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How Elastic Demand Affects Bidding Strategy in Electricity Market: An Auction Approach

Debin Fang [a], Qiyu Ren [a], Qian Yu [b*]

[a] Economics and Management School, Wuhan University, Wuhan 430072, China; dbfang@whu.edu.cn (D.F.); 15994215987@163.com (Q.R.)
[b] School of Economics, Wuhan University of Technology, Wuhan 430070, China
[*] correspondence: yuqian@whut.edu.cn; Tel.: +86-133-4992-6249.

Abstract: The deepening of electricity reform results in increasingly frequent auctions and the surge of generators, it becomes difficult to analyze generators' behaviors. Since it's hard to find analytical market equilibriums, approximate equilibriums were obtained instead in previous studies by market simulations, which are strict to initial estimations and simulation results are chaotic in some cases. In this paper, a multi-unit power bidding model is proposed to reveal the bidding mechanism under clearing pricing rule by employing auction approach, for which initial estimations are non-essential. Normalized bidding price is introduced to construct generator's price-related bidding strategy. Nash equilibriums are derived depend on the marginal cost and the winning probability which are computed from bidding quantity, transmission cost and demand distribution. Furthermore, we propose a comparative analysis to explore the impact of uncertain elastic demand on the performance of the electricity market. The result indicates that, there exists market power among generators leading to social welfare decreases even under competitive conditions but elastic demand is an effective way to restrain generators' market power. The feasibility of the models is verified by a case study. Our work provides decision support for generators and a direction for improving market efficiency.

Keywords: Uniform Clearing Price Auction; Electricity Market; Bidding Strategies; Asymmetric Information; Social Welfare

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1. Introduction

Following the attempts in the USA, Britain, Australia and Russia, many other countries are deepening the massive reforms of the electricity market [1]. As the significant document 'Several Opinions on Further Deepening the Reform of Electric Power System (No.9 document)' was issued in March 2015, China electric power market is gradually introducing competition and establishing a market-oriented power trading platform, then several new kinds of participants have emerged given this new market structure [2]. Consumers exhibit higher price sensitivity and market demand varies from fix demand to uncertain elastic demand with the participation of large bargaining customers [3]. For the supply side, many small and medium-sized generation companies are involved in the competition, making generation companies face fierce competitions and frequent auctions [4]. Various governments, such as China, England, Spanish and other countries' perform wholesale electricity transaction via Uniform Clearing Price (UCP) auction mechanism to balance the supply-demand and improve the economic efficiency [5-7]. In the gradually opened electricity market, it is crucial to analyze the behaviors of electricity generator effectively and seek market equilibrium price accurately.

To address this issue, many scholars focus on modeling and analyzing generators' behavior pattern in a competitive electricity market since 2000 [8-19]. Due to the difficulty to obtain the analytical market equilibrium, the approximate market equilibriums concerning generators' behaviors was sought by market simulations, such as agent-based simulation [9,10], evolutionary simulation [11], hybrid iterative simulation [12-13]. For instance, Wang et al. [11] proposes an evolutionary game approach to analyze the bidding process based on price-responsive demand. The generators update their beliefs of opponents' strategies and optimize its bid based on the updated information. By repeatedly adaptive learning, the electricity market eventually converges to the equilibrium, where no players can increase its profit by changing its strategies unilaterally. However, market simulations do not provide systematic approaches for building bidding strategies [16]. In addition, market simulations have become a complex process to seek market equilibrium in frequent large-scale auction due to the surge of participants and the increase of market uncertainty. Some experiments indicated that the equilibrium may not be found and the results directly relate to initial estimations in some cases [11, 17]. In other words, most iterative algorithms are strict to the initial estimations and the results would be chaotic if the initial estimations were improper.

To fill these gaps, auction models provide analytical rationale and explanation about how market equilibrium can be decided via strategic bidding behavior. Unlike simulation technology which emphasizes learning process, auction model solves the optimal bidding problem by considering the interaction of generators, and achieves the economic equilibrium of power market through Nash equilibrium. Auction approach avoids initial estimations and time limit of simulation technology. The existing researches related to bidding strategies by auction models, always concentrate on bidding behavior based on the assumption that demand is fix and inelastic [20-28], which does not always hold in practice, especially in the market-oriented market. For instance, Hao [20] models bidding behavior and assumes that demand, which is known to all generation companies, is fixed. The results show that those bidders would exert his market power to bid below his marginal cost to maximize the expected profits. Similarly, Li and Shahidehpour [23] proposed a novel bidding model to discuss the Nash equilibrium in electricity markets. Based on this proposed model, Banaei et al. [24] discuss wind generator's bidding strategy and Rahimiyan and Baringo [25] research ISO's scheduling problem. However, the literatures on the key role of the market demand in the bidding strategy are insufficient. Characteristics of electricity demand, such as seasonality, time-fluctuation and price-responsiveness, has received less attention, which is a key factor to strategic behaviors [13]. Compared to other commodities, the demand elasticity of electricity is low, but even a low demand elasticity can result in a noticeable difference of market performance [18].

Given all this above, this paper applies auction theory to model the generators' optimal bidding strategy based on uncertain elastic demand and explore the Nash equilibrium under clearing pricing rule, which initial estimations are non-essential. In UCP auction, all participants submit their bids to Independent System Operators (ISOs) simultaneously and independently according to demand information and expected profit. The low price participants are assigned first and Market Clearing Prices (MCPs) are the highest prices that produce demand. Due to the information asymmetry, the bidding process is a non-cooperative oligopoly game with incomplete information. Anticipated MCPs, transaction cost, cost distribution of opponents (common information) and own true marginal cost (private information), are all considered in our model. With effort to introduce transmission cost into the bidding strategy, normalized bidding price is applied innovatively. It ensures that generators providing a large quantity is more likely to win the auction when bidding prices are equal. Results show, generation companies would exert market power to bid higher than their marginal cost to maximize expected profits. The optimal bidding price is the true marginal cost plus the winning probability which are computed from bidding quantity, transmission cost and market demand distribution. Our work contributes game theoretic models to the auction theory literature and generates novel insights for generation companies seeking profits.

The intended contributions of this paper are listed as follows: (1) To propose a simple and effective auction model that provides a systematic approach for building bidding strategies under uncertain elastic demand. A unique analytical Nash equilibrium is obtained, which solves the time limit and initial estimations problem. (2) To provide a comparative analysis to assess the impact of demand elasticity on the performance of the electricity market (UCP auction VS. complete competition VS. fixed demand auction), which is rare to conventional wisdoms. (3) To disclose market power among generators under competitive conditions and prove that elasticity of market demand is an effective way to restrain generators' market power.

The rest of this paper proceeds as follows. In **Section 2**, the model is established and the optimal bidding strategies are presented. In **Section 3**, we discuss that how uncertain price-responsive demand influences the performance of the electricity market, and case studies to verify the proposed model. The conclusions are given in **Section 4**.

2. The basic model

2.1. Electricity market and MCP

In an electricity market game, due to the information asymmetry exists in the bidding process, such as opponents' marginal cost and opponents' bidding behavior, the bidding process can be described as an asymmetric information game of divisible object. We supposed a model with asymmetric generators i ($i = 1, 2, \ldots, m$), which compete to sell homogenous goods to the market. In period t, the sequence of events in classical UCP can be seem as **Table 1** [29, 30].

Table 1. The timing of the UCP auction proceeds in period t

(1)	.Auctioneer according to market operation rules require release of market information, including the demand information $D_t(p, \varepsilon_t)$ and the history bidding information of participant generators.
(2)	Each generator simultaneously and independently submits a bidding price $b_{i,t}$ at which it is willing to supply its maximum production up to quantity $q_{i,t}$.
(3)	These bids are ranked in terms of their bidding price $b_{i,t}$.
(4)	The low price generator is assigned first. If his quantity $q_{i,t}$ cannot satisfy the demand, the higher price bidder produces the residual demand. If bidders submit equal bids, then bidders split the market equally.
(5)	MCP is the highest prices that produce the demand. Generators would not participate in a bid if the generators' bidding price were higher than the MCP.

The bidding curves are composed of all generators' pair of quantity- price (**Fig. 1**), The day-ahead market demand function in period t is represented by $D_t(p, \varepsilon_t)$, which is a function of random shock ε_t and price p. It is assumed to satisfy the following standard assumptions: $D_t(p, \varepsilon_t)$ is strictly increases in ε_t, and is strictly decrease and concave in p. Demand shock ε_t is a random variable with a differentiable cumulative distribution function $\phi_t(\varepsilon_t)$ and a continuous density function $\varphi_t(\varepsilon_t)$.

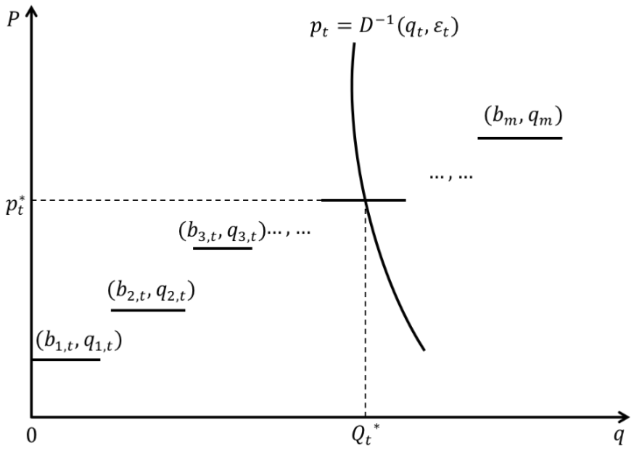

Fig.1. The Bidding Curves in an UCP Electricity Market ($b_{1,t} < b_{2,t} \ldots < b_{m,t}$)

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The generator i's truly variable marginal cost $c_{i,t}$ is considered as constant, which is a private information that only precisely known to itself and independent of each other. The generator can estimate others generators' marginal cost C by probability distribution, which can be described with density function $f(C)$ and cumulative distribution function $F(C)$. In practical, the cost can be estimated from manufactures or from regulatory and market reports [31].

Currently, besides the cost of generating electricity, wheeling cost and auxiliary service cost are also indispensable costs, which account for a large proportion of the whole electricity cost [32, 33]. This paper considers the cost of wheeling cost and auxiliary service as the transmission cost of generators. For a certain quantity electricity, transaction costs will increase as the number of transactions increases. Draws on Fang et al.'s research [32], the standardized bidding price is introduced into the mechanism to ensure that generators with large supply quantity will be more likely to win the auction when the bidding prices are equal. Supposed the transaction cost function is $\beta(q)$, where $\beta'(\cdot) > 0, \beta''(\cdot) < 0$, according to the principle of equivalent profit, the relationship between the actual bidding price $b_{i,t}$ and the virtual bidding price $b'_{i,t}$ by Eq.(1).

$$b_{i,t} = b'_{i,t} - \frac{\beta(q_{i,t}) - q_{i,t}\beta(1)}{q_{i,t}} \tag{1}$$

By normalized bidding price, we convert m generators into $N = \sum_{i=1}^{m} q_{i,t}$ virtual generators, each virtual generator bids 1 units quantity to auctioneer. So the actual bidding information for generator i $(b_{i,t}, q_{i,t})$ is converted to $q_{i,t}$ virtual generators, whose bidding information for each virtual generator is $(b'_{i,t}, 1)$. In the operation of actual electricity market, a generator does not change its production schedule $q_{i,t}$ that frequently, because changing the production quantity leads to excessive operational inefficiencies for the generator [5]. Therefore, although the generation quantity affects the bid price, the generation quantity is not a decision variable. The following work is to solve the virtual optimal bidding price $b'_{i,t}$.

2.2. Winning probabilities

When all generators participate in a bidding game, there exist three type of generators: winning on the margin, who bid the same as MCP, winning below, who bid below the MCP, or losing the game, who bid above the MCP, and the probabilities of the three outcomes are important for deriving the bidding strategy [20]. Assuming the probabilities of generator i win the game under MCP is $R_{i,t}$, the probabilities of win in the margin is $H_{i,t}$, the probabilities of lose the game is $1 - H_{i,t} - R_{i,t}$.

Supposed p_t^* represent the MCP in period t, there are $[D_t(p_t^*, \varepsilon_t)] + 1$ generators win the game, $[\cdot]$ is an integer part of a real number. If generator i win the game in the margin, the generators whose bidding price lower than $b'_{i,t}$ will be assigned first, and generator i produces the residual demand at the MCP p_t^* is equal to $\mathcal{R}_{i,t}(b'_{i,t}, \varepsilon_t) = D_t(b'_{i,t}) - [D_t(p_t^*, \varepsilon_t)]$, which is smaller than one. If generator i win the game below the MCP, he will be assigned first, produces one as his bid quantity. The probabilities of generator i win the game $H_{i,t}(B^{-1}(b'_{i,t}), p_t^*, \varepsilon_t), R_{i,t}(B^{-1}(b'_{i,t}), p_t^*, \varepsilon_t)$ are not only related to his own strategic choice of $b'_{i,t}$, but related to the market demand $D_t(p_t^*, \varepsilon_t)$ and cost distribution of participants.

$$H_{i,t}(B^{-1}(b'_{i,t}), p_t^*, \varepsilon_t) = C_{N-1}^{[D_t(p_t^*,\varepsilon_t)]} F\{C < B^{-1}(b'_i)\}^{[D_t(p_t^*,\varepsilon_t)]} F\{C > B^{-1}(b'_i)\}^{N-[D_t(p_t^*,\varepsilon_t)]-1} \tag{2}$$

$$R_{i,t}(B^{-1}(b'_{i,t}), p_t^*, \varepsilon_t) = \sum_{j=0}^{[D(p_t^*,\varepsilon_t)]-1} C_{N-1}^{j} F\{C < B^{-1}(b'_i)\}^{j} F\{C > B^{-1}(b'_i)\}^{N-1-j} \tag{3}$$

Eq. (2) shows the probability that the generator i winning in the margin, and describes the probability that there are $[D_t(p_t^*, \varepsilon_t)] - 1$ generators bidding price less than generator i. Eq. (3) describes the cumulative probability of at most $[D(p_t^*, \varepsilon_t)] - 2$ generators' bidding price less than generator i and shows the probability that generator i is winning under MCP.

In period t, given any $b'_{-i,t} = (b'_{1,t}, \dots b'_{i-1,t}, b'_{i+1,t} \dots b'_{N,t})$ of other generators, each generator i chooses its bid price $b'_{i,t}$ that achieve its ex post maximum expected profit with respect to period- t

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day-ahead demand. By doing so, generator achieves its maximum expected profit by choosing $b'_{i,t}$ that satisfies the following:

$$\pi_i\left(p_t^*\left(b'_{i,t}, b'_{-i,t}, \varepsilon_t\right); \varepsilon_t, b'_{-i,t}\right) \geq \pi_i\left(p_t^*\left(b'_t, b'_{-i,t}, \varepsilon_t\right); \varepsilon_t, b'_{-i,t}\right) \tag{4}$$

for any realization of bid price b'_t and random shock ε_t. Here, $\pi_i\left(p_t^*\left(b'_{i,t}, b'_{-i,t}, \varepsilon_t\right); \varepsilon_t, b'_{-i,t}\right)$ is generator i's expected profit at MCP $p_t^*\left(b'_{i,t}, b'_{-i,t}, \varepsilon_t\right)$. For example, given a random shock ε_t, the generator i's expected profit by choosing price $b'_{i,t}$ is higher than that by choosing any other price set b'_t, then $b'_{i,t}$ is an equilibrium. The formal definition of such an equilibrium is as follows.

Definition 1. *For any given other profile of bid price $b'_{-i,t} = (b'_{1,t}, \dots b'_{i-1,t}, b'_{i+1,t} \dots b'_{N,t})$, $b'_{i,t}$ is called 'period-t bid price equilibrium' or 'period-t equilibrium' if $b'_{i,t}$ satisfies Eq. (4) for $i = 1,2,\dots,N$.*

By Eq. (4), demand uncertainty is one of the major factors that put MCP at risk. In our analysis, we study an equilibrium in which same-cost generators have same strategies behavior. Given demand random ε_t, MCP p_t and other generators' strategies $b'_{-i,t}$, generator i's expected profit Eq. (4) in period t is equivalent to

$$\pi_i\left(p_t^*; \varepsilon_t, b'_{-i,t}\right) = E_{\varepsilon_t}[R_{i,t}(p_t^* - c_i)\cdot 1 + H_{i,t}(b'_i - c_i)\mathcal{R}_t] \tag{5}$$

2.3. The optimal bidding strategy of the Generators

In this section, we begin our analysis by characterizing generators' equilibrium strategies. Recall from Eq. (5), to maximizes Eq. (5) for any ε_t and $b'_{-i,t}$, generator i must choose a bid price $b'_{i,t}$ inducing a MCP p_t^* that maximizes expected profit Eq. (6).

$$\max_{b'_{i,t}} \pi_i\left(b'_{i,t}, \varepsilon_t\right) = \int_{\varepsilon_t} [R_{i,t}\left(B^{-1}(b'_i)\right)(p_t^* - c_i)\cdot 1 + H_{i,t}\left(B^{-1}(b'_i)\right)(b'_i - c_i)\mathcal{R}_{i,t}(p_t^*, b'_i)]q_{i,t} \tag{6}$$

Note that $D_t(p_t^*, \varepsilon_t) = \sum_{i=1}^{N}{}^{*} q_{i,t}(1|_{b'_{i,t}<p_t^*} + \mathcal{R}_{i,t}|_{b'_i=p_t^*})$, that is, the total demand is equal to the sum of all the bid quantities below the MCP. With such bid behavior, generator i achieves the maximum expected profit it would achieve in period t if it observed the random shock ε_t after its decision in period t. Then, the optimal bid price $b'_{i,t}$ that maximizes excepted profit satisfies the following first-order condition:

$$\begin{aligned}\frac{d\pi_i(b'_{i,t})}{db'_{i,t}} = E_{\varepsilon_t}[H_{i,t}\left(B^{-1}(b'_{i,t})\right)\mathcal{R}_{i,t}(b'_{i,t}) + \frac{d\mathcal{R}_{i,t}(b'_{i,t})}{db'_{i,t}}H_{i,t}\left(B^{-1}(b'_{i,t})\right)\left(b'_{i,t} - c_i\right)\\ + \left(b'_{i,t} - c_{i,t}\right)\frac{dH_{i,t}\left(B^{-1}(b'_{i,t})\right)}{dB^{-1}(b'_{i,t})}\frac{dB^{-1}(b'_{i,t})}{db'_{i,t}}\mathcal{R}_{i,t}(b'_{i,t})\\ + \left(p_t^* - c_{i,t}\right)\left(\frac{dR_{i,t}\left(B^{-1}(b'_{i,t})\right)}{dB^{-1}(b'_{i,t})}\frac{dB^{-1}(b'_{i,t})}{db'_{i,t}}\right)] = 0\end{aligned} \tag{7}$$

Note that $d\mathcal{R}_{i,t}(b'_{i,t})/db'_{i,t} < 0$ and the fact that private cost c and random shock ε are independent of each other. Applied the formula of inverse function differentiation, let $b'_{i,t} = B(c_{i,t})$ to rewrite Eq.(7) as:

$$B'(c_{i,t}) = \frac{[B(c_{i,t})-c_{i,t}]\left[\mathcal{R}_{i,t}(c_{i,t})'H_{i,t}(c_{i,t})-H_i(c_{i,t})'\mathcal{R}_{i,t}(c_{i,t})\right]-(p_t^*-c_i)R(c_{i,t})'}{H_{i,t}(c_{i,t})\mathcal{R}_{i,t}(c_{i,t})} \tag{8}$$

Generator i's optimal bidding strategy $B(c_{i,t})$ satisfied the ordinary differential equation Eq.(8) in interval $[\underline{C}, \bar{C}]$ and the boundary conditions $B(\bar{C}) = \bar{C}$, which the generators with the highest cost cannot bidding higher than the price ceiling $\bar{C}$. Therefore, Eq. (8) has a unique solution. That is to say, a generator with marginal cost $c_{i,t}$ has one and only one optimal bidding strategy $B(c_{i,t})$, which maximizes its expected profits. So we obtained Proposition 1.

Proposition 1. *There exists a unique Nash equilibrium $B_t^* = (b_{1,t}^*, b_{2,t}^*, \dots, b_{N,t}^*)$ satisfies Eq.(4) and the optimal bidding price $b_{1,t}^*$ satisfies Eq.(8).*

Integrating Eq. (8) from $c_{i,t}$ to $\bar{C}$ to yield the following:

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$$\int_{c_{i,t}}^{\bar{C}} \frac{H_{i,t}(c)\mathcal{R}_{i,t}(c)B(c)' - B(c)\mathcal{R}_{i,t}(c)'H_{i,t}(c) + B(c)H_{i,t}(c)'\mathcal{R}_{i,t}(c)}{[\mathcal{R}_{i,t}(c)]^2} = \int_{c_{i,t}}^{\bar{C}} \frac{-[\mathcal{R}_{i,t}(c)'H_{i,t}(c) - H_{i,t}(c)'\mathcal{R}_{i,t}(c)]c - (p^* - c)R_{i,t}(c)'}{[\mathcal{R}_{i,t}(c)]^2} \tag{9}$$

we know the fact that the probability of winning on or below the margin is 0 for the bidder with highest cost $\bar{C}$.

$$H_{i,t}(\bar{C}) = 0,\ R_{i,t}(\bar{C}) = 0 \tag{10}$$

Considered the boundary condition, collected Eq. (9) as canonical forms, we obtain the following formal result from Eq. (9)- (10):

$$B(c_{i,t}) = c_{i,t} + \frac{\mathcal{R}_{i,t}(c_{i,t})}{H_{i,t}(c_{i,t})}\int_{c_{i,t}}^{\bar{C}} \frac{H_{i,t}(c)}{\mathcal{R}_{i,t}(c)} + \frac{(p_t^* - c)R_{i,t}(c)'}{[\mathcal{R}_{i,t}(c)]^2} dc \tag{11}$$

Eq. (11) describes the general bidding strategy given an estimate of the expected MCP p_t^*. This result shows that a bidder's optimal bid is determined by three components: its real marginal cost of production $c_{i,t}$, make-up of the probability of winning below or on the margin $\frac{\mathcal{R}_{i,t}(c_{i,t})}{H_{i,t}(c_{i,t})}\int_{c_{i,t}}^{\bar{C}} \frac{H_{i,t}(c)}{\mathcal{R}_{i,t}(c)} dc$ and the gap between marginal cost and expected MCP $\frac{\mathcal{R}_{i,t}(c_{i,t})}{H_{i,t}(c_{i,t})}\int_{c_{i,t}}^{\bar{C}} \frac{(p_t^* - c)R_{i,t}(c)'}{[\mathcal{R}_{i,t}(c)]^2} dc$. According to Eq. (11), the optimal bid price is related the expected MCP p^*. In practical, a generator who follows this strategy exposes him to additional risk if the expected (ex-ante) winning price is very different from the actual (ex-post) winning price. In 2000, Hao [20]'s study demonstrated that this risk can be mitigated when all bidders act as if they are on the margin, that is, with $p_t{}^* = B(c_{i,t})$, the expect profit to generator i is no worse off than in the best situation in which the ex post winning price is accurately estimated.

Recall the notation ε_t from previous analysis, and to state Proposition 2, we need introduced the following notation:

$$\underline{\varepsilon_t} = \sum_{i=1}^{N} q_{i,t} - D(p_t = 0) \tag{12}$$

$\underline{\varepsilon_t}$ represents period- t day-ahead minimum demand shock that results in MCP equals 0 when each generator's bid quantity is $q_{i,t}$. Then, from Eq. (11) and Eq. (12), we identify generator i's optimal bidding price in period t:

$$B(c_{i,t}) = c_{i,t} + \int_{\underline{\varepsilon_t}}^{\infty} \frac{\mathcal{R}_{i,t}(c_{i,t},\varepsilon_t)\int_{c_{i,t}}^{\bar{C}} \frac{H_{i,t}(c,,\varepsilon_t)}{\mathcal{R}_{i,t}(c,,\varepsilon_t)} - \frac{cR_{i,t}(c,,\varepsilon_t)'}{\mathcal{R}_{i,t}(c,,\varepsilon_t)^2} dc}{H_{i,t}(c_{i,t},\varepsilon_t)[1 - \int_{c_{i,t}}^{\bar{C}} \frac{R_{i,t}(c,\varepsilon_t)'}{\mathcal{R}_{i,t}(c,\varepsilon_t)^2} dc]} d\phi(\varepsilon_t) \tag{13}$$

Proposition 2. *In any period t, generators' optimal strategies are as follows.*

(i) *For $t = 1,2, \ldots T$, each generator i commits a production schedule $S_{i,t} = (b_{i,t}, q_{i,t})$, that satisfies Eq.(1) and Eq.(13).*

(ii) *Period- $t+1$ day-ahead demand shock ε_t is realized and day-ahead MCP p_t^* is determined. Production and obtained profits in period $t+1$.*

Proposition 2 is remarkably simple and yet significant. It obtained the optimal bidding strategy of the generators at each period. Eq. (13) shows that the optimal bidding price of the generator is equal to his cost plus a make-up of winning probability that is computed from bidding quantity, transmission cost and demand distribution. In addition, unlike fixed demand auction which allocated the same quantity to the generators who win the auction, elastic demand auction has a different way to allocated quantity. Specifically, for a generator whose bidding price is equal to the MCP, the ISOs only allocated the residual demand $\mathcal{R}_{i,t}(c_{i,t}, \varepsilon_t)$ to the generator, this allocation rule gives the generators an incentive to lower his bidding price. Specifically, for the generation companies whose bidding price below the MCP, the higher the MCP is, the higher the expected profits they have. However, their bidding price will not affect the MCP. On the other hand, for generators whose bidding price is equal to the MCP, bidding price not only affects the MCP, but also affects his assigned quantity. Therefore, these generators need to find a balance between bidding price and allocated

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quantity. Therefore, compared to inelastic demand, uncertain elastic demand market has an incentive to reduce generators' bidding price (restrain the market power of generators), which can be also confirmed in the numerical examples. None of them has the motivation to deviate from the optimal strategic from Proposition 2.

3. Numerical Examples

3.1. Generators' optimal bidding strategy with uniform cost distribution

Numerical examples are presented to demonstrate applications and salient features of our results in an electricity market. For ease of exposition, hereafter, we consider a linear demand function $D(p,\varepsilon_t) = v_t - \alpha p + \varepsilon_t$ where $v_t > 0$ and $\alpha > 0$ are constants. All problem parameters introduced in this section are common knowledge to all firms. **Table 2** shows the transaction cost function $\beta(q)$, the higher trading quantity q will caused the higher transaction cost $\beta(q)$.

Table 2 Transaction Cost Information (Fang et al., 2012).

q	1	2	3	4
$\beta(q)$	0.120	0.220	0.303	0.372

Table 3 shows the results for a case where 5 generators to participate in the bidding game. The private cost of each generator is a uniform distribution of $[1, 2]$ per MWh. We random select five numbers between $[1, 2]$ to represent five generators' true cost, which is private information that only precisely known to itself. Demand shock ε_t is a random variable with uniform distribution between intervals [-1,1]. Observed **Table 3**, the higher the private true cost $c_{i,t}$, the higher the optimal bidding price $b_{i,t}^{\prime *}$, which is similar as inelastic demand. The optimal the bidding price $b_{i,t}^{\prime *}$ consist of the cost $c_{i,t}$ and probability of winning the game $(H_{i,t}, R_{i,t})$.

Table 3 Bidding Results with demand $D(p,\varepsilon_t) = 4.5 - 0.5p + \varepsilon_t$

Bidder	Private cost	Bid quantity	Optimal bidding price
1	1.1425	1	1.2329
2	1.3510	3	1.4594
3	1.5499	1	1.6151
4	1.6221	2	1.6879
5	1.8530	1	1.8763

3.2. Impact of demand scenario on Generators' optimal bidding behaviors

Demand is one of the major factors that we consider that affects strategic behaviors in electricity markets. In practice, electricity demand is price-responsive, although the demand slope α is fixed and do not changed frequently, but demand scenario v_t is seasonal and time-varying. So in this section, we study the influence of demand scenario v_t on bidding strategy. Similarly, we compute the optimal bidding strategies for each generators according Proposition 2 and the results can be seem based on different demand scenario as **Table 4** (v_t =3.5 to 6.5),

It can be seen from **Table 4** that the optimal bid price increases with the demand scenario increases. From the microeconomic view, the increase in demand scenario will lead to higher MCP and more clearing quantity. When demand scenario varies from 4.5 to 5.5, the MCP is increased from 1.37 to 1.5615 (if $\varepsilon_t = 0$), and the total clearing quantity is increased from 3.1850 to 4.7192. From a macroeconomic perspective, due to demand information released ahead of bid auction, high demand scenario has increased the expectations of generators. In other words, if the demand curve moves to right, the bidding curve will move up.

Table 4 Bidding Results with demand $D(p,\varepsilon_t) = v_t - 0.5p + \varepsilon_t$

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Bidder	Private cost	Bid quantity	Optimal bid price			
			v_t =3.5	v_t =4.5	v_t =5.5	v_t =6.5
1	1.1425	1	1.2487	1.2529	1.2572	1.2617
2	1.3510	3	1.4534	1.4594	1.4667	1.478
3	1.5499	1	1.6093	1.6151	1.6224	1.6312
4	1.6221	2	1.6825	1.6879	1.6947	1.7033
5	1.8530	1	1.8734	1.8763	1.8801	1.8854

3.3. Market power to derive electricity prices and social welfare

In this paper, the social welfare U_t in period t is defined as the sum of the generators surplus $U_{g,t}$ and the consumer surplus $U_{s,t}$. Supposed $q_{i,t}^A$ is actual assigned quantity to generator i in period t, total social welfare U_t is:

$$U_t = U_{g,t} + U_{s,t} = \sum_{i=0}^{n}(p_t^* - c_{i,t})q_{i,t}^A + \frac{(v_t-\alpha p_t^*)^2}{2} \tag{14}$$

This section compares the social welfare under three market structures: UCP auction, complete competition and fixed demand auction. Firstly, we assume a basic scenario of complete information competition. In complete competition market, there is no information asymmetry, every generator adopts cost bidding strategy, and generators surplus $U_{g0,t} = 0$. According to Eq. (14) and proposition 2, we obtained the consumer surplus under complete information competition and UCP auction (**Fig.2**). It can be seen intuitively that complete competition is a very beneficial structure for social welfare compared to UCP auction. On the one hand, complete competition brings more consumer surplus and this surplus increases with the demand scenario increase. On the other hand, generators surplus loss caused by complete competition. But this loss is a drop in the bucket compared to customer surplus increases. However, due to the characteristics such as asymmetric information, transmission constraints and oligopoly structure, electricity markets behaves more like oligopoly markets. Complete competition is not appropriate in the electricity market. But this comparison shows clearly that even under competitive conditions (UCP auction), there exists market power among generators.

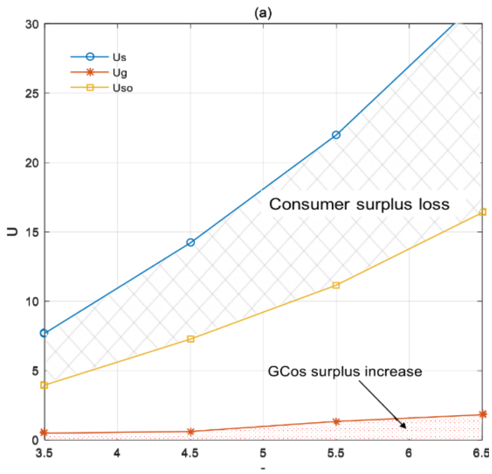

Fig.2. Demand scenario effected on social welfare： UCP auction VS complete competition.

Then, we compared our UCP auction with Hao's fixed demand auction [20]. As shown in **Fig.3**, the impact of demand on social welfare is very huge. Compared to fixed demand, there are more consumer surplus and more social welfare based on price-responsive demand, and this phenomenon is more evident as demand increases. For example, when demand scenario $v_t = 5$, generator surplus equals 0.9806 based on elastic demand and equals 3.113 based on fix demand. One the other hand, market price declines definitely increase consumer surplus. When demand scenario $v_t = 5$, customer surplus equals 18.1 based on elastic demand and equals 7.2771 based on fix demand. Therefore, elasticity of demand is an effective means to restrain the market power of generators. This conclusion

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is similar to results Ruddell K's research [26], who indicated that price-responsive demands is realized to efficiently exploit the available electricity resources.

Proposition 3*. Even under competitive conditions (UCP auction), there exists market power among generators. But price-responsive market demand is an effective way to restrain generators' market power compared to inelastic market demand*.

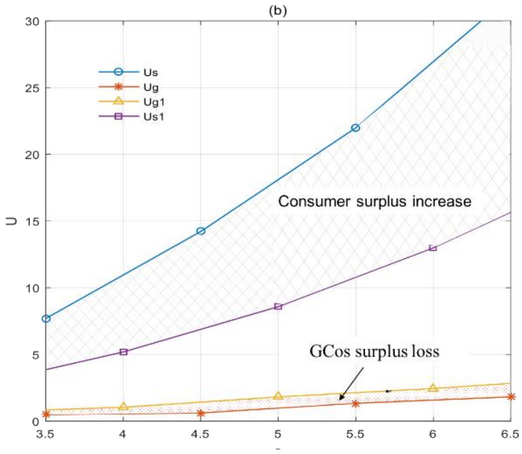

Fig.3. Demand scenario effected on social welfare: UCP auction (price-response demand) VS fixed demand.

4. Conclusions

The openness of the electricity market results in the generators facing fierce competition and frequent auctions and consumers exhibiting higher price sensitivity. However, due to the surge of generators and the increasingly frequent auctions, market simulations may be difficult to seek market equilibrium and the result would be chaos if the initial estimations were not improper. Besides, seasonality, time-fluctuation and price-responsiveness of day-ahead demand, is one of the major factors that affect strategic behaviors, but has received less attention. Given this, based on uncertain price-responsive demand, an auction model is developed to analysis asymmetric companies' bidding strategies, in which initial estimations is not necessary. We derived the unique Nash equilibrium under clearing pricing rule by introducing normalized bidding price into bidding strategy. In particular, we consider the effect of demand on the generators' bidding behavior and numerical examples are provided to show the applicability of the proposed approach.

Our results indicate that, with UCP auction, the optimal bidding price $B(c_{i,t})$ satisfied Proposition 2, which depend on the true private cost and winning probability that are computed from bidding quantity, transmission cost and demand random. The higher the true cost, the higher the optimal bidding price. Besides, this paper compares social welfare under three market structures: UCP auction, complete competition and fixed demand auction. This comparison shows clearly that even under competitive conditions (UCP auction), there exists market power among generators. In addition, we show that price-responsive market demand is effective ways to restrain generators market power than inelastic market demand. The conclusions reached coincidently to Ruddell K's research [26].

In addition, there are some restrictions in our paper. Although we obtained the optimal strategy of the generators, we assume that the cost distribution of all generators is the same to obtain an analyzable Nash equilibrium, which is a relatively strong hypothesis. This hypothesis can be extended in several directions in future work. Future work includes researching the bidding strategy under different sources of power generation and designing an effective auction mechanism to monitor market power.

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