 doi:10.20944/preprints201810.0713.v1

A quantum mechanical relationship between Milgrom's acceleration constant and the Bekenstein-Hawking entropy expression

Engel Roza
Philips Research Labs, Eindhoven, The Netherlands (retired)
Email: engel.roza@onsbrabantnet.nl

Summary
Conceiving vacuum energy as gravitational particles subject to Heisenberg's energy-time uncertainty, modelled as dipoles in a fluidal space at thermodynamic equilibrium, and interpreting the Bekenstein-Hawking entropy as the effective amount of spins of those dipoles enclosed within the event horizon of the universe, allows the calculation of Milgrom's acceleration constant. The result is a quantum mechanical interpretation of gravity, and dark matter in particular.

Keywords: Milgrom's acceleration constant; Bekenstein-Hawking entropy; gravitational dipole; dark matter.

Introduction

One of the most outstanding problems in present physics is the search towards the relationship between quantum mechanics and gravity. This subject has given rise to a discipline in itself, dubbed as quantum gravity. Within this discipline, new concepts are being developed, such as for instance string theory. Those new concepts are often denoted as new physics. It is believed that such new concepts will reveal certain phenomena in cosmology, such as for instance the dark matter mystery. An interesting approach is building new physics emergently from basic principles, such as for instance from the entropy concept as known from thermodynamics. Recent proposals in this direction have been put forward by Verlinde [1,2], inspired by an idea from Jacobson [3] back in 1993. These proposals have resulted in a particular approach to gravity, known as entropic gravity. Verlinde's theory aims to connect entropic gravity with string theoretic principles and information theory. The drawback of this approach for a scientist with an interest in the subject is the need to cope with the highly sophisticated mathematical mask that seems to be needed for the description of the role of informatics and thermodynamic entropy. It is the aim of the author of this article to reverse the argumentation. Rather than showing how Einstein's theory of gravity emerges from basic principles, he wishes to show that the basic principles emerge from Einstein's theory, even to the extent of the role of informatics and thermodynamic entropy. It is the author's belief that this will result in easier understanding of the very same principles and in quantitative results that give a better fit to observational evidence. One of the results, next to understanding the role of informatics in gravity, is a very clear explanation of the dark matter phenomenon, culminating in a quantitative calculation Milgrom's acceleration constant from basic quantum mechanical principles, developed from conceiving the vacuum energy term in Einstein's equation as the Heisenberg fluctuation on the irrelevant thermal dynamical equilibrium state of spatial particles in a virtual fluid. This turns the empirical modification of Newton's gravity law into a fundamental modification with the same, and even more, strength for successful calculations and predictions in cosmology that fit to observational evidence.

Theory

Ultimately, it is the aim in this article to show that Milgrom's acceleration constant can straightforwardly be calculated from the well known entropy expression for black holes as eventually established by Hawking as an improvement of Bekenstein's original formulation. In this respect it is not different from like it is in Verlinde's artcle [2]. The theory, the interpretation and the results, though, are quite different. The Bekenstein-Hawking expression is one of five ingredients for calculation. It reads as [4,5],

$$S = \frac{c^3}{4G\hbar} A \tag{1}$$

where c is the vacuum light velocity, G the gravitational constant, $\hbar$ Planck's (reduced) constant and A the black hole's peripheral area. The peripheral area of a spherical black hole is determined by its Schwarzschild radius as,

$$A = 4\pi R_S^2 \,,\; R_S = \frac{2MG}{c^2} \,, \tag{2}$$

where M is the baryonic mass of the black hole.

The second ingredient is the well known observation that the event horizon ct_H of the visible universe of a "flat" universe equals Schwarzschild radius of the critical mass enclosed within that horizon (t_H is the Hubble time scale), [6]. Hence, from (1), the entropy within the event horizon of the flat universe can be established as

$$S = \frac{c^3}{4G\hbar} 4\pi (ct_H)^2 \,. \tag{3}$$

The third ingredient is a consideration on the possible vacuum energy of the universe. Let us suppose that the origin of gravitational energy is due to the Heisenberg uncertainty of elementary spatial particles. These particles are subject to the constrained time-energy product ΔE x Δt. Usually the Heisenberg relationship is expressed as $\Delta E \Delta t = \hbar / 2$, [7]. In this particular case, ΔE expresses that the spatial particle's vacuum energy is subject to a $\pm \Delta E / 2$ deviation around its thermodynamic equilibrium state. This deviation can be interpreted as the vibration energy of the one-body equivalent of a two-body quantum mechanical oscillator. The vibration energy can then be modelled as the ground state $m_{eff} c^2$ energy of an effective mass in the centre, thereby considering the energetic equilibrium state of the spatial particle as irrelevant. In this model, the two bodies compose a gravitational dipole to which a certain dipole moment can be assigned. Let us assign a mass m to each of the two bodies and let us assign a spacing d between them. This establishes a dipole moment $p = md$ to each elementary vibrating spatial particle. Let the spacing between the poles be determined as $d = c\Delta t$, where Δt is the time uncertainty subject to the Heisenberg constraint, i.e.,

$$d = c\Delta t = c\frac{\hbar}{2}\frac{1}{\Delta E} \rightarrow d = c\frac{\hbar}{2}\frac{1}{m_{eff} c^2} = c\frac{\hbar}{2}\frac{1}{\alpha m c^2} \rightarrow p = md = \frac{\hbar}{2\alpha c} \,, \tag{4}$$

where α is a dimensionless factor that relates the two body masses with the equivalent mass of the vibration energy of the spatial particle. The magnitude of α can be estimated from the relationship

$$m_{eff} = \frac{1}{1/m + 1/m} = \frac{m^2}{2m} = \frac{m}{2} = \alpha m \rightarrow \alpha = \frac{1}{2} \,. \tag{5}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 30 October 2018 doi:10.20944/preprints201810.0713.v1

However, where the effective mass is only halve of the mass of the constituent bodies, the effective spatial coordinate of the effective mass is doubled [8].

Hence, from (3) and (4)

$$p = g\frac{\hbar}{2c}, \tag{6}$$

where I have added a dimensionless *gyrometric* factor g, expected as being $g = 1$, to account for the uncertainty in the Heisenberg relationship. It will be re-discussed later in this text. This implies so far that elementary spatial particles can be modelled by gravitational dipoles with a dipole moment p as described by (6).

The fourth ingredient is the baryonic dipole moment density P_g as shows up in the interpretation of the impact of the Cosmological Constant on the Newtonian gravity as described in [9] (see note), where it is shown that

$$P_g = \frac{a_0}{20\pi G}, \tag{7}$$

where a_0 is Milgrom's empirical acceleration constant [10].

Note: the referenced article is a preprint v3, which has got an update v4. Regrettably, in the update the dipole concept has been omitted, because in the article it was not more than a discussion item that could be regarded as controversial. In the appendix, the concept is explained again.

From (5) and (6), we may calculate the amount N_g of gravitational dipoles in the spatial volume V enclosed by the event horizon of the universe, as

$$N_g = \frac{P_g V}{p};\ V = \frac{4}{3}\pi(ct_H)^3 \tag{8}$$

Not all of these gravitational dipoles are baryonic. In terms of the Lamda-CDM nomenclature the baryonic share is expressed as Ω_B in the relationship

$$1 = \Omega_m + \Omega_\Lambda = (\Omega_B + \Omega_D) + \Omega_\Lambda, \tag{9}$$

where $\Omega_m, \Omega_\Lambda, \Omega_B, \Omega_D$, respectively are the relative matter density, the relative dark energy matter density, the relative baryonic matter density and the relative dark matter density [11]. Where the matter distribution between the matter density Ω_m (= 0.259) and dark energy density Ω_Λ (= 0.741) is largely understood as a consequence from the Friedmann equations [12] that evolve from Einstein's Field Equation under the Friedmann-Lemairtre-Robertson-Walker (FLRW) metric [13], the distribution between the baryonic matter density Ω_B (= 0.0486) and dark matter density Ω_D (= 0.210) is empirically established from observation. The quoted values are those as established by the Planck Collaboration [11]. Taking this account, the amount of baryonic dipoles in a spatial volume V is established as,

$$N_B = \frac{P_g}{\Omega_B p} V = \frac{a_0}{20\pi G} \frac{2c}{g\Omega_B \hbar} (\frac{4}{3}\pi c^3 t_H^3) . \tag{10}$$

These dipoles carry the total amount of baryonic energy. Their spin (= direction of the dipole moment vector) states carry the amount of baryonic information contained in the universe. Because the quantum states of the spin can assume two values only, the number N_B are bits that represent the baryonic information content of a space with volume V. Hence, both S as defined by (1,2) and N_B as defined by (10) express the entropy of the volume enclosed within the event horizon of the universe. Hence, equating S and N_B yields the instrument for the calculation of Milgrom's acceleration constant a_0. Doing so, we find from

$$a_0 = 2\pi g \frac{15}{4} \Omega_B \frac{c}{t_H} . \tag{11}$$

This result is very similar to the one as derived by me before [14] from a quite different perspective, without invoking the Bekenstein-Hawking relationship and without assigning the vacuum energy to the Heisenberg uncertainty. However, making the results identical, imposes to assign a value $1/2\pi$ to the gyrometric factor g. Doing so, we find, as in [11], with Ω_B =0.0486, c = 3 x 10^8 m/s and Hubble time scale t_H =13.8 Gyear, the result $a_0 \approx$ 1.25 x 10^{-10} m/s^2, which corresponds extremely well with observational evidence [15].

Discussion

It will be clear that the beauty of the theory as presented in this text is affected by the required modification of the gyrometric factor from the comprehensible value $g = 1$ to the manipulative g = 1/2π. Although in the quantum theory of electromagnetism some similar manipulation is done with the introduction of the gyromagnetic factor, a more satisfactory explanation would be most welcome. In this discussion paragraph, I'll propose one, albeit with some reluctance. The proposition is to accept a view on the Bekenstein-Hawking entropy as suggested by Susskind (for explanation purpose) in one of his lectures [16]. Susskind has proposed to consider the black hole as a body that captures or releases elementary discrete packages of energy $\Delta M = hf = \hbar\omega$, building its total mass M as a sum of N elementary amounts ΔM. To this end, the Compton wavelength of these elements must equate the peripheral circle $2\pi R_S$ of the black hole, such that

$$\Delta M = hf = h\frac{1}{T} = \frac{hc}{cT} = \frac{hc}{2\pi R_S} = \frac{\hbar c}{R_S} . \tag{12}$$

In Susskind's explanation model the Bekenstein-Hawking entropy is equated with the ratio,

$$S = N = \frac{M}{\Delta M} = \frac{R_S}{\hbar c} M = \frac{R_S}{\hbar c} (\frac{2MG}{c^2}) \frac{c^2}{2G} = \frac{c^3}{2\hbar G} R_S^2 = \frac{c^3}{2\hbar G} \frac{A}{4\pi} = \frac{1}{2\pi} \frac{c^3}{4\hbar G} A . \tag{13}$$

This result shows a $1/2\pi$ difference with the canonical expression (1,2) of the Bekenstein-Hawking entropy. Curiously, if we would accept the Susskind model, there would be no need to modify the gyrometric factor from $g = 1$ to $g = 1/2\pi$. It will be clear that this simplified model is in sharp contrast with the detailed mathematical treatment of Hawking. What kind of conclusion should be drawn in this respect is up to the reader of this text. However, even if most credit has to be given to Hawking's calculation, the simplicity of $1/2\pi$ factor is convincing enough for the conclusion that the quantum mechanical calculation of Milgrom's acceleration constant from the Bekenstein-Hawking entropy confirms the earlier calculation, straightforwardly obtained from the Cosmological Constant in Einstein's Equation, as presented in [9,v3,v4] and [14], that Milgrom's acceleration constant is given by,

$$a_0 = \frac{15}{4}\Omega_B \frac{c}{t_H}, \tag{14}$$

where t_H ($\approx$13.8 Gyear) is Hubble's timescale and Ω_B ($\approx$0.0486) is the baryonic share of the gravitational energy as given in the Lamda-CDM cosmological Standard Model. It is fair to conclude as well that this analysis supports the expected quantum mechanical nature of gravity. This however does not mean that the nature of the gravitational energy is fully understood, for instance because no instrument is available as yet to calculate the baryonic share Ω_B by theory otherwise than assuming that Milgrom's acceleration a_0 is a second gravitational constant next to G.

Appendix: the dipole moment density

It is well known that, as long as the Cosmological Constant Λ is supposed to be zero, the Newtonian potential field Φ can be derived as the weak field limit of Einstein's Field Equation. Although a non-zero value of Λ is a major roadblock to derive an expression for a modified Newtonian potential, it can be done under particular constraints for the spatial validity range. Previous studies [9] show that, under these conditions, the resulting potential Φ of cosmological systems with a central pointlike mass M is the solution of the field equation,

$$\frac{\partial^2}{\partial r^2}(r\Phi) + \lambda^2 (r\Phi) = -r\frac{4\pi GM}{c^2}\delta^3(r), \text{ where } \lambda^2 = 2\Lambda, \tag{A-1}$$

which in an alternative format can be written as,

$$\nabla^2\Phi + \lambda^2\Phi = -\frac{4\pi GM}{c^2}\delta^3(r). \tag{A-2}$$

The constraints mentioned apply to the extreme low end of the spatial range, but also to the extreme far end of it. The first constraint is not different from the weak field limitation that has to be imposed to derive Poisson's equation in the case of $\Lambda =$ 0. The second constraint is required to allow the derivation of Φ from a single metric component in Einstein's metric tensor. As shown in [9,v4], these conditions are met in solar systems as well as in galaxy systems.

By calculating from (A-2) the gravitational acceleration g of objects, it is shown that the Newtonian acceleration g_N is modified in accordance with Milgrom's heuristic expression toward,

$$g = \sqrt{g_N a_0}, \tag{A-3}$$

where a_0 is Milgrom's acceleration constant, such that

$$\lambda^2 = 2\Lambda = \frac{2a_0}{5MG}. \qquad \text{(A-4)}$$

The striking feature of (A-1) is the $+$ sign associated with λ^2. If it were a $-$ sign, the equation would be similar to Debije's equation for the potential of an electric pointlike charge in an electromagnetic plasma [...]. As is well know, the solution of such equation is a shielded Coulomb field, i.e., an electric field with an exponential decay. In the gravitational equivalent (with the $+$ sign) the near field is enhanced ("antiscreened"), because masses are attracting, while electric charges with the same polarity are repelling. The way to solve the equation, though, is similar. Eq. (A-2) can be written as,

$$\nabla^2\Phi = -4\pi G\rho(r), \text{ with } \rho(r) = M\delta^3(r) + \rho_D(r), \qquad \text{(A-5)}$$

where, in the Debije process, $\rho_D(r)$ is known as polarization charge

$$\rho_D(r) = \frac{\lambda^2}{4\pi G}\Phi(r). \qquad \text{(A-6)}$$

The polarization charge can be related with the polarization density vector $\mathbf{P_g}$, [17,18], such that

$$\rho_D(r) = -\nabla\cdot\mathbf{P_g} = \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\{r^2 P_g(r)\}. \qquad \text{(A-7)}$$

The polarization density vector is the density of the dipole moment vectors in the polarization charge.

Assuming that eventually in static condition the space fluid is fully polarized by the field of the pointlike source, $P_g(r)$ is a constant P_{g0}. Hence, from (A-7),

$$\rho_D(r) = 2\frac{P_{g0}}{r}. \qquad \text{(A-8)}$$

Taking into account that to first order,

$$\Phi(r) = \frac{MG}{r}, \qquad \text{(A-9)}$$

we have from (A-8) and (A-9),

$$\rho_D(r) = \frac{2P_{g0}}{MG}\Phi(r). \qquad \text{(A-10)}$$

From (A-10), (A-6) and (A-4) we have

$$P_{g0} = \frac{a_0}{20\pi G}. \qquad \text{(A-11)}$$

The density of the dipole moment vectors, polarized or not, is therefore expressed by the right-hand part of (A-11). Although in this appendix this expression is derived from a cosmological system with central mass, it holds for the cosmological space as whole, with its distributed mass as well, [14].

References

[1] E. Verlinde, JHEP 1104, 029 ; arXiv: 1001.0785v1 [hep-th] (2011)
[2] E. Verlinde, SciPost Phys. 2, 016 (2017); arXiv: 1611.02269v2 [hep-th] (2016)
[3] T. Jacobson, Phys.Rev. Lett., 75, no. 7, 1260 (1995)
[4] J.D. Bekenstein, Phys. Rev. D7, 2333 (1973)
[5] S.W. Hawking, Comm. Math. Phys. 43, 199 (1975)
[6] R.K. Pathria, Nature, 240 (5379, 298 (1972)
[7] L.I. Mandelshtam, I.E. Tamm, J. Phys (USSR) 9, 249 (1945)
[8] http://vergil.chemistry.gatech.edu/notes/ho/node2.html
[9] E. Roza, doi: 10.20944/preprints201705.0164.v3/v4 (2017/2018)
[10] M. Milgrom, The Astrophysical Journal, 270, 365 (1983)
[11] Planck Collaboration, Astronomy and Astrophys., 594: A13 (2015)
[12] A. Friedmann, Zeitschrift fur Physik A, 10(1), 377 (1922)
[13] R. d'Inverno, Introducing Einstein's Relativity, Ch. 23, Oxford Press (1992)
[14] E. Roza, doi:10.20944/preprints201712.0077.v2 (2017)
[15] G. Gentile, B. Famae and W.G.J . de Blok, J. Astronomy and Astrophysics, 527, 76 (2011)
[16] L. Susskind, Inside blackholes, lecture on youtube
[17] D. Hajdukovic, Astrophysics and Space Science, 334, vol.2, 215 (2011)
[18] C.A. Gonano, R.E. Zich, M. Mussetta, Progr. in Electromagn. Res. B, 64, 83 (2015)