Article

Forecasting Events in the Chaotic Dynamics of a Fiber Laser

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Abstract: Being able to forecast events is of great importance in many fields, from brain behavior to earthquakes or stock markets. Because each dynamical system has intrinsic features, different statistical tools have to be used for each system. Here we study the time series of the output intensity of a fiber laser with an ordinal patterns analysis, and we look for temporal correlations in order to statistically forecast the most intense events. We set two thresholds, a low one and a high one, to distinguish between low intensity versus high intensity events. We find that when the time series is performing events below the low threshold it shows some preferred temporal patterns before performing events above a high threshold.

Keywords: forecasting; complex dynamics; fiber laser; chaos; ordinal patterns.

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1. Introduction

Time series analysis is a widespread research field that crosses disciplines. Nature presents many dynamical systems performing complex behavior that are only accessible through the temporal evolution of one or a few of its variables. Understanding the dynamical behavior relies on finding characteristic features that allow us to classify them and make predictions about their immediate future, based on their present demeanor [1–12]. Complex systems constitute a challenge, as the nonlinear interplay between the numerous elements of the system, the presence of time delay, and noise make them impractical for a bottom-up analysis.

Lasers constitute a great example of a dynamical system performing complex behavior. Besides the innumerable technological applications, lasers are a fascinating tool for the study and characterization of nonlinear dynamics from a laboratory-controlled manner [13].

One particular case of laser manifesting complex dynamics are fiber lasers with optical feedback [14]. In previous studies [15,16] it was found that behind the high complexity of the time series of the output intensity of a fiber laser, the system showed some structure that could be unveiled using an ordinal patterns analysis. It was found that the laser executes some preferred intensity and temporal patterns, and those patterns helped to detect a laminar to turbulent transition as the pump power of the laser was increased [14,15]. In Ref. [16], through a simple mathematical model and an ordinal patterns analysis, it was also seen that the transition was related to internal frequencies of the dynamics. In this paper we use the ordinal patterns analysis to study the complex behavior of the output intensity of the fiber laser, to forecast when the laser is changing from emitting low intensity to emit high intensity light.

2. Forecasting method

The ordinal patterns analysis method (OPAM) introduced by Bandt and Pompe in 2012 [17] studies correlations between consecutive events. It defines patterns by comparing the values of n consecutive events in a time series (either magnitudes or time intervals between events), and computes the probabilities of each of the $n!$ possible patterns. For a purely stochastic process, where no correlations exist between events, one would expect to find all patterns equally probable ($P = 1/n!$),

while for systems that present some intensity or temporal correlation one would expect some structure in the probabilities of the patterns. This method has been used to characterize complex dynamics based on the patterns probabilities [9,12,18,19], to distinguish between stochasticity and determinism [20], or to identify transitions in the dynamics of complex networks [21]. In a recent paper [22] the method was used to classify events into two types, and to forecast when the system decides to change from one type of events to the other one.

The OPAM transforms the original series of m events, $X = \{x_1, x_2, ..., x_i, ...\}$, into a series of $m - n$ patterns. To do so, it considers n consecutive events, assigns them correlative integer numbers (0, 1, 2, ..., n), and then orders them from smaller to larger value of the event. For example for dimension $n = 2$, if $x_i < x_{i+1}$, it labels that pattern as 01, if $x_{i+1} < x_i$, it labels that pattern as 10. For dimension $n = 3$ we have six possible patterns (012, 021, 102, 120, 201, 210), i.e., if $x_i < x_{i+1} < x_{i+2}$ it labels that pattern as 012; $x_i < x_{i+2} < x_{i+1}$ is 021 ...

This can be applied to the actual values of the events ($\{x(i)\}$), the magnitudes in the time series, but it can also be applied to the time intervals between events ($\{t(i + 1) - t(i)\}$). The former would give us information about intensity correlations among consecutive events, while the latter about temporal correlations.

For the fiber laser under study both, intensity and temporal correlations were found (see Fig. 2a,b from Ref. [15]), where some patterns where more likely to happen in the time series. The fact that the OPs are not equally probable means that the dynamics is not compatible with a stochastic process, but it is clearly deterministic. The system showed a higher degree of determinism at the laminar-turbulent transition.

3. Results

We analyze the output intensity of a fiber laser with optical feedback. Time series are recorded for 625 ms, with 5×10^7 points and $\Delta t = 12.5\ ps$. The time series are recorded for output powers between 0.80 W and 1.50 W (see Ref. [14] for details). The time series are subtracted the average and normalized to the standard deviation. We detect the peaks in the time series as events and study the time series of these events.
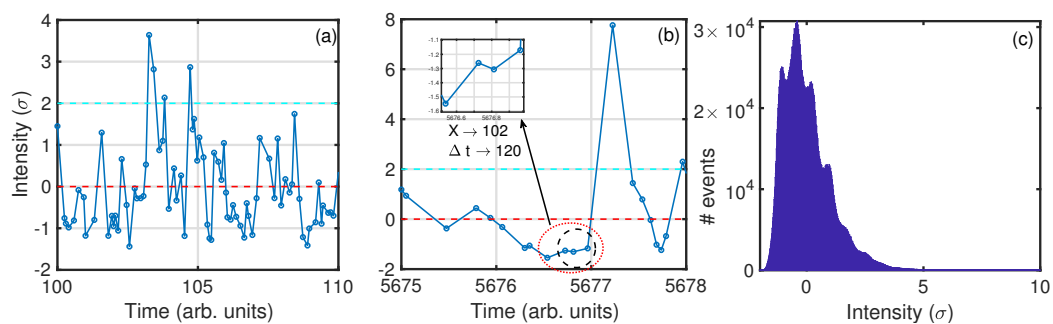

Figure 1. (a) Time series of the events that oscillate around 0 but that present some intensity events larger 2σ. Two thresholds are depicted, a low one at the mean value of the intensities, $th_0 = 0$, and a high one at $th^* = 2\sigma$. **(b)** Detail of the time series where four events are selected, below th_0 that precede a high intensity event, above th^*. The dotted red circle and the dashed black circle indicate the four (three) consecutive events below th_0 used to generate an OP. Three consecutive events when OPs are computed with intensities, and four consecutive events when OPs are computed. **(c)** Histogram of events for power of 1.20 W. The distribution of events shows some structure and a tail that reaches values larger than 6σ.

Figure 1a shows a section of a time series of events. The time series of the output intensity shows a wide distribution with most values around $\pm 2\sigma$ and fewer events reaching values $> 6\sigma$. The insert in Fig. 1b shows four consecutive events and the corresponding OP generated. The black circle in Fig. 1b indicates the three events considered to generate the OP with intensity of the values

68 ($X_{i+1} > X_i > X_{i+2} \rightarrow 102$), and the red circle indicates the four consecutive events considered to
69 generate the OP with their inter-event time-intervals ($\Delta t_{i+1} < \Delta t_{i+2} < \Delta t_i \rightarrow 120$).

70   Figure 1c shows the histogram of the intensity for 1.00 W. This histogram shows a long tail for
71 high intensities. This type of long-tail distribution is present in a wide variety of systems [23–26],
72 that can lead to unwanted phenomena. Hence the importance of understanding and being able to
73 forecast when the system in going to perform events corresponding to the tail, sometimes referred to
74 as extreme events.

75   We define a threshold, th^*, to classify the events as large, or extreme, when they are above that
76 threshold. We are interested in predicting when the system changes from emitting low intensity light
77 (lower than th^*) to emit high intensity light (higher than th^*).

78   To forecast the change from low intensity behavior to high intensity one, we impose that at least
79 three consecutive events are below th^* before the system goes to emit a high intensity event (see
80 figure 1b). We compute the ordinal patterns (OPs) of the three inter-event time-intervals right before
81 the system changes from low to high intensity. This will indicate if there are OPs that are more/less
82 likely to happen before the transition, and which OPs these are.

83   Figure 2 shows the six OPs of dimension three versus the threshold, th^*, for three power outputs,
84 $P = 0.80\ W$ (below laminar to turbulent transition), $P = 0.90\ W$ (at the transition), and $P = 1.10\ W$
85 (after the transition). The top row corresponds to the OPS generated with the intensity of the events.
86 It can be observed that there are OPs that are more likely to happen and OPs that are less likely to
87 happen, before the transition from low to high intensity. This points to some structure in the complex
88 dynamics that manifests as a preference of patterns before performing a high intensity event. The gray
89 region corresponds to the probabilities of an stochastic process of the six OPs equally probable. This
90 region depends on the number of events in each threshold. For high enough threshold the number of
91 high intensity events preceded by low intensity events is too low to have conclusive statistics, and
92 therefore the OPs probabilities lie in the gray region.

93   The second row in Fig. 2 groups the six OPs into three, depending on the last event, to highlight
94 this last event (the event preceding the high intensity event) with respect to the previous two ($XX0 =$
95 $120 + 210$, the latter event is smaller that the previous two; $XX1 = 021 + 201$, the latter event is
96 intermediate; $XX2 = 012 + 102$, the latter event is larger than the previous two). This indicates that
97 between 40% and 50% of the high intensity events will be preceded by pattern $XX2$, i.e., of the three
98 preceding events, the latest one is higher than the other two. In other words this indicates that the
99 change from low intensity to high intensity is not as sharp as it could be. This figure also shows that
100 the OPs probabilities do not depend on the threshold.

101   The bottom row shows the OPs generated with inter-event time-intervals. For low thresholds, th^*,
102 the probabilities are not compatible with a stochastic process but they are mush smaller than the ones
103 computed with intensities. This shows that the dynamics presents less intense temporal correlations
104 than correlations in magnitude. Also, the hierarchy is different from the top to the bottom row. Also in
105 a previous work [15] were found temporal correlations of less relevance than intensity correlations.

106   There is little effect of the power output of the laser on the probabilities of the OPs. For the
107 intensity OPs, the more likely OPs are more probable and the less likely OPs are less probable at the
108 transition power (0.80 W), while for the temporal OPs, all are more equally probable, lying on or very
109 near the gray region.

110   Splitting the peaks into above and below a single threshold does not separate events into those
111 that are much larger than average (events that can be considered as extreme events) and those events
112 around and below average. In Fig. 3 we consider two thresholds, a low one, th_0, and a high one, th^*.
113 We set the lower threshold at $th_0 = 1\sigma$ and scan the high threshold, $2 \leq th^* \leq 4$. We then calculate the
114 OPs probabilities, (see Fig. 3).

115   We find that there are still preferred patterns before jumping from below th_0 to above th^*. The
116 preferred OPs are the same as in Fig. 2. This is interesting as the conditions to select the events to build
117 the OPs are not the same. In the first case it was less restrictive, and low events could be very close
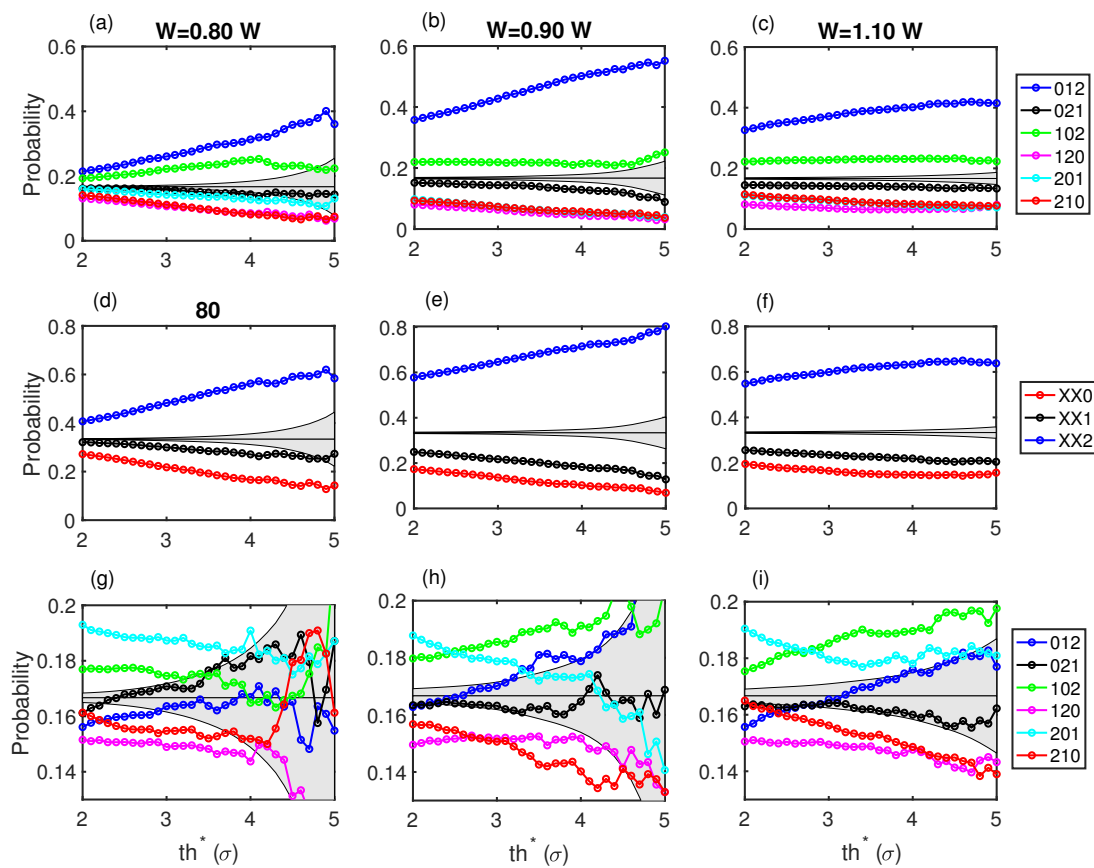
Figure 2. (a-c) Ordinal patterns computed with intensities. Three consecutive events below th^*, that precede an event above th^* are considered for each OP. **(d-f)** The OPs are grouped into $XX0 = 120 + 210$, $XX1 = 021 + 201$, and $XX2 = 012 + 102$. This highlights the fact that the most likely OPs to happen before a jump from low intensity events to high intensity event are those where the last event is larger than the previous two. **(g-i)** OPs computed with inter-event time-intervals. The gray region corresponds to a stochastic process where all six patterns are equally probable.

¹¹⁸ to th^*, while in the second case all events that contribute to the OP are below th_0. This suggests that
¹¹⁹ when the low events are near the low threshold, th_0, there is more chance for a high event to happen.
¹²⁰   With this more restrictive condition of two thresholds the number of events that satisfy that
¹²¹ $x_{i-3} < th_0$, $x_{i-2} < th_0$, $x_{i-1} < th_0$, and $x_i > th^*$ is reduced. This is reflected in the wider gray regions
¹²² in Fig. 3, and in the fact that above $th^* = 4\sigma$ the probabilities are compatible with a stochastic process,
¹²³ so we cannot use this as a forecasting method. In any case, for power outputs of the laser above the
¹²⁴ transition ($P \geq 0.90$ W), and events larger than 3.5σ, we can still see that 50% of these intense events
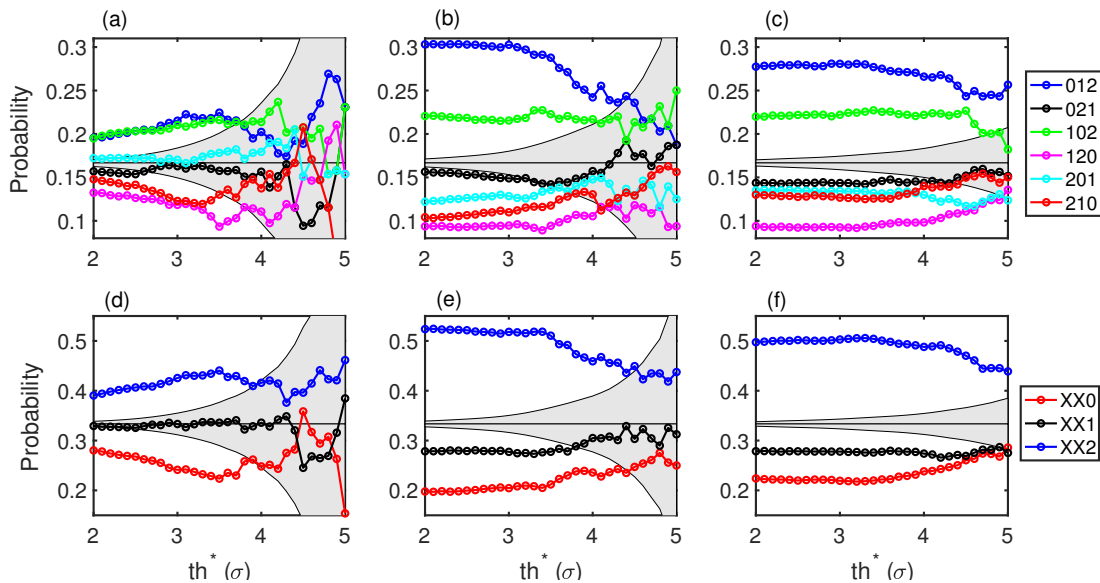¹²⁵ happen after the XX2 pattern.

Figure 3. Ordinal patterns computed with intensities using two thresholds. A low threshold, $th_0 = 1\sigma$, imposes that the consecutive low events that form OPs are below th_0. A high threshold selects the high events that take place after the OP and are above $2\sigma \leq th^* \leq 4\sigma$. **(a-c)** Correspond tp the six OPs of dimension three. **(d-f)** correspond to the OPs combinations where the last of the three event is larger, in between, or shorter than the other two events. This more restrictive condition reduces the number of events and widens the gray area.

4. Conclusions

¹²⁷   We have studied the dynamics of the output light of a fiber laser with optical feedback. By using
¹²⁸ an ordinal patterns approach, we have found signatures of determinism in its dynamics that allow us
¹²⁹ to forecast extreme events, defined as events larger than a threshold ($th^* > 2\sigma$). The system presents
¹³⁰ preferred patterns before it changes from low intensity events to more extreme events. The preferred
¹³¹ pattern before an extreme event is the one where the latter of three consecutive events below the
¹³² threshold is larger than the previous two, with probabilities that reach 80% in some cases, while the
¹³³ less probable pattern is the one where the latter event is less intense that the previous two events,
¹³⁴ which in some cases reaches less than 10%.
¹³⁵   We can compute the patterns with the intensity of the events or with the inter-event time-intervals.
¹³⁶ The former takes advantage of intensity correlations in the system while the latter on temporal
¹³⁷ correlations, both of them present in the dynamics. In this system the more efficient way to compute
¹³⁸ the patterns for forecasting purposes is with the actual intensity of the events. For patterns computed
¹³⁹ with inter-event time-intervals the probabilities, although still not compatible with a stochastic process,
¹⁴⁰ are lower and therefore less efficient to forecast the more extreme events.
¹⁴¹   The fact that there is 70% higher probability of pattern XX2 before a sudden increase in intensity
¹⁴² has to be considered carefully, as there are much less events that are higher than th^* than events below

th_0. This means that not after 70% of all patterns XX2 in the time series will take place an extreme event, but that 70% of extreme events are preceded by pattern XX2.

One interpretation for this preference for specific patterns when the system leaves the low intensity regime and explores the higher intensity regime is that there is a specific region in phase space that triggers extreme events, and those patters are the signature of that region in phase space.

This methodology can be applied to other complex dynamical systems that manifest time series of events, with a long-tail histogram.

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