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Trade Effects Based on Trade Equilibrium

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Abstract - The Rybczynski theorem describes the relationship between commodities and factor supplies, holding output price. By releasing holding commodity price, this paper introduces the trade effects of changes of factor endowments both on factor price and on commodity price. This is a study based on trade equilibrium. Technically, the study shows that change of factor endowments lead to a chain effect that Rybczynski's trade effect triggers the Stolper-Samuelson's trade effect. The analysis of this paper shows that economic activities, such as the change of factor endowments of any factor of any country reward another factor domestically and internationally. This is a tuneful circle. Trade brings a well-balanced development to the world.

Keywords:
IWE; Factor price equalization; Heckscher-Ohlin; Equilibrium price; equalized factor price;

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1. Introduction

The trade effects are very important parts of Heckscher-Ohlin theories. The Rybczynski theorem addressed the trade effect of factor endowments on output. The Stolper-Samuelson theorem addressed the trade effects of factor price on commodity price. There are fewer lectures talk about trade effects of changes of factor endowments on prices. This is due to no consolidated expression of price-trade equilibrium. Dixit-Norman (1980)'s Integrated World Equilibrium (IWE) is remarkable to characterize equalized factor price. They declared that if the allocation of factor endowment of two countries within IWE box, the factor price and commodity price will remain the same. Guo (2005) initial his study on what equalized factor price is. Recently, Guo (2018) proposed a general equilibrium of trade on the Heckscher-Ohlin 2x2x2 model. The factor price in the equilibrium is just also the factor price Dixit-Norman mentioned.

This study processes a trade effect of changes factor endowment on prices, by using price function Guo (2018) proposed. This is an analysis of trade effects based on trade equilibrium.

This paper is divided into four sections. Section 2 review the general equilibrium of trade (Guo 2018). Section 3 presents the trade effects of changes of factor endowments on factor price, commodity price, and commodity output. Section 4 is a discussion.

2. Review of equalized factor price and price-trade equilibrium

With the normal assumptions of the Heckscher-Ohlin model, Guo (2018) denoted a standard 2x2x2 Heckscher-Ohlin model as followings:

a. The production constraint of full employment of resources are

$$AX^h = V^h \qquad (h = H, F) \qquad (2\text{-}1)$$

where A is the 2x2 technology matrix, X^h is the 2x1 vector of commodities of country h, V^h is the 2x1 vector of factor endowments of country h. The elements of matrix A is $a_{ki}, k = K, F, i = 1,2$.

b. The zero-profit unit cost condition

$$A'W^* = P^* \qquad (2\text{-}2)$$

where $W^*$is the 2 x1 vector of factor prices, P^* is the 2x1 vector of commodity prices. Both P^* and W^* are world price when factor price equalization happened.

c. The definition of the country h's share of GNP to world GNP,

$$s^h = P'\,X^h / P'\,X^W \qquad (2\text{-}3)$$

d. The trade balance condition is

$$P'\,T^h = 0 \qquad (2\text{-}4)$$

or

$$W'\,F^h = 0 \qquad (2\text{-}5)$$

where T^h is the 2x1 vector of export of commodity, $F^h$is the 2x1 vector of factor content of trade.

e. The Constraint of the cone of diversification of factor endowments[2]

[2] *We assume that commodity 1 be the relatively capital intensive*.

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$$\frac{a_{K1}}{a_{L1}} > \frac{K^H}{L^H} > \frac{K^F}{L^F} > \frac{a_{K2}}{a_{L2}} \tag{2-6}$$

f. The Constraint of commodity price limits[3]

$$\frac{a_{K1}}{a_{K2}} < \frac{p_1^*}{p_2^*} < \frac{a_{L1}}{a_{L2}} \tag{2-7}$$

g. The home country's GNP share limits

$$s_b^H = \frac{K^H}{K^F+K^H}, \qquad s_a^H = \frac{L^H}{L^F+L^H} \tag{2-8}$$

where s_b^H is the upper limit of the home country GNP, s_a^H is the lower limit of GNP.

By adding the shares of GNP limits on the IWE diagram, using a simple utility function to maximize two countries distributable GNP on trade, Guo (2018) obtained the competitive share of GNP of home country as

$$s = \frac{1}{2}(s_b^H + s_a^H) = \frac{1}{2}\frac{K^H L^W + K^W L^H}{K^W L^W} \tag{2-9}$$

It is just the middle point of the GNP boundaries. He interpreted the result that the best welfares of two countries avoid the hurts of extreme trade at s_b^H *or* s_a^H as far as possible. When taking the share of GNP as s_b^H, then $w^* = 0$; and when taking share of GNP as s_r^H, then $r^* = 0$. The middle point is a good position to reward both factors fairly based on existing factor endowment supplies.

With the share of GNP of home country by equation (2-9), it obtained the general equilibrium of trade of the Heckscher-Ohlin model as

$$\frac{r^*}{w^*} = \frac{L^W}{K^W} \tag{2-10}$$

$$w^* = 1 \tag{2-11}$$

$$p_1^* = a_{k1}\frac{L^W}{K^W} + a_{L1} \tag{2-12}$$

$$p_2^* = a_{k2}\frac{L^W}{K^W} + a_{L2} \tag{2-13}$$

$$F_K^H = \frac{1}{2}\frac{K^H L^W - K^W L^H}{L^W} \tag{2-14}$$

$$F_L^H = \frac{1}{2}\frac{K^H L^W - K^W L^H}{K^W} \tag{2-15}$$

$$X^H = A^{-1}V^H \tag{2-16}$$

$$X^F = A^{-1}V^F \tag{2-17}$$

All the endogenous variables ($p_1^*, p_2^*, w^*, r^*, x_1^H, x_2^H, x_1^F, x_2^F$, and s) in the model are expressed by exogenous variables (K^h, L^h, K^w, L^w).

This is a very interesting result. It shows that Samuelson's Equalized factor price is just the Dixit-Norman's IWE factor price. Dixit and Norman (1980) illustrated that if the allocation of the factor endowments for two countries in the IWE box changes, the factor price and the commodity price will remain.

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[3] *This condition will guarantee all possible factor prices are positive. We may refer equation (5) to constraint of cone of commodity prices, which is a counterpart of the cone of diversification of factor endowments.*

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The equalized relative factor price in (2-10) is not related to technologies. The equalized factor price also illustrates that relative factor price is determined inversely by its resource ratio.

The solution for a giving IWE box is unique since there is only one trade equilibrium point in IWE diagram that made the angle of trade equal to the angle of world factor endowments in the IWE diagram.

3. Trade Effects of changes of factor endowments

The Rybczynski theorem describes the relationship between commodity output and factor supplies, holding output price. With the general equilibrium of trade at the last section, we know equalized factor price and common commodity prices are functions of factor endowments. We can process a trade effect without holding any variables no changes. The trade effects we talk here is an interactive relationship between endogenous variables and exogenous variables.

3.1 Trade Effects of changes of factor endowments on factor price

Differentiating the relative factor price in equations (2-10) with respect to world capital yields,

$$\frac{\partial r^*}{\partial K^w} = -\frac{L^w}{(K^w)^2} \tag{3-1}$$

It shows that

$$\frac{\partial r^*}{\partial K^w} < 0 \tag{3-2}$$

Differentiating equations (2-10) with respect to world capital yields,

$$\frac{\partial r^*}{\partial L^w} = \frac{1}{K^w} \tag{3-3}$$

It shows that

$$\frac{\partial r^*}{\partial L^w} > 0 \tag{3-4}$$

Similarly, we can obtain the trade effect respect to factor endowments of each country K^h, L^h, K^F, L^F as

$$\frac{\partial r^*}{\partial K^h} < 0 \qquad (h = H, F) \tag{3-5}$$

$$\frac{\partial r^*}{\partial L^h} > 0 \qquad (h = H, F) \tag{3-6}$$

r^* is relative price respect to w^* here.

3.2 Trade effect of changes of factor endowments on commodity price

We express a relative commodity price as the following, by using equation (2-12) and (2-13)

$$\frac{p_1^*}{p_2^*} = \frac{a_{1k}L^W + a_{1L}K^W}{a_{2k}L^W + a_{2L}K^W} \tag{3-7}$$

Analysis of relative commodity price will be more convenient for later presentations.

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The changing of factor endowments may have effects both on the commodity price and on factor price. So this has some effect on the technological coefficient a_{ik}. However, by the envelope theorem, any small movement of unit requirement of a factor will not violate the isoquant. We will ignore possible substitutions and elasticities in production caused by the small changes.

Differentiating the relative commodity price in equations (3-7) with respect to world capital yields,

$$\frac{\partial \frac{p_1^*}{p_2^*}}{\partial K^w} = \frac{(a_{1L}a_{2K} - a_{2L}a_{1K})L^W}{(a_{2k}L^W + a_{2L}K^W)^2} \tag{3-8}$$

By the specification of this paper, commodity 1 is the relatively capital intensive as

$$\frac{a_{1k}}{a_{1L}} > \frac{a_{2K}}{a_{2L}} \tag{3-9}$$

so

$$(a_{1L}a_{2K} - a_{2L}a_{1K)} < 0 \tag{3-10}$$

This means

$$\frac{\partial \frac{p_1^*}{p_2^*}}{\partial K^w} < 0 \tag{3-11}$$

Similarly, differentiating equations (2-23) with respect to world labor yields,

$$\frac{\partial \frac{p_1^*}{p_2^*}}{\partial L^w} = -\frac{(a_{1L}a_{2K} - a_{2L}a_{1K)}K^W}{(a_{2k}L^W + a_{2L}K^W)^2} \tag{3-12}$$

We obtain

$$\frac{\partial \frac{p_1^*}{p_2^*}}{\partial L^w} > 0 \tag{3-13}$$

We can also obtain the trade effect respect to factor endowments of each country K^h, L^h as

$$\frac{\partial \frac{p_1^*}{p_2^*}}{\partial K^h} < 0 \qquad (h = H, F) \tag{3-14}$$

$$\frac{\partial \frac{p_1^*}{p_2^*}}{\partial L^h} > 0 \qquad (h = H, F) \tag{3-15}$$

3.3 trade effect of changes of factor endowments on domestic outputs

The changes of factor endowments only effect on domestic commodities. This kind of effect just follows the Rybczynski theorem; we will not add it anymore.

3.4 Tarde effects of factor endowments

The effect of changes of factor endowments is a very comprehensive effect. We summarize above analyses as following (the Generalized Rybczynski theorem).

If both commodities continuous to be produced and continues to be traded, a small increase in the supply of a factor will cause:

(a) an increase in the output of the commodity using its factor intensively and a decrease in the output of the other commodity;

(b) a decrease of the relative price of the commodity intensively using this factor and an increase of the relative price of the commodity intensively using another factor.

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(c) a decrease of relative price of this factor and an increase of the relative price of another factor.

Statement (a) above actually is the trade effects of the Rybczynski theorem.

Statement (c) actually is the trade effect of the Stolper-Samuelson theorem, which is caused by (a).

Statement (b) engages statement (a) and (b) together. The output changes, sourced from changes of factor endowments, lead to commodity price change, so the Stolper-Samuelson trade effect happens.

The trade effects here describe the Rybczynski theorem and the Stolper-Samuelson theorem work together jointly. It is a scenario that Rybczynski trade effect triggers the Stolper-Samuelson trade effect. However, a single effect of either the Rybczynski theorem or the Stolper-Samuelson is very different from the joint effect.

Market responses the (small) increase of any factor endowment in a country by identifying and signaling which factor is scarce worldwide to drive trade participants to realize the maximum productions and consumptions possible in the future.

A small increase of any factor endowment in the home country causes the relative more increase of output of commodity intensively used this factor. Its price should fall to increase competition in the world market to help to realize its sales in the market. Another factor in the country then is a relatively scarce domestically and internationally. The higher relative return of that factor causes the future increases of its supply.

Given a set of exogenous factor endowments L^F , L^H, K^H, and L^H, the commodity price under factor price equalization is best prices for the commodity using intensively with its abundant factor for each country. No way to improve its reward domestically (Such as an increase of any factor, except economic decrease). Meanwhile, it reserved domestic pressure to increase reward another factor. This is a new result from trade equilibrium. It is much different from the analysis from autarky to free trade.

A movement to free trade causes bi-nation economy networked together to share consumptions. The development of a country will cause its counterpart producing more quantities of the commodity using intensively another factor. The relatively scarce factor identified by market worldwide will grow fast due to the rise of its real return.

3.5 The Magnification effect

Jone (1965) proposed a very useful tool, the magnification effect, which can illustrate trade effects more clearly. We use his method to display the trade effect above by the following:

$$\Delta K^W \uparrow: \quad \hat{w}^* \uparrow > 0 > \hat{p}_1^* \downarrow > \hat{r}^* \downarrow$$
$$\Delta L^W \uparrow: \quad \hat{w}^* \downarrow > 0 > \hat{p}_1^* \uparrow > \hat{r}^* \uparrow$$

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$$\begin{array}{lll} \Delta K^H \uparrow: & \hat{x}_1^H \uparrow > \hat{K}^H \uparrow > 0 > \hat{x}_2^H \downarrow & \rightarrow \quad \hat{w}^* \uparrow > 0 > \hat{p}_1^* \downarrow > \hat{r}^* \downarrow \\ \Delta L^H \uparrow: & \hat{x}_1^H \downarrow < 0 < \hat{L}^H \uparrow < \hat{x}_2^H \uparrow & \rightarrow \quad \hat{w}^* \downarrow > 0 > \hat{p}_1^* \uparrow > \hat{r}^* \uparrow \end{array}$$

or

$$\begin{array}{lll} \Delta K^F \uparrow: & \hat{x}_1^F \uparrow > \hat{K}^F \uparrow > 0 > \hat{x}_2^F \downarrow & \rightarrow \quad \hat{w}^* \uparrow > 0 > \hat{p}_1^* \downarrow > \hat{r}^* \downarrow \\ \Delta L^F \uparrow: & \hat{x}_1^F \downarrow < 0 < \hat{L}^F \uparrow < \hat{x}_2^F \uparrow & \rightarrow \quad \hat{w}^* \downarrow > 0 > \hat{p}_1^* \uparrow > \hat{r}^* \uparrow \end{array}$$

Beware that we assumed that commodity 1 is capital-intensive (the first good intensive in the first factor), i.e. $K^H/K^F > L^H/L^F$ and $a_{1K}/a_{2K} > a_{1L}/a_{2L}$.
We refer it to comprehensive magnifications of the trade effects

3.6 A Numerical examples

We present a numerical example here to describe the trade equilibrium and trade effects proposed.

Consider two countries, home and foreign, two commodities, 1 and 2, two factors, capital, and labor. The technological matrix is

$$\begin{bmatrix} a_{1K} & a_{2K} \\ a_{1L} & a_{2L} \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

The factor endowments in two countries are

$$\begin{bmatrix} K^H \\ L^H \end{bmatrix} = \begin{bmatrix} 2400 \\ 1700 \end{bmatrix}, \qquad \begin{bmatrix} K^F \\ L^F \end{bmatrix} = \begin{bmatrix} 1800 \\ 2300 \end{bmatrix}$$

The outputs of two countries by full employment are separate as

$$\begin{bmatrix} x_1^H \\ x_2^H \end{bmatrix} = \begin{bmatrix} 620 \\ 540 \end{bmatrix}, \qquad \begin{bmatrix} x_1^F \\ x_2^F \end{bmatrix} = \begin{bmatrix} 260 \\ 1020 \end{bmatrix}$$

Commodity 1 is K-intensive and commodity 2 is L-intensive. The factor abundant ranking is that home country is capital abundant and foreign country is labor abundant. The trade direction is that home country exports commodity 1 and foreign country exports commodity 2.
The share of GNP of the home country is calculated as 0.4982, based on factor endowments across countries.
The consumption, export, and prices, under free trade, reach the following equilibrium:

$$\begin{bmatrix} c_1^H \\ c_2^H \end{bmatrix} = \begin{bmatrix} 438.42 \\ 777.21 \end{bmatrix}, \qquad \begin{bmatrix} c_1^F \\ c_2^F \end{bmatrix} = \begin{bmatrix} 441.57 \\ 782.78 \end{bmatrix}, \qquad \begin{bmatrix} T_1^H \\ T_2^H \end{bmatrix} = \begin{bmatrix} 181.57 \\ -237.21 \end{bmatrix}$$

$$\begin{bmatrix} p_1^* \\ p_2^* \end{bmatrix} = \begin{bmatrix} 3.8571 \\ 2.9523 \end{bmatrix}, \begin{bmatrix} r^* \\ w^* \end{bmatrix} = \begin{bmatrix} 0.9523 \\ 1.0000 \end{bmatrix}$$

We now analyze trade effects. When the capital endowment in the home country increases 0.5 percent now, the prices and commodity output will be

$$\begin{bmatrix} K^H \\ L^H \end{bmatrix} = \begin{bmatrix} 2412 \uparrow \\ 1700 \end{bmatrix}, \quad \begin{bmatrix} x_1^H \\ x_2^H \end{bmatrix} = \begin{bmatrix} 620.80 \uparrow \\ 539.60 \downarrow \end{bmatrix}, \quad \begin{bmatrix} p_1^* \\ p_2^* \end{bmatrix} = \begin{bmatrix} 3.8557 \downarrow \\ 2.9519 \downarrow \end{bmatrix}, \begin{bmatrix} r^* \\ w^* \end{bmatrix} = \begin{bmatrix} 0.9519 \downarrow \\ 1.0000 \uparrow \end{bmatrix},$$

When the labor endowment in the foreign country increases 0.5 percent, the prices and output will be

$$\begin{bmatrix} K^F \\ L^F \end{bmatrix} = \begin{bmatrix} 1800.00 \\ 2302.00 \uparrow \end{bmatrix}, \quad \begin{bmatrix} x_1^F \\ x_2^H \end{bmatrix} = \begin{bmatrix} 296.0 \downarrow \\ 1021.90 \uparrow \end{bmatrix}, \quad \begin{bmatrix} p_1^* \\ p_2^* \end{bmatrix} = \begin{bmatrix} 3.8585 \uparrow \\ 2.9528 \uparrow \end{bmatrix}, \begin{bmatrix} r^* \\ w^* \end{bmatrix} = \begin{bmatrix} 0.9528 \uparrow \\ 1.0000 \downarrow \end{bmatrix},$$

4. Discussion

The trade effects of this paper can explain some international trade practice. For instance of China economic development, with the interaction with foreign counties, its labor payments increase dramatically those years. The international market incentives its sectors

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using labor intensively. This is its sector with comparative advantage. Its import and exports return world with as a new regional driving force to affect other countries. That will make incentives to other countries' sectors with comparative advantages. A realistic characteristic of the world is that countries conduct international trade, accompanying economic growth. We see the evidence of the development of industries countries benefit the under-developed countries' economies.

Another expression for Stolper-Samuelson theorem is that free international trade benefits the abundant factor and harms the scarce factor. It is about the process from autarky to free trade. It is not true from the view of the above derivation. The analysis of this paper says that economic activities after trade, such as the change of factor endowments after trade, benefits another factor, which is then a relatively scarce factor both domestically and internationally by the market. The trade effects from autarky to tree trade are much different from the trade effects after trade. The trade effects discussed in this paper is for the situations of a continuing trade, under trade equilibrium.

We wish that the studies of this paper could help the Heckscher-Ohlin model stepping in explanations of the dynamics of international trade and economic growth more effectively.

Conclusion

We proposed new trade effects caused by factor endowment changes based on price-trade equilibrium. They are comprehensive trade effects, which engaged the Stolper-Samuelson effect and the Rybczynski effect together. The important conclusion of the trade effects is that any development or changes of a factor endowment of a country, always incentives to another factor both domestically and internationally. Another factor will retune back some incentive to that factor. This is a tuneful circle. Trade brings a well-balanced development to the world.

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