Extended Auxiliary Equation Method to the perturbed Gerdjikov-Ivanov Equation

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ABSTRACT

We apply utilized the extended form of the auxiliary equation method to obtain extensively reliable exact travelling wave solutions of perturbed Gerdjikov–Ivanov equation (GIE) that is widely used as a model in the field theory of quanta and non-linear optics. The method is based on a simple first order second degree ODE. The new form of the approach gives more solutions to the governing equation efficiently.

Keywords:. Perturbed Gerdjikov–Ivanov equation; travelling wave solution; extended auxiliary equation method; complex wave solution.

1. Introduction

Optical soliton perturbation is studied in both physical and experimental phenomena modelled in to linear and non-linear PDEs such as; non-linear optics, electro-magnetics, plasma and solid state physics, water wave propagation in shallow media, fluid problems, and more. However, different approaches have been proposed over the years to handle the analytical solution of non-linear partial differential equations (NPDEs), see [1–18].

The perturbed GIE with full non-linearity is given in [13] of the form

$$iw_t + a\,w_{xx} + b\left|w^4\right| + i\,c\,w^2\,w_x^* = i(\alpha w_t + \lambda(w\left|w\right|^{2m})_x + \mu((\left|w\right|^{2m})_x w), \tag{1.1}$$

herein a, b and c are constants with real values and m denotes all non-linearity effects, α is the dispersion coefficient, μ is another dispersion term with higher effect as λ is the coefficients of short pulses that steepen themselves.

It is remarkable to mention that, quite number of methods have been set for finding the solutions of GIE. Structure in Hamiltonian forms, algebro-geometric solutions approach, tangent expansion technique, solution in Wronskian-types, transformations similar to Darboux. For more details see [19-24]. In this paper, we will propose another scheme for the exact and soliton solutions of GIE with aid of extended

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form of the auxiliary equation method (EAEM) [25].
The remaining article is designed as follows. In the next, we described methodology of the our scheme using EAEM in [25]. Then, in Section 3, some solutions are set to the GIE using the aforementioned methodology. Eventually, a brief conclusion is given in the last section.

2. Description of the method:

Suppose that we have the following PDE

$$F(u,u_t,u_x,u,\dots)=0, \tag{2.1}$$

where $u(x,t)=u$ is the dependent variable, x and t are dependent variables, F is a polynomial-like in terms of u and its derivatives.

EAEM mainly consists of three steps:

Step 1: The first step is the reduction of the governing equation by a compatible wave transformation such as

$$u(x,t)=U(\zeta),\quad \zeta=x-vt, \tag{2.2}$$

where v is arbitrary constant. This transform can be changed to some complex-valued transforms in order to the governing equation into a non-linear ODE with respect to the variable ζ of the form

$$G(U,U',U'',\dots), \tag{2.3}$$

where the prime represents classical derivative for the independent variable ζ .

Step 2. Assume that the predicted solution to Eq. (2.3) is of the form of finite series as

$$U(\zeta)=\sum_{i=0}^{N} n_i\Phi^i(\zeta) \tag{2.4}$$

in which $n_i\,(i=0,1,2,3,\cdots,N)$ are all real constants to be determined, N is a positive integer which can be determined by classical balancing procedure between the highest order derivative terms and the highest degree non-linear terms in Eq. (2.3). One should note that $\Phi(\zeta)$ satisfies the following new first order second degree auxiliary ODE

$$\left(\frac{d\Phi}{d\zeta}\right)^2=m_1\Phi^2(\zeta)+m_2\Phi^4(\zeta)+m_3\Phi^6(\zeta) \tag{2.5}$$

where m_1, m_2, m_3 are real parameters to be found.
Equation (2.5) gives the following solutions:
When $m_1>0$,

$$\Phi_1(\zeta)=\left(\frac{-m_1m_2\,\text{sech}^2(\sqrt{m_1}\zeta)}{m_2^2-m_1m_3(1+\varepsilon\tanh(\sqrt{m_1}\zeta))^2}\right)^{\frac{1}{2}}. \tag{2.6}$$

When $m_1>0$,

$$\Phi_2(\zeta)=\left(\frac{m_1m_2\,\text{csch}^2(\sqrt{m_1}\zeta)}{m_2^2-m_1m_3(1+\varepsilon\coth(\sqrt{m_1}\zeta))^2}\right)^{\frac{1}{2}}. \tag{2.7}$$

When $m_1>0,\Delta>0$,

$$\Phi_3(\zeta)=\left(\frac{2m_1}{\varepsilon\sqrt{\Delta}\cosh(2\sqrt{m_1}\zeta)-m_2}\right)^{\frac{1}{2}}. \tag{2.8}$$

When $m_1<0,\Delta>0$,

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$$\Phi_4(\zeta)=\left(\frac{2m_1}{\varepsilon\sqrt{\Delta}\cos(2\sqrt{-m_1}\zeta)-m_2}\right)^{\frac{1}{2}}. \tag{2.9}$$

When $m_1>0,\Delta<0,$

$$\Phi_5(\zeta)=\left(\frac{2m_1}{\varepsilon\sqrt{-\Delta}\sinh(2\sqrt{m_1}\zeta)-m_2}\right)^{\frac{1}{2}}. \tag{2.10}$$

When $m_1<0,\Delta>0,$

$$\Phi_6(\zeta)=\left(\frac{2m_1}{\varepsilon\sqrt{\Delta}\sin(2\sqrt{-m_1}\zeta)-m_2}\right)^{\frac{1}{2}}. \tag{2.11}$$

When $m_1>0,m_3>0,$

$$\Phi_7(\zeta)=\left(\frac{-m_1\operatorname{sech}^2(\sqrt{m_1}\zeta)}{m_2^2-2\varepsilon\sqrt{m_1m_3}\tanh(\sqrt{m_1}\zeta)}\right)^{\frac{1}{2}}. \tag{2.12}$$

When $m_1<0,m_3>0,$

$$\Phi_8(\zeta)=\left(\frac{-m_1\sec^2(\sqrt{-m_1}\zeta)}{m_2^2+2\varepsilon\sqrt{-m_1m_3}\tan(\sqrt{-m_1}\zeta)}\right)^{\frac{1}{2}}. \tag{2.13}$$

When $m_1>0,m_3>0,$

$$\Phi_9(\zeta)=\left(\frac{m_1\operatorname{csch}^2(\sqrt{m_1}\zeta)}{m_2^2+2\varepsilon\sqrt{m_1m_3}\coth(\sqrt{m_1}\zeta)}\right)^{\frac{1}{2}}. \tag{2.14}$$

When $m_1<0,m_3>0,$

$$\Phi_{10}(\zeta)=\left(\frac{-m_1\csc(\sqrt{-m_1}\zeta)}{m_2^2+2\varepsilon\sqrt{-m_1m_3}\tanh(\sqrt{-m_1}\zeta)}\right)^{\frac{1}{2}}, \tag{2.15}$$

When $m_1>0,\Delta=0,$

$$\Phi_{11}(\zeta)=\left(-\frac{m_1}{m_2}(1+\varepsilon\tanh(\frac{\sqrt{m_1}}{2}\zeta))\right)^{\frac{1}{2}}. \tag{2.16}$$

When $m_1>0,\Delta=0,$

$$\Phi_{12}(\zeta)=\left(-\frac{m_1}{m_2}(1+\varepsilon\coth(\frac{\sqrt{m_1}}{2}\zeta))\right)^{\frac{1}{2}}. \tag{2.17}$$

When $m_1>0,$

$$\Phi_{13}(\zeta)=4\left(\frac{m_1e^{2\varepsilon\sqrt{m_1}\zeta}}{(e^{2\varepsilon\sqrt{m_1}\zeta}-4m_2)^2-64m_1m_3}\right)^{\frac{1}{2}}. \tag{2.18}$$

When $m_1>0,m_2=0,$

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$$\Phi_{14}(\zeta)=4\left(\frac{\pm m_1 e^{2\varepsilon\sqrt{m_1}\zeta}}{(1-64m_1m_3e^{4\varepsilon\sqrt{m_1}\zeta}}\right)^{\frac{1}{2}}, \tag{2.19}$$

where $\Delta=m_2^2-4m_1m_3$ and $\varepsilon=\pm1$.

Step 3. Finally, the ansatz (2.4) is substituted into (2.3) and all the coefficients of all powers of $\Phi(\zeta)$ are equated to zero. The resultants set of algebraic equations are investigated to find the relations among the parameters particularly for unknowns m_1, m_2, m_3, $n_i\,(i=1,2,3,\cdots,N)$.

3. Application:

Consider the complex-valued wave transformation

$$w(x,t)=W(\xi)\exp(i\psi(x,t)), \tag{3.1}$$

with $\xi=x-vt$, $\psi(x,t)=-kx+\varpi t+\theta, W(\zeta)$ representing various features of the wave and the phase $\psi(x,t)$ of the soliton, the other constants k,θ,ϖ,v are frequency, the number of wave, the constant of wave and soliton speed, respectively. Putting (3.1) into (2.2), followed by an uncoupled system of equations derived from the real and imaginary parts of the equation.

The imaginary component reads:

$$v=-\alpha-2a_1k+c_1W^2-(2\mu m+(2m+1)\lambda)W^{2m}, \tag{3.2}$$

which anticipates the soliton speed. By setting m=1 in Eq. (3.2) and equating the coefficients of linearly independent functions to zero forces:

$$v=\alpha-2a_1k\,,c_1=2\mu+3\lambda, \tag{3.3}$$

The real component reads:

$$aW''-(\alpha k+W+a_1k^2)W-k\lambda W^{2m+1}-c_1kW^3+b_1W^5=0, \tag{3.4}$$

which furnishes soliton profile.

Moreover Eq. (3.4) is reduced to following ODE, by employing dependent variable change as $W(\xi)=\sqrt{U(\xi)}$,

$$-4U^2(\alpha k+w+a_1k^2)+4(-c_1kU^3+b_1U^4)-4k\lambda U^{2m+1} \\ +a_1(-(U')^2+2UU'')=0, \tag{3.5}$$

Taking m=1, by homogeneous balancing method we consider U'' or U'^2 with U^4 in equation (3.5), we got $N=1$ and proceeded as

$$U(\xi)=n_0+n_1\Phi(\xi) \tag{3.6}$$

With n_0 and n_1 being constant terms to be determine. Substituting Eq. (3.6) into Eq. (3.5) by the fact that Eq. (2.5) is satisfied and equating the coefficients of $\Phi^i(\xi)$ for $(i=0,1,2,\cdots,N)$ to zero we get the algebraic system of equations as follows:

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$$
\begin{aligned}
&const: \quad -4n_0^2\alpha k - 4n_0^2 ak^2 - 4ckn_0^3 - 4\lambda kn_0^3 - 4n_0^3\varpi + 4bn_0^4 = 0,\\
&\Phi(\xi): \quad -8n_0 n_1 ak^2 - 8n_0 n_1 \varpi - 8n_0 n_1 \alpha k + 16bn_0^3 n_1 - 12\lambda kn_0^2 n_1\\
&\qquad\qquad + 2an_1 n_0 m_1 - 12ckn_0^2 n_1 = 0,\\
&\Phi^2(\xi): \quad -12\lambda kn_0 n_1^2 + 24bn_0^2 n_1^2 - 4n_1^2\varpi - 4n_1^2\alpha k - 4n_1^2 ak^2\\
&\qquad\qquad + an_1^2 m_1 - 12ckn_0 n_1^2 = 0,\\
&\Phi^3(\xi): \quad -4ckn_1^3 - 4\lambda kn_1^3 + 16bn_0 n_1^3 + 4an_1 n_0 m_2 = 0,\\
&\Phi^4(\xi): \quad 4bn_1^4 + 3an_1^2 m_2 = 0,\\
&\Phi^5(\xi): \quad 6an_1 n_0 m_3 = 0,\\
&\Phi^6(\xi): \quad 5an_1^2 m_3 = 0.
\end{aligned}
$$

Solving this set of algebraic equations using *Maple*, we found our solutions to be

$$
n_0 = \frac{3}{8}\frac{k(c+\lambda)}{b}, \quad n_1 = n_1, \quad m_1 = \frac{3}{16}\frac{k^2(c+\lambda)^2}{ab}, \quad m_2 = -\frac{4}{3}\frac{bn_1^2}{a},
$$

$$
m_3 = 0, \quad \varpi = -\frac{1}{64}\frac{k(30\lambda ck + 15\lambda^2 k + 15c^2 k + 64bak + 64\alpha b)}{b}.
$$

When $ab > 0$,

$$
w_1(x,t) = \sqrt{\frac{3k(\lambda+c)}{8b}\left(1+\operatorname{sech}\left(\frac{k(\lambda+c)}{4}\sqrt{\frac{3}{ab}}(x-vt)\right)\right)}
\times \exp\left(i\left(-kx - \frac{1}{64}\frac{k(30\lambda ck + 15\lambda^2 k + 15c^2 k + 64bak + 64\alpha b)}{b}t + \theta\right)\right).
$$

$$
w_2(x,t) = \sqrt{\frac{3k(c+\lambda)}{8b}\left(1+i\,csch\left(\frac{k(c+\lambda)}{4}\sqrt{\frac{3}{ab}}(x-vt)\right)\right)}
\times \exp\left(i\left(-kx - \frac{1}{64}\frac{k(30\lambda ck + 15\lambda^2 k + 15c^2 k + 64bak + 64\alpha b)}{b}t + \theta\right)\right),
$$

$$
w_3(x,t) = \sqrt{\frac{3k(c+\lambda)}{8b}\left(1+\sqrt{\frac{2}{1+\cosh\left(\frac{k(c+\lambda)}{2}\sqrt{\frac{3}{ab}}(x-vt)\right)}}\right)}
\times \exp\left(i\left(-kx - \frac{1}{64}\frac{k(30\lambda ck + 15\lambda^2 k + 15c^2 k + 64bak + 64\alpha b)}{b}t + \theta\right)\right),
$$

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$$w_4(x,t)=\sqrt{\frac{3k(\lambda+c)}{8b}\left(1+\sqrt{\frac{2}{\left(i\sinh\left(\frac{k(\lambda+c)}{2}\sqrt{\frac{3}{ab}}(x-vt)\right)+1\right)}}\right)}$$

$$\times\exp(i(-kx-\frac{1}{64}\frac{k(30\lambda ck+15\lambda^2k+15c^2k+64bak+64\alpha b)}{b}t+\theta)),$$

$$w_5(x,t)=\sqrt{\frac{3k(c+\lambda)}{8b}+n_1k(c+\lambda)\sqrt{\frac{3e^{\frac{k(c+\lambda)}{2}\sqrt{\frac{3}{ab}}(x-vt)}}{ab\left(e^{\frac{k(c+\lambda)}{2}\sqrt{\frac{3}{ab}}(x-vt)}+\frac{16bn_1^2}{3a}\right)^2}}}$$

$$\times\exp(i(-kx-\frac{1}{64}\frac{k(30\lambda ck+15\lambda^2k+15c^2k+64bak+64\alpha b)}{b}t+\theta)),$$

When $ab<0$,

$$w_6(x,t)=\sqrt{\frac{3k(c+\lambda)}{8b}\left(1+\sqrt{\frac{2}{1+\cos\left(\frac{k(c+\lambda)}{2}\sqrt{-\frac{3}{ab}}(x-vt)\right)}}\right)}$$

$$\times\exp(i(-kx-\frac{1}{64}\frac{k(30\lambda ck+15\lambda^2k+15c^2k+64bak+64\alpha b)}{b}t+\theta)),$$

$$w_7(x,t)=\sqrt{\frac{3k(\lambda+c)}{8b}\left(1+\sqrt{\frac{2}{\left(\sin\left(\frac{k(\lambda+c)}{2}\sqrt{-\frac{3}{ab}}(x-vt)\right)+1\right)}}\right)}$$

$$\times\exp(i(-kx-\frac{1}{64}\frac{k(30\lambda ck+15\lambda^2k+15c^2k+64bak+64\alpha b)}{b}t+\theta)).$$

4. Physical Illustrations

In this section, we illustrate some particular forms of the solutions determined above for particular choices of parameters satisfying given constraints.

The real and imaginary components of the solution $w_1(x,t)$ is depicted in Fig 1a-1b for some particular choice of the parameters as $k=1, c=-1, \lambda=-1, a=1, b=1, \upsilon=1, \alpha=1, \theta=0$ in the finite domain of independent variables $x\in[-10,10]\times t\in[0,2]$. Fig 1c represent the modulus of the same solution for the same parameters and in the same domain. Both the real imaginary components represent some periodic solutions as the modulus is a single solitary wave propagating to the right along the space axis.

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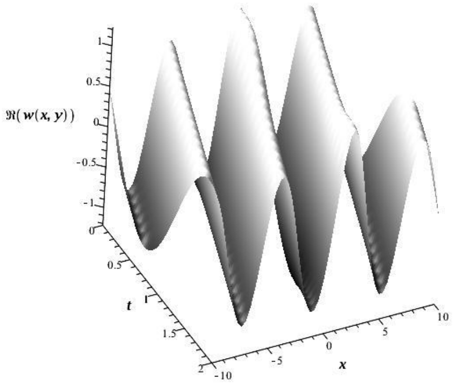
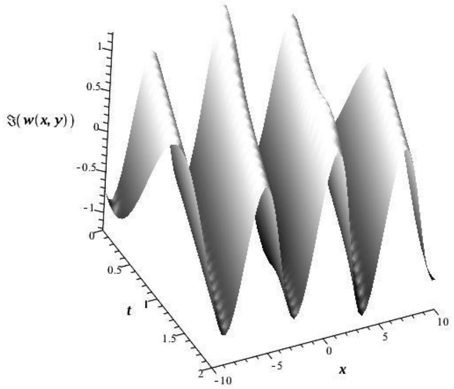
In Fig 2, we plot the solution $w_2(x,t)$ in the finite domain $x \in [-10,10] \times t \in [0,2]$ for the parameters $k = 1, c = -1, \lambda = -1, a = 1, b = 1, \upsilon = 1, \alpha = 1, \theta = 0$. The real and imaginary components of the solutions have some positive and negative explosions at some points of space and time due to their functional nature, Fig 2a-b. The modulus of $w_2(x,t)$ also indicates explosions with huge positive amplitudes time to time owing to the behaviours of the real and imaginary components of the solutions.

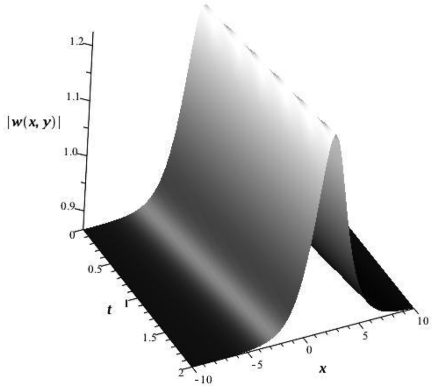
Fig 3 demonstrates the solution $w_6(x,t)$ for the particular parameters $k = 1, c = -1/2, \lambda = -1, a = -1, b = 1, v = 1, \alpha = 1, \theta = 0$. In Fig 3a-b, the real and imaginary components represent multi waves having positive or negative jumps time to time. The modulus of this solution represent periodic long waves with positive amplitudes, Fig 3c.



(a) Real component



(b) Imaginary component



(c) Modulus

Fig 1. The solution $w_1(x,t)$ for $k = 1, c = -1, \lambda = -1, a = 1, b = 1, \upsilon = 1, \alpha = 1, \theta = 0$

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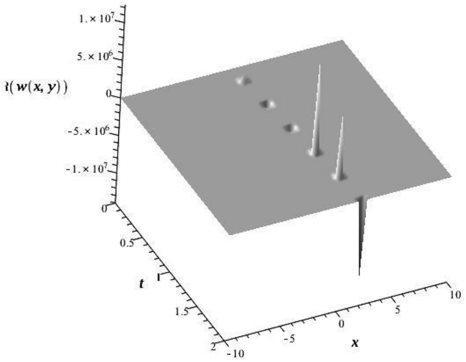
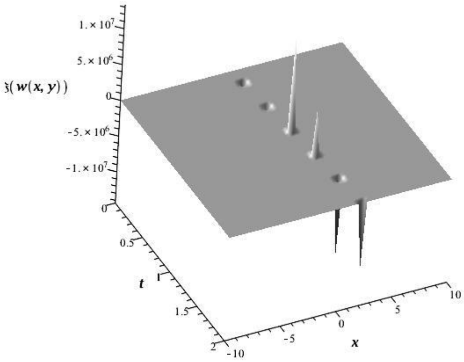
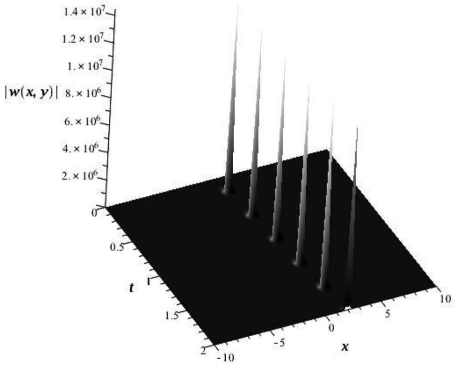


(a) Real component

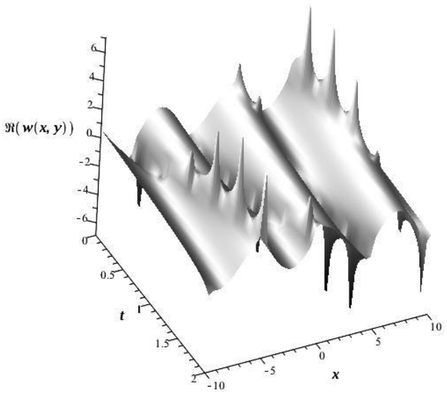


(b) Imaginary component

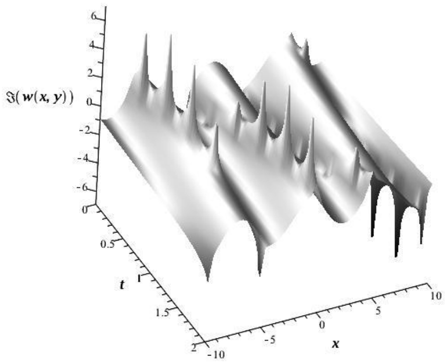


(c) Modulus

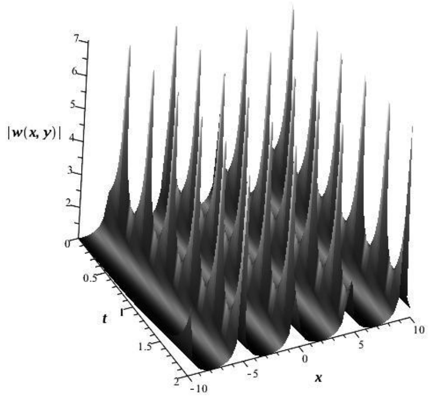
Fig 2. The solution $w_2(x,t)$ for $k = 1, c = -1, \lambda = -1, a = 1, b = 1, \upsilon = 1, \alpha = 1, \theta = 0$



(a) Real component



(b) Imaginary component



(c) Modulus

Fig 3. The solution $w_6(x,t)$ for $k = 1, c = -1/2, \lambda = -1, a = -1, b = 1, \nu = 1, \alpha = 1, \theta = 0$

5.Conclusion:

In this paper, we found exact travelling wave solutions for the non-linear perturbed GIE with the help of new EAEM. The complex valued solutions represent traveling waves in various forms. Even though some are of the form of well-known bell shaped multi waves, the shape of some others are completely different from them.

Eventually, the scheme is simple, direct, efficient and robust for solving perturbed Gerdjikov–Ivanov non-linear equation and it kind that usually arises experimental and mathematical physics.

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