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## 6 **Television Rating Control in the Multichannel** 7 **Environment Using Trend Fuzzy Knowledge Bases** 8 **and Monitoring Results**

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16 **Abstract:** The purpose of the study is to control the ratio of programs of different genres when  
17 forming the broadcast grid in order to increase and maintain the rating of the channel. In the  
18 multichannel environment, television rating control consists of selecting such content, ratings of  
19 which are completely restored after advertising. A hybrid approach combining the benefits of  
20 semantic training and fuzzy relational equations in simplification of the expert recommendation  
21 systems construction is proposed. The problem of retaining the television rating can be attributed  
22 to the problems of fuzzy resources control. The increase or decrease trends of the demand and  
23 supply are described by primary fuzzy relations. The rule-based solutions of fuzzy relational  
24 equations connect significance measures of the primary fuzzy terms. Rules refinement by solving  
25 fuzzy relational equations allows avoiding labor-intensive procedures for the generation and  
26 selection of expert rules. The solution set generation corresponds to the granulation of the  
27 television time, where each solution represents the time slot and the granulated rating of the  
28 content. In automated media planning, generation of the weekly TV program in the form of the  
29 granular solutions provides the decrease of the time needed for the programming of the channel  
30 broadcast grid.

### 31 **Datasets:**

32 TV Channel Inter: Ratings. Available online: <http://inter.ua/uk/about/rating>

33 TV Channel Inter: Weekly TV Program. Available online: <https://inter.ua/uk/tv>

34 IMDb Top Rated Movies. Available online: <https://www.imdb.com>

35 Ukrainian Films Catalog. Available online: <https://kinoafisha.ua/ratings/>

36 **Dataset License:** open access data set

37 **Keywords:** TV channel rating, expert recommendation systems, fuzzy resources control, fuzzy  
38 classification knowledge bases, solving fuzzy relational equations.

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## 42 1. Summary

43 The top priority task of the personnel of TV companies is to control the ratio of programs of  
44 different genres when forming the broadcast grid in order to increase and maintain the rating of the  
45 channel [1]. The rating of a television channel is determined by specialized sociological services and  
46 directly affects the cost of advertising time. Content management is modeled by integrating trials,  
47 expert recommendations and users' preferences [2]. The recommendation accuracy strongly  
48 depends on the mechanisms of the supply and demand regulation [3], which consider time factors  
49 and users' preferences simultaneously [4, 5]. Parametric statistical models are widely used to  
50 evaluate the viewers' demand and the popularity of TV programs [6–9]. These models are adjusted  
51 to fit the distribution of user ratings in video on demand services dealing specifically with TV  
52 content [6, 7]. To enhance the recommendation accuracy, the statistical models aim to learn  
53 audience preferences that follow from the rich user content generated in the social networks [8, 9].  
54 The timing and item recommendations are generated via clustering the common interests of a group  
55 of people [10, 11]. Finally, the cognitive models describe the behavior of viewers when choosing a  
56 TV channel [12, 13]. Such models predict program commitments based on viewer-program  
57 emotional relationships reflecting satisfaction and perception toward alternative programs.

58 The television domain has become increasingly complex due to the multichannel environment.  
59 Television advertising time is purchased on the basis of the projected future ratings [14]. The  
60 problem lies in the paradoxical connection between the programs ratings before and after the ad unit  
61 [15]. In the multichannel environment, the ad skipping behaviors may result in the lower ratings  
62 [16]. Restoration of the program rating after the ad unit depends on the decision to watch or skip the  
63 program, i.e. give preference to the programs broadcast by the concurrent channels. The  
64 conventional recommender systems concentrate on the targeted advertising and do not take the ad  
65 break-factor into account [17, 18]. In this case, the skipping phenomenon stipulated by the  
66 multichannel environment is ignored. Such methods of media planning will inevitably reduce the  
67 rating points because of the content sensitivity to the advertisement insertion [18].

68 The problem of retaining the television rating can be attributed to the problems of fuzzy  
69 resources control [1, 19]. In such models, the "demand-supply" relationships are described by fuzzy  
70 IF-THEN rules. In works [19, 20], it is suggested to build the fuzzy resources control model on the  
71 grounds of the general method of nonlinear dependencies identification by means of fuzzy  
72 knowledge bases. The method [19, 20] implies the stage of tuning the fuzzy control model using  
73 "demand-supply" training data. The tuning stage consists of finding such fuzzy rules weights and  
74 such membership functions forms, which provide maximum proximity of the results of fuzzy logic  
75 inference to the correct managerial decisions.

76 In work [21], the fuzzy control model, which provides the balanced demand and supply for  
77 each content category was proposed. Experienced managers make effective administrative decisions  
78 based on a comparison of the viewers' demand for the programs of different genres with the rating  
79 of the programs offered at the given time. Dependent upon this, a control action is formed, which  
80 consists of increasing or decreasing the rating of the programs in the channel broadcast grid [21].

81 Unlike [21], the fuzzy control model is expanded on the account of the supplementary factors  
82 influencing the demand and supply. The viewers' demand for different content categories is defined  
83 by the time factors, i.e. the time of viewing and the day of the week. In the multichannel  
84 environment, television rating control consists of selecting such content, ratings of which are

85 completely restored after advertising. Therefore, projected ratings after control in each genre  
86 category are evaluated by the content attributes and the ad break-factor. In this case, targeted  
87 advertising is augmented by the viewer intention to watch the program. Finally, the content  
88 category for each time slot is recommended on the grounds of users' preferences.

89 The construction of expert recommendation systems is associated with computational costs.  
90 Constantly changing preferences of different categories of viewers require the selection and  
91 adjustment of the appropriate set of expert rules. At the same time, experts establish the trends of  
92 demand-supply relationships, which are subject to further refinement. Such trend dependencies are  
93 described by primary fuzzy knowledge bases. In this case, the increase or decrease of the television  
94 indices can be considered as the primary fuzzy terms. The solution to the problem of expert rules  
95 refinement may be the use of fuzzy relational equations [22, 23], the solutions of which represent the  
96 linguistic modification of the primary terms. The obtained solutions can be considered as composite  
97 fuzzy rules that connect significance measures of the primary fuzzy terms and reflect the semantic  
98 intensiveness of the increase or decrease trend [24-27]. The number of rules is determined by the  
99 number of solutions. The recommendation accuracy is achieved through the complete solution set,  
100 i.e. the complete rule set of the recommendation knowledge base [28, 29].

101 The composite fuzzy model of the control action balancing the demand and supply was  
102 proposed in [21]. In this paper, the composite fuzzy models for the demand and supply are  
103 proposed. The composite fuzzy model of the demand defined by the time factors allows generating  
104 item and timing recommendations simultaneously. The composite fuzzy model of the supply  
105 defined by the content and advertisement attributes provides the rating restoration in the  
106 multichannel environment. Refinement of the rule set by solving fuzzy relational equations allows  
107 avoiding labor-intensive procedures for the generation and selection of expert rules. The solution set  
108 generation corresponds to the granulation of the television time, where each solution represents the  
109 time slot and the granulated rating of the content. In automated media planning, generation of the  
110 weekly TV program in the form of the granular solutions provides the decrease of the time needed  
111 for the programming of the channel broadcast grid. Therefore, it is important to develop a hybrid  
112 approach combining the benefits of semantic training [19, 20] and fuzzy relational equations [24-27]  
113 in simplification of the process of expert recommendation systems construction. Following the  
114 approach proposed in [24-27], the genetic-neuro algorithm is used for tuning the primary fuzzy  
115 model and solving the primary system of fuzzy relational equations [28, 29].

## 116 2. Data Description

117 The fuzzy model of resources management was constructed using the example of the television  
118 channel Inter, which holds leading positions in the Ukrainian media market [30]. The TV channel  
119 Inter presents programs of such basic genres: political programs and news releases ( $k = 1$ ); TV serials  
120 and documentary projects ( $k = 2$ ); entertaining and sports programs ( $k = 3$ ).

121 For the TV rating control problem, the monitoring and forecast window will be for one week.  
122 We shall denote the time and day of the week for the TV program release as  $\mathbf{t} = (t_1, t_2)$ .  
123 Management is carried out at the level of each air-hour, i.e.  $t_1 \in [0, 24]$ ,  $t_2 \in [1, 7]$ . Let  $p$  be the  
124 number of the current week for media planning. The proportion of TV viewers who watch TV at the  
125 time moment  $(\mathbf{t}, p)$  determines the rating of the TV channel. For the channel Inter, the ratings reach  
126 20%. The analysis of the TV channel rating is carried out according to the results of monitoring of the

127 TV programs ratings obtained for the previous week  $p-1$ . The TV program is compiled for the  
128 forthcoming week  $p$ .

129 The timing and item recommendations in the form of the weekly TV program is presented in  
130 [31]. The weekly TV program is compiled as a grid with the following nodes: the date (the time of  
131 viewing and the day of the week) and the attributes of the program (the genre and title of the  
132 program).

133 The popularity estimates of the TV programs included in the weekly broadcast grid can be  
134 obtained from the Internet Movie Data Base (imdb) [32]. This data set contains nearly 5000 titles of  
135 the top rated TV shows in 26 genre categories. Imdb-ratings of the TV shows produced in Ukraine  
136 are collected in [33]. In developing fuzzy model of resources control, we shall use the imdb-ratings  
137 as the supply attributes weighted using the scale from 0 to 10 [32, 33].

138 Effectiveness of time and content management is evaluated by the weekly top 20 rating [30].  
139 The archive of weekly ratings contains data from 2012-2018 years. The weekly top 20 can be  
140 represented as a table with the following columns: the title of the program, the date and time, the  
141 rating and share. The weekly top 20 list contains the ratings influenced by the ad break-factor. The  
142 ad break-factor can be considered as a proportion of viewers who return to the program after  
143 advertising. TV experts connect the ad break-factor with the lowering down of the possible ratings  
144 up to 40%.

145 Ratings of the programs, which do not fall into the weekly top 20 list are evaluated as follows. It  
146 is obvious, that the ratings of these programs do not exceed the minimum value among the most  
147 popular weekly programs. For the newly launched TV shows, the imdb-ratings can be temporarily  
148 substituted by the actual ratings from the weekly top 20 list.

### 149 3. Method of the Recommendation System Construction

#### 150 3.1. Structure of the TV Rating Control Model

151 The structure of the TV rating control model corresponds to the following hierarchical tree of  
152 logic inference:

153 - for content management

$$154 \quad x_k = f_x^k(t_1, t_2, p), \quad k = \overline{1, n}, \quad (1)$$

$$155 \quad y_k(t, p) = f_y^k(x_k(t, p), z_k(t, p-1)), \quad (2)$$

156 - for rating evaluation

$$157 \quad a_k(t, p) = f_a^k(x_k(t, p)), \quad (3)$$

$$158 \quad v_k(t, p) = f_v^k(z_k(y_k(t, p)), a_k(t, p)), \quad (4)$$

$$159 \quad u(t, p) = f_u(v_1(t, p), \dots, v_n(t, p)), \quad (5)$$

160 where  $n$  is the number of genres of the TV programs;

161  $x_k(t, p)$  is the viewers' demand for the programs of the genre  $k$  at the time moment  $(t, p)$ ;

162  $z_k(t, p-1)$  is the imdb-rating of the program of the genre  $k$  at the time moment  $(t, p-1)$ ;

163  $y_k(t,p)$  is a control action for the time moment  $(t,p)$ , consisting in increasing–decreasing the  
 164 imdb-rating of the program of the genre  $k$ ;

165  $z_k(y_k(t,p))$  is the imdb-rating of the program offered after the control action;

166  $a_k(t,p)$  is the break-factor of the program of the genre  $k$  with the advertisement at the time  
 167 moment  $(t,p)$ ;

168  $v_k(t,p)$  is the rating of the program of the genre  $k$  restored after the ad unit at the time  
 169 moment  $(t,p)$ ;

170  $u(t,p)$  is the rating of the TV channel at the time moment  $(t,p)$ .

171 It is supposed that the control action is determined as the difference between the imdb-rating  
 172 values before and after control, i.e.  $y_k(t,p) = z_k(t,p) - z_k(t,p-1)$ .

173 Variation ranges of the TV indices are defined as follows:  $[0, 10]$  points for  $x_k$  and  $z_k$ ;  $[-10,$   
 174  $10]$  points for  $y_k$ ;  $[0.4, 1]$  for  $a_k$ ;  $[0, 20]$  % for  $v_k$  and  $u$ .

175 We shall describe the trend dependencies with the help of the primary fuzzy terms:

176 - in the morning (M), in the afternoon (A), in the evening (Ev) for  $t_1$ ;

177 - on weekdays (Wd), on weekends (We) for  $t_2$ ;

178 - increased (decreased) (I, D) or stable (St) for  $x_k$ ,  $z_k$ ,  $v_k$  or  $u$ ;

179 - increase (decrease) (I, D) or stay inactive (N) for  $y_k$ .

180 For the composite knowledge base construction, we shall use the linguistic modifiers: sharply  
 181 (sh), moderately (m), weakly (w). These modifiers describe the semantic intensity of the primary  
 182 terms D and I [26, 27].

183 Functional dependencies (1)-(5) are defined by the primary fuzzy relations presented in Tables  
 184 1-5.

185 It is necessary to transfer the primary fuzzy relations into the composite fuzzy rules for the  
 186 modified decision classes of the variables  $x_k(t,p)$ ,  $y_k(t,p)$ ,  $a_k(t,p)$ ,  $v_k(t,p)$  and  $u(t,p)$ . The  
 187 composite rules were built for the seven classes (sh-m-wD, St, w-m-shI) of the variables  $x_k(t,p)$ ,  
 188  $a_k(t,p)$ ,  $v_k(t,p)$ ,  $u(t,p)$ ; for the seven classes (sh-m-wD, N, w-m-shI) of the variable  $y_k(t,p)$ .

189 **Table 1.** Primary fuzzy relations “viewing time - demand”

	IF	THEN $x_k(t,p)$		
		D	St	I
$t_1$	M	w-m	m	w-m
	A	m	m-sh	m
	Ev	w	m-sh	sh
$t_2$	Wd	m-sh	sh	m-sh
	We	m	m-sh	m-sh

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**Table 2.** Primary fuzzy relations “demand and supply – control action”

	IF	THEN $y_k(t,p)$		
		D	N	I
$x_k(t,p)$	D	sh	sh	w
	St	m-sh	m-sh	sh
	I	w	m	sh
$z_k(t,p-1)$	D	w	sh	sh
	St	m-sh	m-sh	m
	I	sh	m	w

195

**Table 3.** Primary fuzzy relations “demand – ad break-factor”

	IF	THEN $a_k(t,p)$		
		D	St	I
$x_k(t,p)$	D	m-sh	w-m	w
	St	w-m	m-sh	w-m
	I	w	m	sh

196

**Table 4.** Primary fuzzy relations “supply with the ad break-factor – restored genre ratings”

	IF	THEN $v_k(t,p)$		
		D	St	I
$z_k(t,p)$	D	sh	w	w
	St	m	m-sh	m
	I	m	m-sh	sh
$a_k(t,p)$	D	sh	w	w
	St	m	m-sh	w-m
	I	w-m	m-sh	sh

197

**Table 5.** Primary fuzzy relations “restored genre ratings – rating of the channel”

	IF	THEN $u(t,p)$		
		D	St	I
$v_k(t,p)$	D	m-sh	w-m	w
	St	w-m	m-sh	m-sh
	I	w	w-m	sh

198

### 199 3.2. The Problem of Tuning the Fuzzy Control Model

200 The fuzzy control model connects the vectors of significance measures of the primary fuzzy  
 201 terms of the variables  $t_1$ ,  $t_2$ ,  $x_k$ ,  $z_k$  and  $y_k$  in correlations (1), (2); variables  $x_k$ ,  $a_k$ ,  $z_k$ ,  $v_k$   
 202 and  $u$  in correlations (3)-(5).

203 Correlations (1)-(5) define the primary fuzzy model in the form:

204

205

206 - for content management

$$207 \quad \mu_x^k = (\mu_t^1 \circ \mathbf{H}_1^k) \cap (\mu_t^2 \circ \mathbf{H}_2^k), \quad (6)$$

$$208 \quad \mu_y^k = (\mu_x^k \circ \mathbf{Q}_1^k) \cap (\mu_z^k \circ \mathbf{Q}_2^k), \quad (7)$$

209

210 - for rating evaluation

$$211 \quad \mu_a^k = \mu_x^k \circ \mathbf{G}^k, \quad (8)$$

$$212 \quad \mu_v^k = (\mu_z^k \circ \mathbf{R}_1^k) \cap (\mu_a^k \circ \mathbf{R}_2^k), \quad (9)$$

$$213 \quad \mu_u = (\mu_v^1 \circ \mathbf{W}^1) \cup \dots \cup (\mu_v^n \circ \mathbf{W}^n), \quad (10)$$

214 where  $\mu_t^1 = (\mu_t^{1,1}, \dots, \mu_t^{1,3})$  and  $\mu_t^2 = (\mu_t^{2,1}, \mu_t^{2,2})$  are the vectors of the fuzzy causes M, A, Ev and Wd,

215 We for the variables  $t_1$  and  $t_2$ ;

216  $\mu_x^k = (\mu_x^{k,1}, \dots, \mu_x^{k,3})$  and  $\mu_z^k = (\mu_z^{k,1}, \dots, \mu_z^{k,3})$  are the vectors of the fuzzy causes D, St, I for the

217 variables  $x_k$  and  $z_k$ ;

218  $\mu_y^k = (\mu_y^{k,1}, \dots, \mu_y^{k,3})$  is the vector of the fuzzy effects D, N, I for the variable  $y_k$ ;

219  $\mu_a^k = (\mu_a^{k,1}, \dots, \mu_a^{k,3})$  and  $\mu_z^k = (\mu_z^{k,1}, \dots, \mu_z^{k,3})$  are the vectors of the fuzzy causes D, St, I for the

220 variables  $a_k$  and  $z_k (y_k)$ ;

221  $\mu_v^k = (\mu_v^{k,1}, \dots, \mu_v^{k,3})$  and  $\mu_u = (\mu_u^1, \dots, \mu_u^3)$  are the vectors of the fuzzy effects D, St, I for the

222 variables  $v_k$  and  $u$ ;

223  $\mathbf{H}_1^k = [h_1^{k,IJ}]$ ,  $\mathbf{H}_2^k = [h_2^{k,LJ}]$  and  $\mathbf{Q}_1^k = [q_1^{k,IJ}]$ ,  $\mathbf{Q}_2^k = [q_2^{k,IJ}]$ ,  $I, J = \overline{1,3}$ ,  $L = \overline{1,2}$ , are the primary

224 fuzzy relational matrices “viewing time  $(t_1, t_2)$  – demand  $x_k$ ” and “demand  $x_k$  and supply  $z_k$  –

225 control action  $y_k$ ”;

226  $\mathbf{G}^k = [g^{k,IJ}]$ ,  $I, J = \overline{1,3}$ , are the primary fuzzy relational matrices “demand  $x_k$  – ad break-factor

227  $a_k$ ”;

228  $\mathbf{R}_1^k = [r_1^{k,IJ}]$ ,  $\mathbf{R}_2^k = [r_2^{k,IJ}]$  and  $\mathbf{W}^k = [w^{k,IJ}]$ ,  $I, J = \overline{1,3}$ , are the primary fuzzy relational matrices

229 “supply  $z_k (y_k)$  with the ad break-factor  $a_k$  – restored genre ratings  $v_k$  – rating of the channel  $u$ ”;

230

231  $\circ$  and  $\circ, \cap$  are the operations of the simplified and extended max-min composition

232 corresponding to the correlations  $f_a^k$ ,  $f_u$  and  $f_x^k$ ,  $f_y^k$ ,  $f_v^k$  [22].

233 We use a bell-shaped membership function model of variable  $\tau$  to arbitrary term  $T$  in the form

234 [19, 20]:

$$235 \quad \mu^T(\tau) = 1 / (1 + ((\tau - \beta) / \sigma)^2),$$

236 where  $\beta$  is a coordinate of function maximum,  $\mu^T(\beta) = 1$ ;  $\sigma$  is a parameter of concentration.

237 In this case, correlations (6)-(10) take the form:

238 - for content management

$$239 \quad \mu_x^k = f_x^k(t_1, t_2, p, \mathbf{H}_1^k, \mathbf{H}_2^k, \Psi_t^1, \Psi_t^2), \quad (11)$$

$$240 \quad \mu_y^k(y_k, \Psi_y) = f_y^k(\mu_x^k, z_k, \mathbf{Q}_1^k, \mathbf{Q}_2^k, \Psi_z^k), \quad (12)$$

241 - for rating evaluation

$$242 \quad \mu_a^k = f_a^k(\mu_x^k, \mathbf{G}^k), \quad (13)$$

$$243 \quad \mu_v^k = f_v^k(\mu_z^k, \mu_a^k, \mathbf{R}_1^k, \mathbf{R}_2^k, \Psi_z^k), \quad (14)$$

$$244 \quad \mu_u(u, \Psi_u) = f_u(\mu_v^1, \dots, \mu_v^n, \mathbf{W}^1, \dots, \mathbf{W}^n), \quad (15)$$

245 where  $\Psi_t^1 = (\beta_t^{1,1}, \sigma_t^{1,1}, \dots, \beta_t^{1,3}, \sigma_t^{1,3})$ ,  $\Psi_t^2 = (\beta_t^{2,1}, \sigma_t^{2,1}, \beta_t^{2,2}, \sigma_t^{2,2})$  and  $\Psi_z^k = (\beta_z^{k,1}, \sigma_z^{k,1}, \dots, \beta_z^{k,3}, \sigma_z^{k,3})$  are  
 246 the vectors of parameters of the primary membership functions of the input variables  $t_1$ ,  $t_2$  and  
 247  $z_k$ ;

248  $\Psi_y^k = (\beta_y^{k,1}, \sigma_y^{k,1}, \dots, \beta_y^{k,3}, \sigma_y^{k,3})$  and  $\Psi_u = (\beta_u^1, \sigma_u^1, \dots, \beta_u^3, \sigma_u^3)$  are the vectors of parameters of the  
 249 primary membership functions of the output variables  $y_k$  and  $u$ .

250 It is assumed that some training data sample can be obtained on the grounds of successful  
 251 managerial decisions

$$252 \quad \langle \hat{p}, \hat{t}_{kl}^p, \hat{z}_{kl}^p(\hat{t}_{kl}^p, \hat{p} - 1), \hat{z}_{kl}^p(\hat{t}_{kl}^p, \hat{p}), \hat{u}_{kl}^p(\hat{t}_{kl}^p, \hat{p}) \rangle, \quad p = \overline{1, P}, \quad l = \overline{1, L_{pk}},$$

253 where  $P$  is the number of weeks in the data sample;

254  $L_{pk}$  is the number of TV programs of the genre  $k$  in the weekly experiment number  $p$ ;

255  $\hat{t}_{kl}^p$  and  $\hat{z}_{kl}^p$  are the control system state parameters in the experimental time slot number  $pkl$ ;

256  $\hat{u}_{kl}^p$  is the TV rating in the experimental time slot  $\hat{t}_{kl}^p$ .

257 The essence of the fuzzy model (11)-(15) tuning is as follows. It is necessary to find the relation  
 258 matrices  $\mathbf{H}_{1-2}^k$ ,  $\mathbf{Q}_{1-2}^k$ ,  $\mathbf{G}^k$ ,  $\mathbf{R}_{1-2}^k$ ,  $\mathbf{W}^k$  and the vectors of the membership functions parameters



259  $\Psi_t^1, \Psi_t^2, \Psi_z^k, \Psi_y^k, \Psi_u$ , which provide the minimum distance between theoretical and  
 260 experimental data:

$$261 \sum_{p=1}^P \sum_{k=1}^n \sum_{l=1}^{L_{pk}} [f_u(\hat{p}, \hat{t}_{kl}^p, \hat{z}_{kl}^p, \mathbf{H}_{l-2}^k, \mathbf{Q}_{l-2}^k, \mathbf{G}^k, \mathbf{R}_{l-2}^k, \mathbf{W}^k, \Psi_t^{l-2}, \Psi_z^k, \Psi_y^k) - \hat{\mu}_u(\hat{u}_{kl}^p, \Psi_u)]^2 = \min_{\mathbf{H}, \mathbf{Q}, \mathbf{G}, \mathbf{R}, \mathbf{W}, \Psi} \quad (16)$$

262 We shall denote:

263  $X_{kj}$  and  $Y_{kj}$ ,  $j = \overline{1,7}$ , are the modified decision classes of the variables  $x_k(t,p)$  and  $y_k(t,p)$ ;

264  $A_{kj}$ ,  $V_{kj}$  and  $U_j$ ,  $j = \overline{1,7}$ , are the modified decision classes of the variables  $a_k(t,p)$ ,  
 265  $v_k(t,p)$  and  $u(t,p)$ , respectively;

266  $N_x^{kj}$  and  $N_y^{kj}$  are the numbers of composite rules in the classes  $X_{kj}$  and  $Y_{kj}$ ;

267  $N_a^{kj}$ ,  $N_v^{kj}$  and  $N_u^j$  are the numbers of composite rules in the classes  $A_{kj}$ ,  $V_{kj}$  and  $U_j$ ,  
 268 respectively.

269 Given the output classes, the solution set of primary fuzzy relational equations (6)-(10) can be  
 270 considered as the set of composite fuzzy rules [24, 25]:

271 - for content management

$$272 \bigcup_{i=1, N_x^{kj}} \left[ \left[ \underline{\mu}_t^{1,ij}, \overline{\mu}_t^{-1,ij} \right] \cap \left[ \underline{\mu}_t^{2,ij}, \overline{\mu}_t^{-2,ij} \right] \right] \rightarrow x_k = X_{kj}; \quad (17)$$

$$273 \bigcup_{i=1, N_y^{kj}} \left[ \left[ \underline{\mu}_x^{k,ij}, \overline{\mu}_x^{-k,ij} \right] \cap \left[ \underline{\mu}_z^{k,ij}, \overline{\mu}_z^{-k,ij} \right] \right] \rightarrow y_k = Y_{kj}; \quad (18)$$

274 - for rating evaluation

$$275 \bigcup_{i=1, N_a^{kj}} \left[ \underline{\mu}_x^{k,ij}, \overline{\mu}_x^{-k,ij} \right] \rightarrow a_k = A_{kj}; \quad (19)$$

$$276 \bigcup_{i=1, N_v^{kj}} \left[ \left[ \underline{\mu}_z^{k,ij}, \overline{\mu}_z^{-k,ij} \right] \cap \left[ \underline{\mu}_a^{k,ij}, \overline{\mu}_a^{-k,ij} \right] \right] \rightarrow v_k = V_{kj}; \quad (20)$$

$$277 \bigcup_{i=1, N_u^j} \left[ \left[ \underline{\mu}_v^{1,ij}, \overline{\mu}_v^{-1,ij} \right] \cup \dots \cup \left[ \underline{\mu}_v^{n,ij}, \overline{\mu}_v^{-n,ij} \right] \right] \rightarrow u = U_j. \quad (21)$$

278 In (17)-(21), the fuzzy solution vectors are presented in the form of the vectors of the lower and  
 279 upper bounds of the fuzzy causes significance measures, where

280  $\underline{\mu}_t^{1,ij} = (\underline{\mu}_t^{11,ij}, \dots, \underline{\mu}_t^{13,ij})$ ,  $\overline{\mu}_t^{1,ij} = (\overline{\mu}_t^{11,ij}, \dots, \overline{\mu}_t^{13,ij})$  and  $\underline{\mu}_t^{2,ij} = (\underline{\mu}_t^{21,ij}, \underline{\mu}_t^{22,ij})$ ,  $\overline{\mu}_t^{2,ij} = (\overline{\mu}_t^{21,ij}, \overline{\mu}_t^{22,ij})$  are  
 281 the lower and upper fuzzy solution vectors for the variables  $t_1$  and  $t_2$ ;

282  $\underline{\mu}_x^{k,ij} = (\underline{\mu}_x^{k1,ij}, \dots, \underline{\mu}_x^{k3,ij})$ ,  $\overline{\mu}_x^{k,ij} = (\overline{\mu}_x^{k1,ij}, \overline{\mu}_x^{k3,ij})$  and  $\underline{\mu}_z^{k,ij} = (\underline{\mu}_z^{k1,ij}, \dots, \underline{\mu}_z^{k3,ij})$ ,

283  $\overline{\mu}_z^{k,ij} = (\overline{\mu}_z^{k1,ij}, \overline{\mu}_z^{k3,ij})$  are the lower and upper fuzzy solution vectors for the variables  $x_k$  and  $z_k$ ;

284  $\underline{\mu}_Z^{k,ij} = (\underline{\mu}_Z^{k1,ij}, \dots, \underline{\mu}_Z^{k3,ij})$ ,  $\overline{\mu}_Z^{k,ij} = (\overline{\mu}_Z^{k1,ij}, \overline{\mu}_Z^{k3,ij})$  and  $\underline{\mu}_a^{k,ij} = (\underline{\mu}_a^{k1,ij}, \dots, \underline{\mu}_a^{k3,ij})$ ,

285  $\overline{\mu}_a^{k,ij} = (\overline{\mu}_a^{k1,ij}, \overline{\mu}_a^{k3,ij})$  are the lower and upper fuzzy solution vectors for the variables  $z_k$  ( $y_k$ ) and

286  $a_k$ ;

287  $\underline{\mu}_v^{k,ij} = (\underline{\mu}_v^{k1,ij}, \dots, \underline{\mu}_v^{k3,ij})$ ,  $\overline{\mu}_v^{k,ij} = (\overline{\mu}_v^{k1,ij}, \overline{\mu}_v^{k3,ij})$  are the lower and upper fuzzy solution vectors

288 for the variables  $v_k$ .

289 Given the primary fuzzy model (6), (7) and the output classes  $X_{kj}$  and  $Y_{kj}$ ,  $j = \overline{1,7}$ , the  
 290 problem of tuning the composite fuzzy model for content management is formulated as follows  
 291 [24–27]. For each output class  $X_{kj}$  and  $Y_{kj}$ ,  $j = \overline{1,7}$ , the solution set (17), (18) should be found  
 292 which provides the least distance between observed and model fuzzy effects vectors in correlations  
 293 (6), (7):

$$294 \quad [f_y^k(\underline{\mu}_x^k, \underline{\mu}_z^k, \mathbf{Q}_{1-2}^k) - \mu_y^k(Y_{kj})]^2 = \min_{\underline{\mu}_x^k, \underline{\mu}_z^k}, \quad (22)$$

$$295 \quad [f_x^k(\underline{\mu}_t^1, \underline{\mu}_t^2, \mathbf{H}_{1-2}^k) - \mu_x^k(X_{kj})]^2 = \min_{\underline{\mu}_t^1, \underline{\mu}_t^2}. \quad (23)$$

296 Given the primary fuzzy model (8)-(10) and the output classes  $A_{kj}$ ,  $V_{kj}$  and  $U_j$ ,  $j = \overline{1,7}$ , the  
 297 problem of tuning the composite fuzzy model for rating evaluation is formulated as follows [24–27].  
 298 For each output class  $A_{kj}$ ,  $V_{kj}$  and  $U_j$ ,  $j = \overline{1,7}$ , the solution set (19)-(21) should be found which  
 299 provides the least distance between observed and model fuzzy effects vectors in correlations (8)-(10):

$$300 \quad [f_u(\underline{\mu}_v^1, \dots, \underline{\mu}_v^n, \mathbf{W}^1, \dots, \mathbf{W}^n) - \mu_u(U_j)]^2 = \min_{\underline{\mu}_v^1, \dots, \underline{\mu}_v^n}, \quad (24)$$

$$301 \quad [f_v^k(\underline{\mu}_z^k, \underline{\mu}_a^k, \mathbf{R}_{1-2}^k) - \mu_v^k(V_{kj})]^2 = \min_{\underline{\mu}_z^k, \underline{\mu}_a^k}, \quad (25)$$

$$302 \quad [f_a^k(\underline{\mu}_x^k, \mathbf{G}^k) - \mu_a^k(A_{kj})]^2 = \min_{\underline{\mu}_x^k}. \quad (26)$$

303 The genetic-neuro algorithm is used for solving the optimization problems (16), (22)–(26) of  
 304 tuning the primary fuzzy model and rule-based solutions of primary fuzzy relational equations [28,  
 305 29].

## 306 3.3. Solving Fuzzy Relational Equations

307 Let us consider the construction of the composite fuzzy rules (17)-(21). The primary system of  
 308 fuzzy relational equations (6)-(10) has the form:

309 - for content management

$$310 \quad \mu_x^{k,1} = ((\mu_t^{1,1} \wedge h_1^{1,11}) \vee \dots \vee (\mu_t^{1,3} \wedge h_1^{1,31})) \wedge ((\mu_t^{2,1} \wedge h_2^{2,11}) \vee (\mu_t^{2,2} \wedge h_2^{2,21}));$$

$$\dots$$

$$\mu_x^{k,3} = ((\mu_t^{1,1} \wedge h_1^{1,13}) \vee \dots \vee (\mu_t^{1,3} \wedge h_1^{1,33})) \wedge ((\mu_t^{2,1} \wedge h_2^{2,13}) \vee (\mu_t^{2,2} \wedge h_2^{2,23}));$$
(27)

$$311 \quad \mu_y^{k,1} = ((\mu_x^{k,1} \wedge q_1^{k,11}) \vee \dots \vee (\mu_x^{k,3} \wedge q_1^{k,31})) \wedge ((\mu_z^{k,1} \wedge q_2^{k,11}) \vee \dots \vee (\mu_z^{k,3} \wedge q_2^{k,31}));$$

$$\dots$$

$$\mu_y^{k,3} = ((\mu_x^{k,1} \wedge q_1^{k,13}) \vee \dots \vee (\mu_x^{k,3} \wedge q_1^{k,33})) \wedge ((\mu_z^{k,1} \wedge q_2^{k,13}) \vee \dots \vee (\mu_z^{k,3} \wedge q_2^{k,33}));$$
(28)

312 - for rating evaluation

$$313 \quad \mu_a^{k,1} = (\mu_x^{k,1} \vee g^{k,11}) \vee \dots \vee (\mu_x^{k,3} \vee g^{k,31});$$

$$\dots$$

$$\mu_a^{k,3} = (\mu_x^{k,1} \vee g^{k,13}) \vee \dots \vee (\mu_x^{k,3} \vee g^{k,33});$$
(29)

$$314 \quad \mu_v^{k,1} = ((\mu_z^{k,1} \wedge r_1^{k,11}) \vee \dots \vee (\mu_z^{k,3} \wedge r_1^{k,31})) \wedge ((\mu_a^{k,1} \wedge r_2^{k,11}) \vee \dots \vee (\mu_a^{k,3} \wedge r_2^{k,31}));$$

$$\dots$$

$$\mu_v^{k,3} = ((\mu_z^{k,1} \wedge r_1^{k,13}) \vee \dots \vee (\mu_z^{k,3} \wedge r_1^{k,33})) \wedge ((\mu_a^{k,1} \wedge r_2^{k,13}) \vee \dots \vee (\mu_a^{k,3} \wedge r_2^{k,33}));$$
(30)

$$315 \quad \mu_u^1 = ((\mu_v^{1,1} \vee w^{1,11}) \vee \dots \vee (\mu_v^{1,3} \vee w^{1,31})) \vee \dots \vee ((\mu_v^{n,1} \vee w^{n,11}) \vee \dots \vee (\mu_v^{n,3} \vee w^{n,31}));$$

$$\dots$$

$$\mu_u^3 = ((\mu_v^{1,1} \vee w^{1,13}) \vee \dots \vee (\mu_v^{1,3} \vee w^{1,33})) \vee \dots \vee ((\mu_v^{n,1} \vee w^{n,13}) \vee \dots \vee (\mu_v^{n,3} \vee w^{n,33})).$$
(31)

316 In (27), (29) and (31), the trend fuzzy relations are tuned for each content category. In (28) and  
 317 (30), the trend fuzzy relations are tuned regardless of the genre. Evaluation of the ratings of TV  
 318 programs in the channel broadcasting network is carried out with the help of the authors'  
 319 monitoring system [34]. The training sample includes data from 2015 to 2018.

320 The results of the primary fuzzy model tuning are presented in Appendix A. Parameters of the  
 321 membership functions for the input and output primary fuzzy terms are presented in Tables A1, A2.  
 322 The primary fuzzy relational matrices after tuning are presented in Tables A3-A7.

323 The results of the composite fuzzy model tuning are presented in Appendix B. In Tables B1-B5,  
 324 the sets of solutions in the form of the fuzzy causes vectors correspond to the fuzzy effects vectors  
 325 obtained for the given decision classes. Bounds of the decision classes  $U_j$  and  $Y_{kj}$ ,  $j = \overline{1,7}$ , were  
 326 defined as follows:

$$327 \quad [\underline{u}, \overline{u}] = \underbrace{[0, 3]}_{shD} \cup \underbrace{[3, 6]}_{mD} \cup \underbrace{[6, 9]}_{wD} \cup \underbrace{[9, 11]}_{St} \cup \underbrace{[11, 13]}_{wI} \cup \underbrace{[13, 17]}_{mI} \cup \underbrace{[17, 20]}_{shI},$$

$$328 \quad [y_k, \bar{y}_k] = \underbrace{[-10, -7]}_{shD} \cup \underbrace{[-7, -3]}_{mD} \cup \underbrace{[-3, -1]}_{wD} \cup \underbrace{[-1, 1]}_N \cup \underbrace{[1, 3]}_{wI} \cup \underbrace{[3, 7]}_{mI} \cup \underbrace{[7, 10]}_{shI}.$$

329 For the given bounds of decision classes, the fuzzy effects vectors  $\mu_u(U_j)$  and  $\mu_y^k(Y_{kj})$  were  
 330 defined with the help of the primary membership functions of the variables  $u$  and  $y_k$  (Table A2).  
 331 For the decision classes  $V_{kj}$ ,  $A_{kj}$  and  $X_{kj}$ ,  $j = \overline{1,7}$ , the fuzzy effects vectors  $\mu_v^k(V_{kj})$ ,  $\mu_a^k(A_{kj})$   
 332 and  $\mu_x^k(X_{kj})$  were defined by the fuzzy solution vectors obtained for the higher levels of the  
 333 hierarchical tree of logic inference.

334 The sets of interval rules corresponding to the sets of solutions from Tables B1-B5 are presented  
 335 in Tables 6-10. Linguistic interpretation of the obtained solutions allows generating the hierarchical  
 336 composite fuzzy rules. For the simplified and extended composition laws [22], the sets of solutions  
 337 are interpreted in the form of the “single input – single output” (Tables 6, 8) and “multiple inputs –  
 338 single output” (Tables 7, 9, 10) rules [28, 29]. The lower and upper bounds of the interval rules were  
 339 obtained with the help of the primary membership functions of the variables  $t_1$ ,  $t_2$  and  $z_k$   
 340 (Table A1).

341 **Table 6.** Composite fuzzy rules for the TV rating

IF (or)			THEN
$v_1(t, p)$	$v_2(t, p)$	$v_3(t, p)$	$u(t, p)$
0–2.0, shD	0–4.0, shD	0–9.0, shD–wD	shD
2.0–4.4, shD–mD	3.8–4.4, mD	0–9.0, shD–wD	mD
4.4–7.4, mD–wD	4.4–7.4, mD–wD	6.2–11.2, wD–St	wD
6.8–9.0, wD	7.6–10.0, wD–St	6.2–11.2, wD–St	St
8.4–12.9, St–wI	10.2–12.9, wI	8.8–14.0, St–wI	wI
10.8–16.7, wI–mI	12.4–15.6, wI–mI	10.8–15.6, wI–mI	mI
16.7–20, shI	15.6–20, mI–shI	15.6–20, mI–shI	shI

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**Table 7.** Composite fuzzy rules for the restored genre ratings

IF (and)		THEN
$z_k(y_k(t, p))$	$a_k(t, p)$	$v_k(t, p)$
0–1.2	shD	shD
1.2–2.2	shD	mD
0–2.2	mD	
2.2–3.7	mD–wD	wD
3.5–4.4	wD–St	
2.9–4.4	St–wI	St
3.7–5.0	wD–wI	
5.0–6.9	wD–St	
5.1–6.9	wI–mI	wI
5.9–7.7	St–wI	
6.9–7.9	wI–mI	mI
5.4–7.6	mI–shI	
7.9–10	shI	shI

350

**Table 8.** Composite fuzzy rules for the ad break-factor

IF (or)			THEN
$x_1(t, p)$	$x_2(t, p)$	$x_3(t, p)$	$a_k(t, p)$
shD–mD	shD	shD	shD
mD	mD	mD	mD
mD–wD	mD–wD	mD–wD	wD
wD–St	wD–wI	St–wI	St
wI–mI	wI	wI–mI	wI
St–mI	mI	wI–mI	mI
shI	shI	shI	shI

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**Table 9.** Composite fuzzy rules for the control action

IF (and)		THEN
$x_k(t, p)$	$z_k(t, p-1)$	$y_k(t, p)$
shD	8.3–10	shD
mD	7.6–10	mD
mD–shD	6.1–7.6	
mD–shD	3.6–5.9	
wD–wI	7.6–10	wD
mD–shD	1.8–2.6	
wD–wI	5.9–6.2	
wD	3.4–6.2	
mI	7.1–10	

shD	0–1.8	
St	3.8–5.0	N
shI	7.9–10	
mD	0–2.1	
wD–wI	3.4–3.5	wI
wI	3.50–5.8	
mI–shI	7.4–7.9	
wD–wI	0–1.9	
mI–shI	3.4–5.3	mI
mI	0–1.9	
mI–shI	1.9–2.8	
shI	0–1.4	shI

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**Table 10.** Composite fuzzy rules for the viewers' demand

IF (and)						THEN
k=1		k=2		k=3		
t <sub>1</sub>	t <sub>2</sub>	t <sub>1</sub>	t <sub>2</sub>	t <sub>1</sub>	t <sub>2</sub>	x <sub>k</sub> (t,p)
12–14	1–5	5–8	1–5	11–14	1–5	shD
10–12	1–5	5–8	6, 7	10–12	1–5	mD
14–16	1–5	8–11	1–5			
11–17	6,7					
5–7	1–5	8–10	6, 7	8–10	1–7	wD
18–20	6, 7	10–12	1–5			
22–24	1–5					
7–8	1–7			5–8	1–7	St
19–21	6, 7	12–16	1–7	12–16	6, 7	
21–22	1–7			14–16	1–5	
8–11	6, 7	16–18	1–7	9–12	6, 7	wI
16–18	1–7			16–18	1–7	
8–10	1–5	17–19	6, 7	17–20	1–7	mI
18–20	1–5	18–24	1–5	18–24	1–5	
20–21	1–5	18–24	6, 7	19–24	6, 7	shI

### 353 3.4. Example of the TV Rating Control: Construction of the Weekly TV Program

354 The values ⟨viewers' demand  $\hat{x}_k(t,p)$ , supply before and after control  $\hat{z}_k(t,p-1)$ ,  $\hat{z}_k(t,p)$ ,  
355 control action  $\hat{y}_k(t,p) = \hat{z}_k(t,p) - \hat{z}_k(t,p-1)$ , rating  $\hat{u}(t,p)$ ⟩, corresponding to the experienced  
356 manager actions were taken as the training data sample. In this case, the TV rating was maintained  
357 at a consistently high level, and the unmet viewers' demand was reduced to a minimum. The unmet  
358 demand after control in each genre category can be defined as  $\Delta_k(t,p) = x_k(t,p) - z_k(t,p)$ . It is  
359 supposed, that for the balanced demand  $x_k(t,p) = z_k(t,p)$ . The experimental values  $\hat{x}_k(t,p)$  for  
360 each genre category can be obtained from Table 10. The experimental values  $\hat{z}_k(t,p-1)$ ,  $\hat{z}_k(t,p)$   
361 and  $\hat{u}(t,p)$  were determined on the basis of the weekly top 20 rating [30].

362 The training sample fragment is presented in Tables 11, 12 in the form of the weekly TV  
 363 program constructed for weekdays and weekends from 1.10.2018 to 7.10.2018. Media planning is  
 364 accompanied with the analysis of the main television indices. Tables 11, 12 reflect the dynamics of  
 365 the demand and supply change for each genre during the day. The experimental demand  
 366  $\hat{x}_k(t,p)$  and supply  $\hat{z}_k(t,p-1)$  are balanced by the model control action  $y_k(t,p)$ . To balance the  
 367 demand, the priority content category is chosen for each time slot. The priority genre  $k$  is in high  
 368 demand compared to other categories. The obtained pairs "time slot  $t$  – imdb-rating  $z_k(t,p)$  after  
 369 control" represent the broadcast grid of the forthcoming week. The characteristics of the generated  
 370 content are evaluated by the ad break-factor  $a_k(t,p)$ . Stability of the content to the ad break-factor  
 371 guarantees the high television ratings  $v_k(t,p)$ . Planning without alternatives is based on the past  
 372 behavior of viewers. In this case, monitoring results depict the rating restored after the ad unit,  
 373 i.e.  $u(t,p) = v_k(t,p)$ . In the case of the alternative propositions  $v_k(t,p)$ , the preliminary selection is  
 374 based on users preferences. Finally, the model  $u(t,p)$  and experimental  $\hat{u}(t,p)$  ratings are compared  
 375 for each time slot.

376 It is shown from Tables 11, 12, that the weekly top 20 list covers each time slot. For the political  
 377 genre, the offered programs have balanced the viewers' demand. On weekdays from 18 to 23 hours,  
 378 some popular serials have been offered instead of the entertainment programs. On weekends, the  
 379 demand for the genre of television serials from 18 to 23 hours has been satisfied with the programs  
 380 of the sports and entertainment genre.

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**Table 11.** Weekdays timetable

Time slot	Television indices for genre $k$							
	$\hat{x}_k(t,p)$	$\Delta_k(t,p)$	$\hat{z}_k(t,p-1)$	$y_k(t,p)$	$z_k(t,p)$	$a_k(t,p)$	$v_k(t,p)/u(t,p)$	$\hat{u}(t,p)$
7-8	1, 3, St	2, shD						
8-9	1, mI;	2, mD	1, 3, 5.6	0	1, 3, 5.6	1, 3, 0.99	1, 3, 14.0	1, 3, 15.0
9-10	3, wD							
10-11	2, wD	1, 3, mD	2, 4.5	0	2, 4.5	2, 0.84	2, 5.5	2, 4.6
11-12								
12-13	1, 2, St	3, shD	1, 3.9	0	1, 3.9	1, 0.89	1, 4.3	1, 4.5
13-14								
14-15	2, 3, St	1, mD	2, 4.8	2, +0.7	2, 5.5	2, 0.88	2, 11.0	2, 9.5
15-16								
16-17	2, 3 wI	1, wI	2, 5.9	2, +0.5	2, 6.4	2, 0.89	2, 12.9	2, 12.1
17-18								
18-19	2, 3, mI	1, mI	2, 8.6	0	2, 8.6	2, 0.99	2, 16.5	2, 14.0
19-20								
20-21	1, shI	2, 3, mI	1, 8.3	0	1, 8.3	1, 0.98	1, 12.0	1, 10.5
21-22	2, 3, mI	1, St	2, 6.7	2, +1.3	2, 8.0	2, 0.87	2, 11.5	2, 7.5
22-23	2, 3, mI	1, wD		2, +1.3 or	2, 7.4 or	2, 0.86 or	2, 9.0 or	2, 6.8
23-24			2, 6.1	3, +8.1	3, 8.1	3, 0.72	3, 8.5	

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**Table 12.** Weekends timetable

Time slot	Television indices for genre k								
	$\hat{x}_k(t,p)$	$\Delta_k(t,p)$	$\hat{z}_k(t,p-1)$	$y_k(t,p)$	$z_k(t,p)$	$a_k(t,p)$	$v_k(t,p)/u(t,p)$	$\hat{u}(t,p)$	
7-8	1, 3, St	2, mD	1, 3, 4.4	0	1, 3, 4.4	1, 3, 0.85	1, 3, 5.5	1, 3, 4.5	
8-9									
9-10	1, wI; 3, wD	2, wD	1, 3, 3.1	1, 3, +1.9	1, 3, 5.0	1, 3, 0.90	1, 3, 10.0	1, 3, 8.1	
10-11	1, 3, wI	2, wD	3, 6.5	2, +4.8 or	2, 4.8 or	2, 0.81 or	2, 7.2 or	3, 6.1	
11-12	2, wD; 3, wI	1, mD		3, 0	3, 6.5	3, 0.84	3, 8.5		
12-13				2, -0.3	2, 5.5	2, 0.79	2, 11.0	2, 9.0	
13-14	2, 3, St	1, mD	2, 5.8						
14-15					2, +0.4	2, 6.2	2, 0.81	2, 11.5	2, 8.6
15-16									
16-17	2, 3, wI	1, mD	2, 6.9	2, +0.5	2, 7.4	2, 0.75	2, 8.2	2, 5.5	
17-18	2, wI	1, 3, wI							
18-19	2, 3, mI	1, wD	3, 8.8	2, +9.1 or	2, 9.1 or	2, 0.82 or	2, 12.5 or	3, 8.2	
19-20	2, shI; 3, mI	1, wD		3, 0	3, 8.8	3, 0.90	3, 12.9		
20-21	1, St;	2, 3, shI	1, 6.7	0	1, 6.7	1, 0.90	1, 8.3	1, 7.6	
21-22	2, 3, shI	1, St	3, 9.1	2, +9.3 or	2, 9.3 or	2, 0.95 or	2, 12.0 or	3, 8.6	
22-23				3, 0	3, 9.1	3, 0.99	3, 12.2		
23-24	2, 3, shI	1, wD	3, 8.7	0	3, 8.7	3, 0.79	3, 9.0	3, 5.6	

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Comparison of the model and experimental ratings for  $p \in [1, 32]$  weeks is shown in Figure 1. Figure 1 depicts the dynamics of the average weekly rating change at the level of each air-hour on weekdays and weekends from autumn 2017 to spring 2018. When compiling the average weekly rating, the time range is  $t_1 \in [8, 23]$  hours, since the programs in the range  $t_1 \in [0, 7]$  do not fall into the weekly top 20 rating.

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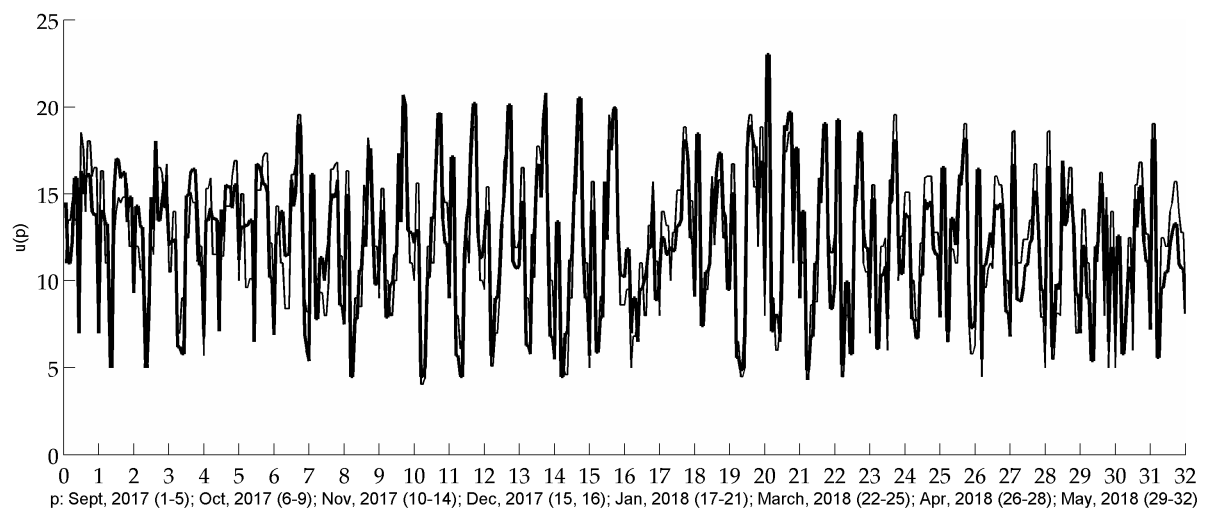
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The fuzzy control model ensures the correctness of rating evaluation at the level of RMSE=1.88 for weekdays; RMSE=2.54 for weekends. On weekdays, the TV ratings are less sensitive to the multichannel environment. The ratings are sufficiently high and the evaluation error decreases. On weekends, the multichannel environment influences the TV ratings significantly. The ratings are unstable and the evaluation error increases.

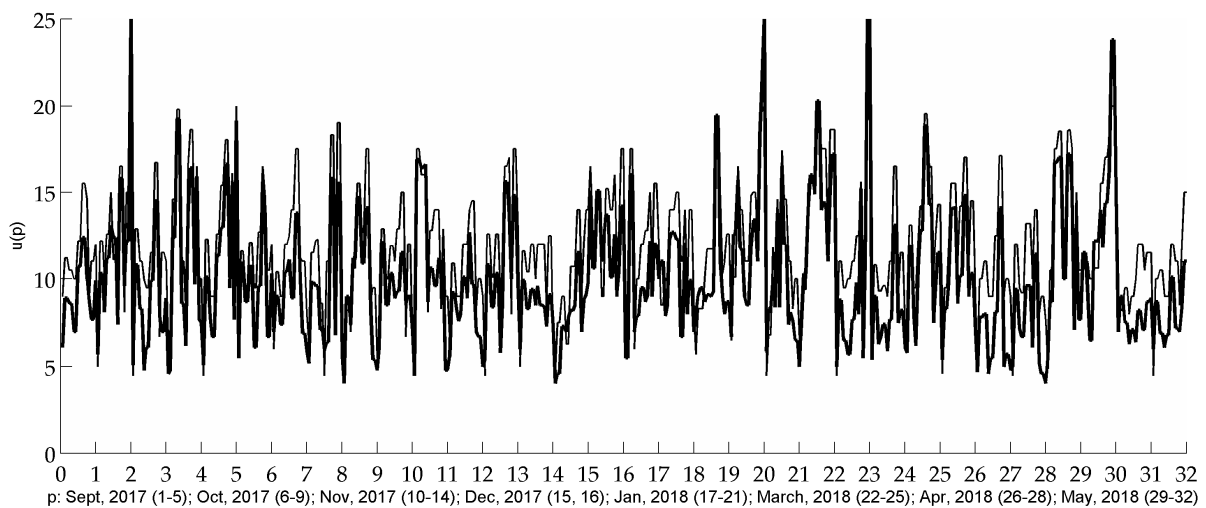




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(a)



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(b)

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**Figure 1.** The dynamics of the average weekly TV rating change for the model (---) and experimental (—) control action: (a) on weekdays; (b) on weekends.

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#### 4. Effectiveness Estimation of the Hybrid Approach

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We shall denote:

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$n$  is the number of input parameters (genres);

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$T$  and  $M$  are the numbers of input and output primary fuzzy terms;

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$Z$  is the number of composite fuzzy rules.

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Candidate rule generation corresponds to the construction of a zero option of the timetable. In semantic training, candidate rule generation requires solving the optimization problem with  $2nZ$  variables for boundaries of interval rules [19, 20]. Application of the hybrid approach allows reducing the number of tuning parameters by solving  $Z$  optimization problems for  $2T$  boundaries of

416 significance measures [24–27]. Rule generator tuning is the optimization problem with 2T+2M+TM  
417 variables for the trend relational matrix and membership functions parameters.

418 Selection, that is, finding the best configuration of zero option terms and rules, corresponds to  
419 the granulation of the television time. As a result, the broadcast grid can be represented in the form  
420 of the time slots and the granulated ratings of the content.

421 In semantic training, selection requires solving the optimization problem with nZ variables. A  
422 term selection sign with the possibility of merging the similar terms, as well as the degree of the rule  
423 relevance are subject to tuning [19, 20]. In the case of Z solutions of the trend system of equations,  
424 selection is reduced to maintaining the level of detail and density of coverage. Application of the  
425 hybrid approach reduces the number of tuning parameters by solving Z optimization problems for T  
426 modified terms in each rule [24–27].

427 The problem of TV rating control is reduced to the parallel programming of the television time  
428 for six genres (news and analytics, TV serials, documentary projects, programs for children, sports,  
429 entertainment). The permissible value of the planning time-frame is 1 hour. The time of tuning the  
430 fuzzy control model according to the method [19, 20] is 1 hour 20 min, which exceeds the planning  
431 time-frame (Intel Core i5 LGA1151 3.5 GHz processor). The tuning time for this method is 45 min,  
432 which allows media planning at the level of each air-hour.

## 433 5. Conclusions

434 1. The fuzzy model for retaining the TV rating is proposed in the framework of fuzzy resources  
435 control. The demand-supply trends are described by the primary fuzzy knowledge bases, where the  
436 increase or decrease of the television indices is described by the primary fuzzy terms. The refined  
437 fuzzy model is built using the linguistic modifiers of the primary fuzzy terms. The time factors,  
438 content and advertisement attributes are taken into account to balance the demand and supply. The  
439 fuzzy model of the demand allows generating item and timing recommendations simultaneously.  
440 The fuzzy model of the supply provides restoration of the ratings in the multichannel environment.

441 2. The method of expert recommendation systems construction for TV domain is proposed. TV  
442 experts define the primary fuzzy relational matrices. Automated media planning is carried out by  
443 solving the primary system of fuzzy relational equations. The granulated television time is  
444 determined by the rule-based solution set. The program attributes for each time slot are determined  
445 by significance measures of the primary fuzzy terms. The recommendation accuracy is achieved  
446 through the complete rule-based solution set. Rule set refinement by solving fuzzy relational  
447 equations allows avoiding labor-intensive procedures for the generation and selection of expert  
448 rules. Generation of the weekly TV program in the form of the granular solutions provides the  
449 decrease of the computational costs needed for the programming of the television time.

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452 Krupelnitsky, L. and Rakytyanska, H.; software, Krupelnitsky, L. and Rakytyanska, H.; validation, Azarov, O.,  
453 Krupelnitsky, L. and Rakytyanska, H.; formal analysis, Rakytyanska, H.; investigation, Krupelnitsky, L. and  
454 Rakytyanska, H.; resources, Krupelnitsky, L.; data curation, Krupelnitsky, L.; writing—original draft  
455 preparation, Rakytyanska, H.; writing—review and editing, Rakytyanska, H.; visualization, Krupelnitsky, L.  
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463 publish the results.

## 464 Appendix A

### 465 Results of the Primary Fuzzy Model Tuning

466 **Table A1.** Parameters of the membership functions for the input primary fuzzy terms

Parameter	$t_1$			$t_2$		$z_k$		
	M	A	Ev	Wd	We	D	St	I
$\beta$	5	14	22	1	6	0.51	4.43	9.12
$\sigma$	3.92	3.19	3.75	4.52	0.94	2.67	1.28	2.54

467 **Table A2.** Parameters of the membership functions for the output primary fuzzy terms

Parameter	$y_k$			$u$		
	D	N	I	D	St	I
$\beta$	-9.53	0.41	9.60	2.14	9.22	16.70
$\sigma$	5.15	2.76	4.89	3.70	2.56	4.39

468 **Table A3.** Primary fuzzy relations “viewing time - demand” after tuning

IF	THEN $x_1(t,p)$			THEN $x_2(t,p)$			THEN $x_3(t,p)$			
	D	St	I	D	St	I	D	St	I	
$t_1$	M	0.11	0.75	0.67	0.64	0.50	0.16	0.35	0.65	0.48
	A	0.79	0.62	0.54	0.22	0.75	0.51	0.59	0.82	0.67
	Ev	0.10	0.51	0.80	0.14	0.83	1.0	0.18	0.77	0.94
$t_2$	Wd	0.80	0.91	1.0	0.61	0.85	0.68	0.64	0.89	0.67
	We	0.56	0.70	0.59	0.27	0.92	1.0	0.19	0.86	1.0

469 **Table A4.** Primary fuzzy relations “demand and supply – control action” after tuning

IF	THEN $y_k(t,p)$			
	D	N	I	
$x_k(t,p)$	D	0.98	0.81	0.12
	St	0.75	0.68	0.95
	I	0.07	0.62	0.90
$z_k(t,p-1)$	D	0.14	0.85	0.97
	St	0.83	0.70	0.65
	I	0.91	0.63	0.18

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**Table A5.** Primary fuzzy relations “demand – ad break-factor” after tuning

IF	THEN $a_k(t,p)$			
	D	St	I	
$x_1(t,p)$	D	0.65	0.52	0.41
	St	0.26	0.82	0.76
	I	0.19	0.75	0.93
$x_2(t,p)$	D	0.88	0.24	0.10
	St	0.43	0.59	0.25
	I	0.16	0.71	0.90
$x_3(t,p)$	D	0.90	0.32	0.09
	St	0.27	0.81	0.67
	I	0.11	0.60	0.99

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**Table A6.** Primary fuzzy relations “supply with the ad break-factor – restored genre ratings” after tuning

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IF	THEN $v_k(t,p)$			
	D	St	I	
$z_k(y_k(t,p))$	D	1.0	0.12	0
	St	0.56	0.80	0.58
	I	0.42	0.78	0.95
$a_k(t,p)$	D	0.93	0.08	0
	St	0.50	0.67	0.33
	I	0.39	0.70	0.81

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**Table A7.** Primary fuzzy relations “restored genre ratings – rating of the channel” after tuning

IF	THEN $u(t,p)$			
	D	St	I	
$v_1(t,p)$	D	0.96	0.20	0
	St	0.45	0.84	0.57
	I	0	0.63	0.90
$v_2(t,p)$	D	0.75	0.26	0
	St	0.34	0.87	0.54
	I	0	0.59	0.83
$v_3(t,p)$	D	0.69	0.30	0
	St	0.33	0.76	0.51
	I	0	0.50	0.82

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479 **Appendix B**480 *Results of the Composite Fuzzy Model Tuning*481 **Table B1.** Solutions of fuzzy relational equations for the TV rating

Genre	IF			THEN
	$\mu_v^k$			$\mu_u(U_j)$
	D	St	I	
k=1	0.96–1			shD, (0.97, 0.40, 0.21)
k=2	0.75–1	0–0.21	0–0.21	
k=3	0.30–1			
k=1	0.72			mD, (0.72, 0.56, 0.25)
k=2	0.25–0.72	0.25	0.25	
k=3	0.30–1			
k=1, 2		0.30–0.80		wD, (0.43, 0.80, 0.27)
k=3	0–0.45	0.51–1	0.27	
k=1		0.84–1		St, (0.30, 1, 0.39)
k=2	0.45	0.30–0.84	0–0.51	
k=3		0.51–1		
k=1		0.76	0–0.51	wI, (0.21, 0.76, 0.54)
k=2	0.34	0.30–0.76	0–0.51	
k=3		0.76–1	0.57	
k=1			0.74	mI, (0.17, 0.50, 0.74)
k=2, 3	0.34	0–0.63	0.34–0.74	
k=1			0.90–1	shI, (0.14, 0.32, 0.91)
k=2, 3	0–0.32	0–0.32	0.82–1	

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**Table B2.** Solutions of fuzzy relational equations for the restored genre ratings

IF						THEN
$\mu_Z^k$			$\mu_a^k$			$\mu_v^k(V_{kj})$
D	St	I	D	St	I	
0.93-1	0.21	0-0.21	0.93-1	0.21	0-0.21	shD, (0.96, 0.21, 0.21)
0.72	0.25	0-0.25	0.72-1	0.25	0-0.25	mD, (0.72, 0.25, 0.25)
0.72-1	0-0.25	0.25	0.72	0-0.25	0.25	
0.45-1	0.80-1 0-0.80	0.27-1 0.78-1	0-0.50	0.67-1	0-0.33	wD, (0.45, 0.80, 0.27)
0.56	0.80-1	0-0.58	0-0.50	0.67-1	0.58-1	St, (0.45, 1, 0.51)
0-0.56	0.80-1	0.58	0-0.50	0.50-1	0.70-1	
0.45	0-0.80	0.78-1	0-1	0.67-1	0.51	
0-1	0.76	0-0.58	0-0.39	0-0.39	0.70-1	wI, (0.34, 0.76, 0.57)
0-0.42	0-0.42	0.76	0-1	0.67-1	0.57	
0-0.42	0-0.42	0.74-1	0-1	0.67	0.74	mI, (0.34, 0.63, 0.74)
0-1	0-0.63	0.74	0-0.39	0-0.39	0.74-1	
0-0.32	0-0.32	0.81-1	0-0.39	0-0.39	0.81-1	shI, (0.32, 0.32, 0.82)

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**Table B3.** Solutions of fuzzy relational equations for the ad break-factor

Genre	IF			THEN
	$\mu_x^k$			$\mu_a^k (A_{kj})$
	D	St	I	
k=1	0.65–1	0–0.21	0–0.21	shD, (0.93, 0.21, 0.21)
k=2	0.88–1	0–0.24	0–0.24	
k=3	0.90–1	0.21	0–0.21	
k=1	0.65–0.72	0.25	0–0.25	mD, (0.72, 0.25, 0.25)
k=2, 3	0.72			
k=1	0.50–0.67	0–0.41	0–0.33	wD, (0.50, 0.67, 0.33)
k=2	0.50	0.59–0.67	0.33	
k=3	0.50	0.67	0–0.33	
k=1	0.50	0.82–1	0–0.41	St, (0.50, 1, 0.51)
k=2		0.67–1	0.51	
k=3		0.81–1	0–0.51	
k=1	0.39	0.57–0.67	0–0.57	wI, (0.39, 0.67, 0.57)
k=2	0–0.43	0.59–0.67	0.57	
k=3	0.39	0.67	0–0.67	
k=1	0.39	0–0.74	0.74	mI, (0.39, 0.59, 0.74)
k=2	0–0.43	0.59		
k=3	0.39	0–0.60		
k=1, 2	0–0.39	0.39	0.81–1	shI, (0.39, 0.39, 0.81)
k=3	0–0.27			

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**Table B4.** Solutions of fuzzy relational equations for the control action

IF						THEN
$\mu_x^k$			$\mu_z^k$			$\mu_y^k (Y_{kj})$
D	St	I	D	St	I	
0.91-1	0-0.12	0-0.12	0-0.14	0-0.14	0.91-1	shD, (0.96, 0.17, 0.11)
0.73-1	0.16	0-0.16	0.38	0-0.38	0.73-1	mD, (0.73, 0.38, 0.16)
0.73-1	0.38	0.16	0.16	0.73-1	0-0.38	
0-0.38	0.73-1	0.38	0-0.16	0.16	0.73-1	
0.62-1	0.20	0-0.20	0.62-1	0.43	0-0.43	wD, (0.43, 0.62, 0.20)
0.43	0.62-1	0-0.20	0.20	0.62-1	0-0.43	
0-0.43	0.43	0.62-1	0-0.20	0.20	0.62-1	
0.81-1	0.27	0-0.27	0.81-1	0.30	0-0.30	N, (0.30, 0.96, 0.27)
0-0.30	0.81-1	0-0.27	0-0.27	0.81-1	0-0.30	
0-0.30	0.30	0.81-1	0-0.27	0.27	0.81-1	
0.74-1	0.45	0-0.45	0.74-1	0.21	0-0.21	wI, (0.21, 0.74, 0.45)
0-0.21	0.68-1	0.45	0-0.45	0.68-1	0.21	
0-0.21	0.21	0.68-1	0-0.45	0.45	0.68-1	
0.16	0.77-1	0-0.40	0.77-1	0.40	0-0.16	mI, (0.16, 0.40, 0.77)
0-0.16	0.40	0.65-1	0-0.40	0.65-1	0.16	
0-0.16	0.16	0.77-1	0.77-1	0-0.40	0.40	
0-0.14	0-0.21	0.90-1	0.90-1	0-0.21	0.14	shI, (0.11, 0.21, 0.94)

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**Table B5.** Solutions of fuzzy relational equations for the viewers' demand

Genre	IF					THEN	
	$\mu_t^1$		$\mu_t^2$			$\mu_x^k (X_{kj})$	
	M	A	Ev	Wd	We		
k=1	0-0.15	0.79-1	0-0.15	0.56-1	0-1	shD, (0.91, 0.16, 0.12)	
k=2	0.61-1	0-0.17	0-0.17	0.61-1	0-1		
k=3	0-0.15	0.59-1	0-0.15	0.81-1	0-0.59		
k=1	0-0.38	0.62-1	0-0.29	0.62-1	0.56-1	mD, (0.62, 0.38, 0.20)	
k=2	0.62	0-0.50	0-0.20	0.61-1	0.61-1		
k=3	0-0.38	0.62	0-0.20	0.62-1	0-1		
k=1	0.80-1	0-0.43	0.80-1	0.73-1	0-0.73	wD, (0.43, 0.73, 0.38)	
k=2	0.43	0.73-1	0-0.14		0-0.73		0.73-1
k=3	0.65	0-0.41	0-0.41				
k=1	0.75-1	0-0.30	0.81-1	0.81-1	0-0.70	St, (0.30, 0.81, 0.27)	
k=2	0.30	0.75-1	0-0.27	0-0.70	0.70-1		
				0.81-1	0.81		
k=3	0.65-1	0.81-1	0-0.27	0-0.81	0-0.81		
k=1	0.68	0-0.54	0-0.51	0.68-1	0-0.68	wI, (0.16, 0.68, 0.45)	
k=2	0-0.22	0.68	0-0.51		0-0.68		0.68-1
k=3	0.45	0.68	0-0.45				
k=1	0-0.65	0-0.40	0-0.65	0.65-1	0-1	mI, (0.21, 0.40, 0.65)	
k=2	0-0.22	0-0.65	0.65-1	0.65	0-0.65		
k=3	0-0.21	0-0.52	0.65-1	0-0.65	0.65-1		
k=1	0-0.18	0.18	0.80-0.90	0.80-1	0-0.59	shI, (0.14, 0.21, 0.80)	
k=2	0-0.22	0-0.51	0.80-1	0-0.80	0.80-1		
k=3	0-0.17	0.21	0.72-1	0-1	0.72-1		

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