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Article

A Distributed Energy-Balanced Topology Control Algorithm based on a Noncooperative Game for Wireless Sensor Networks

Yongwen Du *, Junhui Gong, Zhangmin Wang and Ning Xu

Lanzhou Jiaotong University, The School of Electronics & Information Engineering, Lanzhou 730070, China; 0217603@stu.lzjtu.edu.cn

* Correspondence: duyongwen@mail.lzjtu.cn

Abstract: In wireless sensor networks, there is no a central controller to enforce cooperation between nodes. Therefore, nodes may generate selfish behavior to conserve their energy resources. In this paper, we address the problems of transmission power minimization and energy balance in wireless sensor networks using a topology control algorithm. We considered the energy efficiency and energy balance of the nodes, and an improved optimization-integrated utility function is designed by introducing the Theil index. Based on this, a topological control game model of energy balance is established, and it is proved that the topological game model is an ordinal potential game with Pareto optimality. Additionally, an energy-balanced topology control game algorithm (EBTG) is proposed to construct topologies. The simulation and comparison show that, compared with other topological control algorithms based on game theory, the EBTG algorithm can improve energy balance and energy efficiency while reducing the transmitting power of nodes, thus prolonging the network lifetime.

Keywords: wireless sensor networks; topology control; game theory; energy balanced

1. Introduction

WSN (Wireless sensor networks, WSN) usually consist of a large number of sensor nodes. In general, sensor nodes operate on batteries and are thus limited in their working lifetime [1]; therefore, efficient and balanced energy usage is the key to prolonging the lifetime of a network, which is the primary concern of topology control. The goal of topology control is to optimize the transmitting power of each node and construct a better topology to improve network performance and prolong network lifetime [2].

At present, in the field of wireless sensor networks, many topological control algorithms, which are mainly divided into hierarchical, power control, and game-type topology control algorithms, are proposed. For example, a low-power hierarchical WSN topology control algorithm [3], which is a multilevel topology control algorithm, is designed; this algorithm extends the network level and improves the maintainability of WSN using a combination of the static address and the dynamic address. In another paper [4], an energy-efficient hierarchical topology control method is established in WSN using time slots, in which a cluster-head selecting approach decreases the difference in the cluster size of LEACH and the responsibility mechanism for the active node makes the energy consumption uniform in the cluster. In the literature [5], Kubisch et al. implement dynamic power control to set the node degree of the upper and lower limits, thus resulting in a lower total energy consumption network topology. The power control algorithm proposed in [6] uses a Borel Cayley graph to construct a network topology that has a short average link and low energy consumption. The algorithm does

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not consider the robustness of the network topology and the residual energy of nodes, which affects the operation of the network to some extent.

When the sensor nodes perform data forwarding, the node will show selfish behavior due to energy saving considerations, and competition will occur between nodes [7]. On this basis, the game theory approach can be introduced into the study of WSN topology control. Game theory provides a powerful tool [8] for describing the phenomena competition and individual coping strategies between intelligent rational decision-makers, and it has been used in systems concerning action and payoff. Komali et al. [9], [10] formulated energy-efficient topology control as a noncooperative potential game, which guarantees the existence of at least one Nash equilibrium (NE) and proposes a distributed noncooperative game topology control algorithm based on game theory. In [11], a topology control algorithm based on a link power consumption game is designed to run the minimum MLPT algorithm for the maximum power of the node. To consider network lifetime as well, researchers have proposed two game-based topology control algorithms: the virtual game-based energy-balanced algorithm (VGEB) [12] and the energy welfare topology control algorithm (EWTC) [13]; these algorithms have been developed to improve network lifetime via energy-balanced network topologies. In [14], the adaptive cooperative topological control algorithm (CTCA) based on game theory considers the smallest potential lifetime and degree as the primary and secondary utility functions, respectively. In [15], a topology control algorithm (DEBA) based on the ordinal potential game is proposed by designing a payoff function that considers both network connectivity and the energy balance of nodes. Although some of the abovementioned algorithms based on game theory can achieve network topology control and improve network performance, they cannot guarantee the connectivity and robustness of the network. Additionally, the remaining energy, energy balance and energy efficiency of the nodes are not fully and accurately considered.

Based on the above analysis, this paper takes the energy efficiency and energy balance of the node as the starting point and considers the influence of the residual energy, the transmitting power and the node degree of the node. In addition, by introducing the Theil index to design an improved and optimized integrated utility function, an energy-balanced topology control game algorithm (EBTG) is proposed. The network topology constructed by this algorithm can efficiently guarantee the connectivity and robustness of the network and balance the energy between nodes, which effectively prolongs the network lifetime.

The rest of the paper is organized as follows. Section 2 overviews critical concepts of the network model and the theory of potential games as applicable to our problem. Section 3 presents the topology control game model and provides the game formulation and theoretic analysis. From this model, in Section 4, an energy-balanced topology control algorithm, in which each sensor adaptively adjusts its transmit power according to the residual energy, is proposed. Section 5 validates our EBTG algorithm via simulation. Finally, Section 6 concludes this paper.

2. Preliminaries

In this section, we present a brief overview of some fundamental concepts related to the network model, the ordinal potential game theory and the Theil index.

2.1. Network model

WSN are usually abstracted $\mathcal{G} = (N, L, P)$ as according to graph theory. Let $\mathcal{G} = (N, L, P)$ be an undirected graph, where N denotes the set of nodes, L is a set of two-node communication links in node set N at time t, and P represents the transmit power set of n nodes.

It is assumed that all nodes are randomly deployed in the plane monitoring area and that their maximum transmit power p_i^{max} can be different. When the transmit power $p_i \in [0, p_i^{max}]$ of node i is sufficiently large, the signal received by node j is higher than the receiving threshold p so that node j can respond.

Because most routing and channel studies use bidirectional links, it is assumed that the links in the network topology are bidirectional. When all nodes use their maximum power to communicate, the formed network topology is denoted by $\mathcal{G}_{max}$. In this design, $\mathcal{G}_{max}$ is the connected network.

2.2. Ordinal potential game theory

The ordinal potential game is a kind of strategy game. The strategy game $\mathbf{\Gamma}$ consists of N players, the possible strategy S of the players, and consequences u of the strategy. The following definition is given for the strategy game:

$$\mathbf{\Gamma} = \langle N, S, \{u_i\} \rangle \tag{1}$$

Three definitions are as follows. First, (1) $N = \{1, 2, 3, \cdots, n\}$ represents the set of players, and n is the number of players in the game. Then, (2) S represents the policy space, and S is the Cartesian product of the set of policies $S_i (i \in N)$, where $S_i = \{s_{i1}, s_{i2}, \cdots, s_{ik}\}$ represents an optional set of policies for node i, which is usually abbreviated as $S_i = \{s_1, s_2, \cdots, s_k\}$. In general, we use $s = (s_i, s_{-i}) \in S$ to describe a strategy combination, s_i to represent the strategy choice of node i, and s_{-i} to represent the other node strategy choices except node i. Finally, (3) u represents the utility function $u = \{u_1, u_2, \cdots, u_n\}$, where u_i denotes the maximum utility function that node i can achieve in the policy combination (s_i, s_{-i}).

Definition 1. *In a strategy game $\mathbf{\Gamma} = \langle N, S, \{u_i\} \rangle$, the strategy s_i^* of any game player i is the best strategy response to the strategy combination of s_{-i}^* the remaining game participants. Then, there must be $u_i \{s_1^*, \cdots, s_i^*, \cdots, s_n^*\} \geq u_i \left\{s_1^*, \cdots, s_{ij}^*, \cdots, s_n^*\right\}$, where s_{ij} indicates that the j-th strategy of game player i is valid for any $s_{ij} \in S$; then, $\left\{s_1^*, \cdots, s_n^*\right\}$ is called the "Nash Equilibrium (NE) [16]" of the game.*

A game may possess a large amount of NEs or none at all, but some types of games have been proved to have at least one Nash equilibrium, such as the ordinal potential game used in this paper, which has been proved to be a Nash equilibrium and may not be unique in the literature [17].

Definition 2. *A strategic game $\mathbf{\Gamma} = \langle N, S, \{u_i\} \rangle$ is an ordinal potential game if there exists a function V such that $\forall i \in N$, $\forall s_{-i} \in S_{-i}$ and for $\forall a_i, b_i \in S$*

$$V(a_i, s_{-i}) - V(b_i, s_{-i}) > 0 \Leftrightarrow u_i(a_i, s_{-i}) - u_i(b_i, s_{-i}) > 0 \tag{2}$$

The function V is called the ordinal potential function of the strategy game $\mathbf{\Gamma}$. Then, the strategy combination s^* for the maximum value of the ordinal potential function V is the NE of the game [17].

2.3. Theil index

The Theil index [18] is a statistic primarily used to measure economic inequality and other economic phenomena. It was proposed by econometrician Henri Theil at Erasmus University in Rotterdam. This index measures income inequality through the concept of entropy in information theory [19]. When the concept of the entropy index in information theory is used to measure the income gap, the income gap can be interpreted as the amount of information contained in the message that converts the population share into the income share. The Theil index T is defined as:

$$T = \sum_{i=1}^{N} \left(\frac{x_i}{\sum_{j=1}^{N} x_j} \cdot \ln \frac{x_i}{\overline{x}} \right) \tag{3}$$

where x_i is a characteristic of agent i, $\overline{x}$ represents average income, and N is the population. The range of the Theil index is $[0, \infty)$. The larger the value, the more obvious the difference from the average.

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3. Topology control game model

In this section, a topology control game model is first constructed. Then, it is proved that the game model belongs to the ordinal potential game and the NE is Pareto optimal.

3.1. Utility function

The use environment of wireless sensor networks is relatively complex, and it is difficult to quantify the benefits of nodes. The existing topology control algorithm based on the ordinal potential game [15] does not adequately consider node revenue, and the utility function cannot accurately reflect the competition between nodes and the balance of energy consumption.

This paper uses a power control model based on the utility function [17]. To maximize utility function, each participant adjusts power in a selfish manner, which is typical for noncooperative power control games. In addition, to better balance the load between nodes, energy efficiency is improved. In this paper, the Theil index is introduced in the design of the utility function. In addition, the method of measuring income inequality in the field of social science is used to measure the imbalance of energy consumption between nodes in wireless sensor networks by using the node and its surplus energy as analogues for the group members and their income. Thus, a more accurate node utility function is obtained to describe the competitive relationship between nodes.

Consider a multihop network constituting independent and selfish nodes that adapt the transmit power levels according to their connectivity and energy consumption preferences. By considering the energy efficiency and energy balance, a specific utility function for node $i(\forall i \in N)$ is given by:

$$u_i\left(p_i, p_{-i}\right) = f_{p_i}\left(\lambda p_i^{max} - k_{p_i} p_i + \frac{1}{T+1}\right) + \frac{E_r(i)}{E_0(i) - E_r(i)} + \mu \overline{E_i(p_i)} \tag{4}$$

where, for node i, initial energy, residual energy, current transmitting power and maximum transmitting power are $E_0(i), E_r(i), p_i$ and p_i^{max}, p_{-i} represents the transmit power of the remaining n-1 nodes except node i. In addition, we define a link state variable $f_{p_i}\left(f_{p_i} \geq 0\right)$. If the network is connected, then $f_{p_i} = 1$; otherwise, $f_{p_i} = 0$. Topology control aims to prolong the network lifetime by reducing the node power without destroying the overall network connectivity and robustness. By adding parameter $f_{p_i}\left(f_{p_i} \geq 0\right)$, it is ensured that the network remains connected after repeated iterations of the game. The k_{p_i} represents the degree of node i when the transmitting power is p_i; λ and μ are the weight factors of the utility function and all are positive numbers. $\overline{E_i(p_i)} = \frac{1}{m}\sum\limits_{j=1}^{m}\frac{E_r(j)}{E_0(j)-E_r(j)}$ (node j means that node i is a single hop neighbor node at power p_i, where m represents the number of one-hop neighbor nodes of node i) in equation 4 indicates that more calls to the remaining high-energy nodes participate in the communication link to ensure load balancing [11]. To better balance the load between nodes and improve energy efficiency, the method of measuring income inequality in the field of social science is used to measure the imbalance of energy consumption between nodes in wireless sensor networks using the node and its residual energy as an analogue for group members and their income; the Theil index T is defined as

$$T = \sum_{i=1}^{n}\left(\frac{E_r(i)}{\sum\limits_{j=1}^{n} E_r(j)} \cdot \ln \frac{E_r(i)}{\overline{E_r}}\right).$$

The utility function satisfies the properties described in [20]. With the utility function defined, a game is played with all sensors picking their powers.

3.2. Model proof

We show that the game $\mathbf{\Gamma} = \langle N, S, \{u_i\} \rangle$ with the utility function of each sensor given by (4) is an ordinal potential game; then, the existence of NEs are guaranteed.

Theorem 1. *The game $\mathbf{\Gamma} = \langle N, S, \{u_i\} \rangle$ is an ordinal potential game. The ordinal potential function is given by:*

$$V(p_i, p_{-i}) = \sum_{i \in N} \left\{ f_{p_i} \left(\lambda p_i^{max} - k_{p_i} p_i + \frac{1}{T+1} \right) + \frac{E_r(i)}{E_0(i) - E_r(i)} + \mu \overline{E_i(p_i)} \right\} \tag{5}$$

Proof. We apply the asserted ordinal potential game in (4). First, we have:

$$\begin{aligned} \Delta u_i &= u_i(p_i, p_{-i}) - u_i(q_i, p_{-i}) \\ &= f_{p_i} \left(\lambda p_i^{max} - k_{p_i} p_i + \frac{1}{T+1} \right) + \mu \overline{E_i(p_i)} - f_{q_i} \left(\lambda p_i^{max} - k_{p_i} q_i + \frac{1}{T+1} \right) - \mu \overline{E_i(q_i)} \\ &= (f_{p_i} - f_{q_i}) \left(\lambda p_i^{max} + \frac{1}{T+1} \right) + f_{q_i} k_{q_i} q_i - f_{p_i} k_{p_i} p_i + \mu \left(\overline{E_i(p_i)} - \overline{E_i(q_i)} \right) \end{aligned} \tag{6}$$

Similarly:

$$\begin{aligned} \Delta V &= V(p_i, p_{-i}) - V(q_i, p_{-i}) \\ &= \sum_{i \in N} \left[f_{p_i} \left(\lambda p_i^{max} - k_{p_i} p_i + \frac{1}{T+1} + \mu \overline{E_i(p_i)} \right) \right] \\ &\quad - \sum_{i \in N} \left[f_{q_i} \left(\lambda p_i^{max} - k_{p_i} q_i + \frac{1}{T+1} + \mu \overline{E_i(q_i)} \right) \right] \\ &= (f_{p_i} - f_{q_i}) \left(\lambda p_i^{max} + \frac{1}{T+1} \right) + f_{q_i} k_{q_i} q_i - f_{p_i} k_{p_i} p_i + \mu \left(\overline{E_i(p_i)} - \overline{E_i(q_i)} \right) \\ &\quad + \sum_{j \in N, j \neq i} \left[\lambda \left(f_{p_j} - f_{q_j} \right) p_j^{max} + \mu \overline{E_j(p_j)} \right] \end{aligned} \tag{7}$$

Thus, we have:

$$\Delta V = \Delta u_i + \sum_{j \in N, j \neq i} \left[\lambda \left(f_{p_j} - f_{q_j} \right) p_j^{max} + \mu \overline{E_j(p_j)} \right] \tag{8}$$

For node i, because is monotonic and $\lambda \left(f_{p_j} - f_{q_j} \right) p_j^{max} + \mu \overline{E_j(p_j)} \geq 0$, it follows from (6) that:

$$\Delta u_i = \begin{cases} \geq 0 & if \quad p_i > q_i \quad and \quad f_{p_i} > f_{q_i} \\ \leq 0 & if \quad p_i < q_i \quad and \quad f_{p_i} < f_{q_i} \\ > 0 & if \quad p_i > q_i \quad and \quad f_{p_i} = f_{q_i} \\ < 0 & if \quad p_i > q_i \quad and \quad f_{p_i} = f_{q_i} \end{cases} \tag{9}$$

Therefore, $sgn(\Delta u_i) = sgn(\Delta V)$, the function $V(p_i, p_{-i})$ is the ordinal potential function of the strategy game, and the strategy game $\mathbf{\Gamma} = \langle N, S, \{u_i\} \rangle$ is the ordinal game. □

Theorem 2. *The NE of the topology game model established in this paper is Pareto optimal [16] if the network $\mathcal{G}_{max}$ is connected.*

Proof. Due to the limited number of nodes, a node has a limited number of optional power concentration elements. According to [17], the finite ordinal potential game must converge to the NE. According to the definition of the network model and the description of the utility function, a node maximizes the utility function by adjusting its own policy choice. The concrete manifestation is that

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the node continuously reduces the transmitting power and prolongs the survival time until the power of all nodes no longer changes; that is, the NE state is reached.

When a NE point is reached, if a node reduces its power, network connectivity will be destroyed. As a result, the remaining nodes must increase their power, thus resulting in lower utility function values of other nodes and disruption of the NE. Therefore, according to the Pareto optimal definition, it can be concluded that the NE of the topological control game is Pareto optimal. □

4. Energy-balanced topology control game algorithm

In this section, we propose an energy-balanced topology control algorithm in which each sensor adaptively adjusts its transmit power according to the residual energy.

Nodes in EBTG initiate with the maximum power network $\mathcal{G}_{max}$ and then try to update this topology iteratively according to their increasing unwillingness. EBTG consists of three phases: the initialization phase (topological establishment phase), the adaptation phase (power adjustment phase), and the topology maintenance phase.

4.1. Initialization phase

Every node in topology control game algorithms that makes a topological decision needs to collect some network information. In EBTG, the information required by node i in the topology construction process is the local topology $\mathcal{G}_i$, which is an induced subgraph of $\mathcal{G}_{max}$. Every vertex of the directed graph corresponds to a node in WSN.

To obtain this decision information, node i initializes its transmit power with maximum power p_{max} and discovers its neighbor nodes by broadcasting the "Hello" Message and collecting the responses provided by the receivers at p_{max}. The message contains information such as node ID and remaining energy. By receiving and returning the message, a series of information, such as ID, transmit power and residual energy of its neighbor nodes, is learned and the maximum reachable neighbor set $N_{max}(i)$ of node i and its maximum uplink set $L_{max}(i)$ is determined. By establishing these sets, the largest global network topology view $\mathcal{G}_{max}$ can be learned, thereby establishing a basis for subsequent routing decisions.

4.2. Adaptation phase

Node i in the adaptation phase determines its transmit power according to its current residual energy $E_r(i)$, the current transmitting power and the topology-related information collected during the initialization phase. The procedure of power adaptation is shown in Algorithm 1.

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Algorithm 1 EBTG Power Adaptation

initialization

node i broadcasts "hello" message at p_i^{max}
determine the neighbor set $N_{max}(i)$
Determine the optional power set P_i for node i, descending sort
Broadcast optional power set P_i

Power adjustment

$P_i = \{p_1, p_2, \cdots, p_k\}$, descending sort
while p_i is not NE
    for $i = 1, i \leq N, i++$
        choose power according to $p_i^* = \arg\max_{p_i \in P_i} u_i(p_i, p_{-i})$
        if $u_i^*\left(p_i^*, p_{-i}\right) \geq u_i\left(p_i, p_{-i}\right)$
            if p_i is NE
                $p_i = p_i^*$, update p_i
            end if
        end if
    end for
    broadcast a "hello" message including the new power setting p_i at p_i^{max}
end while

This paper adjusts the network topology structure through the power control method, sets the transmission power of the node as the optimal transmission power, and thus obtains the optimized network topological structure.

In the power adaptation phase, we first need to sort the node's strategy set $P_i = \{p_1, p_2, \cdots, p_k\}$ in descending order. The minimum available transmit power p_i^{min} of node i can be calculated using the free-space model proposed in [21]. Pseudocode is shown in Algorithm 1.

The power adjustment sequence of the EBTG algorithm is based on the node ID, as shown in Fig. 1, the transmit power of one node is adjusted for each round, and the remaining power of the node is unchanged. To ensure convergence to the NE, this algorithm uses the better response strategy update scheme proved in [17], which converges to the NE in the finite ordinal potential game. Given the power p_{-i} of other participants, the optimal response of node i is $r_i\left(p_{-i}\right) = \min\left(p_i^{max}, p_i^*\right)$, which has $p_i^* = \arg\max_{p_i \in P_i} u_i(p_i, p_{-i})$. During the game, when the node selects a power lower than the current transmit power for communication, it is observed whether the corresponding integrated utility function value increases. If it is larger, it indicates that the lower power is more suitable for use as the transmit power; otherwise, the node keeps current transmit power unchanged.

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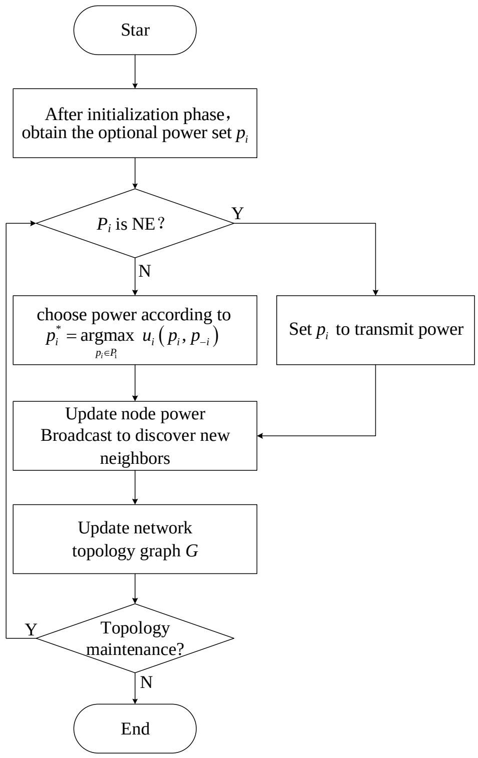

Figure 1. Flowchart of Power Adaptation

When the power of the node is changed, the communication radius, the neighboring node and its related links will change, which leads to the change of the network topology. As shown in Fig. 2, when the transmit power of node i increases, node j will be included in its communication range; then, the nearest neighbor node of node i is changed from the original node k to the current node j. Therefore, node j can reduce its transmit power appropriately under the precondition of guaranteeing full network connectivity.

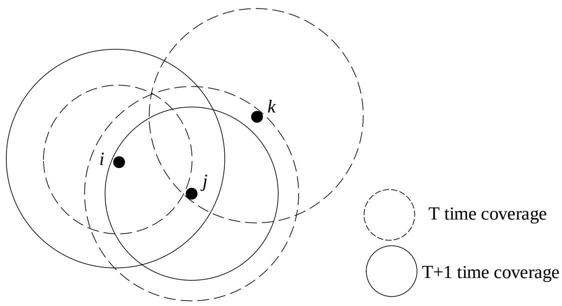

Figure 2. Diagram of Power Adaptation

4.3. Topology maintenance phase

As time flows, the energy consumption of the nodes may become more unbalanced. Therefore, energy consumption between nodes will become unbalanced. In consideration of node failure or death, network topology maintenance must be performed dynamically. For the topology maintenance

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phase, we designed an event-triggered approach that adaptively regenerates a more balanced network topology.

The power game process can be implemented by comparing the residual energy of nodes with the energy threshold or by setting the period to balance the load of nodes and prolong the network lifetime. Pseudocode is shown in Algorithm 2.

Algorithm 2 EBTG Topology Maintenance

initialization

1: Receiving neighbor information
2: if $t \geq T$(T is the set time threshold)
3: Replay the game of power adaptation
4: end if

Theorem 3. *If the $\mathcal{G}_{max}$ is a connected network, the EBTG algorithm converges to the NE state that can maintain the connectivity of network $\mathcal{G}_{max}$.*

Proof. It is known from Theorem 1 that the topological control game model constructed in this paper is an ordinal potential game. In the EBTG algorithm proposed in this paper, the node increases its benefit function value by adjusting the choice of strategy (i.e., reducing the power value of the node) until the selection strategies of all nodes are not changed. Obviously, this state is a NE. It is assumed that the node i obtains greater benefits in the power $p_i < p_i^*$, and the network is disconnected when the power p_{-i} of the other nodes is unchanged; therefore:

$$u_i\left(p_i, p_{-i}\right) = \frac{E_r(i)}{E_0(i) - E_r(i)} + \mu\overline{E_i(p_i)} > \lambda p_i^{max} - k_{p_i} p_i^* + \frac{E_r(i)}{E_0(i) - E_r(i)} + \mu\overline{E_i(p_i^*)} \tag{10}$$

In addition, then:

$$\mu\overline{E_i(p_i)} > \lambda p_i^{max} - k_{p_i} p_i^* + \mu\overline{E_i(p_i^*)} \tag{11}$$

Obviously, equation (10) is not tenable, thus obtaining the connected network at each round of the game execution of the EBTG algorithm. □

5. Simulation results analysis

In this section, computer simulations are provided to illustrate the proposed algorithms. This paper uses MATLAB R2016a as a simulation tool to simulate the EBTG algorithm. In addition, a comparison with the DIA [10], MLPT [11] and DEBA [15] algorithms is conducted with regard to node degree, node transmit power, node hop number and node residual energy. The experiment assumes that all nodes are randomly deployed and cannot be moved, and each sensor sends a packet to other sensors per second, i.e., each sensor transmits n-1 packets per second, the packet size is 1024 bytes, and the transmission rate is 106 bits/s. The remaining emulation parameters are shown in Table 1:

Table 1. Experimental parameter

Parameter name	Parameter size
Monitoring area	$150m \times 150m$
Communication radius	$50m$
Node initial energy	$50J$
Wavelength λ	$0.1224m$
Receiving threshold	$7 \times 10^{-10}w$
Transmit antenna gain G_t	1
Receive antenna gain G_r	1
System loss L	1

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First, weight factor λ and μ in the utility function must be determined; the experiment randomly distributes 50 nodes in the target region, as shown in Fig. 3.

For $\mu = 1$, the influence of λ on the network topology performance is considered in terms of the average transmit power, the average node degree between nodes, the average residual energy of the adjacent node and the average hop number of the shortest path between nodes.

Fig. 3(a) indicates that the average transmit power of the node decreases as λ increases. Fig. 3(b) indicates that the residual energy of the neighboring node decreases as λ increases. Fig. 3(c) indicates that the average node degree of the network decreases as λ increases, but tends to stabilize after $\lambda \geq 2$. Fig. 3(d) indicates that the average hop number of the shortest path between nodes increases as λ increases. The changes after $\lambda \geq 2$ also tended to stabilize. From the general theory of network topology, it can be seen that the topology of the network is perfect when the transmit power of the nodes is low, while there is a moderate node degree and average hop number. By comprehensively considering node computing power and network performance [22], this paper sets $\lambda = 4$ and $\mu = 1$.

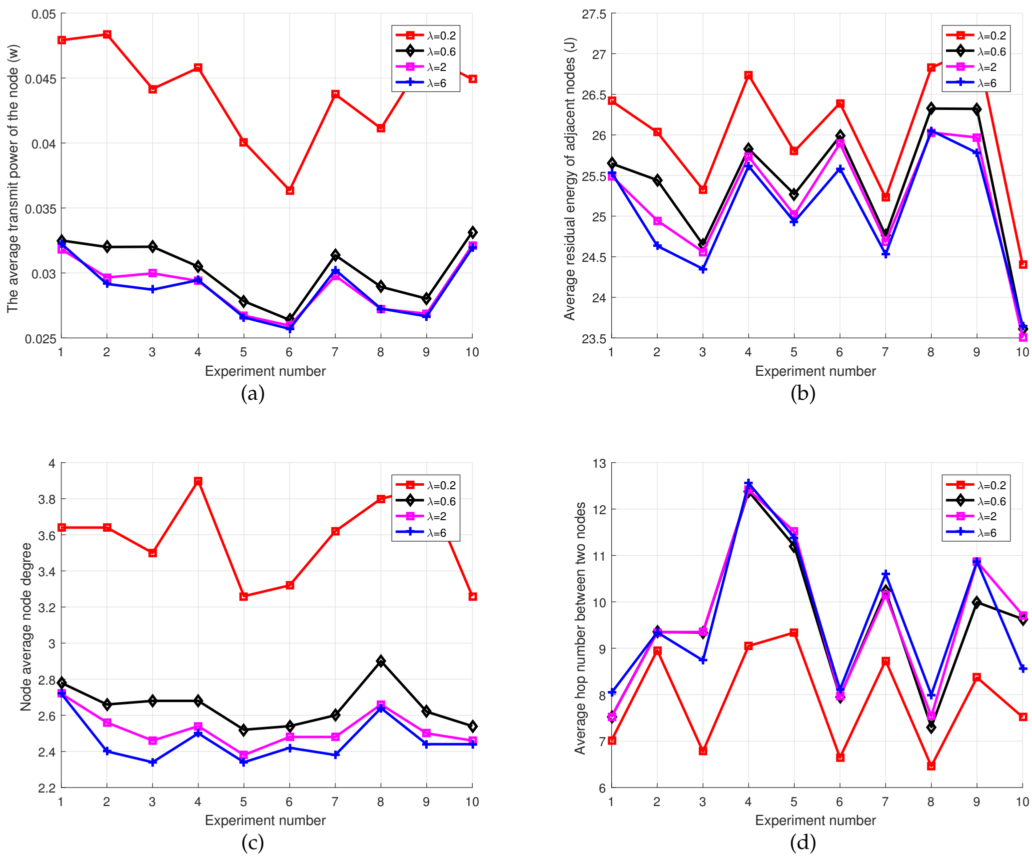

Figure 3. The impact of λ on network performance

Fig. 4 shows the network topology diagram of the four algorithms, i.e., DIA, MLPT, DEBA and EBTG. It can be seen that the network topology built by the DIA algorithm has a large load and low residual energy (the nodes are marked out). The DEBA algorithm has a higher node degree and more redundant nodes, which lead to faster energy consumption. Compared to the other two algorithms, the MLPT and EBTG algorithms have lower node degrees and fewer redundant nodes. The general theory of network topology shows that the EBTG algorithm has moderate nodes and redundant nodes; therefore, its network connectivity and robustness are better than those of the other three algorithms, which can efficiently balance the load between nodes to prolong network lifetime.

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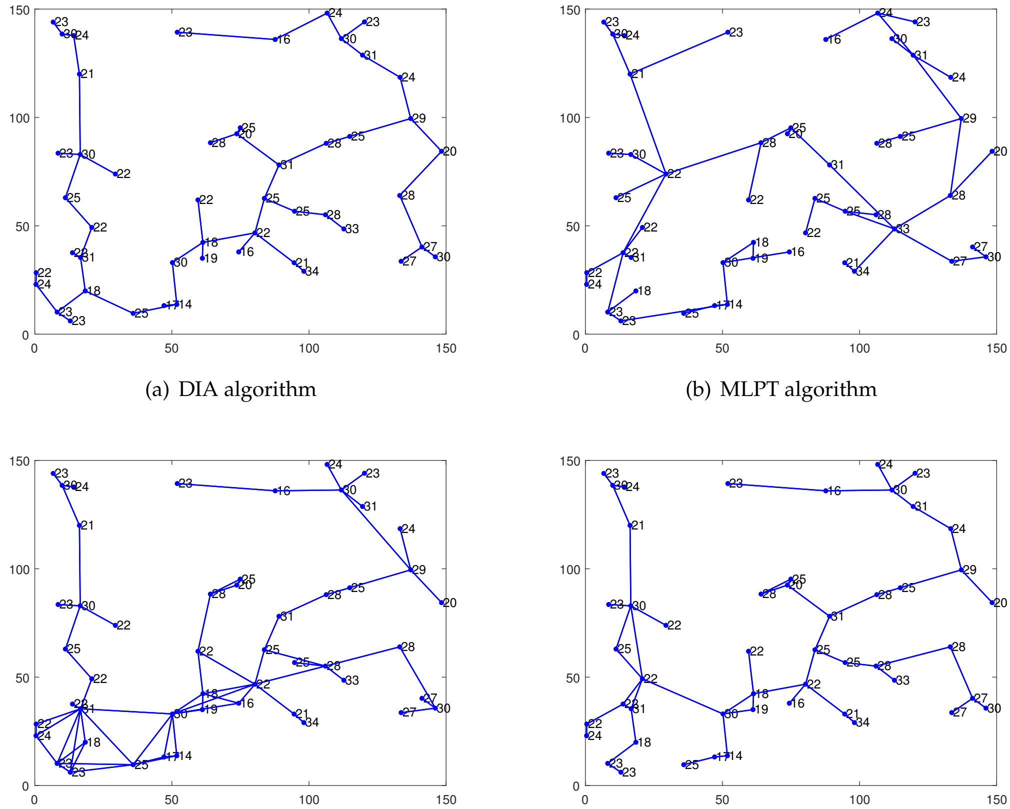

(a) DIA algorithm (b) MLPT algorithm

(c) DEBA algorithm (d) EBTG algorithm

Figure 4. Network topology comparison chart

To make a clearer comparison of the four algorithms, this paper conducted 8 groups of experiments. The specific experimental parameters are set as shown in Table 1, where the number of nodes participating in the experiment is increased from 30 to 100 and the algorithm is compared by calculating the node transmit power of the four algorithms, the hops of the shortest link between nodes and the average value of the four parameters of the node degree.

Fig. 5 is a comparison diagram of the transmission power between nodes. It can be observed from the figure that the transmission power of a node decreases as the number of nodes increases. The EBTG algorithm's node average transmit power is lower than the DIA, MLPT and DEBA algorithms, which can ensure that the EBTG algorithm can establish network topology connections with lower power, which is conducive to extending the network lifetime.

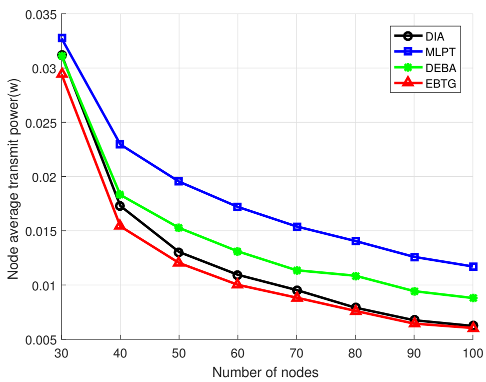

Figure 5. Node average transmit power

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Fig. 6 shows the hop count comparison of the shortest link between nodes. The average hop count of the EBTG algorithm is higher than that of the MLPT algorithm, but it is still lower than the DIA and DEBA algorithms. The MLPT algorithm has higher node transmit power and greater communication coverage, and so its average link hop count is lower. Since the EBTG algorithm operates at lower power and the communication radius is smaller, the average hop count of the link increases. However, the EBTG algorithm still obtains fewer link hops than the DIA and DEBA algorithms when the transmit power is lower than the DIA algorithm.

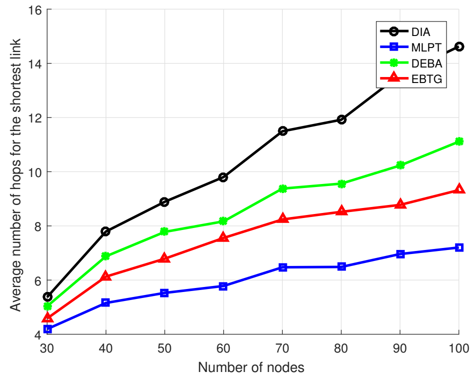

Figure 6. Average number of hops for the shortest link

Fig. 7 shows a comparison of node degrees for the four algorithms. Because nodes with more energy remaining in the EBTG algorithm are more active in node communication, to obtain a more balanced load to prolong the life cycle of the network, the node degree is higher than that of the DIA algorithm but lower than that of the DEBA and MLPT algorithms. The moderate node degree of the EBTG algorithm does occupy too much of the energy resources and obtains relatively good connectivity and robustness, while having fewer redundant nodes can achieve better energy efficiency, improve channel multiplexing and reduce interference.

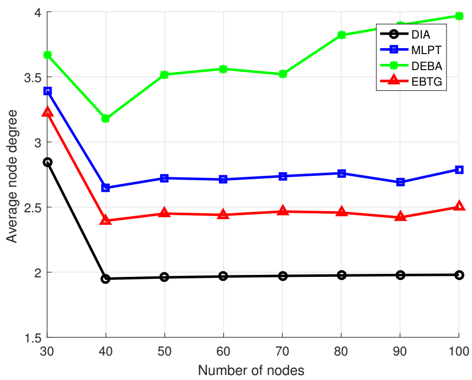

Figure 7. Average node degree

Fig. 8 compares the standard deviations of the node residual energy. It can be seen that the variance of the EBTG algorithm changes slowly. In the network topology constructed by the DIA, MLPT, and DEBA algorithms, the load of some nodes is too high, which affects the network lifetime. If these heavily loaded nodes die prematurely, they will also have a greater impact on the connectivity and robustness of the network. The DIA algorithm overemphasizes that reducing the node transmit power makes the network energy consumption uneven; the MLPT algorithm does not consider the node's residual energy, resulting in poor performance of its energy balance; the DEBA algorithm focuses on energy balance while ignoring the energy efficiency, which leads to the growth of the residual energy standard deviation; the rising trend of the EBTG algorithm is the most gradual. The EBTG algorithm

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not only considers the remaining energy of the node but also transfers the data forwarding task to nodes with more residual energy, effectively balancing the load of the entire network and improving energy efficiency.

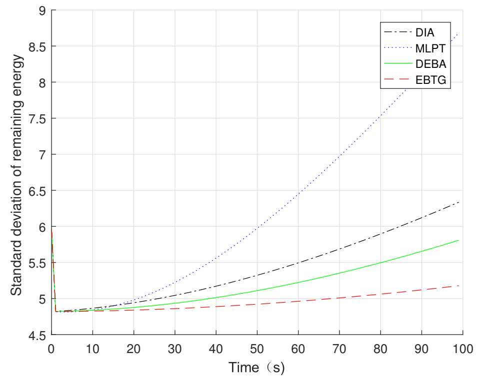

Figure 8. Standard deviation of node residual energy

Fig. 9 is a network lifetime comparison chart. Because topology control is mainly concerned with energy, prolonging the network life cycle is an important index for evaluating the topology control algorithm. The graph shows that the network lifetime of the EBTG algorithm is the longest because the EBTG algorithm reduces the transmit power of the node, expertly balances the load between nodes and improves the energy efficiency; therefore, its network lifetime is much higher than that of the networks constructed using the DIA, MLPT and DEBA algorithms.

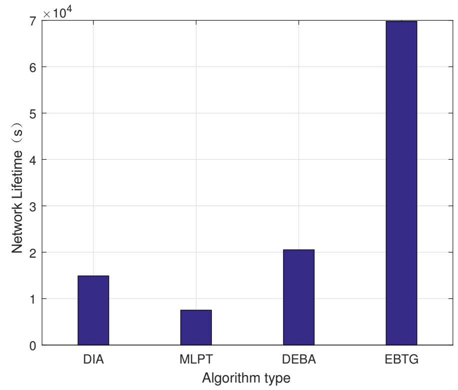

Figure 9. Network lifetime

6. Conclusions

Sensors in wireless sensor networks have been restricted to local communications and make topological decisions selfishly, and the unbalanced energy consumption between nodes is likely to shorten the network lifetime.

Based on the theory of potential games and the Theil index, this paper designs an optimized utility function that considers the residual energy of nodes, the transmitting power of nodes and the connectivity of the network. On this basis, a topological game model is constructed. Additionally, it is proved that a Pareto-optimal NE exists in this model. Thus, an energy-balanced WSN distributed topology game algorithm called EBTG is proposed. From the simulation results, it can be concluded that the EBTG algorithm can effectively reduce the power of the transmitting node, balance the load between nodes, improve the energy efficiency of the network and prolong the network lifetime to ensure network connectivity and robustness. In our future work, we will study the operation of this algorithm in the real-world wireless communication environment to improve the reliability and stability of the algorithm.

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Conflicts of Interest: The authors declare no conflict of interest.

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