 doi:10.20944/preprints201810.0713.v2

A quantum mechanical relationship between Milgrom's acceleration constant and the Bekenstein-Hawking entropy expression

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Summary
Conceiving vacuum energy as gravitational particles subject to Heisenberg's energy-time uncertainty, modelled as dipoles in a fluidal space at thermodynamic equilibrium, and interpreting the Bekenstein-Hawking entropy as the effective amount of spins of those dipoles enclosed within the event horizon of the universe, allows the calculation of Milgrom's acceleration constant. The result is a quantum mechanical interpretation of gravity, and dark matter in particular .

Keywords: Milgrom's acceleration constant; Bekenstein-Hawking entropy; gravitational dipole; dark matter.

Introduction

One of the most outstanding problems in present physics is the search toward the relationship between quantum mechanics and gravity. This subject has given rise to a discipline in itself, dubbed as quantum gravity. Within this discipline, new concepts are being developed, such as for instance the string theory. Those new concepts are often denoted as new physics. It is believed that such new concepts will explain certain phenomena in cosmology, where dark matter and dark energy are still regarded as a mystery. An interesting approach is building new physics emergently from basic principles, such as for instance from the entropy concept as known from thermodynamics. Recent proposals in this direction have been put forward by Verlinde [1,2], inspired by views of Jacobson [3] back in 1995. These proposals have resulted in a particular approach to gravity, known as entropic gravity. Verlinde's theory aims to connect entropic gravity with string theoretic principles and information theory. The drawback of this approach for a scientist with an interest in the subject is the need to cope with the highly sophisticated mathematical mask that covers the theory. It is the aim of the author of this article to reverse the argumentation. Rather than showing how Einstein's theory of gravity emerges from basic principles, he wishes to show that the basic principles emerge from Einstein's theory, even to the extent of the role of informatics and thermodynamic entropy. It is the author's belief that this will result in easier understanding of the very same principles and in quantitative results that give a better fit to observational evidence. One of the results, next to understanding the role of informatics in gravity, is a very clear explanation of the dark matter phenomenon, culminating in a quantitative calculation of Milgrom's acceleration constant [4] from basic quantum mechanical principles, developed from conceiving the vacuum energy term in Einstein's equation as the Heisenberg fluctuation on the irrelevant thermal dynamical equilibrium state of spatial particles in a virtual fluid. This turns the empirical modification of Newton's gravity law into a fundamental modification with the same, and even more, strength for successful calculations and predictions in cosmology that fit to observational evidence.

Theory

Ultimately, it is the aim in this article to show that Milgrom's acceleration constant can straightforwardly be calculated from the well known entropy expression for black holes as eventually established by Hawking as an improvement of Bekenstein's original formulation. In this respect it is not different from like it is in Verlinde's artcle [2]. The theory, the interpretation and the results, though, are quite different. The Bekenstein-Hawking expression is one of five ingredients for calculation. It reads as [5,6],

$$S_H = k_B \frac{c^3}{4G\hbar} A, \tag{1}$$

where c is the vacuum light velocity, G the gravitational constant, $\hbar$ Planck's (reduced) constant, A the black hole's peripheral area and k_B is Boltzmann's constant. The peripheral area of a spherical black hole is determined by its Schwarzschild radius as,

$$A = 4\pi R_S^2, \; R_S = \frac{2MG}{c^2}, \tag{2}$$

where M is the baryonic mass of the black hole. Boltzmann's constant shows up as a consequence of the thermodynamic definition of entropy. In that definition S_H is not dimensionless, because of the thermodynamic interpretation of entropy as a measure for the unrest of molecules due to temperature, which relates the increase ΔS of entropy with an increase molecular energy ΔE due to temperature T, such as expressed by the thermodynamic definition,

$$\Delta E = T\Delta S. \tag{3}$$

Boltzmann's famous conjecture connects entropy with information, by stating

$$S_B = k_B \log(\# \text{ microstates}). \tag{4}$$

This conjecture expresses the expectation that entropy can be expressed in terms of the total number of states that can be assumed by an assembly of molecules. Boltzmann's constant shows up to correct for dimensionality. I would like to emphasize here that (3) and (4) are different definitions for entropy S, and not necessarily identical. Knowing that (1) has been derived from (3) and accepting Boltzmann's conjecture, we would have,

$$\frac{c^3}{4G\hbar} A = \log(\#\text{microstates}). \tag{5}$$

Both sides of this expression are dimensionless. Omitting Boltzmann's constant makes entropy a dimensionless measure of information, which, of course, is very appealing. At this point, I wish to elaborate on a subtlety, which has been shown by Verlinde. According to Boltzmann's conjecture, an elementary step ΔS in entropy would imply $\Delta S = k_B$. Verlinde has proven, however, that an elementary step in entropy from the Hawking-Bekenstein entropy implies $\Delta S = 2\pi k_B$. If not, the Hawking-Bekenstein's formula would violate Newton's gravity law [1]. Because Boltzmann's expression is a conjecture without proof, the problem can be settled by modifying the dimensionless expression of entropy (5) into,

$$S = \frac{1}{2\pi} \frac{c^3}{4G\hbar} A = \log(\#\text{microstates}). \tag{6}$$

It is instructive comparing this result with a view on the Bekenstein-Hawking entropy as suggested by Susskind (for explanation purpose) in one of his lectures [7]. Susskind has proposed to consider the black hole as a body that captures or releases elementary discrete packages of energy $\Delta M = hf = \hbar\omega$, building its total mass M as a sum of N elementary amounts ΔM. Each of these elementary packages add an elementary step on entropy. To this end, the Compton wavelength of these elements must equate the peripheral circle $2\pi R_S$ of the black hole, such that

$$\Delta M = hf = h\frac{1}{T} = \frac{hc}{cT} = \frac{hc}{2\pi R_S} = \frac{\hbar c}{R_S}. \qquad (7)$$

In Susskind's explanation model, the entropy is equated with the ratio,

$$S = N = \frac{M}{\Delta M} = \frac{R_S}{\hbar c}M = \frac{R_S}{\hbar c}(\frac{2MG}{c^2})\frac{c^2}{2G} = \frac{c^3}{2\hbar G}R_S^2 = \frac{c^3}{2\hbar G}\frac{A}{4\pi} = \frac{1}{2\pi}\frac{c^3}{4\hbar G}A. \qquad (8)$$

This result is in agreement with (6).

The second ingredient is the well known observation that the event horizon ct_H of the visible universe equals Schwarzschild radius of the critical mass enclosed within that horizon (t_H is the Hubble time scale), [8]. Hence, from (6), the entropy within the event horizon of the "flat" universe can be established as

$$S = \frac{c^3}{8\pi G\hbar}4\pi(ct_H)^2 . \qquad (9)$$

The third ingredient is crucial novel step. It is a consideration on the vacuum energy of the universe. Let us suppose that the origin of gravitational energy is due to the Heisenberg uncertainty of elementary spatial particles. These particles are subject to the constrained time-energy product ΔE x Δt. Usually the Heisenberg relationship is expressed as $\Delta E\Delta t = \hbar/2$, [9]. I would like to emphasize that this expression is the result of a careful study on the boundary w.r.t. h or $\hbar$, half or integer. In this particular case, ΔE expresses that the spatial particle's vacuum energy is subject to a $\pm\Delta E/2$ deviation around its thermodynamic equilibrium state. This deviation can be interpreted as the vibration energy of the one-body equivalent of a two-body quantum mechanical oscillator. The vibration energy can then be modelled as the ground state $m_{eff}c^2$ energy of an effective mass in the centre, thereby considering the energetic equilibrium state of the spatial particle as irrelevant. In this model, the two bodies compose a gravitational dipole to which a certain dipole moment can be assigned. Let us assign a mass m to each of the two bodies and let us assign a spacing d between them. This establishes a dipole moment $p = md$ to each elementary vibrating spatial particle. Let the spacing between the poles be determined as $d = c\Delta t$, where Δt is the time uncertainty subject to the Heisenberg constraint, i.e.,

$$d = c\Delta t = c\frac{\hbar}{2}\frac{1}{\Delta E} \rightarrow d = c\frac{\hbar}{2}\frac{1}{m_{eff}c^2} = c\frac{\hbar}{2}\frac{1}{\alpha mc^2} \rightarrow p = md = \frac{\hbar}{2\alpha c}, \qquad (10)$$

where α is a dimensionless factor that relates the two body masses with the equivalent mass of the vibration energy of the spatial particle. The magnitude of α can be estimated from the relationship

$$m_{eff} = \frac{1}{1/m + 1/m} = \frac{m^2}{2m} = \frac{m}{2} = \alpha m \rightarrow \alpha = \frac{1}{2}. \tag{11}$$

However, where the effective mass is only halve of the mass of the constituent bodies, the effective spatial coordinate of the effective mass is doubled [10].

Hence, from (11) and (12)

$$p = \frac{\hbar}{2c}, \tag{12}$$

This implies, so far, that elementary spatial particles can be modelled by gravitational dipoles with a dipole moment p as described by (12). Note that, where other authors [11,12] describe gravitational dipoles as structures with positive and negative mass ingredients, the dipole described here is a virtual structure that enables modelling the spin of a one-body vibrating particle as the dipole moment vector of a two-body equivalent. To some readers it may seem that I am introducing here a new kind of matter. This is not true. The vibrating particle is part of the vacuum energy, modelled as an ideal fluid in thermodynamic equilibrium that emerges from the Cosmological Constant as the consequence of the solution of Einstein's Field Equation of the vacuum [13,14,15]. The equilibrium state of the fluid itself is irrelevant. Hence the gravitational molecules show up as a vibration of the vacuum. This is different from novel matter of baryonic nature.

The fourth ingredient is the baryonic dipole moment density P_g as shows up in the interpretation of the impact of the Cosmological Constant on the Newtonian gravity as described in [16] (see note), where it is shown that

$$P_g = \frac{a_0}{20\pi G}, \tag{13}$$

where a_0 is Milgrom's empirical acceleration constant [4].

Note: the referenced article [16] is a preprint v3, which has got an update v4. In the update the dipole concept has been omitted, because in the article it was not more than a discussion item. In the appendix of this article, the concept is explained again.

From (12) and (13), we may calculate the amount N_g of gravitational dipoles in the spatial volume V enclosed by the event horizon of the universe, as

$$N_g = \frac{P_g V}{p};\ V = \frac{4}{3}\pi (ct_H)^3. \tag{14}$$

Not all of these gravitational dipoles are baryonic. In terms of the Lamda-CDM nomenclature the baryonic share is expressed as Ω_B in the relationship

$$1=\Omega_m+\Omega_\Lambda=(\Omega_B+\Omega_D)+\Omega_\Lambda, \quad (15)$$

where $\Omega_m,\Omega_\Lambda,\Omega_B,\Omega_D$, respectively are the relative matter density, the relative dark energy matter density, the relative baryonic matter density and the relative dark matter density [17]. Where the matter distribution between the matter density Ω_m (= 0.259) and dark energy density Ω_Λ (= 0.741) is largely understood as a consequence from the Friedmann equations [18] that evolve from Einstein's Field Equation under the Friedmann-Lemairtre-Robertson-Walker (FLRW) metric [19], the distribution between the baryonic matter density Ω_B (= 0.0486) and dark matter density Ω_D (= 0.210) is empirically established from observation. The quoted values are those as established by the Planck Collaboration [17]. Taking this account, the amount of baryonic dipoles in a spatial volume V is established as,

$$N_B=\frac{P_g}{\Omega_B p}V=\frac{a_0}{20\pi G}\frac{2c}{g\Omega_B\hbar}(\frac{4}{3}\pi c^3 t_H^3)\,. \quad (16)$$

These dipoles carry the total amount of baryonic energy. Their spin states (= direction of the dipole moment vector) carry the amount of baryonic information contained in the universe. Because the quantum states of the spin can assume two values only, the number N_B are bits that represent the baryonic information content of a space with volume V. Hence, S as defined by (1,2,6) and N_B as defined by (16), express the entropy of the volume enclosed within the event horizon of the universe. Hence, equating S and N_B yields the instrument for the calculation of Milgrom's acceleration constant a_0. Doing so, we find from (16) and (7),

$$a_0=\frac{15}{4}\Omega_B\frac{c}{t_H}. \quad (17)$$

This result is very similar to the one as derived by me before [20] from a quite different perspective, without invoking the Bekenstein-Hawking relationship and without assigning the vacuum energy to the Heisenberg uncertainty. From (15), we find, as in [20], with $\Omega_B=0.0486$, $c=$ 3 x 10^8 m/s and Hubble time scale $t_H=13.8$ Gyear, the result $a_0\approx 1.25$ x 10^{-10} m/s^2, which corresponds extremely well with observational evidence [21].

Discussion

The quantum mechanical calculation of Milgrom's acceleration constant from the Bekenstein-Hawking entropy confirms the earlier calculation, straightforwardly obtained from the Cosmological Constant in Einstein's Equation, as presented in [16,v3/v4] and [20], that Milgrom's acceleration constant is given by,

$$a_0=\frac{15}{4}\Omega_B\frac{c}{t_H}, \quad (18)$$

where t_H ($\approx$13.8 Gyear) is Hubble's timescale and Ω_B ($\approx$0.0486) is the baryonic share of the gravitational energy as given in the Lamda-CDM cosmological Standard Model.

It is fair to conclude as well that this analysis supports the expected quantum mechanical nature of gravity. This, however, does not mean that the nature of the gravitational energy is fully understood, for instance, because no instrument is available as yet to calculate the baryonic share Ω_B by theory otherwise than assuming that Milgrom's acceleration a_0 is a second gravitational constant next to G. The curious fact that the factor 4/15 (=0.267) in (16) is almost the same as Ω_m (= 0.259), suggests that further research might reveal more. Another reason for further research is the challenge to describe the vibrating gravitational model more fundamentally than just by the Heisenberg uncertainty. To do so, the principles of QFT have to be invoked, where the Heisenberg uncertainty is formally described in terms of the commutation laws of operators representing momenta.

Appendix: the dipole moment density

It is well known that, as long as the Cosmological Constant Λ is supposed to be zero, the Newtonian potential field Φ can be derived as the weak field limit of Einstein's Field Equation. Although a non-zero value of Λ is a major roadblock to derive an expression for a modified Newtonian potential, it can be done under particular constraints for the spatial validity range. Previous studies [16] show that, under these conditions, the resulting potential Φ of cosmological systems with a central pointlike mass M is the solution of the field equation,

$$\frac{\partial^2}{\partial r^2}(r\Phi) + \lambda^2 (r\Phi) = -r\frac{4\pi GM}{c^2}\delta^3(r) \text{, where } \lambda^2 = 2\Lambda \text{,} \tag{A-1}$$

which in an alternative format can be written as,

$$\nabla^2\Phi + \lambda^2\Phi = -\frac{4\pi GM}{c^2}\delta^3(r). \tag{A-2}$$

The constraints mentioned apply to the extreme low end of the spatial range, but also to the extreme far end of it. The first constraint is not different from the weak field limitation that has to be imposed to derive Poisson's equation in the case of $\Lambda =$ 0. The second constraint is required to allow the derivation of Φ from a single metric component in Einstein's metric tensor. As shown in [16], these conditions are met in solar systems as well as in galaxy systems.

By calculating from (A-2) the gravitational acceleration g of objects, it is shown, in [16], that the Newtonian acceleration g_N, in the relevant spatial range, is modified in accordance with Milgrom's heuristic expression toward,

$$g = \sqrt{g_N a_0}\text{,} \tag{A-3}$$

where a_0 is Milgrom's acceleration constant, such that

$$\lambda^2 = 2\Lambda = \frac{2a_0}{5MG}. \tag{A-4}$$

The striking feature of (A-1) is the $+$ sign associated with λ^2. If it were a $-$ sign, the equation would be similar to Debije's equation for the potential of an electric pointlike charge in an electromagnetic plasma [22]. As is well know, the solution of such equation is a shielded Coulomb field, i.e., an electric field with an exponential decay. In the gravitational equivalent (with the $+$ sign) the near field is enhanced ("antiscreened"), because masses are attracting, while electric charges with the same polarity are repelling. The way to solve the equation, though, is similar. Eq. (A-2) can be written as,

$$\nabla^2\Phi = -4\pi G\rho(r), \text{ with } \rho(r) = M\delta^3(r) + \rho_D(r), \tag{A-5}$$

where, in the Debije process, $\rho_D(r)$ is known as polarization charge

$$\rho_D(r) = \frac{\lambda^2}{4\pi G}\Phi(r). \tag{A-6}$$

The polarization charge can be related with the polarization density vector $\mathbf{P_g}$, [12,23], such that

$$\rho_D(r) = -\nabla\cdot\mathbf{P_g} = \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\{r^2 P_g(r)\}. \tag{A-7}$$

The polarization density vector is the density of the resultant dipole moment vector in the polarization charge.

Assuming that eventually, in static condition, the space fluid is fully polarized by the field of the pointlike source, $P_g(r)$ is a constant P_{g0}. Hence, from (A-7),

$$\rho_D(r) = 2\frac{P_{g0}}{r}. \tag{A-8}$$

Taking into account that to first order,

$$\Phi(r) = \frac{MG}{r}, \tag{A-9}$$

we have from (A-8) and (A-9),

$$\rho_D(r) = \frac{2P_{g0}}{MG}\Phi(r). \tag{A-10}$$

From (A-10), (A-6) and (A-4) we have

$$P_{g0} = \frac{a_0}{20\pi G}. \tag{A-11}$$

The density of the dipole moment vectors, polarized or not, is therefore expressed by the right-hand part of (A-11). Although in this appendix this expression is derived from a cosmological system with central mass, it holds for the distributed mass in the cosmological space as well, [20].

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