Going solo: on the substitutability between paid-employment and self-employment

Emilio Congregado 1, *, Antonio A. Golpe 1 and Vicente Esteve 2

1 University of Huelva; congregado@uhu.es; antonio.golpe@dehie.uhu.es
2 University of Valencia; vicente.esteve@uv.es

* Correspondence: congregado@uhu.es; Tel.: +34-959-217-832

Abstract: This paper provides estimates of the elasticity of substitution between operational and managerial jobs in the US economy covering a period of almost five decades, derived from an aggregate CES production function. Estimating the long-term relationship between (the log of) the aggregate employment/self-employment ratio and (the log of) the returns from paid-employment relative to self-employment and testing for structural breaks, we report different estimates of the elasticity of substitution in each of the two regimes identified. Our results help to understand and interpret one of the most intriguing aspects in the evolution of self-employment rates in developed countries: the reversal of the trend in self-employment rates. Our estimates show that a higher level of development is associated with a greater number of entrepreneurs and smaller firms. Some rationales for understanding the growth of the elasticity between paid-employment and self-employment, including the recent trends in the digital economy– are also suggested.

Keywords: Elasticity of substitution; Cointegration; Self-employment; Structural Breaks.

1. Introduction

In recent years, a growing body of literature has studied the relation between economic development and the aggregate self-employment rate [1-6]. In particular, analysis of the interplay between the economic development phase and the evolution of the independent entrepreneurship rate—or the (inverse) relationship between the wealth of the economy and the related concept of average firm size (i.e., the employment/self-employment ratio)—has become a focus area for scholars because of the observation of a reversal in self-employment rate trends in several developed countries. A handful of works [7-9] documented this reversal trend in the US.

Until the last quarter of the 20th century, economic development was related to the ever increasing importance of economies of scale and scope [10], a switch from agriculture to manufacturing [11]1 and the influence of increasing wage levels on occupational choice [14].2

1Changes in industrial structure should influence independent entrepreneurship rates because some activities lend themselves better to self-employment than others [12]. One could argue that the characteristics of different sectors and industries, in terms of the existence of significant demand for personal (professional) services, jobs with erratic demand, the mix of skills required or low capital requirements, make it more likely that a sector is populated by self-employed workers. These arguments help us to understand the high concentration of self-employed workers in the agriculture and service sectors and the comparatively low concentration in manufacturing. See, e.g., [13] for an analysis of US self-employment by industry.

2Following Lucas’s argument, because capital and labour are substitutes, higher capital stock implies higher returns from working and lower returns from managing. As a result, economic development leads to a higher
Overall, the predominant view was that as economies became wealthier, average firm size should increase; in other words, average firm size should be an increasing function of the wealth of the economy [15]. Therefore, a negative relation between economic development and the self-employment rate was implied. Data regarding the evolution of average firm size during the late nineteenth and first three quarters of the twentieth centuries in most developed countries supported this proposition.

Related to this latter point, in a highly influential paper, [14] developed a model in which firm distribution was the solution to the problem of allocating productive factors among managers of varying ability. The main result of Lucas’s model concerns the effect on average firm size when per capita capital increases. Lucas showed that in the case where the elasticity of substitution between labour and capital is less than one, as the economy becomes wealthier, the wage relative to managerial rents increases, and marginal entrepreneurs prefer to become wage earners rather than manage their own businesses. This causes an increase in the ability threshold that is necessary to become an entrepreneur, which defines the marginal entrepreneur. Then, an increase in wages, relative to a managerial rent increase, induces marginal entrepreneurs to become employees, raising the average size of the firm. Furthermore, an important prediction, given the sustained trend of growth in capital per capita, emerges: ‘the fraction of entrepreneurs will decline over time while average firm size will inexorably increase’ [12]. Development leads to higher average firm size because of a negative relationship between the elasticity of factor substitution and firm size.

Lucas [14] reported that average firm size (using employees per firm as a proxy) was positively related to GNP per capita (used as a proxy for capital per capita) in the US. This positive test of Lucas’s hypothesis reflected not only observed developments in self-employment during the first three quarters of the 20th century but also consistency with estimations of the elasticity of factor substitution between capital and labour.

However, in several developed countries, the trend reversed. The relationship seemed to have changed from a negative relation to a positive one, and the observed recovery in self-employment rates was interpreted as undermining Lucas’s prediction. In fact, the secular decline in self-employment rates experienced by most developed countries was followed by a reversal trend in the last quarter of the twentieth century and in the first decade of the current century. For instance, considering the 23 OECD countries included in COMPENDIA as a reference, the average business ownership rate—i.e., the number of owners of non-agricultural incorporated and unincorporated businesses as a fraction of total labour force—increased from 0.100 in 1972 to 0.112 in 2009. This figure, however, hides huge national disparities in both levels of the average business ownership rate and in their evolution. For example, the sampled business ownership rates in 2009 range from 19.9% in Italy to 4.7% in Luxembourg; analysing the rates’ evolution, business ownership in Japan experienced a decline from 0.125 in 1972 to 0.083 in 2009, while business ownership in the US and the average firm size because of a negative relationship between the elasticity of factor substitution (between capital and labour) and average firm size.

This negative relationship is well documented in the works of [11], [16-18], among others.

By contrast, if the elasticity of substitution is greater than one, then economic increases in per capita capital increase the equilibrium number of entrepreneurs and decrease the average firm size. Note that in the case of a Cobb-Douglas production function, the average firm size is unchanged when per capita capital grows.

Empirical estimates usually converge to an elasticity value—capital-labor—of less than 1 (see [19], ch. 3).

In the US, the self-employment rate began to rise in the 1970s [7].

COMPENDIA is an acronym for COMParative ENtrepreneurship Data for International Analysis. See http://www.entrepreneurship-sme.eu.

Business ownership, self-employment and independent entrepreneurship will be used as interchangeable concepts in this article.
European Union-15 increased from 0.082 to 0.093\(^9\) and from 0.104 to 0.118, respectively, during the same period. The possibility of a U-shaped relationship between entrepreneurship and economic development gradually gained ground, and the re-examination of that relationship became the subject of a large body of empirical and theoretical literature, recently surveyed in [24].

Broadly speaking, at least four arguments have been suggested to explain this reversal.\(^{10}\) The first argument relates to the non-validity of Lucas’s proposition, asking whether something in the proposition itself is amiss or if the proposition depends crucially on some faulty assumption. Using this last argument, [25] extended Lucas’s analysis by utilising a more general aggregate production function (a normalised CES), which allowed them to prove the existence of an inverse relationship between the elasticity of substitution (between capital and labour) and average firm size. From this perspective, the fact that wealthier countries have a higher elasticity of substitution is consistent with the positive association between the growing importance of SMEs in the most developed countries because a high elasticity of substitution value more easily enables individuals to become entrepreneurs. In short, from the model presented in [25], we can confidently state that in economies characterised by higher values of aggregate elasticity of substitution between capital and labour, we should expect higher wealth to be associated with more entrepreneurs and smaller firms. This proposition is supported by the recent evolution of average firm size in developed countries.

In addition to the above arguments, some scholars have suggested that there were also certain changes and mechanisms that can help to understand this trend reversal. One argument is that independent entrepreneurship and average firm size are now decreasing and increasing functions, respectively, of the wealth of the economy due to improvements in information and communication technologies (ICT). It is a well-known fact that the ICT revolution has decreased the importance of scale economies in many industries [26] and has increased opportunities for entrepreneurship and returns to entrepreneurship and managerial talent [27]—managerial works [28].

It has also been suggested that the reversal of the trend in self-employment rates may be the effect of an expansion of the business service sector relative to manufacturing. Several scholars argue that this expansion has attended a shift away from larger corporations and toward entrepreneurial activity. This phenomenon has led to a decline in the average firm size [24].

Finally, one could argue that the reversal in the business ownership rate may be the result of structural changes having strong effects on occupational choice decisions and, therefore, on the elasticity of substitution between paid-employment and self-employment. In particular, we may hypothesise that the above factors, in conjunction with the emergence of incentives schemes, such as subsidies or tax allowances\(^{11}\), and a progressive reduction in the rights and benefits derived from employment protection legislation\(^{12}\) may have introduced substantial changes in the risk-adjusted relative earnings of paid employment and self-employment. Thus, one could argue that higher levels of entrepreneurship may indicate that extant job creators are not creating attractive wage-earning job opportunities\(^{13}\) as a result of a low valuation of the risk associated with self-employment. The loss of rights, in terms of potential severance payments and unemployment benefits, may affect the structure of employment by altering the relative valuation between self-employment and paid-employment.

In short, the importance of several factors—such as the reduction of the extent of scale economies, the existence of more volatile markets or the growing importance of innovation, and the elasticity of substitution between capital and labour—to predicting the progressive decline of the

---

\(^9\) See [7] and [20-23], for a complete picture of the evolution of the self-employment sector in the US.

\(^{10}\) See, e.g. [15] or [24], for a detailed exposition on how these mechanisms operate.

\(^{11}\) See, [29-33].

\(^{12}\) In Botero et al. [34] a measure for labour market regulation is proposed. On the other hand, the works of [35-41] analyse the effects of stricter employment protection legislation on self-employment.

\(^{13}\) Not only in terms of lower wage rates, taking advantage of low union membership rates or segmentation, but also avoiding the costs of compliance of those contracts with higher employment protection rates.
average firm size cannot be denied. This article seeks to test whether changes introduced in some
labour market institutions [34] and labour market dynamics, along with the generalised emergence
of entrepreneurship policy [42], particularly the introduction of different schemes to promote
self-employment, have substantially altered the relative risk-adjusted returns in self-employment
and the elasticity of substitution between them.

This paper investigates this latter hypothesis using US data, testing whether the estimate of the
elasticity of substitution between managerial and operational jobs in a developed economy such as
that of the US is compatible with a fall in average firm size. The aim of this paper is to present
estimates of the elasticity of substitution between entrepreneurship and paid-employment using US
data as a method of testing whether, as recent literature has hypothesised, wealthier and more
developed countries are characterised by a higher elasticity of substitution between
self-employment and paid-employment or if elasticity estimates instead support Lucas’s hypothesis
(in terms of the inexorability of a secular trend of increasing average firm size and decreasing
numbers of entrepreneurs).

Our empirical results are consistent with the existence of a long-term relationship between the
wage-earner/self-employment ratio and the relative earnings of self-employed and paid-employed
workers. However, this relationship is subject to structural changes. In particular, our results report
an elasticity estimate for the first subsample (before the break) that is consistent with Lucas’s
proposition regarding average firm size, while estimates in the second subsample are consistent
with the observed evolution of average firm size. Importantly, the first break date coincides with the
beginning of the rise in American self-employment [7]. Our estimates suggest that at the beginning
of the 1990s, deep changes in the determinants of the substitution rate between self-employed and
paid-employed workers, i.e., between managerial and operative works, should have occurred in
such a manner that, in the most recent regime, self-employment and paid employment are now
gross substitutes instead of complements. These findings are consistent with observed average firm
size development in the US during the covered period.

Technically, our analysis parallels the literature on wage inequality [43] because we consider
self-employment and paid employment as two employment statuses—managerial and operational
works—similar to the literature addressing skilled and unskilled labour. Therefore, we report
estimates of the elasticity of substitution between these two employment statuses by estimating the
linear long-term relationship between the employment/self-employment ratio and the returns from
paid-employment relative to self-employment. After analysis of this relationship, we consider the
possibility that a regression model with multiple structural changes would provide a better
empirical description of the relationship. To that end, instability tests, recently proposed in [44-46],
are performed.

The remainder of the paper is organised as follows. In Section 2, we describe our model and
econometric strategy. In section 3, we present our estimation results. Finally, Section 4 summarises
our main conclusions.

2. Model and econometric strategy

Generalising differences in individual skills in the basic occupational model (see, e.g., pioneer
models of Rees and Shah [47], Borjas and Bronars [48], or Evans and Leighton [49], the choice
between entrepreneurial-managerial and operational jobs is based upon the idea that individuals
respond to the risk-adjusted relative earnings opportunities in each sector (self-employed sector vs.
employed sector).14

The perspective assumed in this paper is that occupational choices of fully informed
individuals are based only on the risk-adjusted relative earnings between self-employment and
paid-employment.

14 See, e.g. [50] and [51]
As mentioned, our empirical strategy parallels the basic framework used by literature addressing wage inequality and skill premiums\textsuperscript{15} because, to some extent, the occupational decision between managerial and non-managerial work is also based on the relative earnings between the two employment statuses. Let us consider a simple closed economy. We begin with an aggregate production framework, where output is described by a constant elasticity of substitution production function of capital $K_t$ and a labour aggregate $L_t$ scaled by a technology parameter $A_t$.

$$Y_t = K_t^\beta (AL_t)^{1-\beta}$$

The labor aggregate is a constant elasticity of substitution combination of wage earners, $E_t$, and self-employed workers, $S_t$, who carry out managerial activities, given by

$$L_t = \left[\theta S_t^{1-\alpha} + (1 - \theta)E_t^{1-\alpha}\right]^{1/\alpha}$$

where $1/\alpha$ represents the elasticity of substitution between wage earners and self-employed workers, and $\theta$ and $(1 - \theta)$ are the distribution parameters that control the intensity with which self-employment and wage earners are used in production, respectively. The elasticity of substitution between the two factor inputs—operational and managerial work—measures the percentage response of the relative marginal products—returns—of the two factors to a percentage change in the ratio of their quantities. Therefore, salaried (operational) and self-employed (managerial) workers are gross substitutes (complements) when the elasticity of substitution is greater than (less than) one. In this framework, the value of the elasticity determines how changes in the relative supply of entrepreneurs and workers affect relative earnings of self-employed and paid-employed workers.

Let us define $W_t$ and $B_t$ as the aggregate incomes from paid-employment and self-employment, respectively. Given competitive markets, the relative returns should equate the relative marginal product of the two labor inputs,

$$\frac{W_t}{B_t} = \frac{\partial Y/\partial E_t}{\partial Y/\partial S_t} = \frac{1 - \theta}{\theta} \left(\frac{E_t}{S_t}\right)^{-\alpha}$$

Assuming that the logarithm of the wage earners and self-employment series are I(1) processes, then a cointegrating regression implied by Eq. (3) is given by

$$\ln \left(\frac{W_t}{B_t}\right) = \mu - \alpha \ln \left(\frac{E_t}{S_t}\right) + \varepsilon_t$$

where $\mu = \ln((1 - \theta)/\theta)$, the error term is an I(0) process with mean zero and $(1, \alpha)$ is the cointegrating vector.

This equation will serve as the basis for our empirical estimates. Our parameter of interest, $\alpha$, will be estimated by analysing the long-term relationship between (the log of) the employment/self-employment ratio and (the log of) the returns from paid-employment relative to self-employment. After confirming that these two variables are non-stationary, we will estimate the linear cointegration relation. However, because we are considering a long period of time, it is possible that the relationship between the two variables changes over time, i.e., it is possible that estimation of linear cointegration relations yields spurious inference results because of the presence of one or more structural breaks in the relation. Therefore, we consider the possibility that a linear cointegrated regression model with multiple structural changes would provide a better empirical description of the elasticity of substitution between self-employment and paid-employment. Our methodology is based on instability tests recently proposed in Kejriwal and Perron [44], as well as

\textsuperscript{15}In particular, see the seminal works of Katz and Murphy [52] or Autor et al. [53]. A selective and critical review of this body of literature can be found in Acemoglu [43].
the cointegration test in Arai and Kurozumi [45] and Kejriwal [46] developed to allow for multiple breaks under a null hypothesis of cointegration.

3. Results

In our empirical analysis, we use US data for the period 1969-2014. As in most previous studies, entrepreneurship is operationalised in terms of self-employment, reflecting available data at the time-series level. We are conscious that entrepreneurship is a multifaceted concept, which encompasses a range of roles and activities, and that any single measure of entrepreneurship is therefore a limited proxy. However, in cross-country comparisons, by far the most common measure used in practice is self-employment rates, reflecting the widespread availability of data. Because the perspective adopted in this paper is close to the Knightian entrepreneur and because alternative (or additional) measures of entrepreneurship, such as those provided by the Global Entrepreneurship Monitor project, neither allow circumvention of these limitations nor provide sufficiently long time series for the analysis of long-term relationships, we recognise these difficulties and bear them in mind during the analysis below\(^{16}\). The variable definitions and their main sources are given below:

\( E_t / S_t \): the paid-employment/self-employment ratio, use the wage and salary employment/proprietorship ratio as a proxy.

\( W_t / B_t \): the relative earning of self-employed and paid-employed workers, i.e., the ratio between wage and salary disbursements and proprietor income.

We use yearly US data from the period 1969-2014, drawn from the Regional Economic Information System (REIS) of the Bureau of Economic Analysis.

3.1. Testing for unit roots

Because estimation of a linear cointegration model requires the series to be non-stationary, we start by testing for a unit root in the employment/self-employment ratio and the returns from paid-employment relative to self-employment. We apply the class of unit root tests developed by Ng and Perron [55] which solve several statistical problems associated with more ‘conventional’ unit root tests.\(^{17}\) All test statistics formally examine the unit root null hypothesis against the stationary alternative. Table 1 reports the results. As shown, the existence of two unit roots is clearly rejected at the usual significance levels for all variables, and the null hypothesis of non-stationarity in levels is clearly rejected at the usual significance levels for both variables. Thus, according to the results of these tests, these two series would be I(1).

---

\(^{16}\)As is well known, self-employment is not a perfect measure of entrepreneurship because it includes many “casual” businesses as well as long-established enterprises. Yet, as noted by entrepreneurship scholars, the self-employment definition has the merits of inclusiveness and convenience. By being residual claimants of their own ventures, the self-employed correspond to the Knightian entrepreneur, who assumes all the risk associated with the firm [54].

\(^{17}\)In general, the majority of the conventional unit root tests such as the Dickey-Fuller tests and the Phillips-Perron tests suffer from three problems. First, many tests have low power when the root of the autoregressive polynomial is close to but less than one [56]. Second, most tests suffer from severe size distortions when the moving-average polynomial of the first-differenced series has a large negative autoregressive root [57, 58]. Third, the implementation of unit root tests often requires the selection of an autoregressive truncation lag \( k \); however, as discussed in Ng and Perron [59], there is a strong association between \( k \) and the severity of size distortions and/or the extent of power loss. Ng and Perron [55] solved these problems, and we refer to their article for further details.
### Table 1. Ng and Perron tests for a unit root

<table>
<thead>
<tr>
<th>Variable</th>
<th>( M_Z^{GLS} )</th>
<th>( M_T^{GLS} )</th>
<th>( M_{SB}^{GLS} )</th>
<th>( M_P^{GLS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_t/S_t )</td>
<td>-16.161***</td>
<td>-2.796***</td>
<td>0.173***</td>
<td>1.691***</td>
</tr>
<tr>
<td>( W_t/B_t )</td>
<td>-13.519**</td>
<td>-2.568**</td>
<td>0.190**</td>
<td>1.936**</td>
</tr>
</tbody>
</table>

### Notes:
- **a**, **b** and *** denote significance at the 10%, 5% and 1% levels, respectively;
- The MAIC information criteria are used to select the autoregressive truncation lag, \( k \), as proposed in Perron and Ng (1996). The critical values are taken from Ng and Perron (2001), table 1.

### Critical values:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Case: ( p = 0, \ ɛ = -7.0 )</th>
<th>Case: ( p = 1, \ ɛ = -13.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_Z^{GLS} )</td>
<td>-5.7</td>
<td>-8.1</td>
</tr>
<tr>
<td>( M_{SB}^{GLS} )</td>
<td>-1.62</td>
<td>-1.98</td>
</tr>
<tr>
<td>( M_P^{GLS} )</td>
<td>4.45</td>
<td>3.17</td>
</tr>
</tbody>
</table>

### Table 2. Kerjiwal-Perron tests for testing multiple structural breaks

<table>
<thead>
<tr>
<th>SupF(l)</th>
<th>( UD_{max} )</th>
<th>Sequential</th>
<th>BIC</th>
<th>LWZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.393</td>
<td>5.393</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( T_b )</td>
<td>( T_b )</td>
<td>( T_b )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1992</td>
<td>1992</td>
<td></td>
</tr>
</tbody>
</table>

### Notes:
- ***, **, and *** denote significance at the 10%, 5% and 1% levels, respectively.

The critical values are taken from Kejriwal and Perron (2010).

3.2. Looking for structural breaks

Having confirmed the non-stationarity of both variables, we now apply the tests for structural change that have been proposed in Kejriwal and Perron [60, 44]. We use a 15% trimming, which limits the maximum number of breaks allowed under the alternative hypothesis to 1. Both the intercept and the slope are allowed to change.

Table 2 shows the results of the stability tests and the number of breaks selected by the sequential procedure proposed by Bai and Perron [61] as well as the Bayesian and the modified Schwarz information criteria (BIC and LWZ, respectively). The supFT (1) test is significant at the 5% level, unlike supFT (2), suggesting that the data do not support a one-break model, although the BIC and LWZ select one break and provide evidence against the stability of the long-term relationship.
Overall, the results of the Kejriwal-Perron tests suggest a model with one break, estimated at 1992, and two regimes: 1969-1992 and 1993-2014.

Table 3. Arai-Kurozumi-Kejriwal cointegration tests with one structural break

<table>
<thead>
<tr>
<th>Test</th>
<th>$\hat{V}(\hat{\lambda})$</th>
<th>$\hat{\lambda}$</th>
<th>$\hat{T}_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.062</td>
<td>0.585</td>
<td>1992</td>
</tr>
</tbody>
</table>

Critical values

<table>
<thead>
<tr>
<th>Test</th>
<th>10%</th>
<th>5%</th>
<th>1%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.108</td>
<td>0.135</td>
<td>0.218</td>
</tr>
</tbody>
</table>

Notes:

a *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.
b Critical values are obtained by simulation using 500 steps and 2000 replications.
The Wiener processes are approximated by partial sums of i.i.d. $N(0, 1)$ random variables.

Because the above stability tests reject the null coefficient stability when the regression is spurious, we need to confirm the presence of cointegration among the variables. We use the residual-based test of the null of cointegration against the alternative of cointegration with unknown multiple breaks proposed in Kejriwal [46], $\hat{V}(\hat{\lambda})$. Arai and Kurozumi [45] show that the limit distribution of the test statistic, $\hat{V}_e(\hat{\lambda})$, depends only upon the timing of the estimated break fraction $\hat{\lambda}$ and the number of I(1) regressors $m$. In our case (one-break model), critical values are obtained for $\hat{\lambda}$=0.585, and $m=1$ by simulation using 500 steps and 2000 replications. The Wiener processes are approximated by partial sums of i.i.d. $N(0, 1)$ random variables. Table 3 shows the results of the Arai-Kurozumi cointegration test, allowing one break. Again, the level of trimming used is 15%. The results show that the test $\hat{V}_e(\hat{\lambda})$ cannot reject the null of cointegration with one structural breaks at 1992. Once the presence of structural breaks has been confirmed, and to compare the coefficients obtained from a one-break model with those reported from a model without any structural break, we proceed with a comparison of the estimates of the elasticity of substitution obtained from a one-break model with those obtained from the full sample.

3.3 Elasticity estimates

For the full sample, we estimate the long-term regression model using the Dynamic Ordinary Least Squares (DOLS) estimation method of Stock and Watson [62], extended by Shin [63]. The Shin [63] approach is similar to the KPSS tests, which, in the case of cointegration, are implemented in two stages.

Therefore, the first step in our estimation strategy consists of the estimation of a long-term dynamic equation, including leads and lags of the explanatory variables in the long-term regression model, i.e., the so-called DOLS regression:

---

18LS estimation of the equation might suffer from two problems: nuisance parameter dependences due to serial correlation in the residuals and possible presence of endogeneity in the explanatory variable.

19 In order to overcome the problem of the low power of classical tests for cointegration under the presence of persistent roots in the residuals of the cointegration regression, Shin [63] suggested a new test where the null hypothesis is cointegration.

20 These tests are called the Kwiatkowski et al. [64] tests and assume the null hypothesis of stationarity.
\[ \ln \left( \frac{W_t}{B_t} \right) = \delta - a \ln \left( \frac{E_t}{S_t} \right) + \sum_{j=-q}^{q} \varphi_j \Delta \ln \frac{E_{t-j}}{S_{t-j}} + \varepsilon_j \]  

(5)

In the second step, we use the statistic \(C_{\mu}\), a LM-type test designed by Shin [63], to test the null of cointegration against the alternative of no cointegration in DOLS regression.\(^{21}\) In Table 4, we report the estimates from the DOLS regression and the results from Shin’s test. The results show that the null of deterministic cointegration is not rejected at the 1% significance level.

**Table 4. Stock – Watson – Shin’s DOLS \(^{a,b,c,d}\) estimation of linear cointegration**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta)</td>
<td>0.923**</td>
<td>2.185***</td>
<td>1.610***</td>
</tr>
<tr>
<td></td>
<td>(0.385)</td>
<td>(0.243)</td>
<td>(0.241)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.359***</td>
<td>1.089***</td>
<td>0.786***</td>
</tr>
<tr>
<td></td>
<td>(0.190)</td>
<td>(0.116)</td>
<td>(0.141)</td>
</tr>
<tr>
<td>(1/\alpha)</td>
<td>2.785</td>
<td>0.918</td>
<td>1.272</td>
</tr>
<tr>
<td>Test: (C_{\mu})</td>
<td>0.117</td>
<td>0.137</td>
<td>0.131</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.617</td>
<td>0.960</td>
<td>0.924</td>
</tr>
<tr>
<td>(\sigma^2)</td>
<td>0.093</td>
<td>0.034</td>
<td>0.049</td>
</tr>
</tbody>
</table>

Notes:

\(^{a}\) Standard Errors (in brackets) are adjusted for long-term variance. The long-term variance of the cointegrating regression residual is estimated using the Barlett window, which is approximately equal to \(\text{INT}(T^{1/2})\), as proposed in Newey and West (1987).

\(^{b}\) We choose \(q = \text{INT}(T^{1/3})\), as proposed in Stock and Watson (1993).

\(^{c}\) \(C_{\mu}\) is a LM statistic for cointegration using the DOLS residuals from deterministic cointegration, as proposed by Shin (1994). A *, ** and *** denote significance at the 10%, 5% and 1% levels, respectively.

\(^{d}\) The critical values are taken from Shin (1994), table 1, from \(m=1\), are as follows:

<table>
<thead>
<tr>
<th>Critical values:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_{\mu})</td>
</tr>
<tr>
<td>10%</td>
</tr>
<tr>
<td>0.231</td>
</tr>
</tbody>
</table>

Because there is strong evidence of the presence of structural breaks in 1992 for the cointegration relationship, we divide our sample into two subsamples to analyse whether the elasticity of substitution changes before and after the breaks. We estimate equation (5) for the two subsamples. The estimates for the subsamples are reported in the last two columns of Table 4. In the two regimes, we cannot reject the null of deterministic cointegration at the 1% level of significance. We obtain significant estimates of \(\alpha\), i.e., estimated values for \(\hat{\alpha} = 1.089\) and 0.786. These parameter estimates imply that the values of the elasticity of substitution are 0.918 and 1.272 for the first, and second subsamples, respectively. Thus, ignoring shifts may cause rejection of the existence of a long-term cointegration relationship between the employment/self-employment ratio and the relative earnings of self-employed and paid-employed workers.

\[^{21}\] \(C_{\mu}\) is the test statistic for deterministic cointegration, i.e., when no trend is present in the regression.
Furthermore, the evolution of the US average firm size (self-employment rate) is consistent with the elasticity estimates for the two identified regimes. In particular, our results report an elasticity estimate for the first subsample (before the first break), which is consistent with Lucas’s proposition regarding average firm size. In contrast, after this first regime, the elasticity experienced drastic growth, and the elasticity reached a value higher than one. Therefore, the estimates suggest that at the beginning of the 1990s, deep changes in the determinants of the substitution rate between self-employed and paid-employed workers, i.e., between managerial and operative works, should have taken place in such a manner that, in the most recent regime, self-employment and paid employment are now gross substitutes instead of complements. These findings are consistent with the evolution of observed average firm size in the US during the covered period.

4. Conclusions

This paper reported estimates of the elasticity of substitution in the US, accounting for the possible existence of structural breaks. Using a methodology based on instability tests recently proposed in Kejriwal and Perron [44] as well as the cointegration tests in Arai and Kurozumi [45] and Kejriwal [46] that were developed to allow for multiple breaks under the null hypothesis of cointegration, our results support the existence of a changing and increasing elasticity of substitution between paid employment and self-employment, supporting both the proposition of Aquilina et al. [25] regarding the decrease in average firm size and the observed evolution of the US self-employment rate.

This change in the elasticity of substitution conforms to the observed relation between average firm size and economic development in advanced economies. However, the relation has been subject to change. Until the last quarter of the twentieth century, the increasing importance in economies of scale and the influence of increasing wage levels on occupational choice implied a growing average firm size (Chandler [10], Wennekers et al., [24]). However, starting in the 1980s, self-employment levels started to increase in many advanced economies, beginning in the US. There are some factors that could explain this structural change in the elasticity of substitution, i.e., some driving forces of this shift toward smallness: i) the fast-growing services sector, with its minor scale and lower entry barriers; ii) an opposite relationship between the elasticity of substitution between labour and capital and average firm size (Aquilina et al.’s proposition); iii) a trend in occupational preferences favouring self-employment following the emergence of incentive schemes; iv) globalisation conforming with the spread of ICT (information and communication technologies), allowing solo entrepreneurs and small firms to reap the fruits of scale economies through loosely organised networks; and finally, v) new technologies’ creation of opportunities for new technology-based business start-ups (Wennekers et al., [24], p. 169).

Recently, Amorós and Cristi [65] presented another argument for economies in which some individuals are ‘pushed’ into entrepreneurship because no better employment options exist, despite the existence of pro-entrepreneurship policies. Most likely, this argument can also be applied to developed countries where the change in the relative response of the employment/self-employment ratio to changes in the relative earnings of self-employed and paid-employed workers has led to a lower average self-employed firm size, as shown by our empirical estimates. This paper reported estimates of the elasticity of substitution with the incorporation of breaks to study how the relationship may have changed over time as well as to estimate the elasticity in every regime in a developed economy.

It is likely that necessity entrepreneurship (Acs et al, [66]), new interactions between labour market institutions and the promotion of self-employment and/or a new risk-adjusted valuation of the relative returns between managerial and operational works in a context of less-protected paid-employment are the key factors explaining the elasticity estimates reported in this study. Further research is needed to determine whether changes in institutional conditions may explain the documented changes in the elasticity of substitution provided in this article.
Funding: V. Esteve wants to acknowledge financial support from the Spanish Ministry of Science and Innovation (Project ECO2008-05908-C02-02), the Department of Education and Science of the regional government of Castilla-La Mancha (Project PEII09-0072-7392), the Generalitat Valenciana (Project GVPMETEO2009-098), and the Fundación Seneca-Regional Agency of the regional government of Murcia, through the project 15363/PHCS/10.

Conflicts of Interest: The authors declare no conflict of interest.

References


