than Planck length for any object if we follow the rules of special relativity. Further, even if we did not assume that the Planck length is the minimum length, but instead came up with an even shorter length as a minimum, we could always get a shorter predicted relativistic length than this simply by letting the Earth or the electron travel closer to the speed of light. We are, all the time, inside the "laws" of special relativity. So, either one must assume there is no minimum length unit or time unit and accept special relativity as a complete theory with respect to its scope, or one must assume something very important is missing in special relativity theory. We will claim the latter – that something critical is missing here.

When the electron is traveling at velocity v_1 relative to the Earth, then the Earth is contracted to the Planck length, and one could argue that this could be possible if the Planck length is the ultimate shortest limit on length. However, not even this velocity makes sense as an upper limit, because if we assume the electron has a length equal to its reduced Compton wavelength $\bar{\lambda}$, then the reduced Compton wavelength of the electron observed at this velocity will be only

$$L = \bar{\lambda}_e \times \sqrt{1-\frac{v_1^2}{c^2}} \approx 4.898 \times 10^{-55} \text{ m} \tag{4}$$

This is much shorter than the Planck length. We can conclude that special relativity in its current form not is consistent with a minimum length unit. This also means that there is no minimum time unit, and, as shown by Haug in a recent paper, there is also no relativistic mass limit (except that it must

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[1] This quote is taken from page 53 in the 1931 edition of Einstein's book *Relativity: The Special and General Theory*. English translation version of Einstein's book by Robert W. Lawson.

[2] A simple way to find this velocity with high precision is using Mathematica: $N[\text{Solve}[12742000 * \text{Sqrt}[1 - v\char`^2] == 1616229 * 10\char`^(-41), v], 100]$ or even simpler $N[\text{Sqrt}[1 - (1616229 * 10\char`^(-41))\char`^2/12742000\char`^2], 100]$, where the value coming out will be in % of the speed of light.

always be below infinity), see [14]. This leads to absurdities such as a case where an electron can have a relativistic mass equal to that of the Sun, the Milky Way, or even the entire observable universe.

However absurd these extrapolations may be, we are already getting an indication of what kind of new speed limit we need to avoid trans-Planckian problems in special relativity. The maximum velocity between the Earth and the electron must be such that the electron's reduced Compton wavelength is not length contracted more than the Planck length. This naturally means that different elementary particles would have different maximum velocities, something we will return to soon.

3 Does Haug's Maximum Velocity of Matter Remove the Trans-Planckian Problem?

Recently, Haug [15–20] has suggested a maximum velocity for all elementary particles given by

$$v_{max} = c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}} \tag{5}$$

where l_p is the Planck length and $\bar{\lambda}$ is the reduced Compton wavelength of the elementary particle for which we are calculating the speed limit. This maximum velocity formula can be derived by setting the maximum length contraction of the reduced Compton wavelength to the Planck length:

$$\begin{aligned} \bar{\lambda}\sqrt{1-\frac{v^2}{c^2}} &\geq l_p \\ 1-\frac{v^2}{c^2} &\geq \frac{l_p^2}{\bar{\lambda}^2} \\ v &\leq c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}} \end{aligned} \tag{6}$$

This means that we never will have a trans-Planckinan problem when we have this speed limit for anything with rest-mass. The maximum velocity formula can also be derived by setting the maximum relativistic mass of an elementary particle to the Planck mass. Further, the same maximum velocity of matter can also be found from Heisenberg's uncertainty principle when assuming the uncertainty in position cannot be smaller than the Planck length [21, 22].

The maximum velocity for an electron would be approximately

$$v_{max} = c\sqrt{1-\frac{l_p^2}{\bar{\lambda}_e^2}} \approx c \times 0.999999999999999999999999999999999999999999999124 \tag{7}$$

In this calculation, we have assumed the reduced Compton wavelength of the electron given by NIST CODATA, that is $\frac{2.4263102367\times10^{-12}}{2\pi}$ m, and a Planck length of 1.616229×10^{-35} m. This speed is below the speed of light, but still considerably higher than achieved in today's particle accelerators, such as the LHC. At this velocity, the Earth would have length contracted to about 5.33×10^{-16} meters, which is far above the Planck length.

This maximum velocity means that different elementary particles will have different maximum velocities. All known elementary particles will have velocities very close to the speed of light. We predict that particles will dissolve (explode) into energy just when they are reaching their maximum velocity. For composite particles such as the proton, this means that in our model they are predicted to start fall apart when the smallest elementary particle reaches its maximum velocity, as suggested by [18].

Consider the case where we have a particle with length equal to the Planck length (we assume the length of an elementary particle is measured by its reduced Compton wavelength). If this particle moves, it will be length contracted relative to the other frame. It must then have a contracted length shorter than the Planck length, as measured from the other frame. Does this mean we cannot have such short particles, as measured by their reduced Compton wavelength? No, it means such a particle must stand still, as observed from any reference frame. This sounds absurd until one understands that such a particle is simply the collision point between two photons. We can also see this from our maximum velocity formula; in the special case of a Planck mass particle, the reduced Compton wavelength is the Planck length, and its maximum velocity is

$$v_{max} = c\sqrt{1-\frac{l_p^2}{l_p^2}} = 0 \tag{8}$$

Again, we have predicted that this is the collision point between two light particles. Indeed, photons always move with the speed of light, with one exception: what is the speed of a light particle just at the instant it collides with another particle? We claim this collision will take one Planck second before the particles are dissolving into light again.

3.1 Generalized maximum velocity for matter formula

Assume we wanted a general minimum length x rather than Planck length limit, then the general maximum velocity formula for matter is

$$v_{max} = c\sqrt{1 - \frac{x^2}{\bar{\lambda}^2}} \tag{9}$$

However, in the next section we point to recent research that strongly supports the idea that $x = l_p$. In October 2015, the author [23, 24] presented the following maximum velocity formula for anything with rest-mass at the Royal Institution in London[3]

$$v_{max} = c\frac{(\bar{\lambda}^2 - x^2)}{(\bar{\lambda}^2 + x^2)} = c\frac{(1 - \frac{x^2}{\bar{\lambda}^2})}{(1 + \frac{x^2}{\bar{\lambda}^2})} \tag{10}$$

where x was a minimum length. The maximum velocity formula that we presented at the Royal Institution was derived from mathematical atomism, where there is an indivisible, minimum-sized particle. Further, in that derivation we did not use Einstein-Poincaré synchronized clocks, therefore we see the small difference in the formulas. When $x = l_p$ and $l_p << \bar{\lambda}$ as is for example the case for all elementary particles "directly" observed so far, then this is approximately equal to $v_{max} \approx 1 - 2\frac{x^2}{\bar{\lambda}^2}$. So, the maximum velocity we presented at the Royal Institution is very close to the formula we have arrived at through further investigation, namely $v_{max} = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}$, which, when $l_p << \bar{\lambda}$, can be approximated by the first term of a series expansion, $v_{max} \approx 1 - \frac{1}{2}\frac{l_p^2}{\bar{\lambda}^2}$. In the special case of a Planck mass particle, where $\bar{\lambda} = l_p$, both formulas give the same prediction, namely that the minimum-sized particle must stand absolutely still. This prediction we think is very essential, as it represents a photon-photon collision. Recent research have been quite clear on the idea that photon-photon collisions indeed can be considered matter [25].

Still, the main point in this paper is that special relativity cannot be consistent with any minimum length, as special relativity only has the requirement of $v < c$. No matter how small one sets the minimum length, special relativity can always give length contraction that makes any length even smaller than this. Our maximum velocity for matter solves this easily without changing the existing equations, only their boundary conditions.

4 Recent Breakthrough in Relation to the Planck Length

Since the time Max Planck introduced the Planck units, it has been assumed that G, c, and $\hbar$ are truly fundamental universal constants, while the Planck length, Planck time, and Planck mass are just derived constants. Over time, a number of physicists have questioned if the Planck length, the Planck time, and the Planck mass are anything more than mathematical artifacts. However, we have recently shown that the Planck length can be found totally independent of both Newton's gravitational constant and the Planck constant. Based on simple gravity observations, we can find the Planck length and given the speed of light, we can complete just about any gravity predictions that may be needed [26], see also [19, 27, 28]. We only need G when we want to find the weight of an object from gravity observations (and even then we can do without G), which is why Cavendish is considered to be the first one to indirectly measure G by weighing the Earth.

One cannot keep special relativity unmodified and at the same time uphold a minimum distance and minimum time. It is therefore useful to examine other theories. The maximum velocity of matter seems to solve a series of infinity challenges in relativity theory. It also provides insight on a series of "mystical" effects such as entanglement, which can suddenly be understood from a different perspective, see [22].

Our maximum velocity formula predicts breaks in Lorentz symmetry. Even after years of thinking hard about the problem it is no big surprise that we have only been able to find the Planck length from gravity observations, either indirectly through G and also $\hbar$ and c (the Max Planck way), or the much more direct approach described by [26] that is independent of G and $\hbar$ or just independent on G, see [**?**

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[3] We then used the symbol h for the minimum length, but as this can be confused with the Planck constant, we use x here. We also used w for the wavelength, as we were not sure if this was the reduced Compton wavelength or not at that point in time.

]. Could the fact that we observe gravity actually be the observation of breaks in Lorentz symmetry? We think so and encourage other researchers to investigate further. As recently demonstrated in a very simple way, [28] has shown that the Schwarzschild radius of any gravity mass can be found without any knowledge of Newton's gravitational constant and no knowledge of the Planck constant, but the speed of light (gravity) is needed. The Schwarzschild radius can be written as

$$r_s = \frac{2GM}{c^2} = Nl_p \tag{11}$$

where N is the number of Planck masses in the mass. Our point is the the Schwarzschild radius is linked to the Planck length, and the Planck length can be found independent of any knowledge of G and $\hbar$, as shown by [26], but based on gravity observations only at this moment. Further, SR is not consistent with such a minimum length. We have shown in a simple way how to extend SR to be compatible with a minimum length limit. However, this leads to Lorentz symmetry breaking at the Planck scale. In other words, we think gravity could be directly link to Lorentz symmetry breaking.

Several researchers have questioned if new Planck scale physics could be weakly detected at lower energies; this is discussed by [29, 30], for example. In a recent review article, [31] on the possibility for Lorentz symmetry breaking in relation to quantum gravity predictions and experiments noted:

> *In conclusion, though no violation of Lorentz symmetry has been observed so far, an incredible number of opportunities still exist for additional investigations.*

But then maybe gravity itself is an indication of Lorentz symmetry breaking. We think this line of thought should be investigated further.

5 Conclusion

We have clearly demonstrated that special relativity predicts that any particle or object can undergo so much length contraction that the contracted object, as observed from the other frame, will be shorter than the Planck length. That is, special relativity leads to a trans-Planckian crisis. One either has to accept that there is no minimum length or time, or one needs to modify special relativity theory. Our suggested formula for the maximum velocity of matter solves the trans-Planckian special relativity problem in an elegant and compelling way.

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