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On the Performance of Garch Family Models in the Presence of Additive Outliers

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Abstract

It is a common practice to detect outliers in a financial time series in order to avoid the adverse effect of additive outliers. This paper investigated the performance of GARCH family models (sGARCH; gjrGARCH; iGARCH; TGARCH and NGARCH) in the presence of different sizes of outliers (small, medium and large) for different time series lengths (250, 500, 750, 1000, 1250 and 1500) using root mean square error (RMSE) and mean absolute error (MAE) to adjudge the models. In a simulation iteration of 1000 times in R environment using rugarch package, results revealed that for small size of outliers, irrespective of the length of time series, iGARCH dominated, for medium size of outliers, it was sGARCH and gjrGARCH that dominated irrespective of time series length, while for large size of outliers, irrespective of time series length, gjrGARCH dominated. The study further leveled that in the presence of additive outliers on time series analysis, both RMSE and MAE increased as the time series length increased.

Key words: Additive Outliers, Models, Simulation, Time Series length, R Software

1.0 Introduction

Response variables are not only affected by exogenous variables but also by themselves from their past behavior. On the basis of this theoretical underpinning, autoregressive models have been invented. Box and Jenkins time series modeling is indispensable in analyzing stochastic processes. Autoregressive and moving average models are used frequently by many disciplines. The autoregressive framework has very useful application in macroeconomics, such as for money supply, interest rate, price, inflation, exchange rates and gross domestic product and in financial time series analysis. The autoregressive heteroskedastic modeling framework is used in

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financial economics, such as asset pricing, portfolio selection, option pricing, and hedging and risk management (Ali, 2013). Studies abound in the financial literature about modeling the return on stocks. Usually, in the financial market, upward movements in stock prices are followed by lower volatilities, while negative movements of the same magnitude are followed by much higher volatilities (Ali, 2013).

Engle (1982) developed the time varying variance model known as autoregressive conditional heteroskedastcity (ARCH) model which was the first model to assume that the volatility is not constant. Bollerslev (1986) extended the model to include the ARMA structure as generalized autoregressive conditional heteroskedasticity (GARCH). Ali (2013) asserted that subsequently, a number of studies have adopted the autoregressive conditional heteroskedasticity (ARCH) or a generalized autoregressive conditional heteroskedasticity (GARCH) models to explain volatility of the stock market; some of these studies have also transformed and developed Engel's basic model to more sophisticated models, such as generalized autoregressive conditional heteroskedasticity (GARCH) model, integrated GARCH (IGARCH), threshold GARCH (TGARCH), exponential GARCH (EGARCH) models, GARCH-in mean (GARCH-M) and others (Atoi, 2014; Grek, 2014); however these sophisticated models, in most case, failed to make the forecast accuracy of the original ARCH model better.

1.1 Outliers

In Statistics, an outlier is an observation point that is distant from the other observations. An outlier may be due to variability in the measurement or it may indicate experimental error; the later are sometimes excluded from the data set. An outlier can cause serious problems in statistical analyses. Outliers can occur by chance in any distribution, but they often indicate either measurement error or that the population has a heavy-tailed distribution. In the former case one wishes to discard them or use statistics that are robust to outliers, while in the later case they indicate that the distribution has high skewness and that one should be very cautious in using tools or intuitions that assume a normal distribution (Wikipedia, 2017).

There are two types of outliers, namely: innovation outlier (IO), in which an outlier affects future values of the series, and additive outlier (AO), in which an outlier affects only the current observation (McQuarrie & Tsai, 2003). It should be noted that additive outliers affects forecast performance of GARCH models such that the sum of squares increases as additive outlier increases to a large number,(McQuarrie & Tsai, 2003).

This study focuses on the impact of additive outliers on performance of GARCH family models. Consequently, some GARCH models are reviewed and the impacts of additive outliers on the GARCH models are examined. Furthermore the study carried out simulation of the GARCH family models in the presence of outliers, assuming three levels of outliers (small, medium and large) at different time series length. The simulation is replicated 1,000 times for each level of outliers and at different time series length, and the performance of the GARCH models is judged using the mean absolute error (MAE) and the root mean square error (RMSE).

1.2 Justification

There is need to have appropriate forecasting models which seek to improve forecast performance in financial time series, especially when there are additive outliers, which sort of violates some assumptions of the model.

Since additive outliers affect forecast performance of GARCH models such that the sum of squares increases as additive outlier increases to a large number, this study will reveal the GARCH model(s) which are more robust in forecasting volatility when additive outliers exist.

The aim of this study is to compare the family of GARCH models when the problem of outliers exists in a financial time series.

2.0 Literature Review

2.1 The GARCH Family Models

The autoregressive conditional heteroskedasticity (ARCH) model introduced by Fredrick Engel in 1982 is the first model that assumed that volatility is not constant. ARCH models are commonly employed in modelling financial time series that exhibit time-varying volatility clustering, that is, period of swings interspersed with periods of relative calm. (Grek, 2014; Wikipedia, 2017).

Over the years the ARCH model has seen several modifications and extensions resulting in different forms of the generalized autoregressive conditional heteroskedasticity (GARCH) models. GARCH model, which is an extension of ARCH model with autoregressive moving average (ARMA) formulation, was proposed independently by Bollerslev (1986) and Tylor (1986) in order to model in a parsimonious way, and to solve some discovered disadvantages of ARCH model, this position was collaborated by Rossi (2004), Ragnarsson (2011) and Kelkay & G/Yohannes (2014).

Exponential generalized autoregressive conditional heteroskedasticity (EGARCH) model proposed by Nelson (1991) to overcome some weaknesses of the GARCH model in handling financial time series. In particular, to allow for asymmetric effects between positive and negative asset returns. The log of the conditional variance in EGARCH signifies that the leverage effect is exponential and not quadratic. And (Tsay, 2005; MªJose, 2010 and Atoi, 2014) assert that the

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transformation of volatility by its logarithm removes the restriction on the parameter to guarantee the positivity of the variance.

The Nonlinear Generalized Autoregressive Conditional Heteroskedasticity (NGARCH) Model which Higgins & Bera (1992), Hsieh & Ritchken (2005) and Duan, *et al* (2006) said is an important modification of the GARCH model as it exhibits the leverage effect, a very attractive feature of stock return data, by shifting the minimum of the news impact curve away from the origin.

Other extensions of the model include the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model proposed by Glosten, *et al* (1993) models asymmetry in the ARCH process. The model assumes a specific parametric form for the conditional heteroskedasticity present in zero mean white noise series which, although being serially uncorrelated, does not need to be serially independent. The Threshold GARCH (TGARCH) model by Zakoian (1994) is similar to GJR GARCH. It is commonly used to handle leverage effects. It allows the conditional standard deviation to depend on the sign of lagged innovation, and it does not show parameter restriction to guarantee that the conditional variance to be positive. The Integrated Generalized Autoregressive Conditional Heteroskedasticity (IGARCH) model is a restricted version of the GARCH model, where the persistent parameters sum up to one, and imports a unit root in the GARCH process. The Quadratic Generalized Autoregressive Conditional Heteroskedasticity (QGARCH) model by Sentana (1995) is also used to model asymmetric effects of positive and negative shocks. Hentschel (1995) proposed the family GARCH (fGARCH) model as an omnibus model that nests a variety of other popular symmetric and asymmetric GARCH models including APARCH, GJR, AVGARCH, NGARCH, and so on. And the Skew-Generalized Autoregressive Conditional Heteroskedasticity (SGARCH) Model was introduced by De Luca, Genton and Loperfido (2005). It is a GARCH structure that takes into account the heteroskedastic nature of financial time series. It allows for parsimonious modeling of multivariate skewness, and according to De Luca and Loperfido (2012), all its elements are either null or negative, consistently with previous empirical and theoretical findings.

3.0 Methodology

This study focuses on the GARCH models that are robust for forecasting the volatility of financial time series data in the presence of outliers; so GARCH model and some of its extensions are presented

3.1 Autoregressive Conditional Heteroskedasticity (ARCH) Family Model

Every ARCH or GARCH family model requires two distinct specifications, namely: the mean and the variance equations (Atoi, 2014). The mean equation for a conditional heteroskedasticity in a return series, y_t is given by

$$y_t = E_{t-1}(y_t) + \varepsilon_t \qquad (1)$$

where $\varepsilon_t = \phi_t \sigma_t$

The mean equation in equation (1) also applies to other GARCH family models. $E_{t-1}(.)$ is the expected value conditional on information available at time *t-1*, while ε_t is the error generated from the mean equation at time t and ϕ_t is the sequence of independent and identically distributed random variables with zero mean and unit variance.

The variance equation for an ARCH(p) model is given by

$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \ldots + \alpha_p a_{t-p}^2 \qquad (2)$$

It can be seen in the equation that large values of the innovation of asset returns have bigger impact on the conditional variance because they are squared, which means that a large shock tends to follow another large shock and that is the same way the clusters of the volatility behave. So the ARCH(p) model becomes:

$$a_t = \sigma_t \varepsilon_t, \qquad \sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + ... + \alpha_p a_{t-p}^2 \tag{3}$$

Where $\varepsilon_t \sim N(0,1)\ iid$, $\omega > 0$ and $\alpha_i \geq 0$ for $i > 0$. In practice, ε_t is assumed to follow the standard normal or a standardized student-*t* distribution or a generalized error distribution (Tsay 2005).

3.2 Asymmetric Power ARCH

According to Rossi (2004), the asymmetric power ARCH model proposed by Ding, Engel & Granger (1993) given below forms the basis for deriving the GARCH family models

Given that:

$$r = \mu + a_t,$$

$$\varepsilon_t = \sigma_t \varepsilon_t,$$

$$\varepsilon_t \sim N(0,1)$$

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i (|a_{t-i}| - \gamma_i a_{t-i})^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta, \tag{4}$$

where

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$\omega > 0$, $\delta \geq 0$,

$\alpha_i \geq 0$ $i = 1,2,\ldots, p$

$-1 < \gamma_i < 1$ $i = 1,2,\ldots, p$

$\beta_j > 0$ $j = 1,2,\ldots, q$

This model imposes a Box-Cox transformation of the conditional standard deviation process and the asymmetric absolute residuals. The leverage effect is the asymmetric response of volatility to positive and negative "shocks".

3.3 GARCH(p, q) Model:

The mathematical model for the GARCH(p,q) model is obtained from equation (4) by letting $\delta = 2$ and $\gamma_i = 0$, $i = 1,\ldots, p$ to be:

$$a_t = \sigma_t \varepsilon_t, \qquad \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{5}$$

Where $a_t = r_t - \mu_t$ (r_t is the continuously compounded log return series), and

$\varepsilon_t \sim N(0,1)$ iid, the parameter α_i is the ARCH parameter and β_j is the GARCH parameter, and $\omega > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, and $\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) < 1$, (Rossi, 2004; Tsay, 2005 and Jiang, 2012).

The restriction on ARCH and GARCH parameters (α_i, β_j) suggests that the volatility (a_i) is finite and that the conditional standard deviation (σ_i) increases. It can be observed that if q = 0, then the model GARCH parameter (β_j) becomes extinct and what is left is an ARCH(p) model.

To expatiate on the properties of GARCH models, the following representation is necessary:

Let $\eta_t = a_t^2 - \sigma_t^2$ so that $\sigma_t^2 = a_t^2 - \eta_t$. By substituting $\sigma_{t-i}^2 = a_{t-i}^2 - \eta_{t-i}$, ($i$ = 0, . . . , q) into Eq. (4), the GARCH model can be rewritten as

$$a_t = \alpha_0 + \sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) a_{t-i}^2 + \eta_t - \sum_{j=1}^{q} \beta_j \eta_{t-j}, \tag{6}$$

It can be seen that $\{\eta_t\}$ is a martingale difference series (i.e., $E(\eta_t)$ = 0 and

$\mathrm{cov}(\eta_t, \eta_{t-j}) = 0$, for $j \geq 1$). However, $\{\eta_t\}$ in general is not an iid sequence.

A GARCH model can be regarded as an application of the ARMA idea to the squared series a_t^2. Using the unconditional mean of an ARMA model, results in this

$$\mathrm{E}(a_t^2) = \frac{\alpha_0}{1 - \sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i)}$$

provided that the denominator of the prior fraction is positive. (Tsay, 2005)

When p =1 and q =1, we have GARCH(1, 1) model given by:

$$a_t = \sigma_t \varepsilon_t,$$
$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{7}$$

3.4 GJR-GARCH(p, q) Model

The Glosten-Jagannathan-Runkle GARCH (GJRGARCH) model, which is a model that attempts to address volatility clustering in an innovation process, is obtained by letting $\delta = 2$.

When $\delta = 2$ and $0 \leq \gamma_i < 1$,

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \qquad (8)$$

$$= \omega + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}|^2 + \gamma_i^2 \varepsilon_{t-1}^2 - 2\gamma_i |\varepsilon_{t-i}| \varepsilon_{t-i}) + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

$$\sigma_t^2 = \begin{cases} \omega + \sum_{i=1}^{p} \alpha_i^2 (1+\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} < 0 \\ \omega + \sum_{i=1}^{p} \alpha_i (1-\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} > 0 \end{cases}$$

i.e; $$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i (1-\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \alpha_i \{(1+\gamma_i)^2 - (1-\gamma_i)^2\} S_i^- \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i (1-\gamma_i)^2 \varepsilon_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-1}^2 + \sum_{i=1}^{p} 4\alpha_i \gamma_i S_i^- \varepsilon_{t-i}^2$$

where $$S_i^- = \begin{cases} 1 \text{ if } \varepsilon_{t-i} < 0 \\ 0 \text{ if } \varepsilon_{t-i} \geq 0 \end{cases},$$

Now define

$$\alpha_i^* = \alpha_i (1-\gamma_i)^2 \text{ and } \gamma_i^* = 4\alpha_i \gamma_i,$$

then

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$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i (1-\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-i}^2 + \sum_{i=1}^{p} \gamma_i^* S_i^- \varepsilon_{t-1}^2 \tag{9}$$

Which is the GJRGARCH model (Rossi, 2004).

But when $-1 \le \gamma_i < 0$,

Then recall Eq. (8)

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

$$= \omega + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}|^2 + \gamma_i^2 \varepsilon_{t-1}^2 - 2\gamma_i |\varepsilon_{t-i}| \varepsilon_{t-i}) + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

$$\sigma_t^2 = \begin{cases} \omega + \sum_{i=1}^{p} \alpha_i^2 (1-\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} > 0 \\ \omega + \sum_{i=1}^{p} \alpha_i (1+\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} < 0 \end{cases}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i (1+\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \{(1-\gamma_i)^2 - (1+\gamma_i)^2\} S_i^+ \varepsilon_{t-i}^2$$

$$= \omega + \sum_{i=1}^{p} \alpha_i (1+\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \{1 + \gamma_i^2 - 2\gamma_i - 1 - \gamma_i^2 - 2\gamma_i\} S_i^+ \varepsilon_{t-i}^2$$

Where

$$S_i^+ = \begin{cases} 1 \text{ if } \varepsilon_{t-i} > 0 \\ 0 \text{ if } \varepsilon_{t-i} \le 0 \end{cases}$$

also define

$\alpha_i^* = \alpha_i(1+\gamma_i)^2$ and $\gamma_i^* = -4\alpha_i\gamma_i$,

then

$$\sigma_t^2 = \omega + \sum_{i=1}^{p}\alpha_i^*\varepsilon_{t-i}^2 + \sum_{j=1}^{q}\beta_j\sigma_{t-i}^2 + \sum_{i=1}^{p}\gamma_i^* S_i^+\varepsilon_{t-1}^2 \qquad (10)$$

which allows positive shocks to have a stronger effect on volatility than negative shocks (Rossi, 2004). But when $p = q = 1$, the GJRGARCH(1,1) model will be written as

$$\sigma_t^2 = \omega + \alpha\varepsilon_t^2 + \gamma S_i\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2. \qquad (11)$$

3.5 IGARCH(1, 1) Model

The integrated GARCH (IGARCH) models are unit- root GARCH models. The IGARCH (1, 1) model is specified in Tsay (2005) and Grek (2014) as

$$a_t = \sigma_t\varepsilon_t; \qquad \sigma_t^2 = \alpha_0 + \beta_1\sigma_{t-1}^2 + (1-\beta_1)a_{t-1}^2 \qquad (12)$$

Where $\varepsilon_t \sim$ N(0, 1) iid, and $0 < \beta_1 < 1$, Ali (2013) used α_i to denote $1-\beta_i$.

The model is also an exponential smoothing model for the $\{a_t^2\}$ series. To see this, rewrite the model as

$$\sigma_t^2 = (1-\beta_1)a_{t-1}^2 + \beta_1\sigma_{t-1}^2$$

$$= (1-\beta_1)a_{t-1}^2 + \beta_1[(1-\beta)a_{t-2}^2 + \beta_1\sigma_{t-2}^2]$$

$$= (1-\beta_1)a_{t-1}^2 + (1-\beta_1)\beta_1 a_{t-2}^2 + \beta_1^2\sigma_{t-2}^2. \qquad (13)$$

By repeated substitutions, we have

$$\sigma_t^2 = (1-\beta_1)(a_{t-1}^2 + \beta_1 a_{t-2}^2 + \beta_1^2 a_{t-3}^3 + \cdots), \tag{14}$$

which is the well-known exponential smoothing formation with β_1 being the discounting factor (Tsay, 2005).

3.6 TGARCH(p, q) Model

The Threshold GARCH model is another model used to handle leverage effects, and a TGARCH(p, q) model is given by the following:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} (\alpha_i + \gamma_i N_{t=i}) a_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 , \tag{15}$$

where N_{t-i} is an indicator for negative a_{t-i}, that is,

$$N_{t-i} = \begin{cases} 1 \text{ if } a_{t-i} < 0, \\ 0 \text{ if } a_{t-i} \geq 0, \end{cases}$$

and α_i, γ_i, and β_j are nonnegative parameters satisfying conditions similar to those of GARCH models, (Tsay, 2015). When $p = 1, q = 1$, the TGARCH(1, 1) model becomes:

$$\sigma_t^2 = \omega + (\alpha + \gamma N_{t-1}) a_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{16}$$

3.7 NGARCH(p, q) Model

The Nonlinear Generalized Autoregressive Conditional Heteroskedasticity (NGARCH) Model has been presented variously in literature by the following scholars: Hsieh & Ritchken (2005), Lanne & Saikkonen (2005), Malecka (2014) and Kononovicius & Ruseckas (2015). The following model can be shown to represent all the presentations:

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \gamma_i \varepsilon_{t-i} + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{17}$$

Where h_t is the conditional variance, and ω, β and α satisfy $\omega > 0$, $\beta \geq 0$ and $\alpha \geq 0$.

Which can also be written as

$$\sigma_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \gamma_i \varepsilon_{t-i} + \sum_{j=1}^{p} \beta_j \sigma_{t-j} \tag{18}$$

3.10 SGARCH(p, q) Model

The SGARCH model can be written as:

$$Y_t = \eta_t \varepsilon_t$$

$$\eta_t^2 = \delta_0 + \sum_{i=1}^{q} \delta_i (\eta_{t-i} \varepsilon_{t-i})^2 + \sum_{j=q+1}^{q+p} \delta_j \eta_{t+q-j}^2 , \tag{19}$$

where Y_t is the leading market return at time *t*, $\{\varepsilon_t\} \sim i.i.d.\, N(0, 1)$ is the innovation (or shock) of the market, and is hypothesized to be Gaussian. δ_0 has to be positive and the remaining parameters nonnegative in order to ensure the positivity of η_t^2, (De Luca & Loperfido, 2012)

3.11 Simulation Procedure

The simulation procedure here considers the following equations of GARCH (1,1):

$$\varepsilon_t = \sigma_t z_t ,$$

$$\sigma_t^2 = a_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{20}$$

The Case simulated is the case of financial time series where there are outliers at three level, namely: small values as 0.000005, 0.00006; medium values as 10, 50 and large values as 100, 500, at the following different time series length: 250, 500, 750, 1000, 1250 and 1500

The rugarch package of the R software was used to execute the simulation.

3.12 Forecast Assessment

The following are the criteria for Forecast assessments used:

1. Mean Absolute Error (MAE) has a formula $MAE_j = \frac{\sum_{i=1}^{n} |e_i|}{n}$. This criterion measures deviation from the series in absolute terms, and measures how much the forecast is biased. This measure is one of the most common ones used for analyzing the quality of different forecasts.

2. The Root Mean Square Error (RMSE) is given as $RMSE_j = \sqrt{\frac{\sum_{i}^{n} (y_i - y^f)^2}{n}}$ where y_i is the time series data and y^f is the forecast value of y (Caraiani, 2010).

For the two measures above, the smaller the value, the better the fit of the model (Cooray, 2008)

In this simulation study, $RMSE = \frac{\sum_{j}^{N} RMSE_j}{N}$ and $MAE = \frac{\sum_{j}^{N} MAE_j}{N}$ where N=1,000, is the number of iterations or replications in the simulation study. Therefore, the model with the minimum RMSE and MAE result will be the preferred model

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4.0 Results and Discussion

4.1 Results

The results of the simulation carried out are presented in Table 1 to Table 8 below.

Table 1: The RMSE and MAE values from the fGARCH family model at different levels of outlier of 0.000005, 0.00006 at different time series lengths

Outlier	0.000005,0.00006											
Time series length (T)	250		500		750		1000		1250		1500	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	4.931399	61.202956	7.021094	123.643002	8.618935	185.975292	9.945382	247.901383	11.15621	310.73484	12.21861	372.94698
gjrGARCH	4.91953	61.17128	7.015205	123.499443	8.60339	185.62731	9.967985	248.371364	11.14907	310.66572	12.22246	372.89526
iGARCH	4.890762	60.848528	7.03993	123.91656	8.57163	184.87379	9.971203	248.298866	11.1452	310.4949	12.20729	372.77983
TGARCH	4.936581	61.365738	6.998018	123.271838	8.626176	186.112062	9.928296	247.502235	11.14536	310.59719	12.22121	372.87911
NGARCH	4.90386	60.89776	7.007123	123.314485	8.598005	185.359426	9.977334	248.561108	11.11676	309.78808	12.21001	372.83915

Table 2: The Ranks of The RMSE and MAE values from the fGARCH family model at different levels of outlier of 0.000005, 0.00006 at different time series lengths

Outliers	0.000005,0.00006											
Time series length (T)	250		500		750		1000		1250		1500	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	4	4	4	4	4	4	2	2	5	5	3	5
gjrGARCH	3	3	3	3	3	3	3	4	4	4	5	4
iGARCH	1	1	5	5	1	1	4	3	2	2	1	1
TGARCH	5	5	1	1	5	5	1	1	3	3	4	3
NGARCH	2	2	2	2	2	2	5	5	1	1	2	2

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Table 3: The RMSE and MAE values from the fGARCH family model at different levels of outlier of 10, 50 at different time series lengths

Outlier	10,50											
Time Series Length (T)	250		500		750		1000		1250		1500	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	70.80231	118.59578	70.46127	118.58901	69.83468	118.25637	70.16223	119.31764	69.81693	119.30811	70.51907	121.11153
gjrGARCH	71.69847	120.10907	70.6713	118.9951	69.96143	118.38869	68.57628	116.64949	67.85942	116.01714	69.43985	119.28377
iGARCH	72.18174	123.33328	72.60727	124.69260	72.2639	123.0646	72.10861	122.67465	72.04126	123.14981	72.10363	123.82272
TGARCH	71.9265	121.2603	72.07572	121.51990	72.07916	122.15585	71.73488	122.01710	72.03237	123.12899	72.10291	123.90638
NGARCH	71.67376	120.60028	71.60875	120.43185	70.08789	118.50482	70.44886	119.71692	69.80573	119.24967	69.94543	120.09876

Table 4: The Ranks of The RMSE and MAE values from the fGARCH family model at different levels of outlier of 10, 50 at different time series lengths

outliers	10,50											
Time series length (T)	250		500		750		1000		1250		1500	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	1	1	1	1	1	1	2	2	3	3	3	3
gjrGARCH	3	2	2	2	2	2	1	1	1	1	1	1
iGARCH	5	5	5	5	5	5	5	5	5	5	5	4
TGARCH	4	4	4	4	4	4	4	4	4	4	4	5
NGARCH	2	3	3	3	3	3	3	3	2	2	2	2

Table 5: The RMSE and MAE values from the fGARCH family model at different levels of outlier of 100, 500 at different time series lengths

OUTLIER	100,500											
	250		500		750		1000		1250		1500	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
SGARCH	723.8332	1223.1663	717.2956	1197.7994	711.0493	1185.5396	709.5818	1183.6321	708.9052	1183.6610	704.5643	1176.5473
GJRGARCH	730.0609	1276.7807	711.0275	1184.9110	704.8452	1195.3627	713.1035	1192.8623	703.415	1175.799	698.0259	1165.7791
IGARCH	767.9728	1500.6277	721.7786	1225.9165	735.1428	1377.8925	768.0416	1503.2967	726.2934	1237.5534	738.1277	1342.4379
TGARCH	718.2462	1211.7168	718.904	1210.781	720.2065	1205.1220	721.221	1227.972	720.7309	1204.4912	721.0888	1204.4408
NGARCH	871.6781	1600.8879	871.6815	1601.6008	721.7176	1241.8651	721.1639	1203.0861	721.1041	1203.5297	721.1119	1204.3342

Table 6: The Ranks of The RMSE and MAE values from the fGARCH family model at different levels of outlier of 100, 500 at different time series lengths

outliers	100, 500											
Time series length (T)	250		500		750		1000		1250		1500	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	2	2	2	2	2	1	1	1	2	2	2	2
gjrGARCH	3	3	1	1	1	2	2	2	1	1	1	1
iGARCH	4	4	4	4	5	5	5	5	5	5	5	5
TGARCH	1	1	3	3	3	3	4	4	3	4	3	4
NGARCH	5	5	5	5	4	4	3	3	4	3	4	3

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Table 7: The Performances of the fGARCH family models at different levels of outliers and at different time series lengths using RMSE

Forecast Statistics: RMSE						
Size of Outlier	Time series length (T)					
	250	500	750	1000	1250	1500
Small	iGARCH	TGARCH	iGARCH	TGARCH	NGARCH	iGARCH
Medium	sGARCH	sGARCH	sGARCH	gjrGARCH	gjrGARCH	gjrGARCH
Large	TGARCH	gjrGARCH	gjrGARCH	sGARCH	gjrGARCH	gjrGARCH

Table 8: The Performances of the fGARCH family models at different levels of outliers and at different time series lengths using MAE

Forecast Statistic: MAE						
Size of Outlier	Time series length (T)					
	250	500	750	1000	1250	1500
Small	iGARCH	TGARCH	iGARCH	TGARCH	NGARCH	iGARCH
Medium	sGARCH	sGARCH	sGARCH	gjrGARCH	gjrGARCH	gjrGARCH
Large	TGARCH	gjrGARCH	sGARCH	sGARCH	gjrGARCH	gjrGARCH

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4.2 Discussion

4.2.1 GARCH models performance in the presence of outliers using the root mean square error (RMSE) from the results of the simulation

When the additive outlier was small, iGARCH outperformed the other models at time series lengths (T) of 250, 750 and 1500, and TGARCH performed better than the other models at time series length (T) of 500 and 1000, while NGARCH performed better than the other models at time series length (T) of 1250.

But for medium level of additive outliers, it can be clearly seen that the GARCH models that dominated were sGARCH and gjrGARCH. Whereas sGARCH performed better at time series lengths (T) = 250, T = 500 and T = 750, gjrGARCH outperformed the other models at T = 1000, T = 1250 and T = 1500.

Also for the large level of outliers, gjrGARCH dominated, performing better at time series lengths (T) of 500, 750, 1250 and 1500, while TGARCH performed better at time series length (T) of 250, and sGARCH outperformed the other models at T = 1000.

4.2.2 GARCH models performance in the presence of outliers using the mean absolute error (MAE) from the results of the simulation

For the small level of additive outliers, iGARCH dominated as it outperformed the other models at time series lengths (T) of 250, 750 and 1500. TGARCH performed better than the other models at time series length (T) of 500 and 1000, while NGARCH outperformed the other models at time series length (T) of 1250.

Coming to the medium level of additive outliers, it can also be seen that sGARCH and gjrGARCH dominated. While sGARCH performed better at time series lengths (T) = 250, T = 500 and T = 750, gjrGARCH on the other hand outperformed the other models at T = 1000, T = 1250 and T = 1500.

Also for the large level of outliers, gjrGARCH dominated again, performing better at time series lengths (T) of 500, 1250 and 1500, sGARCH performed better than the other models at time series length (T) of 750 and 1000, while TGARCH outperformed the other models at T = 250.

5.1 Summary

This study has shown that different models performed better at different levels of outliers and at different time series lengths. This is in line with previous studies: Atoi (2014) modelling the volatility of stock returns using daily closing data of Nigerian Stock Exchange, found that GARCH (1,1), PGARCH (1,1,1) and EGARCH (1,1) with student's t distribution, and TGARCH with GED were the four best fitted models based on Schwarz Information Criterion, and the conclusions in Grek (2014), Chen, Min and Chen (2013), Dijk, Franses and Lucas (1999) and Demos (2000), that different models performed differently under different conditions; in this case, different levels of outliers.

5.2 Conclusion

Finding an optimal model for any time series, so that one can get good forecasting results with less prediction error, is one of the goals of analysis. To this effect, five different GARCH-Type models were simulated. The result of this study showed that with the presence of additive outliers, gjrGARCH dominated, especially for medium and large levels of outliers for large time series lengths, sGARCH dominated for lower time series lengths, irrespective of whether MAE or RMSE was used in the assessment.

5.2 Recommendations

The study therefore recommended that investors, financial analysts and researchers interested in stock prices and asset return should adapt gjrGARCH and sGARCH when outliers exist in their data.

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