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Axial compression model for FRP confined concrete in elliptical cross sections

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Abstract: Most of the existing studies conducted on FRP-confined concrete considered circular and square concrete columns, while limited studies were on columns with rectangular sections. The studies have confirmed that the circular cross-sections exhibited higher confinement effectiveness, whereas in the case of non-circular cross-sections the efficiency of FRP confinement decreases with an increase of the sectional aspect ratio and there is no significant increase, particularly for columns with the aspect ratio of 2.0. As recently suggested by the researchers, to significantly increase the effectiveness of FRP-confinement for these columns is by modifying a rectangular section to an elliptical or oval section. According to the literature, most of the existing confinement models for FRP-confined concrete under axial compression have been proposed for columns with circular and rectangular cross-sections. However, modeling the axial strength and strain of concrete confined with FRP in elliptical cross-sections under compression is most limited. Therefore, this paper provides new expressions based on limited experimental data available in the literature. For a sufficient amount of FRP-confinement, the threshold value was proposed to be 0.02. Finally, the accuracy of the proposed model was verified by comparing its predictions with the same test database, together with those from the existing models.   .

Keywords: FRP confined; axial compression; RC columns; strength; ductility; modeling

1. Introduction

It is now well-recognized that confinement of existing concrete columns in bridges and buildings using fiber-reinforced polymers (FRPs) can significantly increase the strength and ductility of the columns. Over the last 25 years, a large number of experimental tests and analytical models were focused on the axial compressive behavior of FRP-confined concrete (Moran *et al.*, 2002; Albanesi *et al.*, 2007; Vintzileou and Panagiotidou, 2008; Lignola *et al.*, 2008; Micelli *et al.*, 2013; Lim and Ozbakkaloglu, 2015; Cascardi *et al.*, 2016). The majority of the existing studies have focused on modeling the stress-strain behavior of FRP-confined concrete in circular cross-sections subjected to axial compression, while only limited studies considered FRP-confined concrete in rectangular cross-sections (Anselm, 2005; Pham and Hadi, 2014; Isleem *et al.*, 2018a, b, c, d).

Early research studies indicated that FRP confined square and rectangular sections with sharp corners provide only a little enhancement in their axial load capacities, while the confinement effectiveness increases directly with an increase in the corner radius (Wu and Zhou, 2010). Meanwhile, the curvature of the rectangular section's corners could cause stress concentration (Al-Salloum, 2007). Therefore, changing square section to circular section may minimize these stress concentration (Tsai and Lin, 2002; Yan, 2005; Pantelides and Yan, 2007; Yan and Pantelides, 2011; Xu Lei, 2012; Zeng *et al.*, 2017; Jameel *et al.*, 2017). Only very limited studies have been directed to changing a rectangular column section to an elliptical section (Yan, 2005; Pantelides and Yan, 2007; Alsayed *et al.*, 2014). The first study of Yan (2005) included an experimental program involved testing 30 FRP-confined concrete columns of circular, square and rectangular sections subjected to axial compressive loads. It was concluded that the FRP jackets are not able to effectively improve the compressive

behavior of square and rectangular columns exhibiting a softening behavior. Recently it has been confirmed by Isleem *et al*. (2018c, d) for rectangular columns of larger-sized sections that the confinement provided by the FRP wraps resulted in a significant improvement in axial strains but only a little improvement in axial strengths. In their study, the results of tests have shown that only the sufficiently confined specimens with the aspect ratio of 1.5 reached higher strengths as compared with that of the unconfined concrete cylinder, while no strength enhancement was achieved for larger-sized specimens with the aspect ratio of 2.0. All experiments showed that the stress-strain curves of the confined columns exhibited a softening behavior in their response. The key solution to reduce the corner stress concentration that causes the softening behavior for such large-sized sections and thus to improve the strength and deformability of concrete columns with light and moderate confinement level is to modify the shape of square section to circular section and the rectangular cross-section to elliptical or oval cross-section using subsequent steel or composite jacketing (Priestley *et al*., 1994; Priestley and Seible, 1995; Mandal and Mansur, 2001; Teng and Lam, 2002). Further, the shape modification method can also be employed when the corners of the rectangular section can no longer be rounded for fear of infringing on the minimum concrete cover for reinforcing steel bars (Parvin and Schroeder, 2008).

The most economical method of shape modification of concrete columns is adding oval precast concrete segments to the perimeter of the rectangular column with subsequent FRP wrapping (Parvin and Schroeder, 2008). For this technique of modification of section's shape to be widely used for the strengthening of rectangular columns subject to axial compression, analytical expressions for predicting the axial strength and strain of FRP reinforced concrete columns with elliptical sections are needed. Because concretes in rectangular sections confined with FRP behave differently compared with the elliptical sections, if the available models of rectangular columns are directly applied to confined elliptical columns, the strength and strain capacities may not represent the realistic behavior of the columns, and unsafe design may be performed. Therefore, this paper aims to provide expressions for the accurate predictions of the axial strength and axial strain. In addition, based on the existing test database, the threshold for sufficiently confined concrete has been proposed to be equal to 0.02. This can be an important feature of the proposed model being able to predict well the threshold confinement condition that can dictate whether the stress-strain response ascends or descends. Finally, an acceptable correlation was revealed between the predictions of the proposed model and the experimental test data.

2. .Experimental Program

2.1. Overview of specimen details

In order to develop new strength and strain models, the the results of experimental tests performed by Teng and Lam (2002) were used for the calibration of all expressions provided in this paper. Twenty unreinforced concrete specimens were prepared and tested under axial compression loading. The experimental tests included five groups of specimens (S1, S2, S3) divided according to their sectional aspect ratios *a/b* (1.0, 1.28, 1.7, 2.5) and prepared from the same batch of concrete as provided in Table 1. Each group included one specimen with a circular section and three elliptical specimens. The cross-sectional area and height of the elliptical specimens were almost equivalent to those of the circular sections. All of the specimens were 608 mm in height. The unconfined concrete strength was obtained from compressive tests on three cubes of 150. Only the third and fourth groups as control specimens were considered without FRP confinement, while the other groups were confined with different numbers of layers of CFRP wraps. The variables considered in the tests were: (1) the sectional aspect ratio, (2) the batch of concrete, and (3) and the number of CFRP layers. As for the specimens' designation, S5/4L2, as an example, had a cross-sectional aspect ratio of 1.28 and was confined with two CFRP layers. The mechanical properties of the CFRP wraps are also provided in Table 1. Complete details of the tests are not provided in this paper; however, the researchers are directed to their reference.
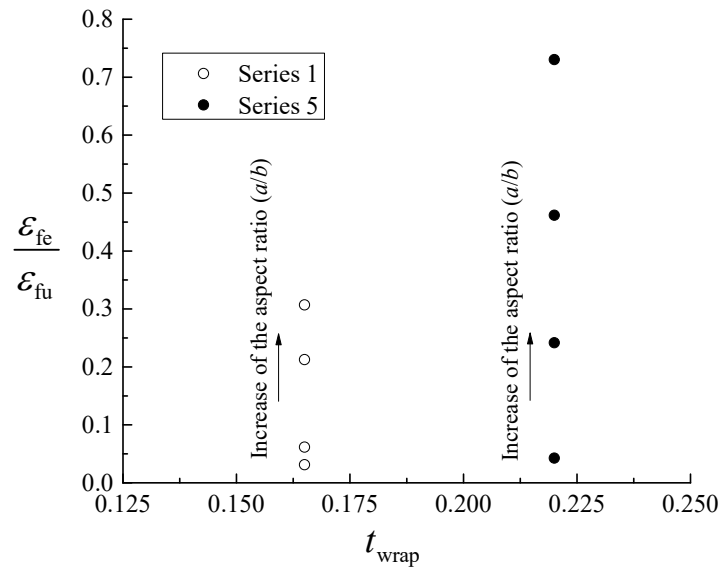
Figure 1. Effect of thickness of FRP wraps ratio on FRP rupture strain (Series 1 &5).

Table 1. Summary of FRP reinforcement, material and mechanical properties of test specimens.

No.	Specimen	Section details				Material Properties				
		a (mm)	b (mm)	a/b	$f_c{'}$ (MPa)	t_{wrap} (mm)	f_f (MPa)	E_f (GPa)	ε_{fu} (%)	
					Series 1					
1	S1.0L1	152.2	152.2	1.00	48.8	0.165	3983	263	1.514	
2	S5/4L1	168.2	131.6	1.28	48.8	0.165	3983	263	1.514	
3	S5/3L1	194.8	115.6	1.69	48.8	0.165	3983	263	1.514	
4	S5/2L1	237.6	94.8	2.51	48.8	0.165	3983	263	1.514	
					Series 2					
5	S1.0L1	151.6	151.6	1.00	47.1	0.110	3824	276	1.386	
6	S5/4L1	168.4	131.6	1.28	47.1	0.165	3983	263	1.514	
7	S5/3L1	194.9	114.8	1.70	47.1	0.165	3983	263	1.514	
8	S5/2L1	236.5	95.0	2.49	47.1	0.165	3983	263	1.514	
					Series 3					
9	S1.0L0	151.9	151.9	1.00	43.5	-	-	-	-	
10	S5/4L0	168.5	131.6	1.28	43.5	-	-	-	-	
11	S5/3L0	194.8	115.9	1.68	43.5	-	-	-	-	
12	S5/2L0	237.8	94.6	2.51	43.5	-	-	-	-	
					Series 4					
13	S1.0L0	152.0	152.0	1.00	44.6	-	-	-	-	
14	S5/4L0	168.7	131.4	1.28	44.6	-	-	-	-	
15	S5/3L0	194.8	115.0	1.69	44.6	-	-	-	-	
16	S5/2L0	236.8	94.6	2.50	44.6	-	-	-	-	
					Series 5					
17	S1.0L2	152.3	152.3	1.00	45.8	0.220	3824	276	1.386	
18	S5/4L2	168.2	131.9	1.28	45.8	0.220	3824	276	1.386	
19	S5/3L2	194.8	115.0	1.69	45.8	0.220	3824	276	1.386	
20	S5/2L2	237.6	94.6	2.51	45.8	0.220	3824	276	1.386	

Note: a = width of a cross-section; b = depth of a cross-section; a/b = aspect ratio of a cross section; t_{wrap} = thickness of FRP composite layers; f_f = maximum tensile strength of FRP composite; E_f = tensile elastic modulus of FRP composite; $\varepsilon_{\mathrm{fu}}$ = tensile strain of FRP at ultimate condition; f_c' = unconfined concrete strength.

2.2. Overview of experimental test results

All the confined specimens failed by the rupture of the FRP (Teng and Lam, 2002). In most cases, the rupture happened at the upper or lower quarters of the specimens. The degree of damage for specimens with smaller aspect ratios was higher than that for specimens with larger aspect ratios. In addition, results have revealed that the axial compressive strength is dependent on the amount of FRP confinement (Figure 1), which is resulted by the dependence of the FRP rupture strain on the confinement stiffness ratio (Rochette and Labossiére, 2000; Wang *et al.*, 2012). To take this parameter's effect into account, Equation 1 was first suggested by Teng *et al.* (2009) for FRP-confined circular concrete columns and later modified by Pham and Hadi (2014) for rectangular columns. In this paper, the following procedure to calculate the strength confinement ratio for the test specimens listed in Table 1 was used.

$$R_s = \frac{0.5\rho_f E_f}{\left(f_c'/\varepsilon_{\mathrm{co}}\right)} \tag{1}$$

$$\rho_f = \frac{\left(3(a+b) - 2\sqrt{ab}\right)t_{\mathrm{wrap}}}{ab} \tag{2}$$

$$\varepsilon_{\mathrm{co}} = (-0.067 f_c'^2 + 29.9 f_c' + 1053) \times 10^{-6} \tag{3}$$

where ρ_f = volumetric ratio of FRP; E_f = modulus of elasticity of FRP; $\varepsilon_{\mathrm{co}}$ = axial strain of unconfined concrete (Tasdemir *et al.*, 1998); t_{wrap} = thickness of FRP composite layers; f_c' = strength of unconfined concrete.

The FRP rupture strain that occurred at the major axes were smaller than the strains measured at the minor axes. In this paper, the effective FRP strain (Equation 4) is related to the strain at the ultimate condition. Besides, it was reported that specimens with higher aspect ratios exhibited smaller FRP strains at peak load. Comparison of the test results of specimens with varying aspect ratios is shown in Figure 2, in which the FRP strain is found to decrease as the sectional aspect ratio increase, as it has been reported in several tests on confined concrete columns with rectangular sections (Anselm, 2005; Yan, 2005; Isleem *et al.*, 2018a, c, d). Table 2 provides a summary of the existing models that predict the strain efficiency factor and Figure 3 provides a comparison of these models against the results. Based on these comparisons and regression of the results of the same specimens, Equation 5 is introduced.

$$\varepsilon_{\mathrm{fe}} = k_\varepsilon \varepsilon_{\mathrm{fu}} \tag{4}$$

where k_ε is a factor that considers the reduction in measured FRP hoop strain at rupture

$$k_\varepsilon = \frac{\varepsilon_{\mathrm{fe}}}{\varepsilon_{\mathrm{fu}}} = 280.59 \left(\frac{b}{a}\right)^{2.53} R_s^{1.84} \tag{5}$$

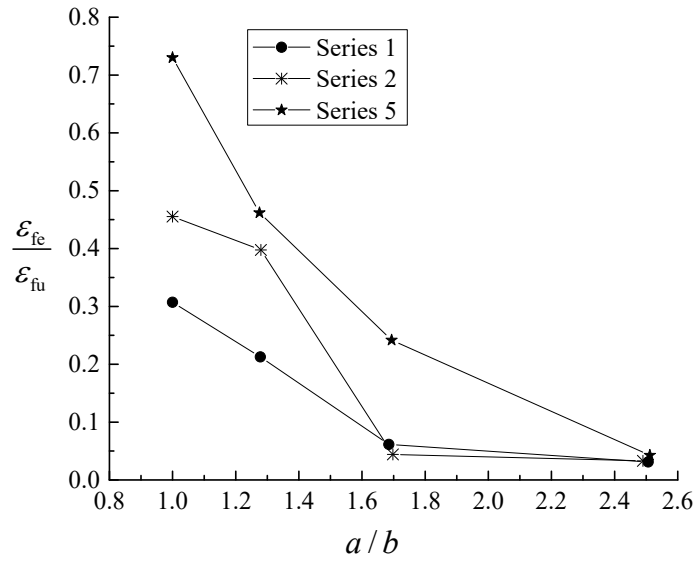In which the terms a and b are the width and depth of an elliptical cross-section, respectively.

Figure 2. Effect of aspect ratio on hoop rupture strain of FRP wraps.

Table 2. Summary of published models for the FRP strain efficiency factor, k_ε.

Research	Analytical expression
ACI Committee 440 (2002)	$k_\varepsilon = \varepsilon_{fe}/\varepsilon_{fu} = 0.004/\varepsilon_{fu} \leq 0.75$
Lam and Teng (2003)	$k_\varepsilon = \varepsilon_{fe}/\varepsilon_{fu} = 0.586$
Yan (2005)	$k_\varepsilon = 0.8\left(a/b\right)^{-0.87}$
Anselm (2005)	$k_\varepsilon = \varepsilon_{fe}/\varepsilon_{fu} = 0.53$
Ilki *et al.* (2008)	$k_\varepsilon = \varepsilon_{fe}/\varepsilon_{fu} = 0.8$
Ozbakkaloglu (2013)	$k_\varepsilon = 0.9 - 2.3 f_c' \times 10^{-3} - 0.75 E_f \times 10^{-6}$ $100000 \leq E_f \leq 640000$
Hany *et al.* (2015)	$k_\varepsilon = \varepsilon_{fe}/\varepsilon_{fu} = 0.6$
Wang *et al.* (2016)	$k_\varepsilon = \dfrac{\varepsilon_{fe}}{\varepsilon_{fu}} = 1 - 0.38\left(\dfrac{b}{100}\right)^{0.41}$ $100\ mm \leq b \leq 400\ mm$

Table 3. Published models of boundary value for sufficiently confined concrete.

Published model	Specimen type	Boundary value
Yan (2005)	Circular, rectangular, elliptical	CR1 ≥ 0.20
Shao *et al.* (2006)	Circular	CR2 ≥ 0.30
Pham and Hadi (2014)	Circular, rectangular	CR3 ≥ 0.15

Table 4. Comparison of published (Table 3) and proposed models for specimens in Table 1.

Source/specimen	S1.0L1	S5/4L1	S5/3L1	S5/2L1	S1.0L1	S5/4L1
f_{cc}' / f_c'	1.240	1.096	0.852	0.770	1.166	1.157

Yan (2005)	0.142	0.118	0.096	0.075	0.098	0.122
Evaluation	U	U	S	S	U	U
Shao *et al*. (2005)	0.125	0.126	0.119	0.105	0.087	0.131
Evaluation	U	U	S	S	U	U
Pham and Hadi (2014)	0.155	0.133	0.132	0.130	0.111	0.138
Evaluation	S	U	S	S	S	U
Proposed MC_R	0.067	0.020	0.011	0.005	0.072	0.021
Evaluation	S	S	S	S	S	S
Source/specimen	S5/3L1	S5/2L1	S1.0L2	S5/4L2	S5/3L2	S5/2L2
$f_{cc}^{'} / f_{c}^{'}$	0.904	0.837	1.563	1.376	0.967	0.755
Yan (2005)	0.099	0.078	0.193	0.161	0.131	0.102
Evaluation	S	S	U	U	S	S
Shao *et al*. (2005)	0.123	0.109	0.171	0.172	0.162	0.144
Evaluation	S	S	U	U	S	S
Pham and Hadi (2014)	0.137	0.135	0.204	0.171	0.175	0.172
Evaluation	U	U	S	S	U	U
Proposed MC_R	0.011	0.005	0.179	0.056	0.028	0.013
Evaluation	S	S	S	S	S	S

Where S = suitable; U = unsatisfied.

3. Effective confinement pressure ratio

As shown in Table 3, three analytical models for quantifying the effectiveness of FRP confinement for specimens with non-circular cross-sections were proposed in previous studies (Yan, 2005; Shao *et al*., 2006; Pham and Hadi, 2014). Based on their results, it was revealed that the strength increases by the increase of the ratio of the estimated confining pressure to the unconfined concrete strength (denoted as *CR* Table 3) being greater than a recommended value (For example 0.3 as reported by Shao *et al*., 2006). Details of the models can be found in their original papers.

Table 4 lists the effective confinement ratios from the proposed MC_R and existing models defined in this paper as *CR* ($CR_{1,2,3}$ = model 1, 2, 3) for a total of 12 FRP-confined specimens selected from the paper of Teng and Lam (2002). As shown the models are not able to represent the actual results of their peak strengths. Therefore, based on the analysis of the database, the following expressions for estimating the FRP-confined peak strength and strain for elliptical columns are proposed in which the coefficient R^2 value is approximately 87 and 89 % (Figure 4).

$$\frac{f_{cc}^{'}}{f_{c}^{'}} = 0.59 + 2.02\left(MC_R\right)^{0.42} \tag{6}$$

$$\frac{\varepsilon_{cc}}{\varepsilon_{co}} = 1 + 28.1\left(MC_R\right)^{0.843} \tag{7}$$

$$MC_R = \frac{k_e \rho_f E_f k_\varepsilon \varepsilon_{fu}}{2 f_c^{'}} \tag{8}$$

where MC_R is a non-dimensionless coefficient used to account for the contributions of FRP confinement on the enhancement in ultimate strength experienced by the specimens; k_e is the FRP efficiency coefficient for elliptical sections (Figure 5). This was calculated using Equation 8, which

was also used by Campione and Fossetti (2006) but for estimating the confinement provided by the internal steel hoops.

$$k_e = \frac{\pi ab - \left[\frac{4}{3} b' \left(a - \frac{a'}{2} \right) + \frac{4}{3} a' \left(b - \frac{b'}{2} \right) + \frac{1}{3} \left(a'^2 + b'^2 \right) \right]}{\pi ab}$$ (9)

where the a' and b' are respectively the depth and width of a rectangular concrete block (Figure 5), referring to Campione and Fossetti (2006).
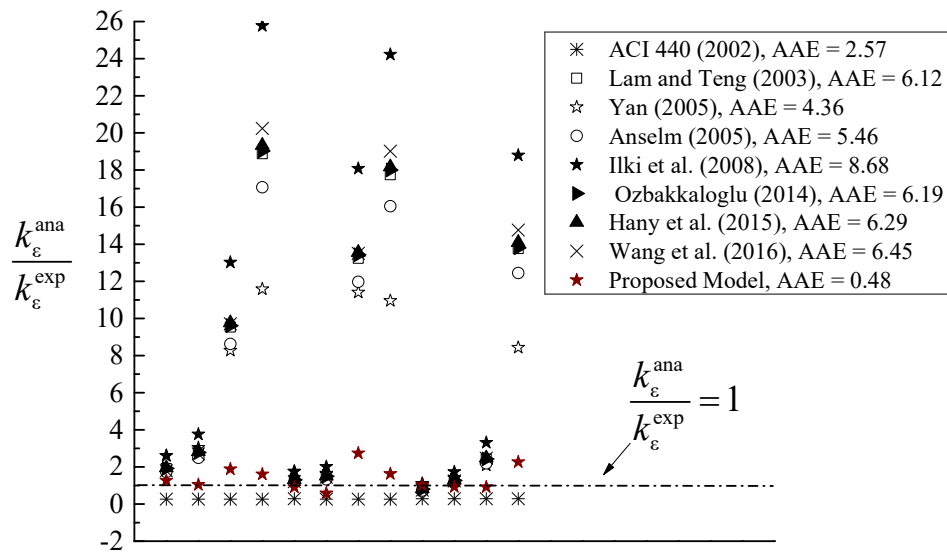
Figure 3. Comparison of existing models of k_ε with the results of tested specimens reported in Table 1.
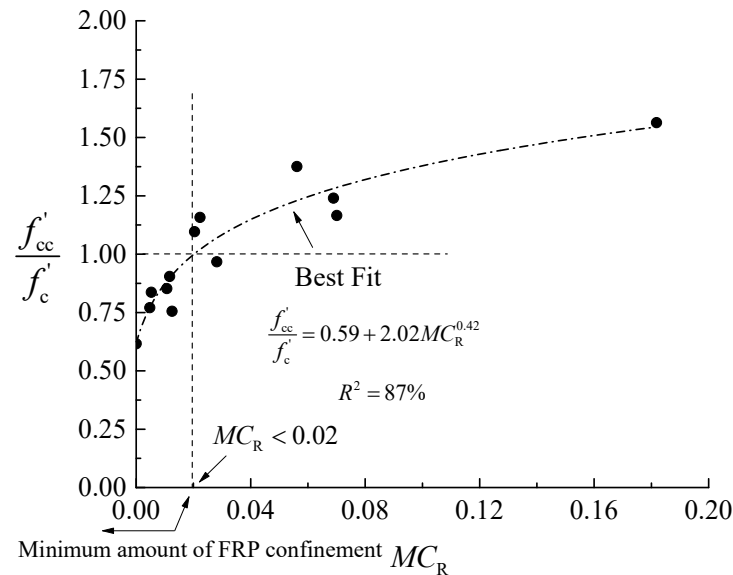
Figure 4. Relationship between effective confinement pressure ratio and ratio of the confined peak strength to the unconfined concrete strength.
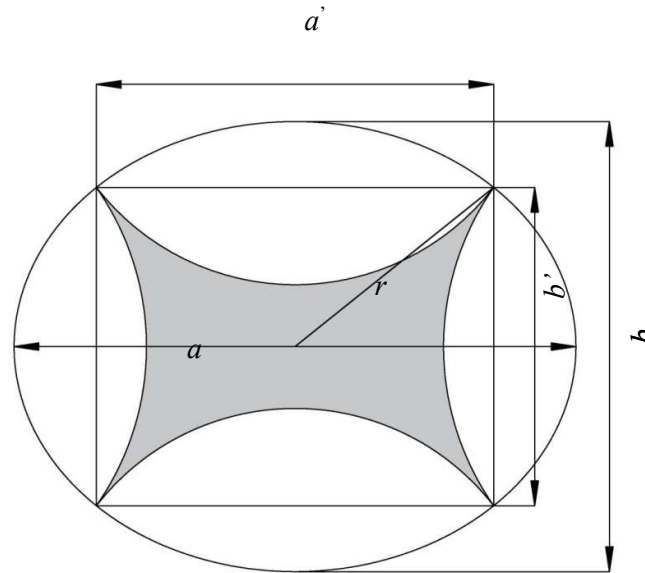
Figure 5. Effective confined area in elliptical-sectioned column.

4. Amount of FRP for sufficiently confined concrete

As stated in a recent by Isleem *et al.* (2018d), in earthquake-prone regions, a large number of existing RC columns, particularly of rectangular cross-sections, that were built based on the out-of-date codes may not have adequate lateral reinforcement to resist high seismic load levels. As a result, they are subjected to major damages causing a total collapse of the building (Ilki *et al.*, 2008). Therefore, this discussion focuses on determining the following indicator that can ensure a sufficient confinement for the existing concrete columns. The relationship between MC_R and the ratio of the test peak strength to the strength of unconfined concrete was presented in Figure 4, in which the regressed line was only based on the results provided in Table 1 due to the unavailability of relative tests in the technical literature. Based on the regressed line, when $f_{cc}' / f_c' = 1$, then the value of MC_R is equal to 0.02. When the MC_R is greater than the 0·02 value, then $f_{cc}' / f_c' > 1$, which means the f_{cc} is greater than the f_c', and as a result, the confined specimen experienced enhancement in their strength and finally an ascending stress-strain response. Conversely, when the value of MC_R is less than 0·02, a second post-peak softening component occurs in the stress-strain response as reported in several tests conducted on columns in the literature (Isleem *et al.*, 2018a).
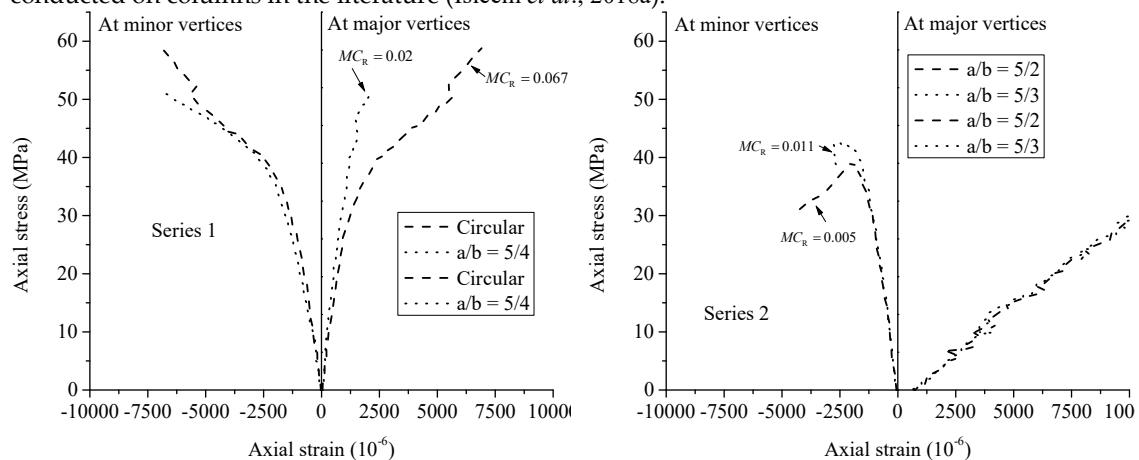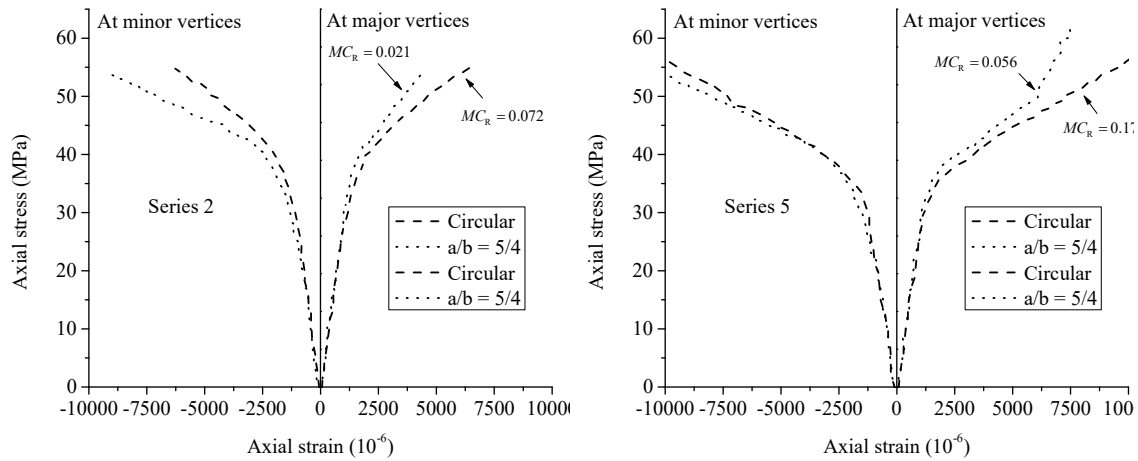
Figure 6. Accuracy of the proposed confinement pressure model MC_R against stress-strain test responses of specimens reported in Table 1.

In the next section, the overall performance of the proposed and existing models of peak strength and strain of FRP-confined rectangular columns were evaluated against the results of columns with elliptical sections (Table 1). In the current discussions, the newly proposed confining pressure ratio is further checked through comparisons between the stress-strain test results of specimens in Table 1 and analytical values obtained from the model. For more details of the selected tests, the readers are motivated to their original sources. These results were selected for the following reasons: (1) they had varying aspect ratios of cross-sections; (2) they had different strength of unconfined concrete; and (3) they had different confinement levels of FRP. In general, comparison of the test results with the predicted MC_R ratios provided in Figure 6 indicates that the confinement pressure model can distinguish between the different ascending and descending responses.

5. Accuracy of the proposed and existing strength and strain models

The performance of existing models that are capable of predicting the confined strength and strain enhancements achieved for rectangular columns due to the FRP confinement are assessed against the results summarized in Table 1. The average absolute error (*AAE*) described by Equation 10 is considered to establish the model accuracy.

$$AAE = \frac{\sum_{i=1}^{n}\left|\dfrac{\text{ana}_i - \text{exp}_I}{\text{exp}_i}\right|}{n} \tag{10}$$

It can be generally observed from the comparisons in Figures 7 and 8 that the analytical values obtained from the proposed Equations 6 and 7 agree well with the test results compared with the models of other researchers.
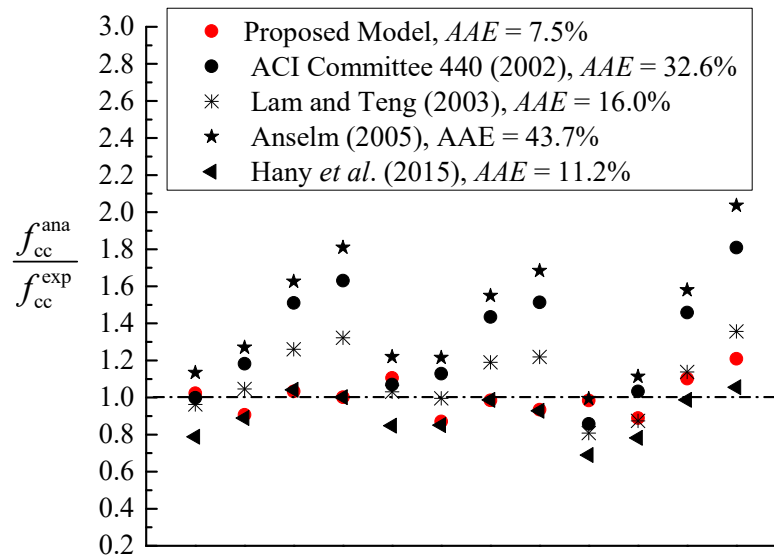
Figure 7. Evaluation of proposed and existing models against the test results of confined axial strength, f_{cc}'.
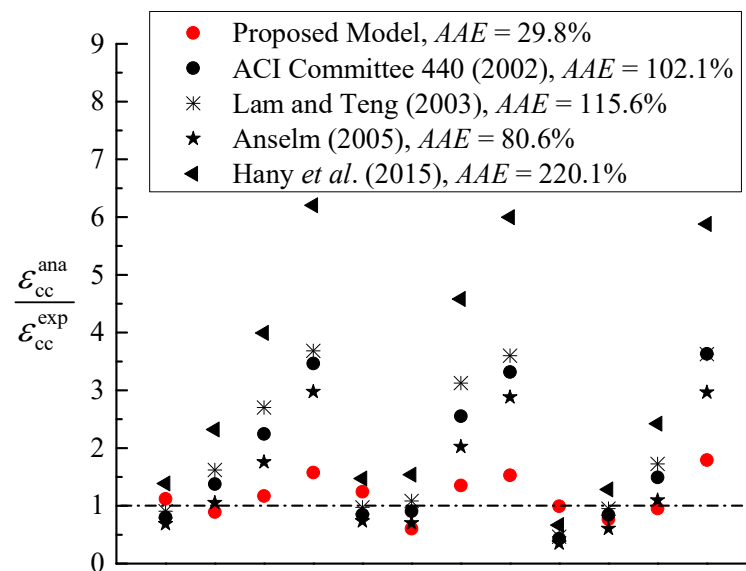
Figure 8. Evaluation of proposed and existing models against the test results of confined axial strain, ε_{cc}.

6. Conclusions

The existing tests and analytical models conducted on the axial compressive behavior of FRP-confined concrete have been largely concerned with columns of circular sections, where the concrete exhibited higher confinement due to uniform distribution of lateral stresses. On the contrary, the stress distribution in the case of a rectangular column varies over its cross section. Generally, the efficiency of FRP wraps decreases as the cross-sectional aspect ratio increases. Significant enhancements in ultimate strengths over the strength of unconfined concrete were achieved for columns with an aspect ratio of of less than 2.0. In particular, columns with the aspect ratio equals to 2.0 were to experience a reduction in their ultimate strengths. The effectiveness of FRP confinement in terms of enhancement in ultimate strength can be significantly improved by the shape modification method. On the basis of the evaluation of existing models, it was revealed that the models available for rectangular FRP-confined specimens do not predict well the results of FRP-confined concrete in

elliptical sections. Based on a regression analysis of the existing test results, a new confinement model was presented to estimate the confined strength and strain of FRP-confined concrete columns having elliptical cross-sections. Based on the proposed model and the assembled test data, the threshold value of 0.02 for sufficiently confined elliptical concrete columns was proposed. Exceeding this value dictates the post-peak curve of the axial stress-strain response exhibits hardening behavior. The proposed model better predicted the test results compared with the predictions from the existing models provided by other researchers for FRP-confined rectangular columns.

Acknowledgments

Notation

$a \& b$ = width and depth of an elliptical section

a/b = aspect ratio of an elliptical cross-section

t_{wrap} = thickness of FRP composite layers

E_f = tensile elastic modulus of FRP composite

f_f = FRP maximum tensile strength

ε_{fu} = FRP ultimate strain at rupture

f_c' = strength of unconfined concrete

ρ_f = volumetric ratio of FRP wraps

ε_{co} = axial strain of unconfined concrete

ε_{fe} = effective FRP rupture strain

k_ε = efficiency factor for determining the FRP rupture strain

k_e = coefficient for effectiveness of FRP confinement

$CR \ or \ MC_R$ = FRP confinement pressure ratio

f_{cc}' = FRP-confined peak strength

ε_{cc} = confined strain of confined concrete

AAE = average absolute error

n = total number of tested specimens

ana = analytical value given by the model

exp = experimental value obtained from tests

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