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# Semi-automatic Algorithms for Estimation and Tracking of AP-diameter of the IVC in Ultrasound Images

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**Abstract:** Medical research has suggested that the anterior-posterior (AP)-diameter of the inferior vena cava (IVC) and its associated temporal variation as imaged by bedside ultrasound is useful in guiding fluid resuscitation of the critically-ill patients. This paper develops semi-automatic active ellipse and rectangle algorithms for measurement and tracking of the AP-diameter. The proposed algorithms are compared with an expert manual measurement and the previous work based on active circle model. It is shown that regardless of the shape of the IVC, the rectangle model always outperforms the two other models and performs very close to manual measurement.

**Keywords:** Inferior vena cava (IVC); ultrasound imaging; anterior posterior (AP) diameter; active ellipse; active rectangle; volume status.

## 1. Introduction

Estimation and monitoring of relative changes in circulating blood volume is critically important for several medical conditions including hemorrhage from trauma, stomach ulcers, blood vessel rupture, septic shock, and volume overload in the setting of congestive heart failure. Fast and accurate monitoring of circulating blood volume is a challenging task as excessive or insufficient fluid administration increases patient morbidity and mortality[1,2]. Clinical research has suggested that the variations in the anterior-posterior (AP) diameter of inferior vena cava (IVC) is correlated with patient's volume status [3–5]. Clinically, the AP-diameter is manually estimated from portable ultrasound imagery which is a tedious and time-consuming task in the setting of poor image quality. Furthermore, artifacts such as shadowing and speckle noise frequently result in indistinct edges and gaps in the vessel walls reducing the accuracy of manual measurement[6,7].

Speckle noise present in ultrasound imagery is theoretically considered to be a Rayleigh distributed multiplicative noise [8] as the envelop of the ultrasound wave reflected from each tissue has a Rayleigh distribution. Hence, Rayleigh mixture models have been proposed as a solution for ultrasound image segmentation [9,10]. However, it has been shown that due to the scattering population and signal processing, the speckle distribution deviates from Rayleigh[11]. In [12], authors proposed a fast algorithm based on optical flow for tracking of the speckles in ultrasound images. This approach fails when the speckle structure is rapidly deformed.

Active contours (ACs) are widely used for segmentation of ultrasound images [13–17]. ACs convert the problem of image segmentation into a minimization of an energy functional with their performance frequently dependent on a manually-defined initialization contour. In order to avoid local minima, the initiating contour needs to be as close as possible to the actual contour. ACs can be combined with other segmentation algorithms as a coarse-to-fine strategy to reduce the impact of the initial contour on segmentation error [18,19]. Researchers have addressed the challenge of IVC segmentation using this strategy by using template matching method as the coarse segmentation and AC as the fine-tuning (TMAC) [20]. Unfortunately, this approach fails when the IVC undergoes large frame-to-frame

36 variations commonly present on portable machines with lower frame rates (e.g. 30 frames-per-second).  
 37 Additionally, ACs continue to perform poorly in the context of fuzzy or unclear boundaries as is  
 38 commonly the case for the IVC.

39 Given that the cross-section of the IVC is largely convex, the IVC contour can be represented in  
 40 polar coordinates and consequently, polar active contours appear as a promising solution for IVC  
 41 segmentation [21]. In [22], a polar AC model based on the third centralized moment (M3) was proposed  
 42 for segmentation of IVC images. Unfortunately, M3 algorithms roughly estimates the cross-sectional  
 43 area (CSA) of the IVC and fails with poor quality images.

44 Clinically the CSA of the IVC is an optimal approach to accurately assess a patient's volume status,  
 45 but all existing approaches fail to accurately estimate the CSA. Hence, clinicians instead of the the  
 46 whole CSA of the IVC, measure its AP-diameter. In [23], AP-diameter was modeled as the diameter of  
 47 a circle fitted inside IVC. This approach, although provides good results, it deviates with the clinical  
 48 definition of AP-diameter. Clinicians measures the AP-diameter as the largest vertical diameter in the  
 49 IVC and obviously, if the CSA of the IVC is deviated from the horizontal angle, the circle diameter  
 50 slightly defers from the clinically measure AP-diameter.

51 In this paper, we propose two algorithms based on ellipse and rectangle models. The height of a thin  
 52 rectangle fitted inside the IVC can efficiently model its clinically measured AP-diameter. We also  
 53 develop another algorithm based on ellipse fitting just for comparison purpose.

54 The remainder of this paper is organized as follows - Section 2 discusses the background and related  
 55 work. The proposed active rectangle and active ellipse algorithms are presented in Section 3 while  
 56 experimental results are in Section 4, the results are discussed in Section 5 and the paper is concluded  
 57 in Section 6.

## 58 2. Background and Related Work

In [23], authors showed that the AP-diameter of the IVC can be accurately modeled with the  
 diameter of a circle fitted in inside the IVC. The active circle algorithm proposed in [23] is based on  
 the following evolution functional:

$$E = \alpha(u - v)(2I - u - v), \quad (1)$$

where  $u$  and  $v$  are the mean of the intensities for the pixels inside and outside the contour, respectively,  
 and  $I$  is the intensity of the pixels on the contour. This functional is used to evolve the parameters of  
 the circle, i.e.,  $R$  as the circle radius and  $(x_c, y_c)$  as its center coordinates. For this, the circle is sampled  
 at  $K$  points with polar angles  $\theta_k = \frac{2k\pi}{N}$ ,  $k = 0, 1, \dots, K - 1$ , where the normal vector and the Cartesian  
 coordinates corresponding to the  $k$ th sampled point are denoted as

$$\vec{n}_k = [\cos(\theta_k), \sin(\theta_k)]^T, \quad (2)$$

and

$$[x_k, y_k]^T = [x_c, y_c]^T + R\vec{n}_k, \quad (3)$$

respectively. The evolution functional generates forces

$$f_k = \alpha(u - v)(2I_k - u - v) \quad (4)$$

along the normal vectors. The forces shift the sampled contour points to new positions governed  
 by

$$[\tilde{x}_k, \tilde{y}_k] = [x_c, y_c]^T + (R + f_k)\vec{n}_k. \quad (5)$$

where  $f_k$  is the value of the evolution functional at  $k$ th contour point. It is shown that the average evolution functional approaches zero when the contour points are on the IVC boundary. In active circle algorithm, the center of the circle is evolved as

$$[\tilde{x}_c, \tilde{y}_c] = [x_c, y_c] + \frac{1}{K} \sum_{k=0}^{K-1} f_k \vec{n}_k, \quad (6)$$

where  $[x_c, y_c]$  and  $[\tilde{x}_c, \tilde{y}_c]$  the center coordinates of the circle before and after evolution, respectively, and  $f_k$  is the evolution force for the  $k$ th contour points obtained from 1, and  $\vec{n}_k$  is its corresponding normal vector that can be obtained as

$$\vec{n}_k = [\cos(\theta_k), \sin(\theta_k)]^T, \quad (7)$$

Eq. (6) indicates that for a given set of forces the circle center is simply shifted by the average of the force vectors  $f_k \vec{n}_k$ .

The center of the circle is evolved as

$$\tilde{R} = R + \frac{1}{K} \sum_{k=0}^{K-1} f_k. \quad (8)$$

59 where  $R$  and  $\tilde{R}$  is the radius of the circle before and after evolution, respectively. This indicates that  
60 for a given set of forces, the circle radius is evolved with the average of the force values  $f_k$ .

### 61 3. Proposed Algorithms

62 In this section, we develop active ellipse and rectangle models based on the evolution functional  
63 proposed in [23].

#### 64 3.1. Active Ellipse Model

In [23], authors showed that the active circle algorithm estimates the IVC AP-diameter much more accurate than the star-Kalman algorithm in [24] which is based on ellipse fitting. But does this indicate that a circular model can estimate the AP-diameter more accurate than an elliptical model? To answer this question, we develop an active ellipse algorithm based on the same evolutionary functional as the one in the active circle algorithm.

In general case, during the evolution, the  $k$ th contour point is evolved as

$$[\tilde{x}_k, \tilde{y}_k] = [x_k, y_k]^T + f_k \vec{n}_k. \quad (9)$$

To fit an ellipse to these  $K$  evolved points, we use the following conic equation.

$$ax^2 + bxy + cy^2 + d * x + e * y = 1, \quad (10)$$

where  $x$  and  $y$  are the coordinate of the points on the conic,  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$  are the conic parameters. Note that with an ellipse, the values of  $a$  and  $b$  must be positive. With  $K$  points with coordinates  $[\tilde{x}_k, \tilde{y}_k]$ , the best ellipse is fitted by minimizing the following cost function.

$$C = \sum_{k=1}^K (a\tilde{x}_k^2 + b\tilde{x}_k\tilde{y}_k + c\tilde{y}_k^2 + d\tilde{x}_k + e\tilde{y}_k - 1)^2 \quad (11)$$

Eq. (11) can be rewritten in matrix form

$$C(A) = A^T X^T X A + 2\mathbf{1}_K^T X A + K, \quad (12)$$

where the vector of conic parameters defined as  $A = [a, b, c, d, e]^T$ ,  $X$  is a matrix with  $[\tilde{x}_k^2, \tilde{x}_k, \tilde{y}_k, \tilde{y}_k^2, \tilde{x}_k, \tilde{y}_k]$  as its  $k$ th row,  $\mathbf{1}_K$  is  $K \times 1$  all-one vector, and superscript  $T$  is the transpose operator. After setting the gradient  $C(A)$  to zero, the vector  $A$  is obtained as

$$\hat{A} = -\mathbf{1}_K^T X (X^T X)^{-1}. \quad (13)$$

65 The active ellipse algorithm is summarized in table 1.

Fig. 1 shows the ellipse evolution versus the number of iterations for a sample IVC frame.

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#### Active ellipse algorithm for estimation and tracking of the IVC AP-diameter

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**Input:** An IVC video and parameter  $\alpha = 10^{-4}$ .

- Read one frame from the input video.
  - If it is the first frame, select one point inside the IVC by a mouse click. This point is assumed as the initial value of the ellipse center. The initial ellipse is created as circle centered at this point and initial radius  $R=8$  pixels.
  - 
  - Compute the forces for the points on each side of the rectangle using (4).
  - Update the ellipse parameters using (13).
  - Repeat the last two steps until a convergence is achieved. We assume the algorithm has reached the convergence when the maximum of the change in the elements of the  $A$  is less than  $10^{-4}$ , i.e., when  $\max(|\hat{A}_n - \hat{A}_{n-1}|)$ , where  $\max$  is the element-wise maximum, and  $\hat{A}_n$  is the vector  $A$  estimated at  $n$ th iteration.
  - Return to the first step for the next frame.
-



**Figure 1.** The rectangle evolution versus number of iterations.

### 67 3.2. Active Rectangle Model

The intuition to use a rectangular model is that the AP-diameter is clinically defined as the largest vertical diameter of the IVC contour which may practically deviate from the actual diameter of a circle or even an ellipse. The AP-diameter can be modeled as the height of a vertical thin rectangle. As a start point, we assume that the fitted rectangle has a fix width  $w=3$ pixels and the only parameters that have to be modified are the center and the height of the rectangle. With the forces defined as (4), either of the upper and lower sides of the rectangle move with the average of the forces applied on that side. Hence, the center of the rectangle is moved as

$$\tilde{x}_c = x_c + \frac{1}{K_l} \sum_{p \in P_l} f_p - \frac{1}{K_r} \sum_{p \in P_r} f_p, \quad (14)$$

$$\tilde{y}_c = y_c + \frac{1}{K_u} \sum_{p \in P_u} f_p - \frac{1}{K_b} \sum_{p \in P_b} f_p, \quad (15)$$

where  $P_l$ ,  $P_r$ ,  $P_u$ , and  $P_b$  are the subsets of the contour points on the left, right, upper, and lower sides of the rectangle, respectively, and  $K_l$ ,  $K_r$ ,  $K_u$ , and  $K_b$  are the number of points in each of these sets. Similarly the height of the rectangle is evolved as

$$\tilde{h} = h + \frac{1}{K_u} \sum_{p \in P_u} f_p + \frac{1}{K_b} \sum_{p \in P_b} f_p. \quad (16)$$

68 Although a thin rectangle model accurately models the clinically measured AP-diameter, it might  
69 be lost if parts of the IVC boundaries are missing as the IVC edges are not detected and hence, the

70 algorithm may diverge. To combat this problem, we modify the active rectangle algorithm by starting  
 71 with a rectangle with a much larger width as  $w=15$  pixels. This rectangle is gradually narrowed to its  
 72 final width, i.e.,  $w=3$  pixels which is narrow enough to model the AP-diameter. The active rectangle  
 algorithm is summarized as

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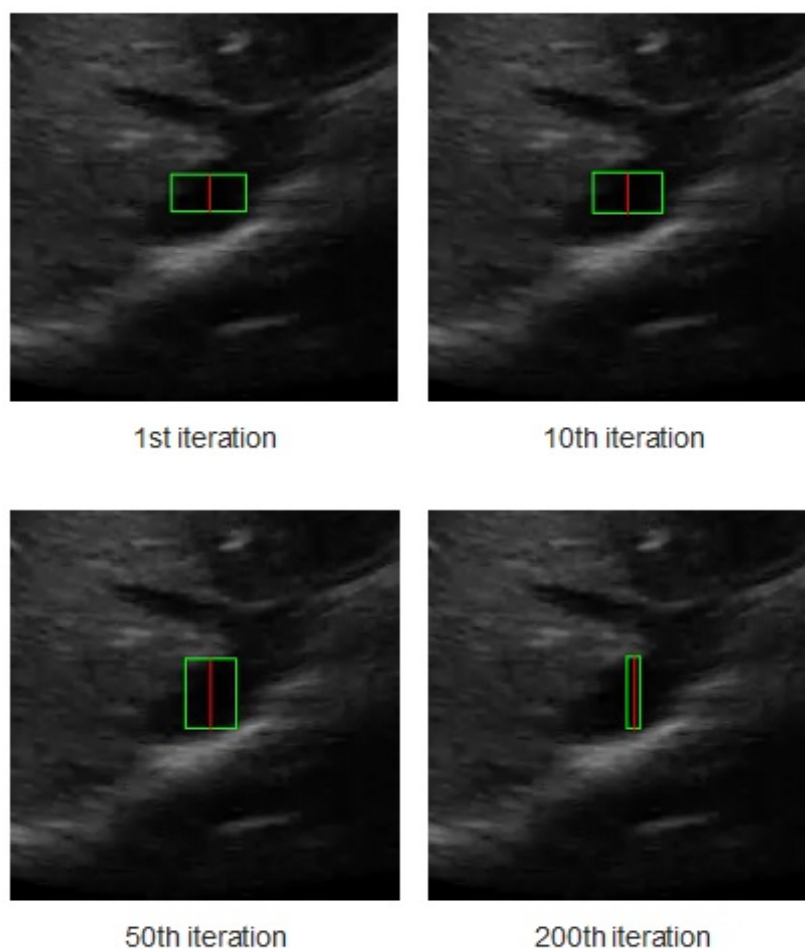
#### Active rectangle algorithm for estimation and tracking of the IVC AP-diameter

---

**Input:** An IVC video and parameter  $\alpha = 10^{-4}$ .

- Read one frame from the input video.
  - If it is the first frame, select one point inside the IVC by a mouse click. This point is assumed as the initial value of the rectangle center.
  - Create a rectangle with initial width  $w = 15$  and height  $h=4$  pixels.
  - Compute the forces for the points on each side of the rectangle using (4).
  - Update the height and center of the IVC using equations (14), (15), and (16) and shrink the rectangle width with factor  $0.2^{1/N}$  where  $N$  is the number of iterations. In this paper, we set  $N=200$  iterations.
  - Repeat the last two steps for  $N=200$  iterations.
  - Return to the first step for the next frame.
- 

73 Fig. 2 shows the rectangle evolution versus number of iterations for the IVC image as in Fig. 1.  
 74 By comparison of Figs. 1 and 2, one can see the active rectangle algorithm not only converges faster  
 75 but also more accurately estimates the AP-diameter than the active ellipse algorithm.  
 76



**Figure 2.** The rectangle evolution versus number of iterations.

## 77 4. Results

78 The experimental data was collected from eight young healthy male subjects. The study protocol  
79 was reviewed and approved by the Health Research Ethics Authority. The IVC was imaged in the  
80 transverse plane using a portable ultrasound (M-Turbo, Sonosite-FujiFilm) with a phased-array probe  
81 (1-5 Mhz). Each video has a frame rate of 30 fps, scan depth of 19cm, and a duration of 15 seconds  
82 (450 frames/clip). Fig. 3 depicts the first frame of all eight subjects. In Fig. 3, we can see that an IVC  
83 image can have different shapes and qualities. For instance, in the clip no. (1), although part of the  
84 image is shadowed, but the IVC edges are almost visible. The IVC videos for the clips no. (3) and  
85 (8) show the lowest quality as the IVC is almost collapsed in the former one and it vanishes after the  
86 initial frames in the latter one (this is not seen in this image).

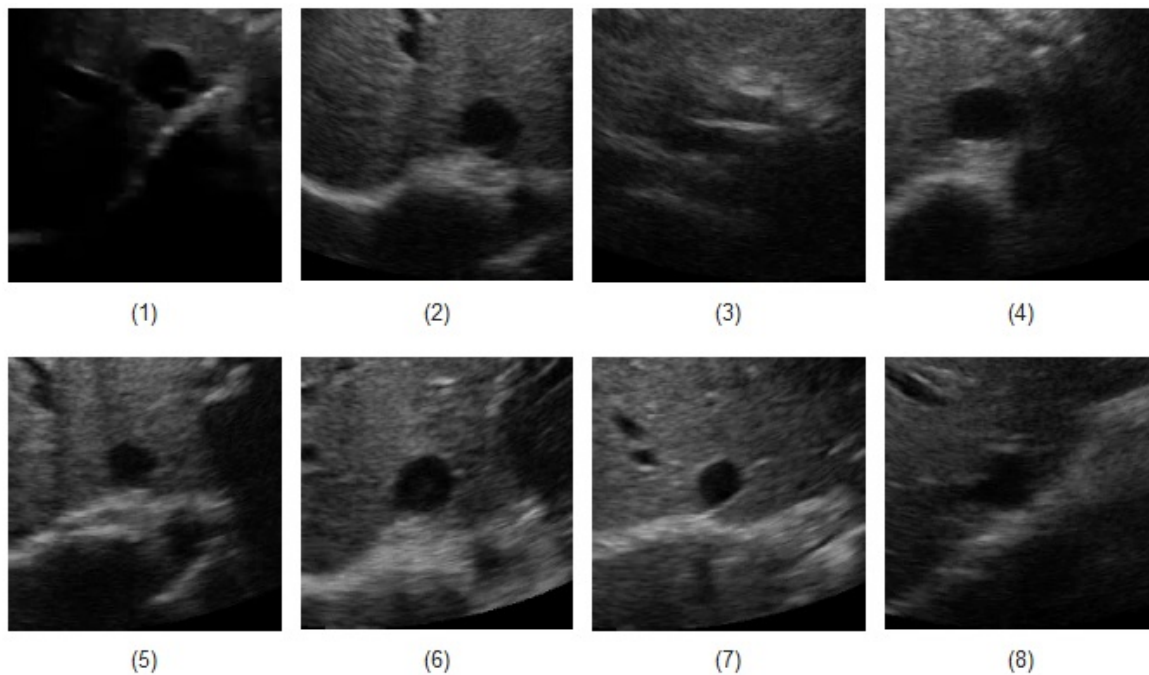
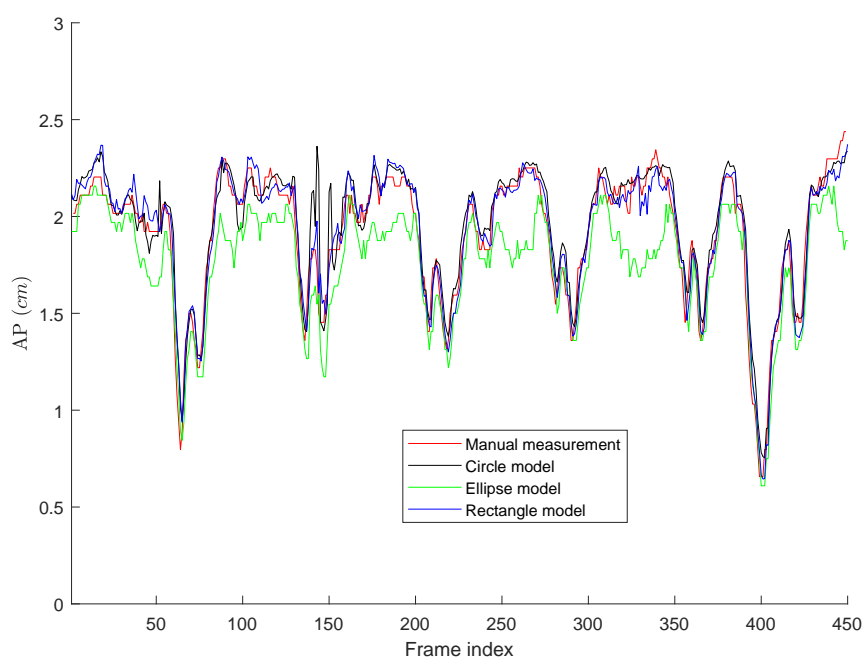


Figure 3. The first frame of the three sample IVC videos.

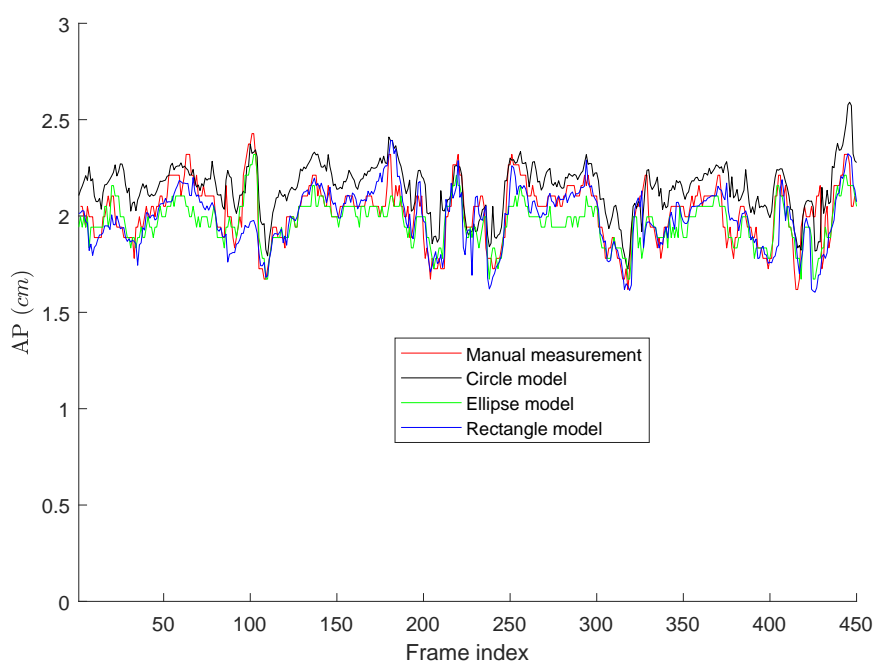
### 87 4.1. Tracking Performance

88 Figs. 4-6 present the AP-diameter manually measured by an expert and the ones that are  
89 semi-automatically estimated by the three shape-based algorithms for the three sample videos depicted  
90 as subjects (1)-(3) in Fig. 3. From Fig. 4, one can see that with the first IVC clip which has a good  
91 quality, both active circle and active rectangle algorithms efficiently track the manual measurement,  
92 although the active ellipse algorithm roughly track the manual measurement and performs poorer  
93 than the other two methods.

94 Fig. 5 presents the tracking results for the second clip where one can easily see that all three algorithms  
95 perform less accurate than the case in Fig. 4. This is mainly due to the fact that although the  
96 second clip seems to have a better quality than the first one, it has a more fuzzy contour, making the  
97 semi-automatic algorithms less accurate than the first clip. Similar to the first clip, we can see that the  
98 active rectangle algorithms performs better than the other two methods.



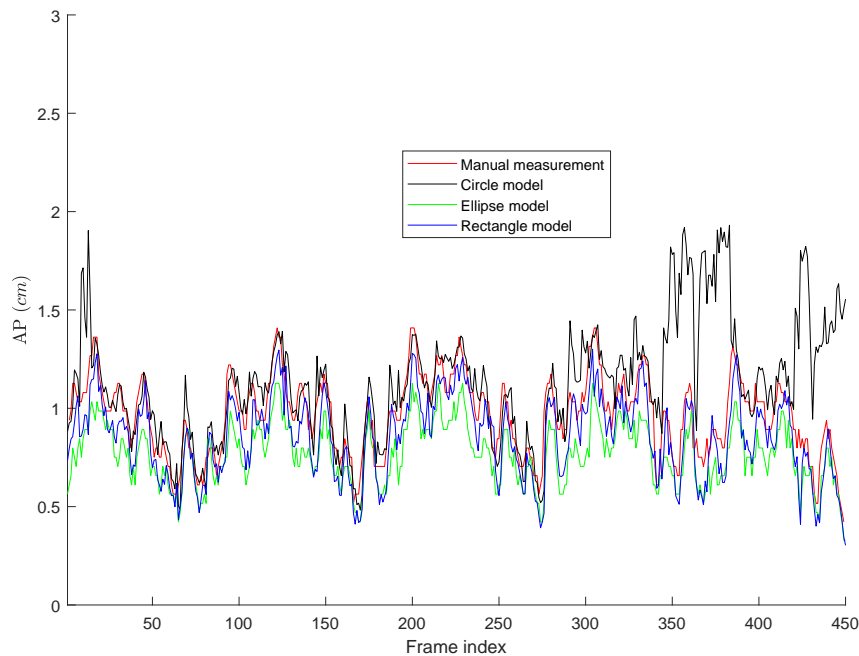
**Figure 4.** AP-diameter for the first sample videos depicted in Fig. 3, as measured by the manual measurement (black line), active circle algorithm (black line), active ellipse algorithm (green line) and active rectangle algorithm (blue line).



**Figure 5.** AP-diameter for the second sample videos depicted in Fig. 3, as measured by the manual measurement (black line), active circle algorithm (black line), active ellipse algorithm (green line) and active rectangle algorithm (blue line).



99 Fig. 5 presents the tracking results for the third clip depicted in Fig. 3 where the IVC is almost  
 100 collapsed and therefore, a smaller AP-diameter is expected. In this case, the active circle algorithm  
 101 loses the tracking after 337 frames, while both active ellipse and active rectangle algorithms efficiently  
 102 track the result obtained from the manual measurement.



**Figure 6.** AP-diameter for the third sample videos depicted in Fig. 3, as measured by the manual measurement (black line), active circle algorithm (black line), active ellipse algorithm (green line) and active rectangle algorithm (blue line).

103 Figs. 4-6 show that in all three investigated clips, the active rectangle algorithm outperforms the  
 104 other two methods. To have a better insight about the results, we present numerical results in tables 1  
 105 and 2. Table 1 present the room mean square (RMS) of the AP-diameter estimation error for the three  
 106 algorithms for the all eight clips depicted in Fig. 3. Note that the error is defined as the difference  
 107 between the AP-diameter estimated by each of the algorithms and the one manually measured by the  
 108 expert. Except with subject no. (8), where manual segmentation is reliable for only the first 150 frames  
 109 (see [23]), for the other seven subjects, the RMS of error is calculated over all 450 frames. From this  
 110 table, one can see that in all eight investigated cases, the active rectangle algorithm outperforms the  
 111 other two methods, while in five out of eight cases, the active circle algorithm performs more accurate  
 112 than the active ellipse algorithm.  
 113 Table 2 present the maximum absolute value of error obtained for the three algorithms for all eight  
 114 clips depicted in Fig. 3. This tables confirms the results obtained in Table 1, i.e., the active rectangle  
 115 algorithms always outperform the other two methods.

**Table 1.** RMS of the AP-diameter estimation error.

Method	Subject no.								Ave.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
Circle model	0.10	0.16	0.16	0.25	0.25	0.10	0.11	0.26	0.17
Ellipse model	0.21	0.19	0.14	0.26	0.18	0.11	0.11	0.20	0.35
Rectangle model	0.08	0.11	0.12	0.23	0.14	0.10	0.10	0.18	0.12

**Table 2.** Maximum absolute value of AP-diameter estimation error.

Method	Subject no.							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Circle model	0.06	0.29	0.48	0.57	0.75	0.41	0.44	0.43
Ellipse model	0.11	0.32	0.35	0.59	0.48	0.54	0.47	0.38
Rectangle model	0.05	0.18	0.19	0.42	0.28	0.37	0.39	0.29

## 116 5. Discussion

### 117 5.1. The Performance of the Proposed Algorithms

118 As it was described earlier in the Results section, for all eight investigated clips, the active  
 119 rectangle algorithm performs closer to the manual measurement than the other two methods. This is  
 120 due to the fact the AP-diameter is clinically measured as the maximum diameter inside the IVC. The  
 121 active circle algorithm finds the largest circle inside the IVC and assumes that the diameter of this  
 122 circle can efficiently model the AP-diameter. This is technically corrected if the IVC is horizontally  
 123 aligned. But based on the angle of the ultrasound prob, the IVC in the ultrasound clip can be rotated  
 124 along the horizontal axis and this makes the results a little different from the clinically measured  
 125 AP-diameter.

126 In five out of the eight cases, the active circle algorithm performed better than the active ellipse  
 127 algorithm. This is mainly due to the fact that a circle has less degree of freedom compared to an ellipse  
 128 and therefore, circle evolution can be performed more accurate than ellipse evolution. Furthermore,  
 129 from Fig. 3, one can see that in most cases, the IVC does not appear to have an elliptical shape making  
 130 the ellipse fitting process inaccurate.

### 131 5.2. Complexity Comparison

132 Obviously, an ellipse has five parameters to estimate while a circle has only three parameters,  
 133 i.e., two for its center coordinates and one for its radius. Consequently, the active ellipse algorithm  
 134 seems to be more complex than the active circle algorithm. The active rectangle algorithm also has  
 135 three parameters to estimate, i.e., two for the rectangle center and one for the rectangle height, but  
 136 its complexity is even much less than the active circle algorithm and it also requires less iterations  
 137 to converge. Furthermore, the main source of the complexity for all these three algorithms is for  
 138 computation of  $u$  and  $v$ , while with the rectangle model,  $u$  and  $v$  can be computed using the integral  
 139 image which is computed only once for each frame.

## 140 6. Conclusions

141 In this paper, two novel algorithms based on elliptical and rectangular models were proposed for  
 142 semi-automatic estimation of AP-diameter of the IVC in ultrasound videos. The proposed algorithm  
 143 were compared with active circle algorithm and was shown that although IVC usually has elliptical

144 CSA, both circular and rectangular models provide a more accurate AP-diameter measurement, while  
145 the rectangular model outperforms both circular and elliptical models. This is due that fact that  
146 the AP-diameter is clinically measured as the maximum vertical diameter of the IVC which can be  
147 modeled better as the vertical side of a rectangle than the diameter of a circle or an ellipse.

148 **Author Contributions:** Andrew Smith provided the data and performed the manual measurement, Ebrahim  
149 Karami designed the algorithms, performed the experiments, and wrote the paper, and Mohamed Shetata and  
150 Andrew Smith edited the paper.

151 **Conflicts of Interest:** The authors declare no conflict of interest.

## 152 Abbreviations

153 The following abbreviations are used in this manuscript:

154  
155 MDPI: Multidisciplinary Digital Publishing Institute

156 AP: Anterior-posterior

157 IVC: Inferior vena cava

158 CSA: Cross sectional area.

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