

THE EFFECT OF HIGH POSITIVE AUTOCORRELATION ON THE PERFORMANCE OF GARCH FAMILY MODELS

Emenogu, G. Ngozi,^{1*} and Adenomon, M. O.^{2*}

1. Department of Statistics, Federal Polytechnic, Bida, Niger State, Nigeria

2. Department of Statistics, Nasarawa State University, Keffi, Nasarawa State, Nigeria

* Corresponding Authors: ngelive4jesus@gmail.com; admonsagie@gmail.com
+2348036759112

Abstract

This study compared the performance of five Family Generalized Auto-Regressive Conditional Heteroscedastic (fGARCH) models (sGARCH, gjrGARCH, iGARCH, TGARCH and NGARCH) in the presence of high positive autocorrelation. To achieve this, financial time series was simulated with autocorrelated coefficients as $\rho = (0.8, 0.85, 0.9, 0.95, 0.99)$, at different time series lengths (as 250, 500, 750, 1000, 1250, 1500) and each trial was repeated 1000 times carried out in R environment using rugarch package. And the performance of the preferred model was judged using Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE). Results from the simulation revealed that these GARCH models performances varies with the different autocorrelation values and at different time series lengths. But in the overall, NGARCH model dominates with 62.5% and 59.3% using RMSE and MAE respectively. We therefore recommended that investors, financial analysts and researchers interested in stock prices and asset return should adapt NGARCH model when there is high positive autocorrelation in the financial time series data.

Keywords: Financial Time Series, Autocorrelation, Models, GARCH, RMSE, MAE.

1.0 Introduction

Financial time series analysis is mainly concerned with the theory and application of asset valuation over time, because of this financial time series analyses are very useful to financial analysts and portfolio managers (Tsay, 2005). Financial time series contains uncertainty, volatility, excess kurtosis, high standard deviation, high skewness and sometimes non normality (Pedroni (2001); Grigoletto & Lisi (2009)).

Volatility is a very important element of financial time series since its introduction by Engle (1982). Models such as Auto-Regressive Conditional Heteroscedastic (ARCH), Generalized Auto-Regressive Conditional Heteroscedastic (GARCH), multivariate GARCH, Stochastic volatility (SV) and various variants of the models have been proposed to handle volatility in financial time series (Lawrance, 2003). In fact the ARCH and GARCH models are now so widely used and they are referred to as the “workhorse of the financial industry” (Lee & Hansen (1994); Lange (2011)).

Many economic time series including financial time series are strongly autocorrelated and can be modeled by linear (near) unit root or $I(1)$ processes such as Threshold Autoregressive (TAR) model (Lanne & Saikkonen, 2002). Autocorrelation leads to serious underestimation of standard error for regression coefficients and makes prediction intervals to be excessively wide and as such the presence of autocorrelation renders inference and decision making about the estimated parameters invalid (Adenomon, et al., 2015). This means that financial time series and its possible modeling techniques such as ARCH and GARCH models may not also be immune against the effects of autocorrelation.

The aim of this study is to compare performance of some family GARCH models in the presence of first order autocorrelation.

2.0 Brief Review of Literature of Modelling Financial Time Series with Autocorrelation

Autocorrelation can be defined as correlation between members of series of observations ordered in time (as in time series data) or space (as in cross-section data) (Adenomon & Micheal, 2017). Gujarati (2003) identified the following as causes of autocorrelation in time series data: inertia, specification Bias, excluded variables, incorrect functional form, cobweb phenomenon, lags, data transformation and manipulation, and non stationarity.

Lanne and Saikkonen (2002) investigated economic time series that are strongly autocorrelated using Threshold Autoregression because TAR model can accommodate time series that are strongly autocorrelated or $I(1)$ time series processes.

Xiao et al. (2003) proposed a modification of local polynomial time series regression estimators that improves efficiency when the innovation process is autocorrelated which was based on pre-whitening transformation of the dependent variable that must be estimated from the time series data. Their proposed method and method was more efficient than the conventional local polynomial method. Their study is similar to the work of Su and Ullah (2006).

Lee and Lund (2004) investigated the properties of ordinary and generalized least squares in a simple linear regression with stationary autocorrelated errors. They derived variances of the parameter estimators for some time series autocorrelation structures which include a first order autocorrelation and general moving averages.

Lanne and Saikkonen (2005) investigated and proposed non-linear GARCH models for highly persistent volatility. They observed that conventional GARCH are inflexible to

simultaneously first order autocorrelation of squares, persistence of shocks to volatility and excess kurtosis prevalent in financial return series.

Lawrance (2013) examined volatility in financial time series using exploratory graphs and observed that volatility can be confused with the effect of negative autocorrelation and can be distorted by positive autocorrelation. Lawrence concluded that volatility can only be visualized and analyzed for linearly uncorrelated or decorrelated series.

3.0 Model Specification

This study focuses on the GARCH models that are robust for forecasting the volatility of financial time series data in the presence of high positive autocorrelation; so GARCH model and some of its extensions are presented in this section

3.1 Autoregressive Conditional Heteroskedasticity (ARCH) Family Model

Every ARCH or GARCH family model requires two distinct specifications, namely: the mean and the variance equations (Atoi, 2014). The mean equation for a conditional heteroskedasticity in a return series, y_t is given by

$$y_t = E_{t-1}(y_t) + \varepsilon_t \quad (1)$$

where

$$\varepsilon_t = \phi_t \sigma_t$$

The mean equation in equation (1) also applies to other GARCH family models. $E_{t-1}(\cdot)$ is the expected value conditional on information available at time $t-1$, while ε_t is the error generated from the mean equation at time t and ϕ_t is the sequence of independent and identically distributed random variables with zero mean and unit variance.

The variance equation for an ARCH(p) model is given by

$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \dots + \alpha_p a_{t-p}^2 \quad (2)$$

It can be seen in the equation that large values of the innovation of asset returns have bigger impact on the conditional variance because they are squared, which means that a large shock tends to follow another large shock and that is the same way the clusters of the volatility behave.

So the ARCH(p) model becomes:

$$a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \dots + \alpha_p a_{t-p}^2 \quad (3)$$

Where $\varepsilon_t \sim N(0,1)$ iid, $\omega > 0$ and $\alpha_i \geq 0$ for $i > 0$. In practice, ε_t is assumed to follow the standard normal or a standardized student- t distribution or a generalized error distribution (Tsay 2005).

3.2 Asymmetric Power ARCH

According to Rossi (2004), the asymmetric power ARCH model proposed by Ding, Engle & Granger (1993) given below forms the basis for deriving the GARCH family models

Given that:

$$\begin{aligned} r &= \mu + a_t, \\ \varepsilon_t &= \sigma_t \varepsilon_t, \\ \varepsilon_t &\sim N(0,1) \\ \sigma_t^\delta &= \omega + \sum_{i=1}^p \alpha_i (|a_{t-i}| - \gamma_i a_{t-i})^\delta + \sum_{j=1}^q \beta_j \sigma_{t-j}^\delta, \end{aligned} \quad (4)$$

where

$$\begin{aligned} \omega &> 0, & \delta &\geq 0, \\ \alpha_i &\geq 0 & i &= 1, 2, \dots, p \\ -1 < \gamma_i < 1 & & i &= 1, 2, \dots, p \\ \beta_j &> 0 & j &= 1, 2, \dots, q \end{aligned}$$

This model imposes a Box-Cox transformation of the conditional standard deviation process and the asymmetric absolute residuals. The leverage effect is the asymmetric response of volatility to positive and negative “shocks”.

3.3 GARCH(p, q) Model:

The mathematical model for the GARCH(p,q) model is obtained from equation (4) by letting $\delta = 2$ and $\gamma_i = 0$, $i = 1, \dots, p$ to be:

$$a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i a_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \quad (5)$$

Where $a_t = r_t - \mu_t$ (r_t is the continuously compounded log return series), and

$\varepsilon_t \sim N(0,1)$ iid, the parameter α_i is the ARCH parameter and β_j is the GARCH parameter, and

$\omega > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, and $\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) < 1$, (Rossi, 2004; Tsay, 2005 and Jiang, 2012).

The restriction on ARCH and GARCH parameters (α_i, β_j) suggests that the volatility (a_i) is finite and that the conditional standard deviation (σ_i) increases. It can be observed that if $q = 0$, then the model GARCH parameter (β_j) becomes extinct and what is left is an ARCH(p) model.

To expatiate on the properties of GARCH models, the following representation is necessary:

Let $\eta_t = a_t^2 - \sigma_t^2$ so that $\sigma_t^2 = a_t^2 - \eta_t$. By substituting $\sigma_{t-i}^2 = a_{t-i}^2 - \eta_{t-i}$, $(i = 0, \dots, q)$ into Eq. (4), the GARCH model can be rewritten as

$$a_t = \alpha_0 + \sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) a_{t-i}^2 + \eta_t - \sum_{j=1}^q \beta_j \eta_{t-j}, \quad (6)$$

It can be seen that $\{\eta_t\}$ is a martingale difference series (i.e., $E(\eta_t) = 0$ and $\text{cov}(\eta_t, \eta_{t-j}) = 0$, for $j \geq 1$). However, $\{\eta_t\}$ in general is not an iid sequence.

A GARCH model can be regarded as an application of the ARMA idea to the squared series a_t^2 .

Using the unconditional mean of an ARMA model, results in this

$$E(a_t^2) = \frac{\alpha_0}{1 - \sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i)}$$

provided that the denominator of the prior fraction is positive. (Tsay, 2005)

When $p=1$ and $q=1$, we have GARCH(1, 1) model given by:

$$\begin{aligned} a_t &= \sigma_t \varepsilon_t, \\ \sigma_t^2 &= \omega + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \end{aligned} \quad (7)$$

3.4 GJR-GARCH(p, q) Model

The Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model, which is a model that attempts to address volatility clustering in an innovation process, is obtained by letting $\delta = 2$.

When $\delta = 2$ and $0 \leq \gamma_i < 1$,

$$\begin{aligned} \sigma_t^2 &= \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \\ &= \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}|^2 + \gamma_i^2 \varepsilon_{t-i}^2 - 2\gamma_i |\varepsilon_{t-i}| \varepsilon_{t-i}) + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \end{aligned} \quad (8)$$

$$\sigma_t^2 = \begin{cases} \omega + \sum_{i=1}^p \alpha_i^2 (1 + \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} < 0 \\ \omega + \sum_{i=1}^p \alpha_i (1 - \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} > 0 \end{cases}$$

i.e;
$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i (1 - \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{i=1}^p \alpha_i \{(1 + \gamma_i)^2 - (1 - \gamma_i)^2\} S_i^- \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2$$

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i (1 - \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p 4\alpha_i \gamma_i S_i^- \varepsilon_{t-i}^2$$

where
$$S_i^- = \begin{cases} 1 & \text{if } \varepsilon_{t-i} < 0 \\ 0 & \text{if } \varepsilon_{t-i} \geq 0 \end{cases}$$

Now define

$$\alpha_i^* = \alpha_i (1 - \gamma_i)^2 \text{ and } \gamma_i^* = 4\alpha_i \gamma_i,$$

then

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i (1 - \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p \gamma_i^* S_i^- \varepsilon_{t-i}^2 \quad (9)$$

Which is the GJR-GARCH model (Rossi, 2004).

But when $-1 \leq \gamma_i < 0$,

Then recall Eq. (8)

$$\begin{aligned} \sigma_t^2 &= \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \\ &= \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}|^2 + \gamma_i^2 \varepsilon_{t-i}^2 - 2\gamma_i |\varepsilon_{t-i}| \varepsilon_{t-i}) + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \end{aligned}$$

$$\sigma_t^2 = \begin{cases} \omega + \sum_{i=1}^p \alpha_i (1 - \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} > 0 \\ \omega + \sum_{i=1}^p \alpha_i (1 + \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} < 0 \end{cases}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i (1 + \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p \alpha_i \{(1 - \gamma_i)^2 - (1 + \gamma_i)^2\} S_i^+ \varepsilon_{t-i}^2$$

$$= \omega + \sum_{i=1}^p \alpha_i (1 + \gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p \alpha_i \{1 + \gamma_i^2 - 2\gamma_i - 1 - \gamma_i^2 - 2\gamma_i\} S_i^+ \varepsilon_{t-i}^2$$

Where

$$S_i^+ = \begin{cases} 1 & \text{if } \varepsilon_{t-i} > 0 \\ 0 & \text{if } \varepsilon_{t-i} \leq 0 \end{cases}$$

also define

$$\alpha_i^* = \alpha_i(1 + \gamma_i)^2 \text{ and } \gamma_i^* = -4\alpha_i\gamma_i,$$

then

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i^* \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-i}^2 + \sum_{i=1}^p \gamma_i^* S_i^+ \varepsilon_{t-i}^2 \quad (10)$$

which allows positive shocks to have a stronger effect on volatility than negative shocks (Rossi, 2004). But when $p = q = 1$, the GJR-GARCH(1,1) model will be written as

$$\sigma_t^2 = \omega + \alpha \varepsilon_t^2 + \gamma \mathcal{S}_t \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2. \quad (11)$$

3.5 IGARCH(1, 1) Model

The integrated GARCH (IGARCH) models are unit-root GARCH models. The IGARCH (1, 1) model is specified in Tsay (2005) and Grek (2014) as

$$a_t = \sigma_t \varepsilon_t; \quad \sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + (1 - \beta_1) a_{t-1}^2 \quad (12)$$

Where $\varepsilon_t \sim N(0, 1)$ iid, and $0 < \beta_1 < 1$, Ali (2013) used α_i to denote $1 - \beta_i$.

The model is also an exponential smoothing model for the $\{a_t^2\}$ series. To see this, rewrite the model as

$$\begin{aligned} \sigma_t^2 &= (1 - \beta_1) a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \\ &= (1 - \beta_1) a_{t-1}^2 + \beta_1 [(1 - \beta_1) a_{t-2}^2 + \beta_1 \sigma_{t-2}^2] \\ &= (1 - \beta_1) a_{t-1}^2 + (1 - \beta_1) \beta_1 a_{t-2}^2 + \beta_1^2 \sigma_{t-2}^2. \end{aligned} \quad (13)$$

By repeated substitutions, we have

$$\sigma_t^2 = (1 - \beta_1)(a_{t-1}^2 + \beta_1 a_{t-2}^2 + \beta_1^2 a_{t-3}^2 + \dots), \quad (14)$$

which is the well-known exponential smoothing formation with β_1 being the discounting factor (Tsay, 2005).

3.6 TGARCH(p, q) Model

The Threshold GARCH model is another model used to handle leverage effects, and a TGARCH(p, q) model is given by the following:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p (\alpha_i + \gamma_i N_{t-i}) a_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2, \quad (15)$$

where N_{t-i} is an indicator for negative a_{t-i} , that is,

$$N_{t-i} = \begin{cases} 1 & \text{if } a_{t-i} < 0, \\ 0 & \text{if } a_{t-i} \geq 0, \end{cases}$$

and α_i , γ_i , and β_j are nonnegative parameters satisfying conditions similar to those of GARCH models, (Tsay, 2005). When $p = 1, q = 1$, the TGARCH(1, 1) model becomes:

$$\sigma_t^2 = \omega + (\alpha + \gamma N_{t-1}) a_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (16)$$

3.7 NGARCH(p, q) Model

The Nonlinear Generalized Autoregressive Conditional Heteroskedasticity (NGARCH) Model has been presented variously in literature by the following scholars: Hsieh & Ritchken (2005), Lanne & Saikkonen (2005), Malecka (2014) and Kononovicius & Ruseckas (2015). The following model can be shown to represent all the presentations:

$$h_t = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \gamma_i \varepsilon_{t-i} + \sum_{j=1}^p \beta_j h_{t-j} \quad (17)$$

Where h_t is the conditional variance, and ω , β and α satisfy $\omega > 0$, $\beta \geq 0$ and $\alpha \geq 0$.

Which can also be written as

$$\sigma_t = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \gamma_i \varepsilon_{t-i} + \sum_{j=1}^p \beta_j \sigma_{t-j} \quad (18)$$

3.8 SGARCH(p, q) Model

The SGARCH model can be written as:

$$Y_t = \eta_t \varepsilon_t$$

$$\eta_t^2 = \delta_0 + \sum_{i=1}^q \delta_i (\eta_{t-i} \varepsilon_{t-i})^2 + \sum_{j=q+1}^{q+p} \delta_j \eta_{t+q-j}^2, \quad (19)$$

where Y_t is the leading market return at time t , $\{\varepsilon_t\} \sim i.i.d. N(0, 1)$ is the innovation (or shock) of the market, and is hypothesized to be Gaussian. δ_0 has to be positive and the remaining parameters nonnegative in order to ensure the positivity of η_t^2 , (De Luca & Loperfido, 2012)

4.0 Materials and Methods

4.1 Simulation Procedure

The simulation procedure here considers the following equations of GARCH (1,1):

$$\begin{aligned}\varepsilon_t &= \sigma_t z_t, \\ \sigma_t^2 &= a_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2,\end{aligned}\quad (20)$$

The Case simulated is the case of financial time series where there are positive high autocorrelation coefficients as $\rho = (0.8, 0.85, 0.9, 0.95, 0.99)$, at different time series lengths (as 250, 500, 750, 1000, 1250, 1500). The experiment is repeated 1000 times. The rugarch package of the R software was used to execute the simulation.

4.2 Forecast Assessment

The following are the criteria for Forecast assessments used:

1. Mean Absolute Error (MAE) has a formula $MAE_j = \frac{\sum_{i=1}^n |e_i|}{n}$. This criterion measures deviation from the series in absolute terms, and measures how much the forecast is biased. This measure is one of the most common ones used for analyzing the quality of different forecasts.

2. The Root Mean Square Error (RMSE) is given as $RMSE_j = \sqrt{\frac{\sum_{i=1}^n (y_i - y^f)^2}{n}}$ where y_i is the time series data and y^f is the forecast value of y (Caraiani, 2010).

For the two measures above, the smaller the value, the better the fit of the model (Cooray, 2008)

In this simulation study, $RMSE = \frac{\sum_j^N RMSE_j}{N}$ and $MAE = \frac{\sum_j^N MAE_j}{N}$ where $N=1,000$, is the number of iterations or replications in the simulation study. Therefore, the model with the minimum RMSE and MAE result will be the preferred model

5.0 Results and Discussion

5.1 Simulation Analysis Results

The results of the simulation carried out are presented in Table 1 to Table 12 below.

Table 1: The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.8 at different time series lengths

Autocorrelation Coefficient	0.8					
Time series length (T)	250		500		750	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	15.71621	198.40292	22.26565	397.21749	27.32206	596.84701
gjrGARCH	15.67723	197.67197	22.26607	397.35969	27.34346	597.34298
iGARCH	15.67723	197.67197	22.28668	397.33560	27.29018	596.33660
TGARCH	*NA	*NA	22.29444	397.54459	27.33909	597.47104
NGARCH	*NA	*NA	22.27423	397.56284	27.24909	595.71351

Table 1 Continued:

Autocorrelation Coefficient	0.8					
Time series length (T)	1000		1250		1500	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	RMSE	MAE	RMSE	MAE	RMSE	MAE
gjrGARCH	31.5664	796.4621	35.31191	996.36279	38.68945	1195.24246
iGARCH	31.54257	795.60796	35.2902	995.3027	38.6903	1195.2416
TGARCH	31.59317	796.72437	35.3148	995.9538	38.66294	1194.47858
NGARCH	31.55313	795.87381	35.32081	996.50479	38.7049	1195.9127
	31.53909	795.63060	35.23728	994.15621	38.59607	1192.49545

Table 2: The Ranks of The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.8 at different time series lengths

Autocorrelation Coefficient	0.8					
Time series length (T)	250		500		750	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	3.0	3.0	1	1	3	3
gjrGARCH	1.5	1.5	2	3	5	4
iGARCH	1.5	1.5	4	2	2	2
TGARCH	*NA	*NA	5	4	4	5
NGARCH	*NA	*NA	3	5	1	1

Table 2 Continued:

Autocorrelation Coefficient	0.8					
Time series length (T)	1000		1250		1500	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	4	4	3	4	3	4
gjrGARCH	2	1	2	2	4	3
iGARCH	5	5	4	3	2	2
TGARCH	3	3	5	5	5	5
NGARCH	1	2	1	1	1	1

Tables 1 and 2 presents the RMSE and MAE values, and their respective ranks from the fGARCH family model for autocorrelation value of 0.8 at different time series lengths

Table 3: The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.85 at different time series lengths

Autocorrelation Coefficient	0.85					
Time series length (T)	250		500		750	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	15.66386	197.70122	22.25821	396.90223	27.28654	595.80040
gjrGARCH	15.68658	197.91230	22.24494	396.80352	*NA	*NA
iGARCH	15.69286	197.90984	22.25257	396.79475	27.34184	597.43763
TGARCH	15.71391	198.18182	22.26876	397.13353	27.23526	595.14723
NGARCH	15.61822	197.17366	22.21827	396.37184	27.25388	595.46976

Table 3 Continued:

Autocorrelation Coefficient	0.85					
Time series length (T)	1000		1250		1500	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	31.55959	796.16952	35.31079	995.68791	38.72317	1196.32629
gjrGARCH	31.5704	796.1239	35.34999	997.37200	*NA	*NA
iGARCH	31.54111	795.49648	35.32417	996.32265	38.69188	1195.64099
TGARCH	31.57198	796.59430	35.33417	996.52832	38.72268	1196.43748
NGARCH	31.50466	795.42553	35.30166	995.55099	38.66997	1194.57390

Table 4: The Ranks of The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.85 at different time series lengths

Autocorrelation Coefficient	0.85					
Time series length (T)	250		500		750	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	2	2	4	4	3	3
gjrGARCH	3	4	2	3	*NA	*NA
iGARCH	4	3	3	2	4	4
TGARCH	5	5	5	5	1	1
NGARCH	1	1	1	1	2	2

Table 4 Continued:

Autocorrelation Coefficient	0.85					
Time series length (T)	1000		1250		1500	

Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	3	4	2	2	4	3
gjrGARCH	4	3	5	5	*NA	*NA
iGARCH	2	2	3	3	2	2
TGARCH	5	5	4	4	3	4
NGARCH	1	1	1	1	1	1

Tables 3 and 4 presents the RMSE and MAE values, and their respective ranks from the fGARCH family model for autocorrelation value of 0.85 at different time series lengths

Table 5: The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.9 at different time series lengths

Autocorrelation Coefficient	0.9					
Time series length (T)	250		500		750	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	15.69056	197.97735	22.22066	396.54629	27.34395	597.45662
gjrGARCH	15.69322	198.21389	22.2961	397.7557	27.36271	598.10197
iGARCH	15.67502	197.83526	22.29554	397.73077	27.23304	594.83387
TGARCH	15.6704	197.9100	22.27206	397.27891	27.34957	597.51156
NGARCH	15.66183	197.72987	22.25393	397.27571	27.25242	595.38001

Table 5 Continued:

Autocorrelation Coefficient	0.9					
Time series length (T)	1000		1250		1500	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	31.5213	795.3229	31.5213	795.3229	38.68276	1195.44711
gjrGARCH	31.5506	796.2314	31.5506	796.2314	38.69959	1196.23093
iGARCH	31.48481	794.08972	31.48481	794.08972	38.65638	1194.28637
TGARCH	31.55911	796.64856	31.55911	796.64856	38.64567	1194.20921
NGARCH	*NA	*NA	*NA	*NA	38.58901	1192.30893

Table 6: The Ranks of The RMSE and MAE values from the fGARCH family model at for autocorrelation value of 0.9 at different time series lengths

Autocorrelation Coefficient	0.9					
Time series length (T)	250		500		750	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	4	4	1	1	3	3
gjrGARCH	5	5	5	5	5	5
iGARCH	3	2	4	4	1	1
TGARCH	2	3	3	3	4	4
NGARCH	1	1	2	2	2	2

Table 6 Continued:

Autocorrelation Coefficient	0.9					
Time series length (T)	250		500		750	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	3.5	2	3.5	2	4	4
gjrGARCH	1	4	1	4	5	5
iGARCH	3.5	1	3.5	1	3	3
TGARCH	2	3	2	3	2	2
NGARCH	*NA	*NA	*NA	*NA	1	1

Tables 5 and 6 presents the RMSE and MAE values, and their respective ranks from the fGARCH family model for autocorrelation value of 0.9 at different time series lengths

Table 7: The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.95 at different time series lengths

Autocorrelation Coefficient	0.95					
Time series length (T)	250		500		750	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	15.68814	197.73295	22.29211	397.51403	27.2964	596.6663
gjrGARCH	15.69865	198.10642	22.2407	396.9439	27.30838	597.00994
iGARCH	15.73973	198.55254	22.2631	397.3768	27.30615	596.77193
TGARCH	15.72409	198.23185	22.26515	397.06924	27.32061	596.60567
NGARCH	15.64798	197.42033	22.22985	396.67303	27.28743	596.51312

Table 7 Continued:

Autocorrelation Coefficient	0.95					
Time series length (T)	1000		1250		1500	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	31.61276	797.63204	35.35245	997.24819	38.64961	1194.17442
gjrGARCH	31.55876	796.28403	35.34023	996.77395	38.69062	1195.48262
iGARCH	31.52512	795.55465	35.32587	996.60224	38.71167	1196.78469
TGARCH	31.52324	795.34252	35.2704	995.3214	*NA	*NA
NGARCH	31.55345	795.67381	35.15163	991.66978	38.58982	1192.56306

Table 8: The Ranks of The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.95 at different time series lengths

Autocorrelation Coefficient	0.95					
Time series length (T)	250		500		750	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE

sGARCH	2	2	5	5	2	3
gjrGARCH	3	3	2	2	4	5
iGARCH	5	5	3	4	3	4
TGARCH	4	4	4	3	5	2
NGARCH	1	1	1	1	1	1

Table 8 Continued:

Autocorrelation Coefficient	0.95					
Time series length (T)	1000		1250		1500	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	5	5	5	5	2	2
gjrGARCH	4	4	4	4	3	3
iGARCH	2	2	3	3	4	4
TGARCH	1	1	2	2	*NA	*NA
NGARCH	3	3	1	1	1	1

Tables 7 and 8 presents the RMSE and MAE values, and their respective ranks from the fGARCH family model for autocorrelation value of 0.95 at different time series lengths

Table 9: The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.99 at different time series lengths

Autocorrelation Coefficient	0.99					
Time series length (T)	250		500		750	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	15.61949	196.78527	22.17187	395.70803	27.28797	596.21018
gjrGARCH	15.5150	195.4712	22.35713	398.76400	*NA	*NA
iGARCH	15.54559	195.92525	22.21787	396.31757	27.25111	595.66820
TGARCH	15.6229	196.6654	22.26547	397.25319	27.2632	595.5894
NGARCH	15.6246	197.1020	22.16014	395.53835	27.25505	595.26933

Table 9 Continued:

Autocorrelation Coefficient	0.99					
Time series length (T)	1000		1250		1500	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	31.49784	794.64301	35.34941	997.45191	38.65248	1194.47871
gjrGARCH	31.58383	796.63904	35.24889	994.76280	38.6506	1194.6269
iGARCH	31.61138	797.79248	35.24748	994.22704	38.6874	1195.7461
TGARCH	31.56081	795.91771	35.21394	993.53380	38.72278	1196.99541
NGARCH	31.48974	794.83949	35.10384	989.91744	38.64695	1193.96249

Table 10: The Ranks of The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.99 at different time series lengths

Autocorrelation Coefficient	0.99					
Time series length (T)	250		500		750	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	3	4	2	2	4	4
gjrGARCH	1	1	5	5	*NA	*NA
iGARCH	2	2	4	4	1	3
TGARCH	4	3	3	3	3	2
NGARCH	5	5	1	1	2	1

Table 10 Continued:

Autocorrelation Coefficient	0.99					
Time series length (T)	1000		1250		1500	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
sGARCH	2	1	5	5	3	3
gjrGARCH	4	4	4	4	2	2
iGARCH	5	5	3	3	4	4
TGARCH	3	3	2	2	5	5
NGARCH	1	2	1	1	1	1

Tables 9 and 10 presents the RMSE and MAE values, and their respective ranks from the fGARCH family model for autocorrelation value of 0.99 at different time series lengths

Table 11: Summary of the Performances of the fGARCH family models at different levels of autocorrelation and time series lengths using RMSE

Forecast Statistics: RMSE						
Autocorrelation Coefficient	Time series length (T)					
	250	500	750	1000	1250	1500
0.8	gjrGARCH iGARCH	sGARCH	NGARCH	NGARCH	NGARCH	NGARCH
0.85	NGARCH	NGARCH	TGARCH	NGARCH	NGARCH	NGARCH
0.9	NGARCH	sGARCH	iGARCH	gjrGARCH	gjrGARCH	NGARCH
0.95	NGARCH	NGARCH	NGARCH	TGARCH	NGARCH	NGARCH
0.99	gjrGARCH	NGARCH	iGARCH	NGARCH	NGARCH	NGARCH

Table 12: Summary of the Performances of the fGARCH family models at different levels of autocorrelation and time series lengths using MAE

Forecast Statistic: MAE						
Autocorrelation Coefficient	Time series length (T)					
	250	500	750	1000	1250	1500

0.8	gjrGARCH iGARCH	sGARCH	NGARCH	gjrGARCH	NGARCH	NGARCH
0.85	NGARCH	NGARCH	TGARCH	NGARCH	NGARCH	NGARCH
0.9	NGARCH	sGARCH	iGARCH	iGARCH	iGARCH	NGARCH
0.95	NGARCH	NGARCH	NGARCH	TGARCH	NGARCH	NGARCH
0.99	gjrGARCH	NGARCH	NGARCH	sGARCH	NGARCH	NGARCH

Table 13: Overall performance rating of the fGARCH models irrespective of the autocorrelation values and time series lengths

Models	RMSE	MAE
sGARCH	6.25%	9.38%
gjrGARCH	12.5%	9.38%
iGARCH	12.5%	15.6%
TGARCH	6.25%	6.25%
NGARCH	62.5%	59.3%

5.2 Discussion of Findings

5.2.1 GARCH models performance in the presence of autocorrelation using the root mean square error (RMSE) from the results of the simulation in Table 11.

For autocorrelation value of 0.8, iGARCH and gjrGARCH performed better than other models when the time series length (T) is 250, but at time series length (T) is 500, sGARCH performed better. However, NGARCH dominated as it outperformed the other models at the time series lengths (T) of 750, 1000, 1250 and 1500.

For autocorrelation value of 0.85, NGARCH outperformed the other models irrespective of the time series (T) length except at time series length of 750 where TGARCH performed better than others.

Coming to the autocorrelation value of 0.9, it can be seen that while NGARCH performed better at T = 250 and T = 1500, gjrGARCH performed better at T = 1000 and T = 1250, and sGARCH and iGARCH, respectively, outperformed the other models at T = 500 and T = 750.

For autocorrelation value of 0.95, it can clearly be observed that NGARCH outperformed the other models irrespective of the time series (T) length except at time series length of 1000 where TGARCH performed better than others.

For autocorrelation value of 0.99, while gjrGARCH performed better than other models at T = 250, iGARCH performed better than other models at T = 750, NGARCH outperformed the other

models at the time series (T) lengths of 500, 1000, 1250 and 1500; thereby being the dominant model for that autocorrelation value.

5.2.2 GARCH models performance in the presence of autocorrelation using the root mean square error (MAE) from the results of the simulation in Table 12.

For autocorrelation value of 0.8, gjrGARCH and iGARCH performed better than other models when the time series length (T) is 250, gjrGARCH again performed better than other models at T = 1000, sGARCH outperformed the other models at T = 500, while NGARCH performed better than the other models at the time series lengths (T) of 750, 1250 and 1500.

For autocorrelation value of 0.85, NGARCH dominated as it outperformed the other models irrespective of the time series (T) length except at time series length of 750 where TGARCH performed better than others.

And for the autocorrelation value of 0.9, it can be seen that whereas NGARCH performed better at time series length (T) = 250 and T = 1500, sGARCH performed better than the other models at T = 500, while iGARCH dominated at the other time series lengths, performing better at T = 750, T = 1000 and T = 1250.

For autocorrelation value of 0.95, it can be observed that NGARCH was the preferred model as it outperformed the other models irrespective of the time series (T) length except at time series length (T) of 1000 where TGARCH performed better than others.

And for autocorrelation value of 0.99, while NGARCH dominated the other models, performing better at time series lengths (T) = 500, T = 750, T = 1250 and T = 1500, gjrGARCH performed better than other models at T = 250, and sGARCH performed better than other models at T = 1000. Using RMSE and MAE criteria, this study has shown that different models performed better at different autocorrelation coefficients and at different time series lengths. This is in line with previous studies: Atoi (2014) modelling the volatility of stock returns using daily closing data of Nigerian Stock Exchange, found that GARCH (1,1), PGARCH (1,1,1) and EGARCH (1,1) with student's t distribution, and TGARCH with GED were the four best fitted models based on Schwarz Information Criterion, and the conclusions in Grek (2014), Chen, Min and Chen (2013), Dijk, Franses and Lucas (1999) and Demos (2000), that different models performed differently under different conditions; in this case, different autocorrelation coefficients and different time series lengths. Results are however in contrast to Mikosch and Starica (2000) using GARCH (1,1) model to estimate log return of foreign exchange, said that

the level of autocorrelation is unreliable means for model selection . In summary in Table 13, modeling financial time series with high positive autocorrelation values is dominated by NGARCH model which is in line with results obtained by Lanne and Saikkonen (2005).

6.0 Conclusion and Recommendation

This study investigated the performance of fGARCH family models in the presence of high positive autocorrelation values, the results concluded the dominance of NGARCH model. We therefore recommended that investors, financial analysts and researchers interested in stock prices and asset return should adapt NGARCH model when there is high positive autocorrelation in the financial time series data.

Reference

- Adenom, M. O. and Micheal, V. A. (2017): Introduction Econometrics with Examples in EViews. Niger State: Jube-Evans Book & Publication.
- Adenom, M. O.; Micheal, V. A. and Evans, P. O.(2015): A Simulation Study on the Performances of Classical VAR and Sims-Zha Bayesian VAR Models in the Presence of Autocorrelated Errors. *Open Journal of Modeling and Simulation*, 3:146-158.
- Ali, G. (2013) “EGARCH, GJR-GARCH, TGARCH, AVGARCH, NGARCH, IGARCH, and APARCH Models for Pathogens at Marine Recreational Sites”. *Journal of Statistical and Econometric Methods*, 2 (3): 57-73.
- Atoi, N. V. (2014) “Testing volatility in Nigeria stock market using GARCH models”. *CBN Journal of Applied Statistics*, 5: 65-93.
- Chen, S., Min, W. & Chen, R. (2013) “Model identification for Time Series with dependent innovations”. *Statistica Sinica*, 23: 873 - 899.
- De Luca, G. & Loperfido, N. (2012) “Modeling multivariate skewness in financial returns: a SGARCH approach”. *The European Journal of Finance*, 1 -19 iFirst. <http://dx.doi.org/10.1080/1351847X.2011.640342>
- Demos (2000) “Autocorrelation Function of Conditionally Heteroskedastic in Mean Models”. *Journal of Athens University of Economics and Business, Department of International and European Economic Studies*, pp 1- 52.
- Dijk, D. V., Franses, P. H. & Lucas, A. A. (1999) “Testing for ARCH in the Presence of Additive Outliers”. *Journal of Applied Econometrics*, 14 (5) 539-562.

- Ding, Z, Granger, C. W. J. & Engle, R. F. (1993) "A long memory property of stock market returns and a new model", *Journal of Empirical Finance*, 1, 83-106.
- Greks, A. (2014) "Forecasting accuracy for ARCH models and GARCH(1,1) family which model does best capture the volatility of the Swedish stock market"? *Statistics Advance Level Theses 15hp*; Örebro University.
- Greks, A. (2014) "Forecasting accuracy for ARCH models and GARCH(1,1) family which model does best capture the volatility of the Swedish stock market"? *Statistics Advance Level Theses 15hp*; Örebro University.
- Grigoletto, M. and Lisi, F.(2009): Looking for Skewness in Financial Time Series. *The Econometrics Journal*, 12(2):310-323.
- Gujarati, D. N.(2003): Basic Econometrics, 4th edition. New Delhi: The McGraw-Hill co,
- Hsieh, K. C. & Ritchken, P. (2005) "An Empirical Comparison of GARCH Option Pricing Models". *Review of Derivatives Research*, 8 (3): 129 -150.
- Jiang, W. (2012) "Using the GARCH model to analyse and predict the different stock markets" *Master Thesis in Statistics*, Department of Statistics, Uppsala University Sweden.
- Kononovicius, A. & Ruseckas, J. (2015) "Nonlinear GARCH Model and 1/f noise". ARXIV:1412.6244V2[Q-FIN.ST].
- Lange, T.(2011): Tail Behavior and OLS Estimating in AR-GARCH Models. *Statistica Sinica*, 21(3):1191-1200.
- Lanne, M. & Saikkonen, P. (2005) "Nonlinear GARCH Models for Highly Persistent Volatility". *Econometrics Journal*, 8 (2): 251-276.
- Lanne, M. and Saikkonen, P.(2002): Threshold Autoregression for Strongly Autocorrelated Time Series, 20(2): 282-289.
- Lawrance, A. J.(2013): Exploration Graphics for Financial Time Series Volatility. *Journal of the Royal Statistical Society Series C (Applied Statistics)*, 62(5): 669-686.
- Lee, J. and Lund, R.(2004): Revisiting Simple Linear Regression with Autocorrelated Errors. *Biometrika*, 91(1):240-245.
- Lee, S. and Hansen, B.(1994): Asymptotic Theory for the GARCH(1,1) Quasi-Maximum Likelihood Estimator. *Econometric Theory*, 10, 29-53.
- Malecka, M. (2014) "GARCH Class Models Performance in Context of High Market Volatility". *ACTA Universitatis Lodziensis Folia Oeconomica*, 3: 253 -266

- Mikosch, T. & Starica, C. (2000) “Limit theory for the sample autocorrelations and extremes of a GARCH(1,1) process”. *Annals of Statistics*, 28 (5): 1427 - 1451.
- Pedroni, P.(2001): The Econometric Modeling of Financial Time Series by Terence Mills. *Journal of the American Statistical Association*, 96(453): 345-346.
- Rossi, E. (2004) “Lecture notes on GARCH models”. University of Pavia, March.
- Su, L. and Ullah, A.(2006): More Efficient Estimation in Non Parametric Regression with Non Parametric Autocorrelated Errors: *Econometric Theory*, 22(1):98-126.
- Tsay, R. S. (2005) *Analysis of financial time series*, 2nd Edition. New Jersey: John Wiley & Sons.
- Xiao, Z; Linton, O. B; Carroll, R. J. and Mammen, E. (2003): More Efficient Local Polynomial Estimation in Non Parametric Regression with Autocorrelated Errors. *Journal of the American Statistical Association*, 98(464):980-992.