

1 Article

2 **Going solo: on the substitutability between 3 paid-employment and self-employment**

4 **Emilio Congregado** ^{1,*}, **Antonio A. Golpe** ¹ and **Vicente Esteve** ²

5 ¹ University of Huelva; congregado@uhu.es; antonio.golpe@dehie.uhu.es

6 ² University of Valencia; vicente.esteve@uv.es

7 * Correspondence: congregado@uhu.es; Tel.: +34-959-217-832

8

9 **Abstract:** This paper provides estimates of the elasticity of substitution between operational and
10 managerial jobs in the US economy covering a period of almost five decades, derived from an
11 aggregate CES production function. Estimating the long-term relationship between (the log of) the
12 aggregate employment/self-employment ratio and (the log of) the returns from paid-employment
13 relative to self-employment and testing for structural breaks, we report different estimates of the
14 elasticity of substitution in each of the two regimes identified. Our results help to understand and
15 interpret one of the most intriguing aspects in the evolution of self-employment rates in developed
16 countries: the reversal of the trend in self-employment rates. Our estimates show that a higher level
17 of development is associated with a greater number of entrepreneurs and smaller firms. Some
18 rationales for understanding the growth of the elasticity between paid-employment and
19 self-employment, including the recent trends in the digital economy— are also suggested.

20 **Keywords:** Elasticity of substitution; Cointegration; Self-employment; Structural Breaks.
21

22 **1. Introduction**

23 In recent years, a growing body of literature has studied the relation between economic
24 development and the aggregate self-employment rate [1-6]. In particular, analysis of the interplay
25 between the economic development phase and the evolution of the independent entrepreneurship
26 rate—or the (inverse) relationship between the wealth of the economy and the related concept of
27 average firm size (i.e., the employment/self-employment ratio)—has become a focus area for
28 scholars because of the observation of a reversal in self-employment rate trends in several developed
29 countries. A handful of works [7-9] documented this reversal trend in the US.

30 Until the last quarter of the 20th century, economic development was related to the ever
31 increasing importance of economies of scale and scope [10], a switch from agriculture to
32 manufacturing [11]¹ and the influence of increasing wage levels on occupational choice [14].²

¹Changes in industrial structure should influence independent entrepreneurship rates because some activities lend themselves better to self-employment than others [12]. One could argue that the characteristics of different sectors and industries, in terms of the existence of significant demand for personal (professional) services, jobs with erratic demand, the mix of skills required or low capital requirements, make it more likely that a sector is populated by self-employed workers. These arguments help us to understand the high concentration of self-employed workers in the agriculture and service sectors and the comparatively low concentration in manufacturing. See, e.g., [13] for an analysis of US self-employment by industry.

²Following Lucas's argument, because capital and labour are substitutes, higher capital stock implies higher returns from working and lower returns from managing. As a result, economic development leads to a higher

33 Overall, the predominant view was that as economies became wealthier, average firm size should
34 increase; in other words, average firm size should be an increasing function of the wealth of the
35 economy [15]. Therefore, a negative relation between economic development and the
36 self-employment rate was implied.³Data regarding the evolution of average firm size during the late
37 nineteenth and first three quarters of the twentieth centuries in most developed countries supported
38 this proposition.

39 Related to this latter point, in a highly influential paper, [14] developed a model in which firm
40 distribution was the solution to the problem of allocating productive factors among managers of
41 varying ability. The main result of Lucas's model concerns the effect on average firm size when per
42 capita capital increases. Lucas showed that in the case where the elasticity of substitution between
43 labour and capital is less than one, as the economy becomes wealthier, the wage relative to
44 managerial rents increases, and marginal entrepreneurs prefer to become wage earners rather than
45 manage their own businesses. This causes an increase in the ability threshold that is necessary to
46 become an entrepreneur, which defines the marginal entrepreneur. Then, an increase in wages,
47 relative to a managerial rent increase, induces marginal entrepreneurs to become employees, raising
48 the average size of the firm.⁴ Furthermore, an important prediction, given the sustained trend of
49 growth in capital per capita, emerges: 'the fraction of entrepreneurs will decline over time while
50 average firm size will inexorably increase' [12]. Development leads to higher average firm size
51 because of a negative relationship between the elasticity of factor substitution and firm size.

52 Lucas [14] reported that average firm size (using employees per firm as a proxy) was positively
53 related to GNP per capita (used as a proxy for capital per capita) in the US. This positive test of
54 Lucas's hypothesis reflected not only observed developments in self-employment during the first
55 three quarters of the 20th century but also consistency with estimations of the elasticity of factor
56 substitution between capital and labour.⁵

57 However, in several developed countries, the trend reversed. The relationship seemed to have
58 changed from a negative relation to a positive one, and the observed recovery in self-employment
59 rates was interpreted as undermining Lucas's prediction. In fact, the secular decline in
60 self-employment rates experienced by most developed countries was followed by a reversal trend in
61 the last quarter of the twentieth century and in the first decade of the current century.⁶ For instance,
62 considering the 23 OECD countries included in COMPENDIA⁷ as a reference, the average business
63 ownership rate⁸—i.e., the number of owners of non-agricultural incorporated and unincorporated
64 businesses as a fraction of total labour force—increased from 0.100 in 1972 to 0.112 in 2009. This
65 figure, however, hides huge national disparities in both levels of the average business ownership
66 rate and in their evolution. For example, the sampled business ownership rates in 2009 range from
67 19.9% in Italy to 4.7% in Luxembourg; analysing the rates' evolution, business ownership in Japan
68 experienced a decline from 0.125 in 1972 to 0.083 in 2009, while business ownership in the US and the

average firm size because of a negative relationship between the elasticity of factor substitution (between capital and labour) and average firm size.

³ This negative relationship is well documented in the works of [11], [16-18], among others.

⁴ By contrast, if the elasticity of substitution is greater than one, then economic increases in per capita capital increase the equilibrium number of entrepreneurs and decrease the average firm size. Note that in the case of a Cobb-Douglas production function, the average firm size is unchanged when per capita capital grows.

⁵Empirical estimates usually converge to an elasticity value—capital-labor—of less than 1 (see [19], ch. 3).

⁶ In the US, the self-employment rate began to rise in the 1970s [7].

⁷COMPENDIA is an acronym for COMParative ENtrepreneurship Data for International Analysis. See <http://www.entrepreneurship-sme.eu>.

⁸ Business ownership, self-employment and independent entrepreneurship will be used as interchangeable concepts in this article.

69 European Union-15 increased from 0.082 to 0.093⁹ and from 0.104 to 0.118, respectively, during the
70 same period. The possibility of a U-shaped relationship between entrepreneurship and economic
71 development gradually gained ground, and the re-examination of that relationship became the
72 subject of a large body of empirical and theoretical literature, recently surveyed in [24].

73 Broadly speaking, at least four arguments have been suggested to explain this reversal.¹⁰ The
74 first argument relates to the non-validity of Lucas's proposition, asking whether something in the
75 proposition itself is amiss or if the proposition depends crucially on some faulty assumption. Using
76 this last argument, [25] extended Lucas's analysis by utilising a more general aggregate production
77 function (a normalised CES), which allowed them to prove the existence of an inverse relationship
78 between the elasticity of substitution (between capital and labour) and average firm size. From this
79 perspective, the fact that wealthier countries have a higher elasticity of substitution is consistent
80 with the positive association between the growing importance of SMEs in the most developed
81 countries because a high elasticity of substitution value more easily enables individuals to become
82 entrepreneurs. In short, from the model presented in [25], we can confidently state that in economies
83 characterised by higher values of aggregate elasticity of substitution between capital and labour, we
84 should expect higher wealth to be associated with more entrepreneurs and smaller firms. This
85 proposition is supported by the recent evolution of average firm size in developed countries.

86 In addition to the above arguments, some scholars have suggested that there were also certain
87 changes and mechanisms that can help to understand this trend reversal. One argument is that
88 independent entrepreneurship and average firm size are now decreasing and increasing functions,
89 respectively, of the wealth of the economy due to improvements in information and communication
90 technologies (ICT). It is a well-known fact that the ICT revolution has decreased the importance of
91 scale economies in many industries [26] and has increased opportunities for entrepreneurship and
92 returns to entrepreneurship and managerial talent [27]—managerial works [28].

93 It has also been suggested that the reversal of the trend in self-employment rates may be the
94 effect of an expansion of the business service sector relative to manufacturing. Several scholars argue
95 that this expansion has attended a shift away from larger corporations and toward entrepreneurial
96 activity. This phenomenon has led to a decline in the average firm size [24].

97 Finally, one could argue that the reversal in the business ownership rate may be the result of
98 structural changes having strong effects on occupational choice decisions and, therefore, on the
99 elasticity of substitution between paid-employment and self-employment. In particular, we may
100 hypothesise that the above factors, in conjunction with the emergence of incentives schemes, such as
101 subsidies or tax allowances¹¹, and a progressive reduction in the rights and benefits derived from
102 employment protection legislation¹² may have introduced substantial changes in the risk-adjusted
103 relative earnings of paid employment and self-employment. Thus, one could argue that higher
104 levels of entrepreneurship may indicate that extant job creators are not creating attractive
105 wage-earning job opportunities¹³ as a result of a low valuation of the risk associated with
106 self-employment. The loss of rights, in terms of potential severance payments and unemployment
107 benefits, may affect the structure of employment by altering the relative valuation between
108 self-employment and paid-employment.

109 In short, the importance of several factors—such as the reduction of the extent of scale
110 economies, the existence of more volatile markets or the growing importance of innovation, and the
111 elasticity of substitution between capital and labour—to predicting the progressive decline of the

⁹ See [7] and [20-23], for a complete picture of the evolution of the self-employment sector in the US.

¹⁰ See, e.g. [15] or [24], for a detailed exposition on how these mechanisms operate.

¹¹ See, [29-33].

¹² In Botero et al. [34] a measure for labour market regulation is proposed. On the other hand, the works of [35-41] analyse the effects of stricter employment protection legislation on self-employment.

¹³ Not only in terms of lower wage rates, taking advantage of low union membership rates or segmentation, but also avoiding the costs of compliance of those contracts with higher employment protection rates.

112 average firm size cannot be denied. This article seeks to test whether changes introduced in some
113 labour market institutions [34] and labour market dynamics, along with the generalised emergence
114 of entrepreneurship policy [42], particularly the introduction of different schemes to promote
115 self-employment, have substantially altered the relative risk-adjusted returns in self-employment
116 and the elasticity of substitution between them.

117 This paper investigates this latter hypothesis using US data, testing whether the estimate of the
118 elasticity of substitution between managerial and operational jobs in a developed economy such as
119 that of the US is compatible with a fall in average firm size. The aim of this paper is to present
120 estimates of the elasticity of substitution between entrepreneurship and paid-employment using US
121 data as a method of testing whether, as recent literature has hypothesised, wealthier and more
122 developed countries are characterised by a higher elasticity of substitution between
123 self-employment and paid-employment or if elasticity estimates instead support Lucas's hypothesis
124 (in terms of the inexorability of a secular trend of increasing average firm size and decreasing
125 numbers of entrepreneurs).

126 Our empirical results are consistent with the existence of a long-term relationship between the
127 wage-earner/self-employment ratio and the relative earnings of self-employed and paid-employed
128 workers. However, this relationship is subject to structural changes. In particular, our results report
129 an elasticity estimate for the first subsample (before the break) that is consistent with Lucas's
130 proposition regarding average firm size, while estimates in the second subsample are consistent
131 with the observed evolution of average firm size. Importantly, the first break date coincides with the
132 beginning of the rise in American self-employment [7]. Our estimates suggest that at the beginning
133 of the 1990s, deep changes in the determinants of the substitution rate between self-employed and
134 paid-employed workers, i.e., between managerial and operative works, should have occurred in
135 such a manner that, in the most recent regime, self-employment and paid employment are now
136 gross substitutes instead of complements. These findings are consistent with observed average firm
137 size development in the US during the covered period.

138 Technically, our analysis parallels the literature on wage inequality [43] because we consider
139 self-employment and paid employment as two employment statuses—managerial and operational
140 works—similar to the literature addressing skilled and unskilled labour. Therefore, we report
141 estimates of the elasticity of substitution between these two employment statuses by estimating the
142 linear long-term relationship between the employment/self-employment ratio and the returns from
143 paid-employment relative to self-employment. After analysis of this relationship, we consider the
144 possibility that a regression model with multiple structural changes would provide a better
145 empirical description of the relationship. To that end, instability tests, recently proposed in [44-46],
146 are performed.

147 The remainder of the paper is organised as follows. In Section 2, we describe our model and
148 econometric strategy. In section 3, we present our estimation results. Finally, Section 4 summarises
149 our main conclusions.

150 2. Model and econometric strategy

151 Generalising differences in individual skills in the basic occupational model (see, e.g., pioneer
152 models of Rees and Shah [47], Borjas and Bronars [48], or Evans and Leighton [49], the choice
153 between entrepreneurial-managerial and operational jobs is based upon the idea that individuals
154 respond to the risk-adjusted relative earnings opportunities in each sector (self-employed sector vs.
155 employed sector).¹⁴

156 The perspective assumed in this paper is that occupational choices of fully informed
157 individuals are based only on the risk-adjusted relative earnings between self-employment and
158 paid-employment.

¹⁴ See, e.g. [50] and [51]

159 As mentioned, our empirical strategy parallels the basic framework used by literature
 160 addressing wage inequality and skill premiums¹⁵ because, to some extent, the occupational decision
 161 between managerial and non-managerial work is also based on the relative earnings between the
 162 two employment statuses. Let us consider a simple closed economy. We begin with an aggregate
 163 production framework, where output is described by a constant elasticity of substitution production
 164 function of capital K_t and a labour aggregate L_t scaled by a technology parameter A_t .

$$Y_t = K_t^\rho (AL_t)^{1-\rho} \quad (1)$$

165 The labor aggregate is a constant elasticity of substitution combination of wage earners, E_t , and
 166 self-employed workers, S_t , who carry out managerial activities, given by

$$L_t = [\theta S_t^{1-\alpha} + (1-\theta)E_t^{1-\alpha}]^{\frac{1}{1-\alpha}} \quad (2)$$

167 where $1/\alpha$ represents the elasticity of substitution between wage earners and self-employed
 168 workers, and θ and $(1-\theta)$ are the distribution parameters that control the intensity with which
 169 self-employment and wage earners are used in production, respectively. The elasticity of
 170 substitution between the two factor inputs—operational and managerial work—measures the
 171 percentage response of the relative marginal products—returns—of the two factors to a percentage
 172 change in the ratio of their quantities. Therefore, salaried (operational) and self-employed
 173 (managerial) workers are gross substitutes (complements) when the elasticity of substitution is
 174 greater than (less than) one. In this framework, the value of the elasticity determines how changes in
 175 the relative supply of entrepreneurs and workers affect relative earnings of self-employed and
 176 paid-employed workers.

177 Let us define W_t and B_t as the aggregate incomes from paid-employment and
 178 self-employment, respectively. Given competitive markets, the relative returns should equate the
 179 relative marginal product of the two labor inputs,

$$\frac{W_t}{B_t} = \frac{\partial Y / \partial E_t}{\partial Y / \partial S_t} = \frac{1-\theta}{\theta} \left(\frac{E_t}{S_t} \right)^{-\alpha} \quad (3).$$

180 Assuming that the logarithm of the wage earners and self-employment series are I(1) processes,
 181 then a cointegrating regression implied by Eq. (3) is given by
 182

$$\ln \left(\frac{W_t}{B_t} \right) = \mu - \alpha \ln \left(\frac{E_t}{S_t} \right) + \varepsilon_t \quad (4),$$

183 where $\mu = \ln[(1-\theta)/\theta]$, the error term is an I(0) process with mean zero and $(1,\alpha)$ is the
 184 cointegrating vector.

185 This equation will serve as the basis for our empirical estimates. Our parameter of interest, α ,
 186 will be estimated by analysing the long-term relationship between (the log of) the
 187 employment/self-employment ratio and (the log of) the returns from paid-employment relative to
 188 self-employment. After confirming that these two variables are non-stationary, we will estimate the
 189 linear cointegration relation. However, because we are considering a long period of time, it is
 190 possible that the relationship between the two variables changes over time, i.e., it is possible that
 191 estimation of linear cointegration relations yields spurious inference results because of the presence
 192 of one or more structural breaks in the relation. Therefore, we consider the possibility that a linear
 193 cointegrated regression model with multiple structural changes would provide a better empirical
 194 description of the elasticity of substitution between self-employment and paid-employment. Our
 195 methodology is based on instability tests recently proposed in Kejriwal and Perron [44], as well as

¹⁵In particular, see the seminal works of Katz and Murphy [52] or Autor et al. [53]. A selective and critical review of this body of literature can be found in Acemoglu [43].

196 the cointegration test in Arai and Kurozumi [45] and Kejriwal [46] developed to allow for multiple
197 breaks under a null hypothesis of cointegration.

198 **3. Results**

199 In our empirical analysis, we use US data for the period 1969-2014. As in most previous studies,
200 entrepreneurship is operationalised in terms of self-employment, reflecting available data at the
201 time-series level. We are conscious that entrepreneurship is a multifaceted concept, which
202 encompasses a range of roles and activities, and that any single measure of entrepreneurship is
203 therefore a limited proxy. However, in cross-country comparisons, by far the most common measure
204 used in practice is self-employment rates, reflecting the widespread availability of data. Because the
205 perspective adopted in this paper is closed to the Knightian entrepreneur and because alternative (or
206 additional) measures of entrepreneurship, such as those provided by the Global Entrepreneurship
207 Monitor project, neither allow circumvention of these limitations nor provide sufficiently long time
208 series for the analysis of long-term relationships, we recognise these difficulties and bear them in
209 mind during the analysis below¹⁶. The variable definitions and their main sources are given below:
210

211 E/S_t : the paid-employment/self-employment ratio, use the wage and salary
212 employment/proprietorship ratio as a proxy.
213

214 W_t/B_t : the relative earning of self-employed and paid-employed workers, i.e., the ratio between wage
215 and salary disbursements and proprietor income.
216

217 We use yearly US data from the period 1969-2014, drawn from the Regional Economic
218 Information System (REIS) of the Bureau of Economic Analysis.

219 *3.1. Testing for unit roots*

220 Because estimation of a linear cointegration model requires the series to be non-stationary, we
221 start by testing for a unit root in the employment/self-employment ratio and the returns from
222 paid-employment relative to self-employment. We apply the class of unit root tests developed by Ng
223 and Perron [55] which solve several statistical problems associated with more 'conventional' unit
224 root tests.¹⁷ All test statistics formally examine the unit root null hypothesis against the stationary
225 alternative. Table 1 reports the results. As shown, the existence of two unit roots is clearly rejected at
226 the usual significance levels for all variables, and the null hypothesis of non-stationarity in levels is
227 clearly rejected at the usual significance levels for both variables. Thus, according to the results of
228 these tests, these two series would be I(1).

¹⁶As is well known, self-employment is not a perfect measure of entrepreneurship because it includes many "casual" businesses as well as long-established enterprises. Yet, as noted by entrepreneurship scholars, the self-employment definition has the merits of inclusiveness and convenience. By being residual claimants of their own ventures, the self-employed correspond to the Knightian entrepreneur, who assumes all the risk associated with the firm [54].

¹⁷In general, the majority of the conventional unit root tests such as the Dickey-Fuller tests and the Phillips-Perron tests suffer from three problems. First, many tests have low power when the root of the autoregressive polynomial is close to but less than one [56]. Second, most tests suffer from severe size distortions when the moving-average polynomial of the first-differenced series has a large negative autoregressive root [57, 58]. Third, the implementation of unit root tests often requires the selection of an autoregressive truncation lag k ; however, as discussed in Ng and Perron [59], there is a strong association between k and the severity of size distortions and/or the extent of power loss. Ng and Perron [55] solved these problems, and we refer to their article for further details.

229

Table 1. Ng and Perron,a,b tests for a unit root

I(2) vs. I(1)		Case: $p = 0, \bar{c} = -7.0$		
Variable	$\bar{M}Z_{\alpha}^{GLS}$	$\bar{M}Z_t^{GLS}$	$\bar{M}SB_{\alpha}^{GLS}$	$\bar{M}P_T^{GLS}$
E_t/S_t	-16.161***	-2.796***	0.173***	1.691***
W_t/B_t	-13.519**	-2.568**	0.190**	1.936**
I(1) vs. I(0)		Case: $p = 1, \bar{c} = -13.5$		
E_t/S_t	-4.469	-1.457	0.326	20.084
W_t/B_t	-5.106	-1.597	0.313	17.843

230

Notes:

231

a*, ** and *** denote significance at the 10%, 5% and 1% levels, respectively;

232

b The MA/C information criteria are used to select the autoregressive truncation lag, k , as proposed in Perron and Ng (1996). The critical values are taken from Ng and Perron (2001), table 1.

233

234

Critical values: Case: $p = 0, \bar{c} = -7.0$ Case: $p = 1, \bar{c} =$

-13.5

Variable	10%	5%	1%	10%	5%	1%
$\bar{M}Z_{\alpha}^{GLS}$	-5.7	-8.1	-13.8	-14.2	-17.3	-23.8
$\bar{M}SB_{\alpha}^{GLS}$	0.275	0.233	0.174	0.185	0.168	0.143
$\bar{M}Z_t^{GLS}$	-1.62	-1.98	-2.58	-2.62	-2.91	-3.42
$\bar{M}P_T^{GLS}$	4.45	3.17	1.78	6.67	5.48	4.03

235

236

3.2. Looking for structural breaks

237

Having confirmed the non-stationarity of both variables, we now apply the tests for structural change that have been proposed in Kejriwal and Perron [60, 44]. We use a 15% trimming, which limits the maximum number of breaks allowed under the alternative hypothesis to 1. Both the intercept and the slope are allowed to change.

240

241

Table 2. Kejriwal-Perron tests for testing multiple structural breaks

Number of breaks selected				
$SupF_i(1)$	UD_{\max}	<i>Sequential</i>	<i>BIC</i>	<i>LWZ</i>
5.393	5.393	0	1	1
		T_b	T_b	T_b
			1992	1992

Notes:

*, **, and *** denote significance at the 10%, 5% and 1% levels, respectively.

The critical values are taken from Kejriwal and Perron (2010).

242

243

Table 2 shows the results of the stability tests and the number of breaks selected by the sequential procedure proposed by Bai and Perron [61] as well as the Bayesian and the modified Schwarz information criteria (BIC and LWZ, respectively). The supFT (1) test is significant at the 5% level, unlike supFT (2), suggesting that the data do not support a one-break model, although the BIC and LWZ select one break and provide evidence against the stability of the long-term relationship.

244

245

246

247

248 Overall, the results of the Kejriwal-Perron tests suggest a model with one break, estimated at 1992,
 249 and two regimes: 1969-1992 and 1993-2014.

Table 3. Arai-Kurozumi-Kejriwal cointegration tests with one structural break

Test $\hat{V}(\hat{\lambda})$	$\hat{\lambda}$	\hat{T}_1	
0.062	0.585	1992	
Critical values	10%	5%	1%
$\hat{V}_k(\hat{\lambda})$	0.108	0.135	0.218

250 Notes:

251 a *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

252 b Critical values are obtained by simulation using 500 steps and 2000 replications.

253 The Wiener processes are approximated by partial sums of i.i.d. $N(0, 1)$ random variables.

254

255 Because the above stability tests reject the null coefficient stability when the regression is spurious,
 256 we need to confirm the presence of cointegration among the variables. We use the residual-based
 257 test of the null of cointegration against the alternative of cointegration with unknown multiple
 258 breaks proposed in Kejriwal [46], $\hat{V}(\hat{\lambda})$. Arai and Kurozumi [45] show that the limit distribution of
 259 the test statistic, $\hat{V}_k(\hat{\lambda})$, depends only upon the timing of the estimated break fraction $\hat{\lambda}$ and the
 260 number of I(1) regressors m . In our case (one-break model), critical values are obtained for $\hat{\lambda}=0.585$,
 261 and $m=1$ by simulation using 500 steps and 2000 replications. The Wiener processes are
 262 approximated by partial sums of i.i.d. $N(0,1)$ random variables. Table 3 shows the results of the
 263 Arai-Kurozumi cointegration test, allowing one break. Again, the level of trimming used is 15%. The
 264 results show that the test $\hat{V}_1(\hat{\lambda})$ cannot reject the null of cointegration with one structural breaks at
 265 1992. Once the presence of structural breaks has been confirmed, and to compare the coefficients
 266 obtained from a one-break model with those reported from a model without any structural break,
 267 we proceed with a comparison of the estimates of the elasticity of substitution obtained from a
 268 one-break model with those obtained from the full sample.

269 3.3 Elasticity estimates

270 For the full sample, we estimate the long-term regression model using the Dynamic Ordinary
 271 Least Squares (DOLS)¹⁸ estimation method of Stock and Watson [62], extended by Shin [63].¹⁹ The
 272 Shin [63] approach is similar to the KPSS²⁰ tests, which, in the case of cointegration, are implemented
 273 in two stages.

274 Therefore, the first step in our estimation strategy consists of the estimation of a long-term
 275 dynamic equation, including leads and lags of the explanatory variables in the long-term regression
 276 model, i.e., the so-called DOLS regression:

¹⁸ LS estimation of the equation might suffer from two problems: nuisance parameter dependences due to serial correlation in the residuals and possible presence of endogeneity in the explanatory variable.

¹⁹ In order to overcome the problem of the low power of classical tests for cointegration under the presence of persistent roots in the residuals of the cointegration regression, Shin [63] suggested a new test where the null hypothesis is cointegration.

²⁰ These tests are called the Kwiatkowski et al. [64] tests and assume the null hypothesis of stationarity.

$$\ln\left(\frac{W_t}{B_t}\right) = \delta - \alpha \ln\left(\frac{E_t}{S_t}\right) + \sum_{j=-q}^q \varphi_j \Delta \ln \frac{E_{t-j}}{S_{t-j}} + \varepsilon_j \quad (5)$$

277

278 In the second step, we use the statistic C_μ , a LM-type test designed by Shin [63], to test the null
 279 of cointegration against the alternative of no cointegration in DOLS regression.²¹ In Table 4, we report
 280 the estimates from the DOLS regression and the results from Shin's test. The results show that the
 281 null of deterministic cointegration is not rejected at the 1% significance level.
 282

283 **Table 4.** Stock –Watson-Shin's DOLS ^{a,b,c,d} estimation of linear cointegration

Parameter estimates	Full sample	First regime	Second regime
	1969-2014	1969-1992	1993-2014
δ	0.923** (0.385)	2.185*** (0.243)	1.610*** (0.241)
α	0.359*** (0.190)	1.089*** (0.116)	0.786*** (0.141)
$1/\alpha$	2.785	0.918	1.272
Test: C_μ^c	0.117	0.137	0.131
R^2	0.617	0.960	0.924
$\hat{\sigma}^2$	0.093	0.034	0.049

Notes:

^aStandard Errors (in brackets) are adjusted for long-term variance. The long-term variance of the cointegrating regression residual is estimated using the Barlett window, which is approximately equal to $INT(T^{1/2})$, as proposed in Newey and West (1987).

^bWe choose $q = INT(T^{1/3})$, as proposed in Stock and Watson (1993).

^c C_μ is a LM statistic for cointegration using the DOLS residuals from deterministic cointegration, as proposed by Shin (1994). A * ** and *** denote significance at the 10%, 5% and 1% levels, respectively.

^dThe critical values are taken from Shin (1994), table 1, from m=1, are as follows:

Critical values:

	10%	5%	1%
C_μ	0.231	0.314	0.533

284

285 Because there is strong evidence of the presence of structural breaks in 1992 for the
 286 cointegration relationship, we divide our sample into two subsamples to analyse whether the
 287 elasticity of substitution changes before and after the breaks. We estimate equation (5) for the two
 288 subsamples. The estimates for the subsamples are reported in the last two columns of Table 4. In the
 289 two regimes, we cannot reject the null of deterministic cointegration at the 1% level of significance.
 290 We obtain significant estimates of α , i.e., estimated values for $\hat{\alpha} = 1.089$ and 0.786 . These parameter
 291 estimates imply that the values of the elasticity of substitution are 0.918 and 1.272 for the first, and
 292 second subsamples, respectively. Thus, ignoring shifts may cause rejection of the existence of a
 293 long-term cointegration relationship between the employment/self-employment ratio and the
 294 relative earnings of self-employed and paid-employed workers.

²¹ C_μ is the test statistic for deterministic cointegration, i.e., when no trend is present in the regression.

295 Furthermore, the evolution of the US average firm size (self-employment rate) is consistent with
296 the elasticity estimates for the two identified regimes. In particular, our results report an elasticity
297 estimate for the first subsample (before the first break), which is consistent with Lucas's proposition
298 regarding average firm size. In contrast, after this first regime, the elasticity experienced drastic
299 growth, and the elasticity reached a value higher than one. Therefore, the estimates suggest that at
300 the beginning of the 1990s, deep changes in the determinants of the substitution rate between
301 self-employed and paid-employed workers, i.e., between managerial and operative works, should
302 have taken place in such a manner that, in the most recent regime, self-employment and paid
303 employment are now gross substitutes instead of complements. These findings are consistent with
304 the evolution of observed average firm size in the US during the covered period.
305

306 4. Conclusions

307

308 This paper reported estimates of the elasticity of substitution in the US, accounting for the
309 possible existence of structural breaks. Using a methodology based on instability tests recently
310 proposed in Kejriwal and Perron [44] as well as the cointegration tests in Arai and Kurozumi [45]
311 and Kejriwal [46] that were developed to allow for multiple breaks under the null hypothesis of
312 cointegration, our results support the existence of a changing and increasing elasticity of substitution
313 between paid employment and self-employment, supporting both the proposition of Aquilina et al.
314 [25] regarding the decrease in average firm size and the observed evolution of the US
315 self-employment rate.

316 This change in the elasticity of substitution conforms to the observed relation between average
317 firm size and economic development in advanced economies. However, the relation has been subject
318 to change. Until the last quarter of the twentieth century, the increasing importance in economies of
319 scale and the influence of increasing wage levels on occupational choice implied a growing average
320 firm size (Chandler [10], Wennekers et al., [24]). However, starting in the 1980s, self-employment
321 levels started to increase in many advanced economies, beginning in the US. There are some factors
322 that could explain this structural change in the elasticity of substitution, i.e., some driving forces of
323 this shift toward smallness: i) the fast-growing services sector, with its minor scale and lower entry
324 barriers; ii) an opposite relationship between the elasticity of substitution between labour and capital
325 and average firm size (Aquilina et al.'s proposition); iii) a trend in occupational preferences
326 favouring self-employment following the emergence of incentive schemes; iv) globalisation
327 conforming with the spread of ICT (information and communication technologies), allowing solo
328 entrepreneurs and small firms to reap the fruits of scale economies through loosely organised
329 networks; and finally, v) new technologies' creation of opportunities for new technology-based
330 business start-ups (Wennekers et al., [24], p. 169).

331 Recently, Amorós and Cristi [65] presented another argument for economies in which some
332 individuals are 'pushed' into entrepreneurship because no better employment options exist, despite
333 the existence of pro-entrepreneurship policies. Most likely, this argument can also be applied to
334 developed countries where the change in the relative response of the employment/self-employment
335 ratio to changes in the relative earnings of self-employed and paid-employed workers has led to a
336 lower average self-employed firm size, as shown by our empirical estimates. This paper reported
337 estimates of the elasticity of substitution with the incorporation of breaks to study how the
338 relationship may have changed over time as well as to estimate the elasticity in every regime in a
339 developed economy.

340 It is likely that necessity entrepreneurship (Acs et al, [66]), new interactions between labour
341 market institutions and the promotion of self-employment and/or a new risk-adjusted valuation of
342 the relative returns between managerial and operational works in a context of less-protected
343 paid-employment are the key factors explaining the elasticity estimates reported in this study.
344 Further research is needed to determine whether changes in institutional conditions may explain the
345 documented changes in the elasticity of substitution provided in this article.
346

347 **Funding:** V. Esteve wants to acknowledge financial support from the Spanish Ministry of Science and
348 Innovation (Project ECO2008-05908-C02-02), the Department of Education and Science of the regional
349 government of Castilla-La Mancha (Project PEII09-0072-7392), the Generalitat Valenciana (Project
350 GVPROMETEO2009-098), and the Fundación Seneca-Regional Agency of the regional government of Murcia,
351 through the project 15363/PHCS/10.

352 **Conflicts of Interest:** The authors declare no conflict of interest.

353 References

1. Carree, M., Van Stel, A., Thurik, R., and Wennekers, S. (2002) Economic Development and Business Ownership: An Analysis Using Data of 23 OECD Countries in the Period 1976-1996. *Small Business Economics* 19(3), 271-290.
2. Carree, M., Van Stel, A., Thurik, R., and Wennekers, S. (2007) The Relationship Between Economic Development and Business Ownership Revisited, *Entrepreneurship and Regional Development* 19(3), 281-291.
3. Kelley, D.J., Bosma, N. and Amorós, J.E. (2011) Global Entrepreneurship Monitor, 2010 Global Report, Wellesley, MA: Babson College and Santiago, Chile: Universidad del Desarrollo.
4. Audretsch, D. B., Belitski, M., & Desai, S. (2015). Entrepreneurship and economic development in cities. *The Annals of Regional Science*, 55(1), 33-60.
5. Acs, Z. J., Estrin, S., Mickiewicz, T., & Szerb, L. (2018). Entrepreneurship, institutional economics, and economic growth: an ecosystem perspective. *Small Business Economics*, 1-14.
6. Dhahri, S., & Omri, A. (2018). Entrepreneurship contribution to the three pillars of sustainable development: What does the evidence really say?. *World Development*, 106, 64-77.
7. Fain, T.S. (1980) Self-employed Americans: their number has increased. *Monthly Labour Review* 103, 3-8.
8. Blau, D. (1987) A time series analysis of self-employment. *Journal of Political Economy* 95, 445-467.
9. Acs, Z. J., Audretsch, D. B. and Evans, D. S. (1994). The determinants of variation in the self-employment rates across countries and over time. *Discussion paper DP871* (London: Centre for Economic Policy Research).
10. Chandler, A.D. (1990) *Scale and Scope: The Dynamics of Industrial Capitalism*, Cambridge, MA: Harvard University Press.
11. Kuznets, S. (1971) *Economic Growth of Nations, Total Output and Production Structure* (Cambridge, MA: Harvard University Press/Belknap Press).
12. Parker, S.C. (2004) *The Economics of Self-Employment and Entrepreneurship*. Cambridge University Press, Cambridge.
13. Aronson, R.L. (1991) *Self-employment: A Labour Market Perspective*. Ithaca, NY, ILR Press.
14. Lucas, R. E. (1978) On the size distribution of business firms. *The Bell Journal of Economics* 9, 508-523.
15. Acs, Z.J. (2006) How Is Entrepreneurship Good for Economic Growth? *Innovations*, 1(1), 97-107
16. Schultz, T.P. (1990) Women's changing participation in the labor force: a world perspective, *Economic Development and Cultural Change*, 38, 457-488.
17. Yamada, G. (1996) Urban informal employment and self-employment in developing countries: theory and evidence, *Economic Development and Cultural Change*, 44, 289-314.
18. Iyigun, M. F. and Owen, A. L. (1998) Risk, Entrepreneurship and Human-Capital Accumulation, *American Economic Review*, 88(2), 454-457.
19. Hamermesh, D.S. (1996) *Labour Demand*, Princeton University Press, US.
20. Bregger, J.E. (1996) Measuring Self-employment in the United States, *Monthly Labor Review*, 119, 3-9.
21. Hippel, S.F. (2004) Self-employment in the United States: an update, *Monthly Labor Review*. January/February, 24-47.
22. Hippel, S.F. (2010) Self-employment in the United States. *Monthly Labor Review*, September, 17-32.
23. Karoly L.A. and Zissmopoulos, J. (2004) Self-Employment Trends and Patterns Among Older U.S. Workers. *Monthly Labor Review*, 24-47.
24. Wennekers, S., van Stel, A., Carree, M., and Thurik A.R. (2010) The relationship between entrepreneurship and economic development: is it U-shaped. *Foundations and Trends in Entrepreneurship* 6(3), 167-237.
25. Aquilina, M., Klump, R. and Pietrobelli, C. (2006) Factor Substitution, Average Firm Size and Economic Growth. *Small Business Economics*, 26, 203-214.

400 26. Carlsson, B. (1989) The evolution of manufacturing technology and its impact on industrial structure: an
401 international study, *Small Business Economics* 1, 21-38.

402 27. Thompson, N., and Eijkemans, R. (2018). Why Do Sustainable Ventures Fail to Attract Management
403 Talent?. *Sustainability*, 10(11), 4319.

404 28. Parker, S. C. and Robson, M.T. (2004) Explaining international variations in self-employment: evidence
405 from a panel of OECD countries, *Southern Economic Journal*, 7, 287-301

406 29. Robson, M. T. and Wren, C. (1999) Marginal average tax rates and the incentive for self-employment.
407 *Southern Economic Journal*, 65, 757-773.

408 30. Schuetze, H.J. (2000) Taxes, economic conditions and recent trends in male self-employment: a Canada-US
409 comparison. *Labour Economics*, 7, 507-44.

410 31. Parker, S.C. (2003) Does tax evasion affect occupational choice?. *Oxford Economic Bulletin of Economics
411 and Statistics*, 65, 379-394.

412 32. Bruce, D. and Schuetze, H. J. (2004) Tax Policy and Entrepreneurship. *Swedish Economic Policy Review*, 2
413 (11), 233-265.

414 33. Schuetze, H.J. (2008) Tax Incentives and Entrepreneurship: Measurement and Data Considerations. In:
415 Congregado, E. (ed.), *Measuring Entrepreneurship*, New York, Springer, pp.205-225.

416 34. Botero J., Djankov S., La Porta R., Lopez-de-Silanes, R., and Shleifer A. (2004) The Regulation of Labor,
417 *Quarterly Journal of Economics* 119, 1339-82.

418 35. Grubb, D. and Wells, W. (1993) Employment regulation and patterns of work in EC countries. *OECD
419 Economic Studies* 21, 7-58.

420 36. Robson, M.T. (2003) Does stricter employment protection legislation promote self-employment? *Small
421 Business Economics*, 21, 309-319.

422 37. Kannaiainen, V. and Vesala, T. (2005) Entrepreneurship and labour market institutions. *Economic
423 Modelling* 22, 828-847.

424 38. Torrini, R., 2005. Cross-country differences in self-employment rates: the role of institutions. *Labour
425 Economics*, 12, 661-683.

426 39. van Stel, A., Storey, D. and Thurik, A. R. (2007) The Effect of Business Regulations on Nascent and Young
427 Business Entrepreneurship. *Small Business Economics*, 28(2-3), 171-186.

428 40. Nyström, K. (2008) The institutions of economic freedom and entrepreneurship: evidence from panel data.
429 *Public Choice*, 136(3), 269-282.

430 41. Roman, C., Congregado, E., Millán, J.M. (2011) Dependent self-employment as a way to evade
431 employment protection legislation. *Small Business Economics*, 37(3), 363-392.

432 42. Hart, D.M. (2003) *The Emergence of Entrepreneurship Policy*. Cambridge University Press, Cambridge,
433 UK.

434 43. Acemoglu, D. (2002) Technical Change, Inequality, and the Labor Market. *Journal of Economic Literature*,
435 40(1), 7-72.

436 44. Kejriwal, M. and Perron, P., (2010) Testing for multiple structural changes in cointegrated regression
437 models. *Journal of Business and Economic Statistics* 28, 503-522.

438 45. Arai, Y. and Kurozumi, E. (2007) Testing for the null hypothesis of cointegration with a structural break.
439 *Econometric Reviews* 26, 705-739.

440 46. Kejriwal, M. (2008) Cointegration with structural breaks: an application to the Feldstein-Horioka Puzzle.
441 *Studies in Nonlinear Dynamics & Econometrics* 12(1), 1-37.

442 47. Rees, H., and Shah A. (1986) An empirical analysis of self-employment in the U.K. *Journal of Applied
443 Econometrics*, 1(1), 95-108.

444 48. Borjas, G.J. and Bronars, S.G. (1989) Consumer Discrimination and Self-employment. *Journal of Political
445 Economy*, 97(3), 581-605.

446 49. Evans, D. and Leighton L.S. (1989) Some Empirical Aspects of Entrepreneurship. *American Economic
447 Review*, 79(3), 519-535.

448 50. Kihlstrom, R. and Laffont J. (1979) A General Equilibrium Entrepreneurial Theory of the Firm. *Journal of
449 Political Economy*, 87(4), 719-748.

450 51. Banerjee, A.V. and Newman A.F. (1993) Occupational Choice and the Process of Development. *Journal of
451 Political Economy*, 101(2), 274-298.

452 52. Katz, L. And Murphy, K. (1992) Changes in Relative Wages: Supply and Demand Factors. *Quarterly
453 Journal of Economics*, CVII, 35-78.

454 53. Autor, D., Katz, L. and Krueger, A. (1998) Computing Inequality: Have Computers Changed the Labor
455 Market?. *Quarterly Journal of Economics*, CXIII, 1169-1214.

456 54. Iversen J., Jorgensen R. and Malchow-Moller N. (2008) Defining and Measuring Entrepreneurship,
457 Foundations and Trends in Entrepreneurship 4(1), 1-63.

458 55. Ng, S. and Perron P. (2001) Lag length selection and the construction of unit root tests with good size and
459 power. *Econometrica* 69, 1529-1554.

460 56. DeJong, D.N.J., Nankervis, J.C., Savin, N.E. and Whiteman C.H. (1992) Integration versus trend
461 stationary in time series. *Econometrica* 60, 423-433.

462 57. Schwert, G.W. (1989) Tests for unit roots: A Monte Carlo investigation. *Journal of Business and Economic
463 Statistics* 7, 147-159.

464 58. Perron, P. and Ng S. (1996) Useful modifications to some unit root tests with dependent errors and their
465 local asymptotic properties. *Review of Economic Studies* 63, 435-65.

466 59. Ng, S., and Perron P. (1995) Unit Root Tests in ARMA Models with Data Dependent Methods for the
467 Selection of the Truncation Lag. *Journal of the American Statistical Association* 90, 268-81.

468 60. Kejriwal, M. and Perron, P. (2008) The limit distribution of the estimates in cointegrated regression models
469 with multiple structural changes. *Journal of Econometrics* 146, 59-73.

470 61. Bai J., Perron P. (2003) Critical Values for Multiple Structural Change Tests. *Econometrics Journal* 6, 72-78.

471 62. Stock, J. H., and Watson M.W. (1993) A simple estimator of cointegrating vectors in higher order
472 integrated systems', *Econometrica* 61, 783-820.

473 63. Shin, Y. (1994) A residual-based test of the null of cointegration against the alternative of no cointegration.
474 *Econometric Theory* 10, 91-115.

475 64. Kwiatkowski, D., Phillips, P.C.B., Schmidt, P. and Shin Y. (1992) Testing the null hypothesis of stationarity
476 against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal
477 of Econometrics* 54, 159-178.

478 65. Amorós, J.E., and Cristi, O. (2008) Longitudinal analysis of entrepreneurship and competitiveness
479 dynamics in Latin America. *International Entrepreneurship and Management Journal*, 4(4), 381-399.

480 66. Acs, Z.J., Desai, S., Hessels, J. (2008) Entrepreneurship, economic development and institutions. *Small
481 Business of Economics*, 31, 219-234.