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Article

Exploring how Homophily and Accessibility can Facilitate Polarization in Social Networks

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Abstract: Polarization in online social networks has gathered a significant amount of attention in the research community and in the public sphere due to stark disagreements with millions of participants in topics surrounding politics, climate, the economy and other areas where an agreement is required. There are multiple approaches to investigating the scenarios in which polarization occurs and given that polarization is not a new phenomenon but that its virality may be supported by the low cost and latency messaging offered by online social media platforms; an investigation into the intrinsic dynamics of online opinion evolution is presented for complete networks. Extending a model which utilizes the Binary Voter Model (BVM) to examine the effect of the degree of freedom for selecting contacts based upon homophily, simulations show that different opinions are reinforced for a period of time when users have a greater range of choice for association. The facility of discussion threads and groups formed upon common views further delays the rate in which a consensus can form between all members of the network. This can temporarily incubate members from interacting with those who can present an alternative opinion where a voter model would then proceed to produce a homogeneous opinion based upon pairwise interactions.

Keywords: political polarization; echo-chambers; social networks; binary voter model; discussion dynamics; opinion dynamics model

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1. Introduction

Ideological polarization has been addressed as a potential problem for healthy societies. There has been an increase in the attention given to the subject in recent years with a particular focus on various political disagreements with a challenge towards finding resolution. These considerations have been modeled in various paradigms such as the spatial segregation model of [1] (Schelling model), direct survey analysis [2], ideological exchanges in [3–5] and other approaches also exist. Given the recent discussion of a possible association between this increase in polarization and the use of online social networks, questions about the particular features that might have caused this change are becoming extremely important. Online social networks provide a means for the exploration of a wider community with reduced costs of connecting over longer distances and if this ease of connection discovery can produce a side effect not given attention, it may potentially grow. The question explored here is whether there is valid concern that a combination of access to a larger size of potential friends in combination with tendencies towards homophily (ideological) can produce a network which is more polarized than if the accessibility was more constrained to a set of random set of associations (local view).

The work presented here addresses some of the shortcomings and extends the model produced in [6]. Referred to in this paper as the Davies model, [6] provides a methodology for producing social networks using the concepts of local associates, accessibility, homophily [7] and ideological label assignments. The results of this reseaerch are important to thoroughly investigate as they provide evidence against the conclusions widely accepted in the well known work of [8]. The deficiencies of

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the model and presentation are addressed in order to support the change of perspective regarding the association of accessibility and homophily with the production of polarization in the ideological separation between members of a network. The necessary features lacking the model of Davies is the representation of a group/organisation which is presumed and this work presents the *extended Davies model* that incorporates this. A full presentation of the results allows some of the previous question and comments on the previous approach to be answered. Mainly the question about the final state and significance of the polarization increases through a full trajectory plot can be answered.

It will be assumed that the ideologies are a discrete label set similar to the discussion threads organized by hashtags rather to assume a more complex model which relies upon a placement within a continuous domain of ideological memberships, as in [9]. The model does not account for the particularities of the means of communication or the dynamics of exchange between certain ideological labels. This may at first appear to be a deficiency and in terms of real world applicability for understanding particular dynamics related to polarization it may be but here the question of the role that social networks may play is the main focus of the investigation. It is a question as to whether there is an intrinsic feature of online social networks which could facilitate an increase of polarization. One of the core revolutionary features of online social networks is that it provides greater access to people around the world in terms of breadth of potential candidate friendships and depth of the information.

The Davies model produces a set of local associates which correspond to members of society that a person is randomly allocated through work affiliations or spatial proximities of pairs of people. Then a graph of friends for each node is produced where a local associate is chosen at random, or one node uniformly (randomly) from the rest of the nodes in the graph. This chocie between the sets of nodes is then added according to 2 parameters of *accessibility* and *homophily*. In summary the results from the Davies model show that an increase in the accessibility of a user, a greater range of choice of contacts, in the presence of homophily will produce an increase the measure of assortativity [10] (polarization of ideologies within the network structure). This goes against the findings of previous research that associates greater accessibility with a reduction of polarization and therefore the topic warrants a careful examination as well as confirmatory efforts. Since online platforms facilitate communication at a reduced cost and latency, if there is a byproduct of this efficiency over time and space in way that polarization can be reinforced; further research along this direction in an applied sense would be necessary.

An independent implementation of the Davies model has been produced and simulations produced from it. The provided code on Github was used to ensure that the description in the paper was understood correctly but but the implementation was produced independently to avoid duplication of mistakes that may have been present in the original implementation (one measure of independence was to choose a different programming language). Figure 1 presents the results of the independent simulations, which confirm the results shown in the first panel of Figure 2 in [6]. The main feature is that an increase of accessibility can be associated with an increase in polarization (measured by assortativity). Subfigure a) shows a panel of plots for the initial state the friendship graphs begin in after a selection of friends based upon the choices of local associates or the external pool based upon the parameter of homophily. It can be seen that larger values of homophily and accessibility can produce a greater value of polarization (measured via the Newman assortativity). Subfigure b) and c) display the results of using the Binary Voting Model (BVM) where there is an exchange of ideological labels. Subfigure b) shows the mean value of the polarization for the iterations 50 to the final iteration and c) the mean through till the last iteration minus the values from the initial state. From these figures it can be seen that the model creates an initially polarized arrangement of connections so that those 'users' which had homophilic preferences and a high degree of accessibility produce more polarization than those with less accessibility. Over a standard manner of ideological exchange this effect can be reduced when the edge connectivity remains constant. The methodology for the creation of the network and the simulation details are presented in Section 2.

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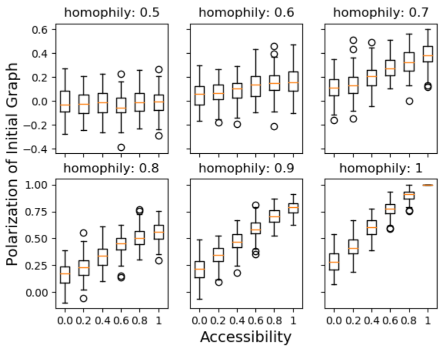
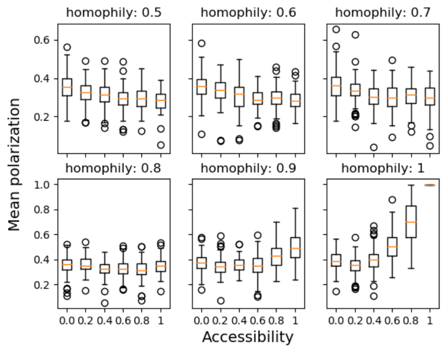


a)



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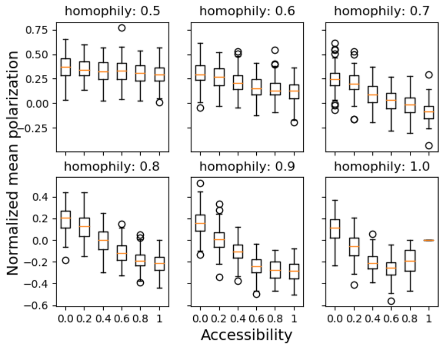

c)

Figure 1. These subfigures of plot panels display an independent reproduction of the results of [6] which demonstrate that in the presence of homophily, greater accessibility for choice of friendships, can produce polarized networks. The manner in which nodes produce the edges between them and exchange labels during a simulation of ideological discourse is discussed in Section 2. The main parameters is the value of the *accessibility* which affects the chance of friendships being created outside of a set of local associates and the value of *homophily* for which a potential friend is chosen based upon common ideological label assignments. Subfigure a) displays the results of the polarization following an initialization from the Davies model. It shows that greater accessibility with homophily will generate more polarized networks. Subfigure b) shows the results from the mean polarized value of the network from the 50th iteration till the final iteration and Subfigure c) the values of b) minus that of a). This shows that the polarization can be reduced post initialization when the labels are changed according to the classic Binary Voter Model (BVM).

The reproduced results from the Davies model demonstrate that accessibility can facilitate nodes to develop a greater proportion of homogeneous friendships in the graphs when label associations exist, and there are homophilic preferences in connections according to the labels. This provides a counterargument to the conclusions of the model in [8] that accessibility reduces polarization and that this conclusion may not always hold; brings into question whether negative effects from boundless accessibility exist. What is absent from [6] is an examination of the trace from the simulations to examine the stability of the network polarization values where accessibility can be shown to increase polarization. In [6] analogues from society are used to motivate the difference between local associates and those that are outside the affiliations from being a functional member of society. Those outside yet accessible depend upon a parameter value which controls the probability that such a connection can occur and certain platforms change that probability. From the societal analogues described as example from where the facility to access non local associates can arise from, examples such as Twitter communities, rotatary clubs and others are mentioned. These aggregations of users provide a meeting place for the choice process of new friendships to occur outside of the pre-existing locality and provide the means for the accessibility but are not explicitly modeled. It therefore necessary to explore the effect of the introduction of such a feature to ensure that the process which produces the results just shown continues to support the same conclusions. Even if the qualitative results that, the increase in

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accessibility is able produce more polarization is maintained, it is a requirement to see if the presence of groups/community organizations invalidates the results or whether any effect is produced. Section 2 provides a more thorough presentation of the methodology and the extended model that addresses the gap in the model paradigm with a clear overview of the simulation trajectories.

2. Methodology

The Davies model (as referred to here) is presented in [6], and aims to investigate the effect of 3 parameters in the creation of networks where a single ideology (from a set of ideologies) can be held by each node. These 3 parameters are *accessibility*, *homophily* and *dynamic balancing*. Using these parameters, networks of a fixed number of nodes (hypothetical users) are able to produce a friendship network from a set of local associates and those outside based upon the amount of accessibility offered which is increased through online services/platforms. Each network consists of 40 nodes in the Davies model and the same number is used here; presumably taken as the second Dunbar number [11] (which is more recently confirmed in an analysis of microblogs [12]). The *dynamic rebalancing* is not presented in this work, as the Davies results and those produced indepedently showed no new qualitative insights from its use. Two different ideologies are considered to label users in the initialization of the first stages of the network creation and that users change these labels according to the classic Binary Voter Model (BVM) [13]. An extension of the Davies model is developed and presented in order to incorporate the facility of the networks referred to in [6] of group memberships and organisations. In that work, rotary clubs, political parties, Twitter communities and others are mentioned as memberships which provide means to access those with similar ideologies, although that effect is not accounted for, but is brought into the model presented here.

The parameter of *accessibility*, A, regulates the probability of choosing between the local associates produced in the generation of the ER graph for initial neighbors and the chance of using a medium such as the internet to sample connections outside of that initial locality provided. Sampling a friendship from the local neighbor set is done with probability $1 - A$ and draws a neighboring edge within the ER graph that is not already present. For a node i, v_{a_i} is that node's position in the ER graph and $deg(v_{a_i})$ represents the number of edges (friendships) i will produced that is distributed between those in $\{v_{a_j}, v_{a_i}\} \in e_{a_i}$ (given local associates) and those sampled outside of that set $\{v_{a_j}, v_{a_i}\} \notin e_{a_i}$. This probability regulates the ratio of the friendships which are associated with a process of regular association with surrounding individuals rather than those that can be indepedently searched for. The friendship graph G_f, will maintain the number of edges for each vertex, $deg(v_{f_i}) = deg(v_{a_i})$, so that the accessibility replaces friendships that would otherwise be included from the local associate set supplied by default. It can be considered that the local associates still exist and should still be considered explicitly but those in E_a that are no longer included in E_f will have no ideological influence in the simulation (it is possible to be an associate but not a friend with ideological influence).

Given the set of nodes to sample from (local associates or outside of that group), that set is then differentiated into those with the same or different ideological labels. *Homophily*, H, is the probability that an edge is produced between nodes with the same ideology, and $1 - H$ for choosing a node with the opposite label (type of heterophily). For a homophilic friendship assignment, $p(v_{f_j}) = p\left(v_{f_j} | v_{f_j}(I) = v_{f_i}(I)\right) = H$ (same ideology), otherwise probability $1 - H$ a different ideological node is chosen, $p(v_{f_j}) = p\left(v_{f_j} | v_{f_j}(I) \neq v_{f_i}(I)\right) = 1 - H$. An edge in the friendship graph is added $e_f \cup p\left(v_{f_j} | \{v_{f_j}, v_{f_i}\} \in e'_{f_i}\right)$ according to:

$$e_f \cup \begin{cases} \{v_{f_j}, v_{f_i}\} & if \quad \left(\mathcal{U}(0,1) \leq p(v_{f_j})\right) \wedge \left(\{v_{f_j}, v_{f_i}\} \in e'_{f_i}\right) \\ & \text{else} \quad \emptyset \end{cases}.$$

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Here the reference to e'_{f_i} is the intermediate set of nodes that differentiates between the choices of local associates and those outside that group.

In Table 1 the steps of the Davies model is presented in a different manner to what is originally provided with the aim that the model can more easily be understood. Steps 1-4 initialize the network where the main components of accessibility and homophily determine the ratio of friend candidates chosen from local associates or those outside of that group uniformly and from those candidates the homophily parameter is then used to choose according to that probability that such a node with the same or different ideology is included in their friendship graph G_f. The selection of candidate nodes as friends may end up being void for certain iterations; such as when the initial G_a associates graph produces zero edges for a node then the addition of edges is bypassed. Step 5 corresponds to the operation of the classic BVM which draws uniform samples of the nodes in a friendship graph, which then allows a node to propagate its ideology deterministically. The main feature which relates the accessibility to the homophily is that the accessibility allows for a greater number of homogeneous nodes to be discovered when creating the friendship graph.

1. Generate an undirected Erdos-Reyni graph according to a chosen p, to produce a *Local Associates Graph*, ($G_a = (V_a, E_a)$ where $|V_a| = N$)
2. Uniformly assign each node an ideology from set of ideologies **I**, $v_{a_i}(I) = \mathcal{U}(I \in \mathbf{I}) \, \forall i$
3. Generate corresponding *Friendship Graph*, $G_f = (V_f, E_f)$ with $|V_f| = N$ and initialize for every $v_f \in V_f$ to have $deg(v_f) = 0$, $\left(\{E_f\} = \emptyset\right)$
4. For $\forall v_f$ ***add friends to*** v_{f_i} ***while*** $\left(deg(v_{f_i}) <= deg(v_{a_i})\right)$
   Select:
   (a) With probability A (*accessibility*), sample a candidate edge uniformly among non-current associates (non-neighbors), $e'_{f_i} \cup \mathcal{U}\left(\{v_{f_j}, v_{f_i}\} : v_{f_j} \notin v_f \wedge \left(\{v_{a_j}, v_{a_i}\} \notin e_{a_i}\right)\right)$, and with probability $1 - A$ choose an edge uniformly from the Local Associates Graph $e'_{f_i} \cup \mathcal{U}\left(\{v_{f_j}, v_{f_i}\} : v_{f_j} \notin v_f \wedge \left(\{v_{a_j}, v_{a_i}\} \in e_{a_i}\right)\right)$
   (b) Assign to each potential friendship node probability H, if $p(v_{f_j}) = p\left(v_{f_j} | v_{f_j}(I) = v_{f_i}(I)\right) = H$ (same ideology), otherwise probability $1 - H$, $p(v_{f_j}) = p\left(v_{f_j} | v_{f_j}(I) \neq v_{f_i}(I)\right) = 1 - H$
   (c) Include edges between candidates and v_{f_i} according to the probability $e_f \cup p\left(v_{f_j} | \{v_{f_j}, v_{f_i}\} \in e'_{f_i}\right)$

$$e_f \cup \begin{cases} \{v_{f_j}, v_{f_i}\} & if \quad \left(\mathcal{U}(0,1) \leq p(v_{f_j})\right) \wedge \left(\{v_{f_j}, v_{f_i}\} \in e'_{f_i}\right) \\ & \text{else} \quad \emptyset \end{cases}$$

5. Simulate the ***Binary Voter Model (BVM)*** for T iterations
   while $(t < T)$
   (a) $i \leftarrow \mathcal{U}(1, N)$
   (b) $\mathcal{U}\left(v_{f_j} : \{v_{f_j}, v_{f_i}\} \in e_f\right)$
   (c) Change ideology of v_{f_i} according to:

$$v_{f_i} = \begin{cases} v_{f_j}(I) & if \quad v_{f_i}(I) \neq v_{f_j}(I) \\ v_{f_i}(I) & if \quad v_{f_i}(I) = v_{f_j}(I) \end{cases}$$

Table 1. Outline of the methodology of the Davies model for the initialization of graphs of social inconnectivity based upon the parameters of accessibility A and homophily H. The purpose it to examine whether accessibility and homophily can produce unexpected changes in the assortativity (polarization) amongst nodes with specific ideologies. The interplay of accessibility and homophily can produce different results when interactions are found and developed through online platforms which provide a greater means for users to fill their social circle with a greater proportion of like minded individuals.

The *assortativity* coefficient [10] is used to examine the magnitude of polarization since it quantifies the lack of connectivity between users of different ideological labels. Its values range between -1 and +1, and for the edge pairs in the friendship graph, e_f where the node pairs are $\{v_{f_i}, v_{f_j}\} \in e_f$, those edges between identical ideologies is denoted as $e_{I_{ij}}$ when $v_{f_i}(I) = v_{f_j}(I)$. We consider e_I to be represented as a matrix which allows the assortativity to then be given by:

$$r = \frac{Tr(e_I) - ||e_I^2||}{1 - ||e_I^2||}$$

with $Tr(e_I)$ being the trace of the ideology matrix between nodes that are connected. A value close to 1 is indicative of strong homophily and -1 that of heterophily with 0 corresponding to a lack of bias.

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2.1. Extension of Davies model

This subsection describes the alterations of the Davies model which accounts for the presence of discussion groups and describes the changes in assortativity values in the simulation. The modeling design does not affect the dynamics of the network initializations and has an effect when nodes interact in an ideological exchange.

The procedural elements of the *extended Davies model* are presented in Alg 1 and Alg 2. In Alg 1 the steps to initialize the network based upon the accessibility A, homophily H, and the introduction of the new feature where users are assigned to discussion groups based upon similar ideologies are described. This new feature can be seen as a uniform sample across the *Ideologies* available and the number of *Discussions* where a user of a particular Ideology can be placed subsequently. This discussion group membership in the initialization stage does not affect the edge creation between nodes as the accessibility and homophily do. This membership will play a role in the similation of the ideological exchange in the iterations post initialization. As the edge productions are not affected by the presence of a discussion group membership or absence of membership, the assortativity (polarization) measurement at initialization will not be altered. Here the discussion groups for the simulations conducted will follow that of the Davies investigation of 2 groups of ideologies, but this effect can easily take a different number of group memberships for equality testing. This premise is also comparable to the group similarity verification which is done at the micro level in models of residential dynamics such as the Schelling model [1].

Algorithm 1 Extended Davies Initialization Algorithm

```
procedure INITIALIZEGRAPH(N, A, H, p, Ideologies, Discussions)
    A ⇐ accessibility_parameter
    L ⇐ ER_Graph(N, p)
    F ⇐ Empty_Graph()
    for all Node ∈ F do
        Node.Ideology ⇐ SampleOne(Ideologies)
        Node.Discussion ⇐ SampleOne(Discussions)
    end for
    for all Node ∈ F do
        while degree(F[Node]) < degree(L[Node]) do
            if U(0,1) < A then
                Friends ⇐ Set(L.nodes)
            else
                Friends ⇐ Set (Neighbors(L[Node]))
            end if
            Weights = ArrayInit()
            for all Friend ∈ Friends do
                if Friend.Ideology == Node.Ideology then
                    Append(Weights, {H, 1})
                else
                    Append(Weights, {(1 − H), 0})
                end if
            end for
            AddEdge(F, Node, weightedChoice(Friends, Weights))
        end while
    end for
return L, F
end procedure
```

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Alg 2 shows the procedures of the extension of the BVM which accounts for the presence of the discussion groups introduced in Alg 1. The standard BVM is a direct assymetric label propagation between non-identical ideological associations in which structural information of the network is not taken into account in terms of the neighbor influences. It is considered that the neighboring label set will play a role in the ability for a node to change the ideology of another node as noted in [14,15]. This information is brought into the model by the membership of a node in a discussion group based upon similar ideologies. It is also considered that it is through these discussion groups that the accessibility for edge creations, outside of the locality, makes it possible to facilitate a search based upon the parameter of Homophily. Since the simple random sample is not representative of the intelligent optimal association searches that platforms offer services for. The quantitative effect of the membership on the BVM is that the probability of a successful ideological conversion will then be inversely proportional to the size of the group.

Algorithm 2 Binary Voter Model (BVM) extended to consider discussion group effects

procedure BVM(L, F, T)
  for $t \in T$ **do**
    $X \Leftarrow SampleOne(F.nodes)$
    $Y \Leftarrow SampleOne(neighbors(X))$
    if $\mathcal{U}(0,1) < \left({}^{1}/_{size(X.Discussion)}\right)$ **then**
      $X.Ideology \Leftarrow Y.Ideology$
      $X.Discussion \Leftarrow None$
    end if
  end for
end procedure

3. Results

Here the results of the simulations of the Davies model, shown in Table 1, and the extended Davies model described in Alg 1 and Alg 2 are presented. The main feature that differentiates the two models is that the extension accounts for the membership of users in discussion groups (communities/echo-chambers/clubs etc) in which the participation is associated with a homophilic interaction. From the simulations a comparison is made as to what impact this can make upon the trajectories of the simulation which, are based upon the Binary Voter Model (BVM).

Figure 2 displays a set of trajectories of the Davies model for two different values of the homophilic affinity of ideological connectivity beyond a node's locality with values $H = 0.6$ and $H = 0.9$ (subfigures a) and b) respectively) for a range of accessibility values $A = [0, 0.2, 0.4, 0.6, 0.8, 1]$. These plots are meant to expand upon the results presented in Figure 1 which are presented in the Davies paper and are averages over 500 independent simulations for the different (A, H) parameter values. The iteration number for the simulation is plotted against the assortativity that provides a quantification for the amount of 'polarization' between nodes of different ideologies in the network. What is interesting to point out is that there are parameterizations for which there can be an increase in the polarization from the initial values and also that the increase in the accessibility values introduces greater initial polarization prior to the process of ideological homogenization which begins with iterations of the BVM. In subfigure a) the lower A values produce increases in the assortativity early in the simulations, and this is seen as well in subfigure b) for $A = [0, 0.2]$. The reason this feature has increased presence with lower A values and H is that the initial networks are less interconnected due to members having fewer 'long-range' associations so that the communities/clusters in a first stage of ideological label agreements do so without a network wide consensus. The cluster homogeneity forms first before the non-local associations become the predominate cause for the label switching for the BVM. Therefore, this can be seen as a dual phase process with the local community associations

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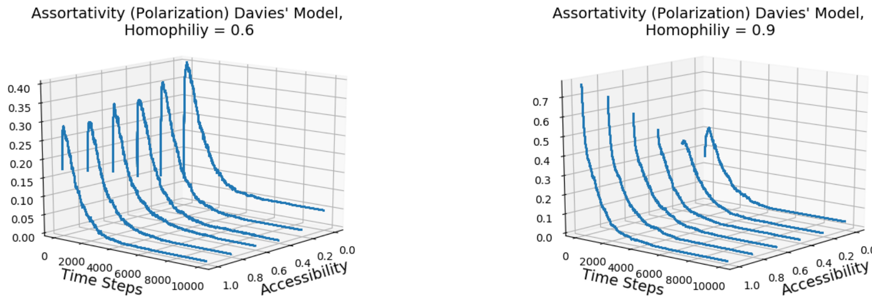
conforming to a uniform ideological adoption prior to the full network. In the second phase, the networks proceed towards homogenization with a monotonic decrease in the polarization along the iterations (ignoring the stochasticity inherent of the process which may produce sporadic irregularities in that pattern). The main feature is that the polarization is increased at the initialization merely by permitting users to explore beyond their local associates when homophilic interactions are a choice.



a) b)

Figure 2. These 2 subfigures present the trajectories of the simulations based upon the Davies model described in Table 1. Subfigure a) and b) produced the initial networks with homophily values $H = 0.6$ and $H = 0.9$ respectively where the values of the accessibility chosen for independent simulations are $A = [0, 0.2, 0.4, 0.6, 0.8, 1]$ (shown are averages over 500 simulations). The iterations are plotted against the assortativity values which measures the polarization during the simulation. The BVM does manage to remove the polarization that is produced at the initial stages over the simulation. The increase in initial polarization accessibility with accessibility and homophily is that user are not required to accumulate friendships based upon a limited number of users with similar ideologies and the ability to look beyond that increases the density of edges between users with the same ideology.

Figure 3 provides the traces for the simulations on the same parameter values (A, H) applied the extended Davies model which includes the creation of discussion groups as outlined in alg 2. The 2 subfigures a) and b) look at the the initialization parameters for the network, $H = 0.6$ and $H = 0.9$ respectively with a range of accessibility values $A = [0, 0.2, 0.4, 0.6, 0.8, 1]$. Each of the trajectories is an average over 500 independent runs. As noted in the Davies model simulation, for both chosen values of homophily in the network creation, shown in Figure 2, there is a decrease in the assortativity measure towards the end of the simulation as the BVM stochasticity ends up producing a monolithic ideology amongst users. The initial values are comparable and the same modes seen in $H = 0.6$ (subfigure a)) and for $H = 0.9$ (subfigure b)) with $A = [0, 0.2]$ are also seen with the original Davies model results, Figure 2. The main feature which showcases the difference between the simulation with the discussion groups is that iteraction assortativity values are greater with the extension. Since the initial values are similar the discussion groups introduce a dampening effect upon the reduction of the ideological homogenization process. This corresponds to the expected effects of ’echo chambers’, ’group think’ and incubation of ideas which isolate users from independent ideological exchange.

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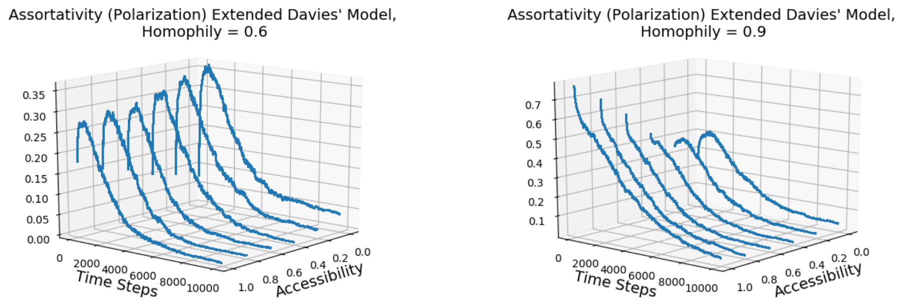

Figure 3. The subfigures a) and b) ($H = 0.6$ and $H = 0.9$) present the simulation traces of the assortativity (polarization) values over iterations where the extended Davies model (described in alg 2) is used to change the ideologies of users based upon their interactions while accounting for the membership in discussion groups of users with the same ideological labels. Similar trend shapes as with the original Davies model can be seen in Figure 2. In the comparison, the initial assortativity values are comparable but a delay in the reduction of the polarization appears to be present.

Figure 4 displays the results of the investigation into the difference between values of the network assortativity between the Davies model and the extended Davies model which accounts for discussion groups (clusters). Of the 2 simulations presented the homophily parameter values used in the initializations are $H = 0.6$ and $H = 0.9$ in subfigure a) and b) respectively. A range of accessibility values are chosen and shown in the legend, $A = [0, .2, .4, .6, .8, 1]$. Both models for each parameter pair values are run for 10K iterations and the differences are plotted at each 10 iteration step. For both the simulation with $H = 0.6$ and $H = 0.9$ the differences at the initialization are low which is expected since the discussion groupings/clusters based upon homophily did not effect the edge construction process. Each subfigure trace set for different accessibility values, shows an increase in values which then diminishes towards the end of the simulations. That both simulations begin at approximately the same point and then end at the same point is expected, but what is important to notice is the increase in relative assortativity values of the extended Davies model in the first half. Looking at the results from Figure 3 and Figure 2 it becomes evident that the addition of the discussion groups to the formation of the initial network delays the process of the BVM in reducing the assortativity (polarization). Although the accessibility increases this disparity between the models at their maximum difference, it is seen even for low parameter values of A. The reason the dynamics of the BVM result in an altered trajectory is because the influence of the single points of contact for a change in opinion are outweighted by the aggregate of a discussion group which can mimick the 'echo-chamber' and require more messages (iterations) to shift an ideology. These curves having the right tail skew are akin to the complex phenomena observed in many systems.

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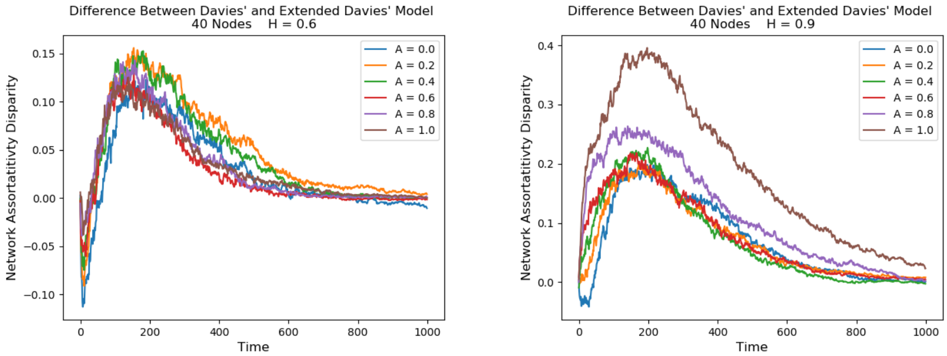

Figure 4. These subfigures show the difference in values between the simulations presented in Figure 2 and Figure 3 (Davies model and extended model respectively). From initialization till a point of reduced polarization for different accessibility values it can be seen that the BVM succeeds in reducing any disparity between the models but for each A (accessibility) value there is a delayed decrease seen in the extended Davies model which is attributed to the presence of dicussion groups. The Binary Voter Model (BVM) has a reduced efficiency upon the rate in which users change ideologies due to their membership in discussion groups (echo-chambers) where it is expected that a greater number of trials is necessary to create the ideological change than the single interaction in the previous approaches that ignores group memberships or identities.

4. Discussion

The work presented by the authors explores the results of a recent study [6] that investigates the choice mechanism of establishing a friendship graph and the assortativity of ideologies when they are mutually exclusive labels per individual. This mechanism of developing friendship networks relies upon each member having an initial set of local associates which are provided through a random allocation as being a functioning member of society. Subsequently, a set of friends are chosen based upon the accessibility that a member can access potential friends outside of their local associates and the choice within these 2 pools is biased according to the value of homophily. After the friendship graph is generated it is assumed that only this set can influence the ideologies of a node regardless of whether a node is a member of the local associates and not in the group of designated friends. This work investigates the relationship of *accessibility* and *homophily* with the change of values of *assortativity* that quantifies polarization. These associations will have potentially large impacts on society given that there is an increase in friendship developments online [16,17] which have been provided to users of platforms and there is also research indicating an increase of polarization surrounding sensitive societal choices in recent years that can be associate with certain types of isolation [18].

The results of the previous research (Davies model) provides support for an idea that an increase in accessibility can increase the assortativity values due to users having access to a large pool of candidate friends with whom homophilic interactions can produce friendships at an increased rate. In that model the exchange of ideological labels due to friendship interactions is governed by the classic Binary Voting Model (BVM) [13]. Since these results can alter the conclusions in previous publications [8], the work here presents an independent implementation of the model after careful examination of its description provided and the code on github. This independent implementation is written in a different programming language, and can confirm the validity of those results shown in Figure 1.

There are 2 important explorations in [6] which are required in order appreciate the insight it can bring towards understanding the impact of accessibility in online social networks upon mesaures of polarization. The first is that the although the initialization of a network, given incereased accessibility can produce an increase in polarization and that during the simulation of the BVM these polarized states can be found; the full trajectory of the BVM simulation upon these networks is not provided. Such a set of plots will provide insight into states of convergence and different modes of the model. These are provided in Figure 2 and show that the initial networks follow the pattern that increased

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accessibility in the presence of homophily creates increases in assortativity and that early stages of the simulation produce temporary increases in the assortativity mostly for lower homophily. This is due to the relatively lower initial assortativity that exists in comparison to larger H, as the BVM homogenizes local clusters before the macroscopic homogenization process begins. The most important feature to note in the inspection of a full run of the BVM on the Davies model is that the BVM does proceed to reduce the heterogeneous clusters. The second feature explored in this work is that in [6] the increased accessibility offered is motivated by the presence of offline or online clubs, communities and organizations where members can find these pools of homophilic affiliations in ideological labels. This is not present in the Davies model and it is uncertain whether their introduction would disrupt the results presented. An extension of the Davies model is developed in Subsection 2.1 where discussion groups memberships are represented, and the results are shown in Figure 3. It can be seen that the presence of discussion groups reinforces the results of the Davies model and adds to the degree in which polarization can be manifested with accessibility increases and even the discussion groups. Figure 4 provides an indication of the increase in assortativity during the simulation in the initial phases of the BVM in a manner. As both models do provide an increase and then homogenization it can be said that the results of [6] are reliable and can provide insight into a possible negative effect that accessibility may have in the presence of homophily.

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