Precision Permittivity Measurement for Low-Loss Thin Planar Materials Using Large Coaxial Probe from 1 to 400 MHz

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Abstract: This paper focuses on the non-destructive dielectric measurement for low-loss planar materials with a thickness of less than 3 mm using a large coaxial probe with an outer diameter of 48 mm. The aperture probe calibration procedure required only to make a measurement of the half-space air and three offset shorts. The reflection coefficient for the thin material is measured using a Keysight E5071C network analyzer from 0.3 MHz to 650 MHz and then converted to a relative dielectric constant and tangent loss via closed form capacitance model and lift-off calibration process. Measurement error of dielectric constant, \( \Delta \varepsilon_r \) is less than 2.5 \% from 1 MHz to 400 MHz and the resolution of loss tangent, \( \tan \delta \) measurement is capable of achieving \( 3 \times 10^{-3} \).

Keywords: large coaxial probe, thin planar materials, low-loss materials, relative permittivity, reflection coefficient, calibration.

1. Introduction

The introduction should briefly place the study in a broad context and highlight why it is important. It should define the purpose of the work and its significance. The current state of the research field should be reviewed carefully and key publications cited. Please highlight controversial and diverging hypotheses when necessary. Finally, briefly mention the main aim of the work and highlight the principal conclusions. As far as possible, please keep the introduction comprehensible to scientists outside your particular field of research. References should be numbered in order of appearance and indicated by a numeral or numerals in square brackets, e.g., [1] or [2,3], or [4–6]. See the end of the document for further details on references. Sensitivity, sustainability and simplicity in operations are important requirements for sensing devices in large-scale material processing industries. To reduce the uncertainty of the measurements, the sensors should have large sensing area, high concentration of sensing field and high sensitivity to the slight changes in material under test (MUT). The operating frequency of the permittivity, \( \varepsilon_r \) measurements in most of the material processing industries are up to a few hundred MHz [1]-[3].

Open-ended coaxial probe technique is a simple, broadband and non-destructive way to measure the relative permittivity, \( \varepsilon_r \) of a material. Coaxial probes have been commercialized and used commonly since 1990 [4]. Recently, several probes have been produced by manufacturers such as SPEAG Inc. [5], KEYCOM [6], and APREL Inc. [7]. However, coaxial probes (N-type’s or SMA’s diameter size) are less sensitive to small changes in the MUT especially for thin and low-loss materials in permittivity, \( \varepsilon_r \) measurement at MHz frequency. This causes the measurement results for low-loss materials at low frequencies to be highly scattered and less precise [8]. In fact, most
coaxial probe is only suitable for half-space infinite lossy material with $\varepsilon_r' > 5$ and $\tan \delta > 0.05$ [4]-[11].

In this study, a large open-ended coaxial probe was designed to overcome those issues, which is capable of measuring the $\varepsilon_r$ of low-loss materials having thickness of 1 mm precisely from 1 to 400 MHz. The probe design of the probe and its performance were analyzed in detail. An explicit formulation for the prediction of $\varepsilon_r$, which does not involve numerical inversion routines (iterative method) as in [8]-[11],[13] is used. The differences in this study compared with previous works are summarized in Table 1.

2. Large Coaxial Probe Design

2.1. Dimensions and Structure

The designed coaxial probe as shown in Figure 1 (a) is made of brass, since the material is relatively low cost and has slower surface oxidation process when it is exposed to the moisture in the air. The dimensions of the coaxial probe with outer radius of inner conductor, $a = 7.5$ mm and inner radius of outer conductor, $b = 24.0$ mm, were designed based on the characteristic impedance, $Z_0 = [60\ln(b/a)/\varepsilon_{c}] = 50$ $\Omega$. The maximum limit of the operating frequency, $f_{\text{max}}$ propagating in the coaxial line of the probe is determined using TE$_{11}$ cut-off as: $f_{\text{max}} = (3\times10^8)/[\pi(b+a)\varepsilon_{c}] \approx 2.1$ GHz. The symbol $\varepsilon_c$ (Air: $\varepsilon_c = 1$, Teflon: $\varepsilon_c = 2.06$) represents the relative permittivity of material filling in the coaxial line in between the inner and outer conductor. The total weight of the coaxial probe is 2.6 kg. It has been divided into three sections: (I) N-type connector, which is used to connect the coaxial probe with the network analyzer via cable. (II) Transition section, which is a 50 mm of air-filled conical taper. The radius $a$ and $b$ of the conductors are increased along the transition length with a constant ratio, $b/a = 2.3$. Ratio, $b/a = 2.3$ is required to maintain $Z_0 = 50$ $\Omega$ along the transition length to achieve low return loss and the lowest standing wave ratio (SWR) during the transformation from small to large coaxial line. (III) Large coaxial line section, which is a 100 mm length of 50 $\Omega$ teflon-filled coaxial line with $b/a = 3.3$. The Teflon isolation block is used to prevent the MUT from getting into the coaxial line. In addition, Teflon has high flexural strength, excellent chemical resistance, and high stability over a wide temperature range.

![Figure 1](image-url)

Figure 1. (a) Cross-sectional side view and dimensions (in millimeter) of the coaxial sensor. (b) Side view of the large coaxial sensor. (c) Internal configuration of the coaxial sensor.
Table 1. Comparative study of large probe.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Probe size (cm)</th>
<th>$f$ (MHz)</th>
<th>Transition section</th>
<th>Sample contact</th>
<th>Sample size/shape</th>
<th>Calibration standards</th>
<th>Measured $\varepsilon_r'$ range</th>
<th>Inverse method</th>
</tr>
</thead>
<tbody>
<tr>
<td>[8]</td>
<td>$2a = 1.00$</td>
<td>100–900</td>
<td>without</td>
<td>Aperture probe</td>
<td>Half-space infinite</td>
<td>Air, short, NaCl solution.</td>
<td>$-5$–$80$</td>
<td>Iterative</td>
</tr>
<tr>
<td></td>
<td>$2b = 3.25$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$L = 5.00$</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>[9]</td>
<td>$2a = 1.00$</td>
<td>1–10</td>
<td>without</td>
<td>Aperture probe</td>
<td>Half-space infinite</td>
<td>Short cavity or Air, short, short cavity</td>
<td>$-30$–$80$ (Lossy)</td>
<td>Iterative</td>
</tr>
<tr>
<td></td>
<td>$2b = 3.25$</td>
<td>10–3000</td>
<td></td>
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<tr>
<td></td>
<td>$L = 5.00$</td>
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</tr>
<tr>
<td>[10]</td>
<td>$2a = 1.18$</td>
<td>200–1500</td>
<td>with</td>
<td>Aperture probe</td>
<td>Half-space infinite</td>
<td>Air, copper plate, Teflon plate.</td>
<td>$-5$–$35$ (Lossy)</td>
<td>Iterative</td>
</tr>
<tr>
<td></td>
<td>$2b = 4.00$</td>
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<td></td>
<td>$L \approx 13.0$</td>
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<tr>
<td>[11]</td>
<td>$2a = 4.50$</td>
<td>50–1000</td>
<td>with</td>
<td>Filled in coaxial line</td>
<td>Toroid-shaped along the coaxial line</td>
<td>$-7$–$80$ (Lossy)</td>
<td>Iterative</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2b = 10.3$</td>
<td></td>
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<tr>
<td></td>
<td>$L \approx 41.0$</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>This</td>
<td>$2a = 1.50$</td>
<td>1–400</td>
<td>with</td>
<td>Aperture Probe</td>
<td>Thin planar backed by metal plate</td>
<td>Air, 3 offset shorts</td>
<td>1–20 (Lossless)</td>
<td>Non-iterative</td>
</tr>
<tr>
<td>study</td>
<td>$2b = 4.80$</td>
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<tr>
<td></td>
<td>$L = 15.0$</td>
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* $L$ is the coaxial length of the probe.

2.2. Probe Characterization Test

The complex reflection coefficient, $\Gamma_{AA'} = |\Gamma_{AA'}| \exp(j\phi_{AA'})$ at plane AA’ for four MUTs were measured using Keysight E5071C network analyzer in the frequency ranging from 0.3 MHz to 650 MHz at 25 °C. Calibration was done at the AA’ plane, as shown in Figure 2, using Keysight 85032F kit.

![Experimental set-up and measurement.](image-url)

The sensing distance, $h$ of the probe is 30 mm, which was estimated based on the distance at which the measured phase shift, $\phi_{AA'}$ (rad) as shown in Figures 3 (a) and (b) starting to become constant when the metallic plate was being moved away from the probe aperture in air. Figure 4 shows the time-domain measurements of the coaxial probe, which is operated with minimum windowing and bandpass mode. Clearly, coaxial probe of 15 cm length is sufficient to avoid the interference between plane-AA’ and -BB’ for the frequency-domain $\Gamma_{AA'}$ measurement.
Figure 3. Variation in phase shift $\phi_{AA'}$ with air thickness, $h$ backed by metal plate at (a) 50 MHz and (b) 400 MHz.

Figure 4. The time-domain response for the end of cable at plane-$AA'$ with and without probe.

3. Calibrations

3.1. Probe Aperture Calibration

The actual normalized admittance, $\tilde{Y}_{\text{meas}}$, was measured with the aperture of the probe placed on a two-layer media, in which the first layered medium was a thin sample to be tested with thickness, $h$ and the second layered medium was the conducting plate. The value of the $\tilde{Y}_{\text{meas}}$ of the thin sample is obtained after de-embedding using (2). The effective relative permittivity, $\varepsilon_{\text{eff}}$ of the sample can be estimated as:

$$
\varepsilon_{\text{eff}} = \left( \frac{\tilde{Y}_{\text{meas}}}{f_0 C} \right) \frac{C_f}{C} = \frac{\tilde{Y}_{\text{meas}}}{f_0 C} \frac{C_f}{C} \left( \frac{\tilde{Y}_{\text{meas}}}{f_0 C} \right)
$$

(1)

where

$$
\tilde{Y}_{\text{meas}} = \tilde{Y}_{AA'} \left( \frac{\tilde{Y}_{BB'}_{\text{Air}}}{\tilde{Y}_{AA'_{\text{Air}}}} \right)
$$

(2)

where $\tilde{Y}_{AA'_{\text{Air}}}$ is the measured admittance for air at plane $AA'$, and $\tilde{Y}_{BB'_{\text{Air}}}$ is the simulated standard value of admittance for air at aperture probe (plane $BB'$) using the COMSOL simulator. The relationship between $\Gamma_{AA'}$ and $\tilde{Y}_{AA'}$ is given as:
\[ 
\tilde{Y}_{s, d'} = \frac{1 - \Gamma_{s, d'}}{1 + \Gamma_{s, d'}} 
\]  
(3)

Symbol \( Y_s \) = 0.02 S, \( C \) = 2.38e-\((b-a)\) \[11\], \( C_p \) and \( \omega \) are the characteristic admittance, aperture probe capacitance, fringing field capacitance, and the angular frequency, respectively.

3.2. Effective Permittivity Calibration

For thin solid planar material measurements, the scattering of the wave from probe’s aperture would penetrate the thin planar material and impinge on the other layer-interface media. In this situation, the effective permittivity, \( \varepsilon_{eff} \) of the thin specimen will be measured, but not the actual permittivity, \( \varepsilon_r \) of the material \[10\]. In this study, the relationship between the actual relative permittivity, \( \varepsilon_r \) and effective relative permittivity, \( \varepsilon_{eff} \) for a finite thickness planar specimen was empirically expressed as:

\[ 
\varepsilon_r = \varepsilon_{eff} \left( a_1 + a_2 e^{-b/M} + a_3 e^{-2b/M} \right) 
\]  
(4)

where \( h \) is the thickness of the specimen. The empirical coefficient, \( M \) was found to suite the large probe, which can be roughly represented by single value as 0.006 \[10\]. The unknown complex coefficients \((a_1, a_2, \text{ and } a_3)\) values in (4), were found by using three offset-short terminators \((\varepsilon_r = 1)\), yield:

\[ 
1 = \varepsilon_{eff, 1} \left( a_1 + a_2 e^{-h_1/M} + a_3 e^{-2h_1/M} \right) 
\]  
(5a)

\[ 
1 = \varepsilon_{eff, 2} \left( a_1 + a_2 e^{-h_2/M} + a_3 e^{-2h_2/M} \right) 
\]  
(5b)

\[ 
1 = \varepsilon_{eff, 3} \left( a_1 + a_2 e^{-h_3/M} + a_3 e^{-2h_3/M} \right) 
\]  
(5c)

The unknown values of \( a_1, a_2, \text{ and } a_3 \) in equations (5a)-(5c) were explicitly determined using Cramer’s rule as:

\[ 
\begin{align*}
  a_1 &= \frac{\left( \frac{1}{\varepsilon_{eff, 1}} \left( e^{-h_2+2h_1/M} - e^{-2(h_2+h_1)/M} \right) - \left( \frac{1}{\varepsilon_{eff, 2}} \left( e^{-(h_1+2h_2)/M} - e^{-h_1+2h_2/M} \right) \right) \right) + \left( \frac{1}{\varepsilon_{eff, 3}} \left( e^{-(h_1+2h_2)/M} - e^{-(h_1+2h_2)/M} \right) \right)}{D} \\
  a_2 &= \frac{\left( \frac{1}{\varepsilon_{eff, 2}} \left( e^{-2h_1/M} - \left( \frac{1}{\varepsilon_{eff, 1}} \right) \left( e^{-(h_1+2h_2)/M} - e^{-h_1+2h_2/M} \right) \right) \right) + \left( \frac{1}{\varepsilon_{eff, 3}} \left( e^{2h_1/M} - \left( \frac{1}{\varepsilon_{eff, 2}} \right) e^{-2h_1/M} \right) \right)}{D} \\
  a_3 &= \frac{\left( \frac{1}{\varepsilon_{eff, 3}} \left( e^{-h_1/M} - \left( \frac{1}{\varepsilon_{eff, 2}} \right) e^{-h_1/M} - \left( \frac{1}{\varepsilon_{eff, 1}} \right) \left( e^{h_1/M} - \left( \frac{1}{\varepsilon_{eff, 3}} \right) e^{h_1/M} \right) \right) \right) + \left( \frac{1}{\varepsilon_{eff, 1}} \left( e^{h_1/M} - \left( \frac{1}{\varepsilon_{eff, 3}} \right) e^{-h_1/M} \right) \right)}{D}
\end{align*} 
\]  
(6a)  
(6b)  
(6c)

The determinant, \( D \) of the equations (6a)-(6c) are given as:

\[ 
D = \left\{ e^{(h_1+2h_2)/M} - e^{-(h_2+2h_1)/M} - e^{-(h_1+2h_2)/M} + e^{-(h_1+2h_2)/M} - e^{-(h_1+2h_2)/M} \right\} 
\]  
(7)
4. Results and Discussion

Figures 5 (a) and (b) show the measured $\varepsilon'_r$ and $\tan\delta$ of four thin low-loss MUTs which are in good agreement with expected values as tabulated in Table 2. The measured scattered data in Figures 5 (a) and (b) have been smoothed by Local Polynomial Regression (Loess) algorithm, which is available in built-in MATLAB “smooth” command. The smoothed data are represented by the black solid lines in Figures 5 (a) and (b).

![Graphs showing $\varepsilon'_r$ and $\tan\delta$ vs. frequency for MUTs at room temperature.]

**Figure 5.** Variation in (a) measured $\varepsilon'_r$ and (b) measured $\tan\delta$ with frequency for the MUTs at room temperature.

<table>
<thead>
<tr>
<th>MUT</th>
<th>$\varepsilon'_r$ (Typ)</th>
<th>$\tan\delta$ (Max)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teflon</td>
<td>2.06</td>
<td>0.0004</td>
</tr>
<tr>
<td>Acrylic</td>
<td>2.75</td>
<td>0.019</td>
</tr>
<tr>
<td>FR-4</td>
<td>4.4</td>
<td>0.022</td>
</tr>
<tr>
<td>Glass</td>
<td>6.1</td>
<td>0.0036</td>
</tr>
</tbody>
</table>

Figure 6 shows the percentage of the relative error between the smoothed values of $\varepsilon'_r$ in Figure 5 (a) and the expected values of $\varepsilon'_r$ in Table 2. The relative errors are less than 2.5 % for overall measurements from 1 MHz to 400 MHz. In addition, the resolution for measurement $\tan\delta$ is capable of achieving $3\times10^{-3}$. 
5. Conclusion

One-port measurement using a large coaxial probe is an easy, durable and cost effective measurement method in large-scale material processing industries. In this study, the dielectric properties for thin low-loss materials (a few millimeter thickness) at very low frequencies (below 400 MHz) have been precisely measured using the open-ended coaxial probe techniques. This accuracy level of the measurement is rarely achieved by previous studies, which used the same technique for low-loss material at very low frequencies.


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**Conflicts of Interest:** The authors declare no conflict of interest.

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