Unification of Thermo Field Kinetic and Hydrodynamics Approaches in Theory of Dense Quantum–Field Systems

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Abstract: A formulation of nonequilibrium thermo field dynamics has been performed using the nonequilibrium statistical operator method by D.N.Zubarev. Generalized transfer equations for a consistent description of kinetics and hydrodynamics of the dense quantum-field system with strongly coupled states are derived.

Keywords: nonequilibrium thermo field dynamics, kinetics, hydrodynamics, kinetic equations, transport coefficients, coupled states, quark-gluon plasma

1. Introduction

A problem of accounting for the bound states (clusters) [15,16] formed by particles is particularly important in the development of the theories of nonequilibrium processes of temperature quantum-field systems, such as nuclear matter [1–14]. Kinetic and hydrodynamic processes in a hot, compressed nuclear matter, which appears after ultrarelativistic collisions of heavy nuclei [5,12,14,17–21] are mutually connected, and, therefore, the coupled states between nucleons should be consider. This is of great importance for the analysis and correlation of final reaction products. Obviously, a nucleon interaction investigation based on a quark-gluon plasma is a sequential microscopic approach to the dynamical description of reactions in a nuclear matter. The problems of a dense quark-gluon matter were discussed in detail in [2,3,10,11,23–27].

In his recent works [15,16,19] G. Röpke noted the importance of constructing a nonequilibrium theory in which along with hydrodynamic parameters a cluster distribution function are taken into account, similarly to the case of the classical theory of non-equilibrium processes of dense gases and liquids [28–31].

In modern theoretical studies of the nonequilibrium properties of quark-gluon plasma [10,11,23,24], which is one of the states of nuclear matter, one of the most widely used statistical concept is the entropy of Tsallis and Renyi [32–41]. At the same time, the important problem of the construction of kinetic and hydrodynamic equations for nuclear matter of high density and high temperature is not sufficiently addressed for these systems. However, within the framework of the Gibbs statistics, the equations of hydrodynamics and thermodynamics were already considered in many papers using the method of the Zubarev’s nonequilibrium statistical operator [42–51], the projection operator method [52,53] and kinetic equations [54–57]. Thus, we propose an approach to solve these problems based on the the nonequilibrium thermofield dynamics [58–60] in the formulation of the method of nonequilibrium statistical operator [61–63]. Below, in the second section of this paper, we consider the nonequilibrium thermo field dynamics in the formulation of the nonequilibrium statistical operator method [64–66] in Renyi statistics. Next, in the third section, generalized equations for the consistent description of kinetic and hydrodynamic processes which take into account the bound states that emerge in the temperature quantum-field system will be presented.
2. Nonequilibrium Statistical Operator in Thermo Field Space

We use the nonequilibrium statistical operator method in the thermofield formulation [61,62], where the mean values corresponding to the observables can be found using the nonequilibrium thermovacuum state vector $|q(t)\rangle$:

$$\langle A \rangle^t = \langle 1 | A | q(t) \rangle = \langle 1 | \hat{A} | q(t) \rangle,$$

(1)

where $\hat{A}$ is a superoperator acting on the state $|q(t)\rangle$. The nonequilibrium thermovacuum state vector $|q(t)\rangle$ satisfies the Schrödinger equation [61]:

$$\frac{\partial}{\partial t} |q(t)\rangle - \frac{1}{i\hbar} [H_{\text{rel}}, |q(t)\rangle] = 0,$$

(2)

or

$$\frac{\partial}{\partial t} |q(t)\rangle - \frac{1}{i\hbar} \hat{H} |q(t)\rangle = 0.$$

(3)

Here, the total Hamiltonian $\hat{H}$ takes the form:

$$\hat{H} = H - \hat{\hat{H}},$$

(4)

where $\langle 1 | \hat{H} | 0 \rangle = 0$, and $H = H(\hat{a}^+, \hat{a}), \hat{H} = H(\hat{a}^+, \hat{a})$ are superoperators constructed from the creation and annihilation of superoperators of the thermal Liouville space [61,68,69]. The superoperators $H$ and $\hat{H}$ are defined in [61].

In the nonequilibrium statistical operator method in the thermofield formulation [61,62], the nonequilibrium thermovacuum state vector as a solution of the Schrödinger equation (3) with a source $-\varepsilon (\langle q(t)\rangle - \langle q_{\text{rel}}(t)\rangle)$, with the projection taken into account can be found in the form

$$|q(t)\rangle = |q_{\text{rel}}(t)\rangle + \int_{-\infty}^t dt' e^{i(t'-t)} T(t, t') \left[1 - \mathcal{P}_{\text{rel}}(t')\right] \frac{1}{i\hbar} \hat{H} |q_{\text{rel}}(t')\rangle.$$

(5)

Here $T(t, t') = \exp_+ \left\{ \int_t^{t'} dt' [1 - \mathcal{P}_{\text{rel}}(t')] \frac{1}{i\hbar} \hat{H} \right\}$ is the evolution operator with the projection taken into account, where $\exp_+$ is the ordered exponential, $\varepsilon \to +0$ after the thermodynamic limit transition.

$$\mathcal{P}_{\text{rel}}(t) \langle \ldots \rangle = |q_{\text{rel}}(t)\rangle + \sum_n \frac{\delta |q_{\text{rel}}(t)\rangle}{\delta \langle 1 | \hat{p}_n | q(t) \rangle} \langle 1 | \hat{p}_n | \ldots \rangle - \sum_n \frac{\delta |q_{\text{rel}}(t)\rangle}{\delta \langle 1 | \hat{p}_n | q(t) \rangle} \langle 1 | \hat{p}_n | \ldots \rangle \langle 1 | \ldots \rangle$$

(6)

is the Kawasaki-Ganton projection operator, which acts only on the state vectors $|\ldots\rangle$ and has the operator properties $\mathcal{P}_{\text{rel}}(t)|q(t')\rangle = |q_{\text{rel}}(t)\rangle$, $\mathcal{P}_{\text{rel}}(t)|q_{\text{rel}}(t')\rangle = |q_{\text{rel}}(t)\rangle$, $\mathcal{P}_{\text{rel}}(t)\mathcal{P}_{\text{rel}}(t') = \mathcal{P}_{\text{rel}}(t)$. The relevant thermovacuum state vector $|q_{\text{rel}}(t)\rangle = |q_{\text{rel}}(t)|1\rangle$, is normalized in accordance with the relation $\langle 1 | q_{\text{rel}}(t) \rangle = \langle 1 | q_{\text{rel}}(t) \rangle |1\rangle = 1$, where $q_{\text{rel}}(t)$ is the relevant statistical superoperator. The relevant thermovacuum state vector of the system can be defined as follows. We assume that $\langle p_n \rangle^t = \langle 1 | \hat{p}_n | q(t) \rangle$ is the set of observed variables describing the nonequilibrium system state, where $p_n$ are the operators constructed on the respective creation and annihilation operators $a_i^+$ and $a_j$. The relevant statistical operator $q_{\text{rel}}(t)$ is determined from the extremum of the Renyi entropy functional

$$L_E(t) = \frac{1}{1 - q} \ln \langle 1 | q(t) \rangle^q - a \langle 1 | q(t) \rangle - \sum_n F_n^r(t) \langle 1 | \hat{p}_n | q(t) \rangle$$
under the additional condition that the mean values $\langle p_n \rangle^t$ are given with the normalization condition

$$\langle 1 | \hat{q}(t) | 1 \rangle = 1$$

The Lagrange parameters $\alpha$ and $F_n(t)$ are determined from the respective normalization condition and self-consistency conditions:

$$\langle \ldots |_{\text{rel}} = \langle 1 | \ldots | e_{\text{rel}}(t) \rangle, \quad \langle p_n \rangle^t = \langle p_n \rangle^t_{\text{rel}} = \langle 1 | \hat{p}_n | e_{\text{rel}}(t) \rangle.$$  \hspace{1cm} (7)

The relevant statistical operator $e_{\text{rel}}(t)$ then becomes

$$e_{\text{rel}}(t) = \frac{1}{Z_R(t)} \left[ 1 - \frac{q-1}{q} \sum_n F_n^*(t) \delta \hat{p}_n(t) \right]^{\frac{1}{q-1}},$$  \hspace{1cm} (8)

where $q$ is the Renyi parameter, $\delta \hat{p}_n(t) = \hat{p} - \langle 1 | \hat{p}_n | e(t) \rangle$, and

$$Z_R(t) = \left\langle 1 \right| \left[ 1 - \frac{q-1}{q} \sum_n F_n^*(t) \delta \hat{p}_n(t) \right]^{\frac{1}{q-1}} \rangle,$$  \hspace{1cm} (9)

is the partition function. The sum over $n$ can denote the summation over the wave vector $k$, the kind of particles and a whole series of quantum numbers, such as spin. From (8) at $q = 1$, we obtain the relevant statistical operator corresponding to Gibbs statistics [61]:

$$e_{\text{rel}}(t) = \exp \left\{ - \Phi(t) - \sum_n F_n^*(t) p_n \right\},$$  \hspace{1cm} (10)

where $\Phi(t) = \ln \text{Sp} \exp \left\{ - \sum_n F_n^*(t) p_n \right\}$ is the Massieu-Planck functional. Substituting (8) in (5), we now obtain the nonequilibrium thermovacuum vector

$$|\varrho(t)\rangle = |e_{\text{rel}}(t)\rangle + \sum_n \int_0^t dt' e^{i(t'-t) T(t,t')} \left[ \int_0^1 d\tau \varrho_{e_{\text{rel}}}(t') F_n(t') |e_{\text{rel}}(t')\rangle \right] F_n^*(t'),$$  \hspace{1cm} (11)

where $J_n(t) = [1 - \mathcal{P}(t)] t \varphi^{-1} \hat{p}_n$ are the operators of the generalized flows describing the dissipative processes $\hat{p}_n = -\frac{i}{\hbar} \hat{H} \hat{p}_n$ in the system. The projection operator $\mathcal{P}(t)$ acts on operators and has the structure

$$\mathcal{P}(t) \ldots = \langle 1 | \ldots | e_{\text{rel}}(t) \rangle + \sum_m \delta \left[ \int_0^1 d\tau \varrho_{e_{\text{rel}}}(t) \varphi^{-1}(t) F_m(t) \right]$$

$$\left. + \sum_n f_{mn}^{-1} \delta \hat{p}_n \right] \langle \ldots | \left[ \int_0^1 d\tau \varrho_{e_{\text{rel}}}(t) \delta \hat{p}_n \varphi^{-1}(t) e_{\text{rel}}(t) \rangle,$$  \hspace{1cm} (12)

where $\delta[\ldots] = [\ldots] - \langle 1 | \ldots | e_{\text{rel}}(t) \rangle$ and $f_{mn}(t) = \frac{\delta \langle 1 | \hat{p}_n | e_{\text{rel}}(t) \rangle}{\delta e_{\text{rel}}(t)}$. The operator $\varphi(t)$ has the form

$$\varphi(t) = 1 - \frac{q-1}{q} \sum_n F_n^*(t) \delta \hat{p}_n(t).$$

Using the nonequilibrium thermovacuum state vector $|\varrho(t)\rangle$ given by (11), we obtain the transport equations for the nonequilibrium means $\langle 1 | \hat{p}_n | e(t) \rangle$ in the thermofield representation. For this, we use the identity

$$\frac{\partial}{\partial t} \langle 1 | \hat{p}_n | e(t) \rangle = \langle 1 | \hat{p}_n | e_{\text{rel}}(t) \rangle + \langle J_n(t) | e(t) \rangle.$$  \hspace{1cm} (13)

Averaging the last term in the right-hand side with $|\varrho(t)\rangle$ given by (11), we obtain the transport equations for the means $\langle 1 | \hat{p}_n | e_{\text{rel}}(t) \rangle$.
\[
\frac{\partial}{\partial t} \langle 1| \hat{p}_n | q(t) \rangle = \langle 1| \hat{p}_n | \varrho_{\text{rel}}(t) \rangle 
+ \sum_{n'} \int_{-\infty}^{t} dt' \, e^{i(t-t')} \langle \hat{p}_n T(t,t') \rangle \int_{0}^{1} d\tau \varrho_{\text{rel}}(t') \rangle \langle \hat{p}_n^{\dagger} (t') \rangle F_{n'}^{n'}(t').
\] (14)

Transport equations (14) take the memory effects into account and can be used to describe nonequilibrium processes in quantum Bose and Fermi systems in concrete cases in the framework of the nonequilibrium thermofield dynamics of nonextensive statistics. In particular, a system of relativistic transport equations for a consistent description of the kinetic and hydrodynamic processes in a quark-gluon system was derived in [62] using the nonequilibrium statistical operator method in the thermofield representation in Gibbs statistics. The advanced approach in terms of Renyi statistics can be generalized to the case of relativistic systems, and this observation is important [32,34–41]. This subject will be described in forthcoming works.

3. Thermo field transport equation with taking into account coupled states

We will consider a quantum field system in which coupled states can appear between the particles. Let us introduce annihilation and creation operators of a coupled state \((A,\alpha)\) with \(A\)-particle:

\[
a_{A,\alpha}(p) = \sum_{l=1}^{\infty} \Psi_{A,\alpha l}(1, \ldots, A) a(l) \ldots a(A),
\]
\[
a_{A,\alpha}^{\dagger}(p) = \sum_{l=1}^{\infty} \Psi_{A,\alpha l}^{\dagger}(1, \ldots, A) a^{\dagger}(1) \ldots a^{\dagger}(A),
\] (15)

where \(\Psi_{A,\alpha l}(1, \ldots, A)\) is a self-function of the \(A\)-particle coupled state, \(\alpha\) denotes internal quantum numbers (spin, etc.), \(p\) is a particle momentum, the sum covers the particles. Annihilation and creation operators \(a(j)\) and \(a^{\dagger}(j)\) satisfy the following commutation relations:

\[
[a(l), a^{\dagger}(j)]_\sigma = \delta_{l,j}, \quad [a(l), a(j)]_\sigma = [a^{\dagger}(l), a^{\dagger}(j)]_\sigma = 0,
\] (16)

where \(\sigma\)-commutator is determined by \([a, b]_\sigma = ab - ba\) with \(\sigma = \pm 1\): for bosons and \(-1\) for fermions.

The Hamiltonian of such a system can be written in the form:

\[
H = \sum_{A,A} \int \frac{dpdq}{(2\pi)^6} \frac{p^2}{2m_A} a_{A,\alpha}^{\dagger} (p - \frac{q}{2}) a_{A,\alpha} (p + \frac{q}{2})
+ \frac{1}{2} \sum_{A,B} \sum_{\alpha,\beta} \int \frac{dpd\rho dq}{(2\pi)^9} V_{AB}(q) a_{A,\alpha}^{\dagger} (p + \frac{q - q'}{2}) \hat{n}_{B\beta}(q) a_{A,\alpha} (p - \frac{q - q'}{2}),
\] (17)

where \(V_{AB}(q)\) is interaction energy between \(A\)- and \(B\)-particle coupled states, \(q\) is a wavevector. Annihilation and creation operators \(a_{A,\alpha}(p)\) and \(a_{A,\alpha}^{\dagger}(p)\) satisfy the following commutation relations:

\[
[a_{A,\alpha}(p), a_{B,\beta}^{\dagger}(p')]_\sigma = \delta_{A,B} \delta_{\alpha,\beta} \delta(p - p'),
\]
\[
[a_{A,\alpha}(p), a_{B,\beta}(p')]_\sigma = [a_{A,\alpha}^{\dagger}(p), a_{B,\beta}^{\dagger}(p')]_\sigma = 0.
\] (18)

\(\hat{n}_{B\beta}(q)\) in (17) is a Fourier transform of the \(B\)-particle density operator:

\[
\hat{n}_{B\beta}(q) = \int \frac{dp}{(2\pi)^3} \frac{q^2}{2} \frac{a_{B,\beta}(p) a_{B,\beta}^{\dagger}(p)}{p^2 + \frac{q^2}{2}}.
\]

As parameters of a reduced description for the consistent description of the kinetics and hydrodynamics of a system, where coupled states between the particles can appear, let us choose nonequilibrium distribution functions of \(A\)-particle coupled states in thermo field representation.
where Lagrange multipliers $q$ with (8) from (18). Following [61], one can rewrite relevant statistical operator $\hat{\rho}_{\text{rel}}(t)$, $|\varphi_{\text{rel}}(t)\rangle\rangle$ and with (8) from $q = 1$ for the mentioned parameters of a reduced description in the form:

$$\vartheta_{\text{rel}}(t) = \exp \left\{ -\Phi^*(t) - \int d\varphi \beta(\varphi; t) \left( \hat{H}(\varphi) - \sum_{\alpha} \int \frac{d\varphi}{(2\pi \hbar)^3} \mu_{\alpha}(\varphi; t) \hat{\rho}_{\alpha}(\varphi) \right) \right\} ,$$

where Lagrange multipliers $\beta(\varphi; t)$ and $\mu_{\alpha}(\varphi; t)$ can be found from the self-consistency conditions, correspondingly:

$$\langle \langle 1 | \hat{H}(\varphi)|\varphi(t)\rangle \rangle = \langle \langle 1 | \hat{H}(\varphi)|\varphi_{\text{rel}}(t)\rangle \rangle ,$$

$$\langle \langle 1 | \hat{\rho}_{\alpha}(\varphi)|\varphi(t)\rangle \rangle = \langle \langle 1 | \hat{\rho}_{\alpha}(\varphi)|\varphi_{\text{rel}}(t)\rangle \rangle .$$

$\Phi^*(t)$ is the Massieu-Planck functional and it can be defined from the normalization condition:

$$\Phi^*(t) = \ln \left\langle \left| \int d\varphi \beta(\varphi; t) \left( \hat{H}(\varphi) - \sum_{\alpha} \int \frac{d\varphi}{(2\pi \hbar)^3} \mu_{\alpha}(\varphi; t) \hat{\rho}_{\alpha}(\varphi) \right) \right| \right\rangle .$$

Using now the general structure of nonequilibrium thermo field dynamics (14), one can obtain a set of generalized transport equations for $A$-particle Wigner distribution functions and the average interaction energy:

$$\frac{\partial}{\partial t} \langle \langle 1 | \hat{\rho}_{\alpha}(x)|\varphi(t)\rangle \rangle = \langle \langle 1 | \dot{\hat{\rho}}_{\alpha}(x)|\varphi(t)\rangle \rangle$$

$$+ \int d\varphi' \int_{-\infty}^{t} dt' e^{\varepsilon(t'-t)} \varphi_{\alpha B}(x, x', t, t') \beta(r'; t') + \sum_{B, \beta} \int d\varphi' \int_{-\infty}^{t} dt' e^{\varepsilon(t'-t)} \varphi_{\alpha B}(x, x', t, t') \beta(r'; t') $$
\[
\frac{\partial}{\partial t} \langle \langle 1|\hat{H}(r)|\varphi(t) \rangle \rangle = \langle \langle 1|\hat{H}(r)|\varphi(t) \rangle \rangle \\
+ \int dr' \int dt' \ e^{i(t-t')} \varphi_{HHH}(r, r'; t, t') \beta(r', t') \\
+ \sum_{\beta, \beta} \int dx' \int dt' \ e^{i(t-t')} \varphi_{BBH}(r, x'; t, t') \beta(r', t') \mu_{BB}(x', t'),
\]

where \( x' = \{ r', p' \} \), \( dx' = (2\pi\hbar)^{-3}dr'dp' \). Here

\[
\varphi_{HHH}^{\rho}(x, x', t, t') = \langle \langle 1|\hat{J}_{HH}(x, t)T(t, t')|\frac{1}{1-t} \int d\tau \varphi_{rel}(t')I_{n_{\rho}}(x; t')\varphi_{rel}(t') \rangle \rangle,
\]

\[
\varphi_{HHH}^{\rho}(x, r', t, t') = \langle \langle 1|\hat{J}_{HH}(x, t)T(t, t')|\frac{1}{1-t} \int d\tau \varphi_{rel}(t')I_{n_{\rho}}(r'; t')\varphi_{rel}(t') \rangle \rangle,
\]

\[
\varphi_{HHH}^{\rho}(r', x', t, t') = \langle \langle 1|\hat{J}_{HH}(r, t)T(t, t')|\frac{1}{1-t} \int d\tau \varphi_{rel}(t')I_{n_{\rho}}(x'; t')\varphi_{rel}(t') \rangle \rangle,
\]

\[
\varphi_{HHH}(r, r', t, t') = \langle \langle 1|\hat{J}_{HH}(r, t)T(t, t')|\frac{1}{1-t} \int d\tau \varphi_{rel}(t')I_{n_{\rho}}(r'; t')\varphi_{rel}(t') \rangle \rangle.
\]

are generalized transport cores which describe dissipative processes. In these formulae

\[
I_{HH}(r, t) = (1 - P(t'))\hat{H}(r),
\]

\[
I_{n_{\rho}}(r, p; t) = (1 - P(t'))\hat{n}_{\rho}(r, p)
\]

are generalized flows, \( \hat{H}(r) = -\frac{i}{\hbar}[H, H(r)] \), \( \hat{n}_{\rho}(r, p) = -\frac{i}{\hbar}[H, n_{\rho}(r, p)] \), \( P(t) \) is a generalized Mori projection operator in thermo field representation. It acts on operators

\[
P(t)P = \langle \langle |\varphi_{rel}(t)\rangle \rangle + \int dr \ \delta\langle \langle 1|\hat{P}|\varphi_{rel}(t)\rangle \rangle \left( H(r) - \langle \langle 1|\hat{H}(r)|\varphi(t)\rangle \rangle \right)
\]

\[
+ \sum_{\rho, \rho} \int dr dp \ \delta\langle \langle 1|\hat{P}|\varphi_{rel}(t)\rangle \rangle \left( n_{\rho}(x) - \langle \langle 1|\hat{n}_{\rho}(x)|\varphi(t)\rangle \rangle \right)
\]

and has all the properties of a projection operator:

\[
P(t)H(r) = H(r), \quad P(t)P(t') = P(t),
\]

\[
P(t)n_{\rho}(r, p) = n_{\rho}(r, p), \quad (1 - P(t))P(t) = 0.
\]

The obtained transport equations have the general meaning and can describe both weakly and strongly nonequilibrium processes of a quantum system with taking into consideration coupled states.

In the next step we will construct such annihilation and creation superoperators, for which the relevant thermo vacuum state vector is a vacuum state. Analysing the structure of relevant statistical superoperator (22), one can mark out some part which would correspond to the system of noninteracting quantum \( A \)-particles. Let us write \( \varphi_{rel}(t) \) in an evident form and separate terms which are connected with the interaction energy between the particles:
\[ \varrho_{\text{rel}}(t) = \exp \left\{ -\Phi^*(t) - \int \mathrm{d}\mathbf{r} \beta(t, \mathbf{r}) \right\} \]
\[ \times \sum_{A,a} \int \frac{\mathrm{d}d\mathbf{p}}{(2\pi\hbar)^3} \left[ \frac{\mathbf{p}^2}{2m_A} \hat{h}_{Aa}(x) - \mu_{Aa}(x; t)\hat{n}_{Aa}(x) \right] - \int \mathrm{d}r \beta(t, \mathbf{r}) \hat{H}_{\text{int}}(\mathbf{r}) \right\} \].

Using operator equality (A and B are some operators)
\[ e^{A+B} = \left[ 1 + \int_0^1 \mathrm{d}\tau e^{\tau(A+B)} B e^{-(A+B)} \right] e^A, \]
the relation for \( \varrho_{\text{rel}}(t) \) can be rewritten in the following form:
\[ \varrho_{\text{rel}}(t) = \left[ 1 - \int \mathrm{d}\mathbf{r} \beta(t, \mathbf{r}) \int_0^1 \mathrm{d}\tau \varrho^0_{\text{rel}}(t, \mathbf{r}) \varrho_{\text{rel}}(t) \right]^{-\tau} \varrho^0_{\text{rel}}(t), \]

where
\[ \varrho^0_{\text{rel}}(t) = \exp \left\{ \Phi(t) - \int \mathrm{d}\mathbf{r} \beta(t, \mathbf{r}) \sum_{A,a} \int \frac{\mathrm{d}d\mathbf{p}}{(2\pi\hbar)^3} \left[ \frac{\mathbf{p}^2}{2m_A} \hat{h}_{Aa}(x) - \mu_{Aa}(x; t)\hat{n}_{Aa}(x) \right] \right\}, \]

or
\[ \varrho^0_{\text{rel}}(t) = \exp \left\{ \Phi(t) - \int \mathrm{d}\mathbf{r} \beta(t, \mathbf{r}) \sum_{A,a} \int \frac{\mathrm{d}p}{(2\pi\hbar)^3} b_{Aa}(x; t)\hat{n}_{Aa}(x) \right\}, \]

where \( b_{Aa}(x; t) = \left[ \frac{\mathbf{p}^2}{2m_A} \hat{h}_{Aa}(x) - \mu_{Aa}(x; t)\hat{n}_{Aa}(x) \right] \). Relevant statistical superoperator \( \varrho^0_{\text{rel}}(t) \) is bilinear on annihilation and creation superoperators \( \hat{a}_{Aa}(\mathbf{P}) \) and \( \hat{a}^+_A(\mathbf{P}) \), as well as on the non-perturbed part of Hamiltonian \( \hat{H}_0 \). One can write the total relevant superoperator as some non-perturbed part of \( \varrho^0_{\text{rel}}(t) \) and the part which describes interaction of quantum particles in the relevant state. Further, we introduce the following designation:
\[ \varrho_{\text{rel}}(t) = \varrho^0_{\text{rel}}(t) + \varrho^\dagger_{\text{rel}}(t), \]

where
\[ \varrho^\dagger_{\text{rel}}(t) = -\int \mathrm{d}\mathbf{r} \beta(t, \mathbf{r}) \int_0^1 \mathrm{d}\tau \varrho^\dagger_{\text{rel}}(t, \mathbf{r}) \varrho_{\text{rel}}(t) \right\]^{-\tau} \varrho^0_{\text{rel}}(t). \]

Relevant (relevant) thermo vacuum states \( |\varrho^0_{\text{rel}}(t)\rangle \rangle \) and \( |\varrho^\dagger_{\text{rel}}(t)\rangle \rangle \) are not vacuum states for annihilation and creation superoperators \( \hat{a}_{Aa}(\mathbf{P}), \hat{a}^+_A(\mathbf{P}), \hat{a}_{Aa}(\mathbf{P}), \hat{a}^+_A(\mathbf{P}) \). But for \( |\varrho^0_{\text{rel}}(t)\rangle \rangle \) one can construct new superoperators \( \hat{\gamma}_{Aa}(\mathbf{P}), \hat{\gamma}^+_A(\mathbf{P}), \hat{\gamma}^-_{Aa}(\mathbf{P}) \) as a linear combination of superoperators \( \hat{a}_{Aa}(\mathbf{P}), \hat{a}^+_A(\mathbf{P}), \) and \( \hat{a}_{Aa}(\mathbf{P}), \hat{a}^+_A(\mathbf{P}) \) in order to satisfy the conditions:
\[ \hat{\gamma}_{Aa}(\mathbf{P}; t)|\varrho^0_{\text{rel}}(t)\rangle \rangle = 0, \]
\[ \langle (1|\hat{\gamma}^+_A(\mathbf{P}; t) = 0, \]
\[ \hat{\gamma}^-_{Aa}(\mathbf{P}; t)|\varrho^0_{\text{rel}}(t)\rangle \rangle = 0, \]
\[ \langle (1|\hat{\gamma}^-_{Aa}(\mathbf{P}; t) = 0. \]

To achieve this let us consider an action of annihilation superoperators \( \hat{a}_{Aa}(\mathbf{P}; t), \hat{a}^+_A(\mathbf{P}; t) \) on relevant state \( |\varrho^0_{\text{rel}}(t_0)\rangle \rangle \):
where superoperators $\hat{a}_{A\alpha}(p; t)$, $\hat{a}^+_{A\alpha}(p; t)$, $\hat{a}_{A\alpha}(p; t)$, $\hat{a}^+_{A\alpha}(p; t)$ are in the Heisenberg representation

\[
\hat{a}_{A\alpha}(p; t) = e^{-\frac{i}{\hbar} p \cdot \vec{r}} \hat{a}_{A\alpha}(p) e^{\frac{i}{\hbar} p \cdot \vec{r}}, \quad \hat{a}^+_{A\alpha}(p; t) = e^{-\frac{i}{\hbar} p \cdot \vec{r}} \hat{a}^+_{A\alpha}(p) e^{\frac{i}{\hbar} p \cdot \vec{r}},
\]

and satisfy commutation relations:

\[
\left[ \hat{a}_{A\alpha}(p; t), \hat{a}^+_{B\beta}(p'; t) \right]_{\sigma} = \delta_{\alpha,\beta} \delta_{\sigma,\delta}(p - p'),
\]

\[
\left[ \hat{a}_{A\alpha}(p; t), \hat{a}_{B\beta}(p'; t) \right]_{\sigma} = \delta_{\alpha,\beta} \delta_{\sigma,\delta}(p - p'),
\]

\[
\left[ \hat{a}_{A\alpha}(p; t), \hat{a}^+_{B\beta}(p'; t) \right]_{\sigma} = \left[ \hat{a}^+_{A\alpha}(p; t), \hat{a}_{B\beta}(p'; t) \right]_{\sigma} = 0.
\]

It is necessary to note that superoperators $\hat{H}(\vec{r})$, $\hat{a}_{A\alpha}(x)$ are built on superoperators $\hat{a}_{A\alpha}(p + \frac{\vec{q}}{2})$, $\hat{a}^+_{A\alpha}(p - \frac{\vec{q}}{2})$, $\hat{a}_{A\alpha}(p + \frac{\vec{q}}{2})$, $\hat{a}^+_{A\alpha}(p - \frac{\vec{q}}{2})$. Therefore, for convenience here a unit denotation was introduced for arguments like $\vec{p} = p \pm \frac{\vec{q}}{2}$. This should be taken into account in further calculations where obvious expressions are needed.

According to general relations of [61,62], we can introduce new operators $\hat{\gamma}_{A\alpha}(p; t)$, $\hat{\gamma}^+_{A\alpha}(p; t)$, $\hat{\gamma}_A(p; t)$, $\hat{\gamma}^+_A(p; t)$ via superoperators $\hat{a}_{A\alpha}(p; t)$, $\hat{a}^+_{A\alpha}(p; t)$, $\hat{a}_{A\alpha}(p; t)$, $\hat{a}^+_{A\alpha}(p; t)$:

\[
\hat{\gamma}_{A\alpha}(p; t) = \sqrt{1 + \sigma n_{A\alpha}(p; t, t_0)} \left[ \hat{a}_{A\alpha}(p; t) - \frac{n_{A\alpha}(p; t, t_0)}{1 + \sigma n_{A\alpha}(p; t, t_0)} \hat{a}^+_{A\alpha}(p; t) \right],
\]

\[
\hat{\gamma}^+_{A\alpha}(p; t) = \sqrt{1 + \sigma n_{A\alpha}(p; t, t_0)} \left[ \hat{a}^+_{A\alpha}(p; t) - \sigma \hat{a}_{A\alpha}(p; t) \right].
\]

Relations (42) satisfy conditions (40). Here

\[
n_{A\alpha}(p, q; t, t_0) = n_{A\alpha}(p; t, t_0) = \langle \langle 1 | \hat{a}^+_{A\alpha}(p; t) \hat{a}_{A\alpha}(p; t) | \psi^0_{rel}(t_0) \rangle \rangle = \langle \langle 1 | \hat{a}^+_{A\alpha}(p - \frac{\vec{q}}{2}; t) \hat{a}_{A\alpha}(p + \frac{\vec{q}}{2}; t) | \psi^0_{rel}(t_0) \rangle \rangle,
\]

is a relevant distribution function of $A$-particle coupled states in momentum space $p, q$, which is calculated with the help of relevant thermo vacuum state vector $\psi^0_{rel}(t_0)$ (37). Function $f_{A\alpha}(p; t - t_0)$ in formulae (41) is connected with $n_{A\alpha}(p; t, t_0)$ by the relation

\[
f_{A\alpha}(p; t - t_0) = \frac{n_{A\alpha}(p; t, t_0)}{1 + \sigma n_{A\alpha}(p; t, t_0)}.
\]

Superoperators $\hat{\gamma}_{A\alpha}(p; t)$ and $\hat{\gamma}^+_{A\alpha}(p; t)$, $\hat{a}^+_{A\alpha}(p; t)$ and $\hat{\gamma}^+_A(p; t)$ satisfy the “canonical” commutation relations:

\[
\left[ \hat{\gamma}_{A\alpha}(p; t), \hat{a}^+_{B\beta}(p'; t) \right]_{\sigma} = \delta_{\alpha,\beta} \delta_{\sigma,\delta}(p - p'),
\]

\[
\left[ \hat{\gamma}_{A\alpha}(p; t), \hat{a}_{B\beta}(p'; t) \right]_{\sigma} = \delta_{\alpha,\beta} \delta_{\sigma,\delta}(p - p'),
\]

\[
\left[ \hat{\gamma}^+_A(p; t), \hat{a}^+_{B\beta}(p'; t) \right]_{\sigma} = \left[ \hat{a}^+_{A\alpha}(p; t), \hat{\gamma}^+_{B\beta}(p'; t) \right]_{\sigma} = 0.
\]
Inversed transformations to superoperators $\hat{a}_{Aa}(\mathbf{P}; t), \hat{a}_{Aa}^+(\mathbf{P}; t)$ are easily obtained from (42):

\[
\begin{align*}
\hat{a}_{Aa}(\mathbf{P}; t) &= \sqrt{1 + \sigma n_{Aa}(\mathbf{P}; t_0)} \left[ \hat{\gamma}_{Aa}(\mathbf{P}; t) + \frac{n_{Aa}(\mathbf{P}; t_0)}{1 + \sigma n_{Aa}(\mathbf{P}; t_0)} \hat{\gamma}_{Aa}^+(\mathbf{P}; t) \right], \\
\hat{a}_{Aa}^+(\mathbf{P}; t) &= \sqrt{1 + \sigma n_{Aa}(\mathbf{P}; t_0)} \left[ \hat{\gamma}_{Aa}^+(\mathbf{P}; t) + \sigma \hat{\gamma}_{Aa}(\mathbf{P}; t) \right].
\end{align*}
\]

$\hat{\gamma}_{Aa}(\mathbf{P}; t), \hat{a}_{Aa}^+(\mathbf{P}; t), \hat{\gamma}_{Aa}(\mathbf{P}; t), \hat{\gamma}_{Aa}^+(\mathbf{P}; t)$ could be defined as some operators of annihilation and creation of $A$-quasiparticle coupled states, for which relevant thermo vacuum state $|\phi_{rel}(t_0)\rangle\rangle$ (37) is a vacuum state. In such a way, we obtained relations of dynamical reflection of superoperators $\hat{a}_{Aa}(\mathbf{P}; t), \hat{a}_{Aa}^+(\mathbf{P}; t)$, $\hat{\gamma}_{Aa}(\mathbf{P}; t), \hat{\gamma}_{Aa}^+(\mathbf{P}; t)$ to new superoperators of “quasiparticles” $\hat{\gamma}_{Aa}(\mathbf{P}; t), \hat{a}_{Aa}^+(\mathbf{P}; t), \hat{\gamma}_{Aa}(\mathbf{P}; t), \hat{\gamma}_{Aa}^+(\mathbf{P}; t)$.

A set of transport equations (26), (27) together with dynamical reflections (42), (44) of superoperators in the thermo field space constitute the basis for a consistent description of the kinetics and hydrodynamics of a dense quantum system with strongly coupled states. Both strongly and weakly nonequilibrium processes of a nuclear matter can be investigated using this approach, in which the particle interaction is characterized by strongly coupled states, taking into account their nuclear nature [1–4].

Weakly nonequilibrium processes can be described when the fluctuations of the parameters $\delta \bar{r}(\mathbf{r}; t) = \bar{r}(\mathbf{r}; t) - \bar{r}, \delta \bar{\mu}_{Aa}(x; t) = \bar{\mu}_{Aa}(x; t) - \mu_{Aa}$ are small, where $\bar{r}$ and $\bar{\mu}_{Aa}$ are equilibrium values for temperature and chemical potential respectively. In this case, the system of equations (26), (27) will have a similar structure, but is closed with respect to $\langle (1|\delta \hat{h}_{Aa}(x)|\mathbf{e}(t)) \rangle, \langle (1|\delta \hat{H}(\mathbf{r})|\mathbf{e}(t)) \rangle$, where $\delta \hat{h}_{Aa}(x) = \hat{h}_{Aa}(x) - \langle (1|\hat{h}_{Aa}(x)|\mathbf{q}_0) \rangle, \delta \hat{H}(\mathbf{r}) = \hat{H}(\mathbf{r}) - \langle (1|\hat{H}(\mathbf{r})|\mathbf{q}_0) \rangle$ is the equilibrium thermo vacuum state vector of the systems.

In addition, by designing a system of equations on moments $1, \mathbf{P}$ of distribution function we obtain, respectively, the equation of the thermo field hydrodynamic for the dense quantum-field systems.

These questions require separate consideration and will be investigated in future work.

4. Conclusions

We generalized the nonequilibrium thermo field dynamics in the frames of Zubarev’s nonequilibrium statistical operator method [61] within the framework of Renyi statistics. Based on this approach and Gibbs statistics the generalized equations of consistent description of kinetics and hydrodynamics for dense quantum field systems with strongly bound states were obtained. Using this approach, one can investigate both strongly and slightly nonequilibrium processes of nuclear matter, when the interaction between particles of the latter is characterized by strongly bound states of internucleon nature [2,3].

DOI: 10.1007/978-3-642-02286-9_4.
DOI: 10.1007/978-3-642-02286-9_4.
DOI:10.1007/BF01019063.
DOI:10.1007/s10955-014-0980-4.


