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The vacuum polarization paradox and its solution

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Summary
In this article various heuristic approaches are discussed to solve the dark matter phenomenon by the concept of vacuum polarization. They are compared with a more fundamental approach, based upon an entropy model of the visible universe. They all make use of some kind of gravitational dipole. These dipoles seem to violate Einstein's equivalence principle between inertial mass and gravitational mass. It is shown how the paradox can be solved by a quantum mechanical principle.

Keywords: quantum gravity; vacuum polarization; gravitational dipole; Majorana particle

Introduction
In the search toward a solution of the dark matter phenomenon, various authors have proposed to consider the vacuum as a medium subject to polarization of its constituents, under influence of baryonic sources. All these proposals are struggling with the principle of equivalence between inertial mass (Newton's first law) and gravitational mass (gravitational force). In these approaches, it became clear that if there would exist some kind of a gravitational dipole, dark matter would show up as a virtual amount of matter, due to polarization of the medium that increases the gravitational field strength from a baryonic source, similarly as the electric field strength is enhanced in dielectric media. This has been shown in articles by Blanchet and Tiec [1,2], Hajdukovic [3], Raymond Penner [4] and by me [5]. The approaches, though, are different, albeit that the gravitational dipole concept is common. Its most simple configuration is a linear structure with a positive pole spaced from a negative pole with the same absolute value of its strength. Naively, the positive pole would have a strength $+m$ and the negative pole would have a strength $-m$ and the baryonic source would direct the orientation of the dipole. This would introduce the concept of negative mass and that would mean a conflict with the principle of equivalence. To escape from this problem, the various authors propose different solutions. The most simple solution is to ignore the equivalence principle and to make a distinction between gravitational mass and inertial mass. This approach has been chosen by Hajdokovic, by defining a antiparticle in his dipole that has positive inertial mass, but unlike its particle, has a negative gravitational mass. Blanchet and Tiec try to escape from the problem by making a distinction between active gravitational mass (from the source) and passive gravitational mass (from the object under acceleration). Where the passive intertial mass is equal to its gravitational mass, those of the active ones are not. Raymond Penner has chosen for an easy way out, by simply hypothesizing a new unknown particle, dubbed as virtual graviton, characterized by zero mass and a finite gravitational dipole moment. The three approaches are highly heuristic, lacking a firm theoretical basis. They are justified by fits between heuristic theory and evidence from cosmological observations. Because these fits cannot be denied, there must be credit given to the concept of vacuum polarization. It is my aim in this article showing that it is possible to a give a more satisfying theoretical basis to it, by solving the vacuum polarization paradox that seems to arise from the equivalence principle.

From entropy to vacuum polarization
To this end, I would like to invoke two different, but complementary, views as outlined in a triptych of articles [6,7,5]. The first two of these explain the dark matter phenomenon without making use of the gravitational dipole concept. The third one relies upon some kind of gravitational dipole of a quite different nature than so far described in literature. In the first of these three articles, it has been shown that a positive value of the Cosmological Constant in Einstein's Field Equation, under the weak limit

constraint and under particular constraints for the spatial validity range, results into a modification of Poisson's equation, such that,

$$\nabla^2\Phi + \lambda^2\Phi = -\frac{4\pi GM}{c^2}\delta^3(r), \tag{1}$$

where Φ is the gravity potential, G the gravitational constant, c the vacuum light velocity, M the mass of a pointlike baryonic source, $\delta(r)$ Dirac's delta function and where λ is related with Einstein's Cosmological Constant Λ, such that

$$\lambda^2 = 2\Lambda = \frac{2a_0}{5MG}, \tag{2}$$

where a_0 is Milgrom's acceleration that characterizes the dark matter phenomenon.

The striking feature of (1) is the $+$ sign associated with λ^2. If it were a $-$ sign, the equation would be similar to Debije's equation for the potential of an electric pointlike charge in an electromagnetic plasma [8]. As is well know, the solution of such equation is a shielded Coulomb field, i.e., an electric field with an exponential decay. In the gravitational equivalent (with the $+$ sign) the near field is enhanced ("antiscreened"), because masses are attracting, while electric charges with the same polarity are repelling. The equation holds for galaxies. Eq. (1) is, in fact, a formal expression for the Debije process that has been heuristically copied by Raymond Penner for vacuum polarization, in analogy with electromagnetism and that forced him to define his virtual graviton. However, (1) is straightforwardly been derived in [6] from Einstein's Equation. To calculate a numerical value for a_0, it appeared required to extend the analysis of a cosmological system with central mass to the visible universe enclosed by the Hubble time event horizon. This has been done in [7]. This has resulted into an expression for a_0 that reads as,

$$a_0 = \frac{\Omega_B}{\Omega_m}\frac{c}{t_H}, \tag{3}$$

where t_H ($\approx$13.8 Gyear) represents the Hubble time scale, and where Ω_B ($=$0.0486) and Ω_m ($=$ 0.259), respectively, are the relative shares of baryonic matter and true matter of universe, as defined in and known from the Lamda-CDM model. The numerical value $a_0 \approx 1.25 \times 10^{-10}$ m/s^2 corresponds with observational evidence.

These results (1-3) have been obtained without invoking the gravitational dipole concept. Hence the results don't suffer from the vacuum polarization paradox. However in the third triptych article, the problem pops up. In that article, Milgrom's acceleration constant has been calculated by applying the Bekenstein-Hawking entropy expression to the visible universe. This has been done by describing the vacuum as a fluid in thermodynamic equilibrium, where the molecules are subject to Heisenberg's energy-time uncertainty. The vibration of the molecules is modelled as the motion of an quantum mechanical oscillator with an effective mass that embodies the energy of the vibration. Note that the energy of this mass is in fact a modulation on top of the massive energy of the thermodynamic state of the fluidal molecules. To model the spin of those molecules associated with the motion, the quantum mechanical harmonic oscillator is modelled as a two-body equivalent for the purpose of assigning an dipole moment vector to the vibrating molecules. This will allow a polarization of the dipole moment vector (= directing the spin) under influences of baryonic sources. Hence, the molecules are modelled as virtual gravitational dipoles, similarly as in Hajdukovic 's theory. However, rather than a-priori identifying these virtual gravitational dipoles as pions and copying their attributes, like Hajdokovic has done, the properties of the gravitational dipoles are derived by theory. First of all, invoking the Heisenberg relationship enables the assignment of a magnitude to the dipole moment vector $\mathbf{p}$ of the vibrating molecules, such that

$$|\mathbf{p}| = \frac{\hbar}{2c} , \tag{4}$$

where $\hbar$ is the reduced Planck constant and c the vacuum light velocity.

The dipole moment density P_g amounts to

$$P_g = \frac{a_0}{20\pi G} , \tag{5}$$

Hence, from (4) and (5),

$$N\Omega_B p = P_g V \rightarrow \frac{N}{V} = \frac{a_0}{20\pi G\Omega_B}\frac{2c}{\hbar} = \frac{3}{8}\frac{a_L c}{\pi G\hbar} . \tag{6}$$

The total number of spins of the N particles in the volume V determine the information content (= entropy) in the volume V, which be calculated from (6). It is proven in [5] that, by taking V as the volume enclosed by the event horizon of the visible universe, and equating the result with the Bekenstein-Hawking expression, Milgrom's acceleration constant a_0 is found as in (3) indeed. The agreement between the two approaches support the viability of both.

From (6), the cell size ΔV of the vibrating gravitational molecule is found as,

$$\Delta V = \frac{V}{N} = \frac{8}{3}\frac{\pi G\hbar}{a_L c} . \tag{7}$$

Hence, the cell radius R of a spherical cell is established from,

$$\frac{4}{3}\pi R^3 = \frac{8}{3}\frac{\pi G\hbar}{a_L c} \rightarrow R = (\frac{2G\hbar}{a_L c})^{1/3} . \tag{8}$$

Inserting G =6.67 10-11 m^3 kg^{-1} s^{-2}, $\hbar =$ 1.05 10^{-34} kg m^2 s^{-1} , a_L =7.1 10^{-10} m s^{-2} and c = 3 10^8 m s^{-1}, gives $R \approx$ 4.1 fm. Let us now calculate the energy of the gravitational dipole in the cell by equating its Compton wavelength with the diameter of the grain cell. Hence, from (6),

$$hf = h\frac{1}{T} = hc\frac{1}{cT} = \frac{2\pi\hbar c}{2R} = \pi(\frac{a_L c}{2G\hbar})^{1/3}(\hbar c) . \tag{9}$$

With $\hbar c = 197$ MeV fm and the other values just given, the gravitational dipole in would have an energy to the amount of

$$\hbar\omega \approx 150 \text{ MeV}. \tag{10}$$

This is pretty close to the 140 MeV massive energy of a pion. Let u compare this dipole energy with the gravitational energy contained in the cell, which amounts to

$$\Delta mc^2 = \rho c^2 \Delta V , \tag{11}$$

where ΔV is given by (7) and where ρ is the critical mass density of the flat universe, which can be found from textbooks as,

$$\rho = \frac{3}{8\pi G}\frac{a_L^2}{c^2} . \tag{12}$$

Hence, from (11) and (12),

$$\Delta mc^2 = \frac{a_L}{c^2}(\hbar c) . \tag{13}$$

This amount is extremely small and is, by far, inadequate to deliver the energy of the pion in real state. It has to be taken into account, though, that gravitational energy is not more than a tiny modulation on the energic equilibrium state of the fluidal background energy in space. The energy conflict can be avoided by taking into account that the poles in a dipole are not bound by a particle in real state, but, instead by a particle in virtual state. Considering that the energy of the gravitational dipole, calculated from theory, is close to the rest mass of a pion, Hadjuovic's heuristic assumption of virtual pions does not seem bad at all.

Now however, it may seem that we are back to the polarization paradox. That is not quite true. The gravitational dipole described in the context of the analysis by entropy is not a dipole in the sense of a construct of a particle with positive mass and an antiparticle with negative mass. Instead it is a vibrating gravitational molecule to which, by certain type of modelling, a gravitational dipole moment, is assigned. It can't be a pion, virtual or not, because the pion is a boson and the gravitational molecule is a fermion. Another issue is that, while two-body quantum mechanical oscillators can be very well modelled by a one-body equivalent, the opposite is not obvious, because it is not clear how to justify the equilibrium between an attractive force and a repelling force between the two bodies. At this point, we could choose now for the Raymond Penner solution by identifying this gravitational molecule as a hypothetical elementary particle (a virtual graviton), with a certain gravitational dipole moment as the only relevant attribute. That does not satisfy me after having shown the very basic relationships with fundamental theories as Einstein's Field Equation and the Bekenstein-Hawking entropy.

Solution of the paradox

So, let us ask ourselves what an elementary particle could be with spin (representing the gravitational dipole) that satisfies the conditions for vacuum polarization. First of all, it must be a fermion, it must be electrically neutral, it must almost stay in rest, it must have a very tiny mass only and the orientation of its spin should be subject to a baryonic gravitational field. None of the known elementary electromagnetic or nuclear particles comply with these requirements. There is, however, an exotic neutral elementary fermion that fulfils the requirements. It is a fermion, known as the Majorana particle, theorized by Ettori Majorana back in 1937 [9], so far not identified in free state. In 2012, the particle has been identified as an electromagnetic particle in solid state by a research group headed by Leo Kouwenhoven [10]. Such a particle has two orthogonal real wave functions, which, similarly as bosons, with the same amplitude, required to meet the condition of positive definiteness. Hence, similarly as a dipole, it has two components, in spite of its monopole manifestation. This particle has, similarly as Dirac fermions, non-integer spin. In spite of its neutral charge, this spin can be influenced. In the case of electromagnetic Dirac fermions, the spin can be conceived as an elementary angular momentum. In particle physics, Dirac's theory, including Majorana's particle, not only applies to electromagnetic particles, but to all physical particles. There is no reason whatsoever why a gravitational equivalent could not exist. Hence, the vacuum polarization paradox can be solved by conceiving the vacuum particles that vibrate because of the Heisenberg uncertainty, as gravitational Majorana particles. The quantum mechanical two body equivalent of a one-body quantum mechanical oscillator is just a comprehensible physical illustration how the spin of the gravitational Marjona particle is influenced by a baryonic gravitational field.

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