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On the role of large nuclear gravity in understanding strong coupling constant, nuclear stability range, binding energy of isotopes and magic proton numbers – A critical review

Seshavatharam.U.V.S[1] and S. Lakshminarayana[2]

[1]Honorary faculty, I-SERVE, Survey no-42, Hitech city, Hyderabad-84,Telangana, INDIA
[2] Dept. of Nuclear Physics, Andhra University, Visakhapatnam-03,AP, INDIA
emails: seshavatharam.uvs@gmail.com and sln@auvsp.edu.in

Abstract: With reference to our earlier published views on large nuclear gravitational constant G_s, nuclear elementary charge e_s and strong coupling constant $\alpha_s \cong (e/e_s)^2$, in this paper, we present simple relations for nuclear stability range, binding energy of isotopes and magic proton numbers.

Summary: Probable range of stable mass numbers can be estimated with $A_s \cong \left[Z+\sqrt{(1/\alpha_s)}\pm 1\right]^x$ where $x \cong 1.2$ for $Z \approx (3 \text{ to } 100)$ and $x \cong 1.19$ for $Z \geq 100$. A_s can also be expressed as, $A_s \cong 2Z + kZ^2$ where $k \cong \left[4\pi\varepsilon_0 (\hbar/2)^2 m_e c^2 / e^2 G_s m_p^3\right] \cong 0.006333$. Energy coefficient being $\left[e_s^2/8\pi\varepsilon_0 \left(G_s m_p/c^2\right)\right] \approx 10.06$ MeV, for $Z \approx (5 \text{ to } 118)$, nuclear binding energy can be understood/fitted with two terms as, $B_{A_s} \cong \left\{A_s - \left[(kA_s Z/2.531)+3.531\right]\right\} \times 10.06$ MeV where $\ln\left(1/\sqrt{k}\right) \cong \left(m_n - m_p/m_e\right) \cong 2.531$. By considering a third term of the form $\left[(A_s - A)^2/A_s\right]$, binding energy of isotopes of Z can be fitted approximately. It needs further investigation.

Keywords: Strong nuclear gravity, nuclear elementary charge, strong coupling constant, nuclear stability range, binding energy of isotopes, magic proton numbers.

1. Introduction

With reference to the figure of 'Strong (nuclear) gravity' [1-6], if $G_f \approx 10^{38} G_N$ and with reference to our recent symposium proceedings and journal publications [7-22], we try to refine our proposed concepts with the following three assumptions for a better understanding on nuclear stability range, binding energy of isotopes and magic proton numbers. We consider,

$$G_f \cong G_s \cong \frac{4\pi\varepsilon_0 \hbar^2 c^2 m_e}{e^2 m_p^3} \cong 3.329561\times 10^{28} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2}.$$

2. Three simple assumptions

1) Nuclear charge radius can be addressed with,
$$R_0 \cong \frac{2G_s m_p}{c^2} \cong 1.23929083 \text{ fm}$$

2) Strong coupling constant can be expressed with,
$$\alpha_s \cong \left(\frac{\hbar c}{G_s m_p^2}\right)^2 \cong 0.1151937353$$

3) There exists a nuclear elementary charge,
$$e_s \cong \frac{e}{\sqrt{\alpha_s}} \cong \left(\frac{G_s m_p^2}{\hbar c}\right) e \cong 4.720586027\times 10^{-19} \text{ C}$$

3. Semi empirical relations and applications

1) Proton magnetic moment can be addressed with
$$\mu_p \cong \frac{e_s \hbar}{2m_p} \cong \frac{eG_s m_p}{2c} \cong 1.488142\times 10^{-26} \text{ J.T}^{-1}$$

2) Neutron magnetic moment can be addressed with
$$\mu_n \cong \frac{(e_s - e)\hbar}{2m_n} \cong 9.817102\times 10^{-27} \text{ J.T}^{-1}.$$

3) Nuclear unit radius can be expressed as,
$$R_0 \cong \frac{2G_s m_p}{c^2} \cong \left(\frac{e_s}{e}\right)\left\{\frac{\hbar}{m_p c}+\frac{\hbar}{m_n c}\right\}$$
4) Root mean square nuclear charge radii can be addressed with,
$$R_{(Z,A)} \cong \left\{Z^{1/3}+\left(\sqrt{Z(A-Z)}\right)^{1/3}\right\}\left(\frac{G_s m_p}{c^2}\right)$$
5) Nuclear potential energy can be understood with ,
$$\cong \frac{e_s^2}{4\pi\varepsilon_0\left(G_s m_p/c^2\right)} \cong 20.1734 \text{ MeV}$$
6) Nuclear binding energy can be understood with ,
$$\frac{e^2 G_s m_p^3}{8\pi\varepsilon_0\hbar^2} \cong \frac{e_s e}{8\pi\varepsilon_0\left(\hbar/m_p c\right)} \cong \frac{e_s^2}{8\pi\varepsilon_0\left(G_s m_p/c^2\right)}$$
$$\cong 10.0867 \text{ MeV}$$
7) With reference to $(\hbar/2)$, a useful quantum energy constant can be expressed with,
$$E_{(\hbar/2)} \cong \left(\frac{e^2 G_s m_p^3}{4\pi\varepsilon_0 (\hbar/2)^2}\right) \cong 80.6934 \text{ MeV}$$
8) Close to magic and semi magic proton numbers, nuclear binding energy seems to approach
$$\left[2.531\left(n+\frac{1}{2}\right)\right]^2 10.0 \text{ MeV where } n = 0,1,2,3,...$$
and $\left(m_n - m_p/m_e\right) = 2.531$.
9) Characteristic melting temperature associated with proton can be expressed with,
$$T_{proton} \cong \frac{\hbar c^3}{8\pi k_B G_s m_p} \cong 0.15\times 10^{12} \text{ K}$$
10) Characteristic nuclear neutral mass unit [6,18] can be addressed with, $\sqrt{\frac{\hbar c}{G_s}} \cong 546.7 \text{ MeV}/c^2$

4. To fit neutron-proton mass difference

Neutron-proton mass difference can be understood with:
$$\left(\frac{m_n c^2 - m_p c^2}{m_e c^2}\right) \cong \ln\sqrt{\frac{E_{(\hbar/2)}}{m_e c^2}} \cong \ln\sqrt{\frac{4e^2 G_s m_p^3}{4\pi\varepsilon_0\hbar^2 m_e c^2}} \quad (4)$$

5. To fit neutron life time

Neutron life time t_n can be understood with the following relation:
$$t_n \cong \exp\left(\frac{E_{(\hbar/2)}}{(m_n - m_p)c^2}\right)\times\left(\frac{\hbar}{m_n c^2}\right) \cong 877.3 \text{ sec} \quad (5)$$
This value can be compared with recommended value of (878.5 ± 0.8) sec.

6. Understanding proton-neutron stability

$$\text{Let, } \left(\frac{m_e c^2}{E_{(\hbar/2)}}\right) \cong \left(\frac{4\pi\varepsilon_0\hbar^2 m_e c^2}{4e^2 G_s m_p^3}\right) \cong k \cong 0.0063326 \quad (6)$$

Quantitatively, we noticed that,
$$\left.\begin{array}{l}\sqrt{\dfrac{e_s^2}{4\pi\varepsilon_0 G_s m_p m_e}} \cong 2\pi \text{ and} \\ \dfrac{4\pi\varepsilon_0 G_s m_p m_e}{(2e_s)^2} \cong \dfrac{1}{16\pi^2} \cong \left(\dfrac{1}{4\pi}\right)^2 \cong k\end{array}\right\} \quad (7)$$
The new factor k needs a clear interpretation and we are working on that for its scope and applicability. It can be considered as a result oriented number connected with nuclear stability and binding energy.

Stable mass number A_s of Z can be estimated with the following simple relations [23],
$$A_s \cong (N_s + Z) \cong 2Z + kZ^2 \cong 2Z + 0.0063326(Z)^2 \quad (8)$$
$$A_s \cong \left[Z + \sqrt{(1/\alpha_s)}\right]^{1.2} \cong [Z + 2.9463]^{1.2} \quad (9)$$
where $(e/e_s)(1/k)^{1/4} \cong (\alpha_s)^{1/2}(1/k)^{1/4} \cong 1.2$. It can be called as 'power factor of stability'.

Proton number Z associated with stable A_s can be estimated with the following simple relations,
$$\left.\begin{array}{l}Z \cong \dfrac{\sqrt{1+kA_s}-1}{k} \\ Z \cong \dfrac{A_s}{1+\sqrt{1+kA_s}}\end{array}\right\} \quad (10)$$

7. Understanding proton-neutron stability range

Considering relation (8), it seems possible to find the best possible range of A_s. We noticed that,
$$\left.\begin{array}{l}(A_s)_{mean} \cong \left[Z + \sqrt{(1/\alpha_s)}\right]^{1.2} \\ (A_s)_{low}^{up} \cong \left[Z + \left(\sqrt{(1/\alpha_s)} \pm 1\right)\right]^{1.2}\end{array}\right\} \quad (11)$$

Lower stable A_s can be estimated with,
$$(A_s)_{low} \cong \left[Z + \left(\sqrt{(1/\alpha_s)} - 1\right)\right]^{1.2} \cong [Z + 1.9463]^{1.2} \quad (12)$$

Upper stable A_s can be estimated with,

$$(A_s)_{up} \cong \left[Z+\left(\sqrt{(1/\alpha_s)}+1\right)\right]^{1.2} \cong [Z+3.9463]^{1.2} \quad (13)$$

See table-1 for the estimated range of stable mass numbers for Z=3 to 100. With even-odd corrections data can be refined.

Considering a factor of 1.19 in place of 1.2, stable mass numbers of super heavy elements can be fitted. For Z=116, estimated stable mass number range seems to be 292 to 298 and its experimental mass range is 291 to 294 [24]. See table 2 for a comparison.

Table-1: Estimated range of stable mass numbers for Z=3 to 100 with a power factor of 1.20

Z	$(A_s)_{low}$	$(A_s)_{mean}$	$(A_s)_{up}$	Main Isotope range
3	7	8	10	6 to 7
4	8	10	12	7 to 10
5	10	12	14	10 to 11
6	12	14	16	11 to 14
7	14	16	18	13 to 15
8	16	18	20	16 to 18
9	18	20	22	18 to 19
10	20	22	24	20 to 22
11	22	24	26	22 to 24
12	24	26	28	24 to 26
13	26	28	30	26 to 27
14	28	30	32	28 to 32
15	30	32	34	31 to 33
16	32	34	36	32 to 36
17	34	36	38	35 to 37
18	36	38	41	36 to 42
19	38	41	43	39 to 41
20	41	43	45	40 to 48
21	43	45	47	44 to 48
22	45	47	50	46 to 50
23	47	50	52	48 to 51
24	50	52	54	50 to 54
25	52	54	57	52 to 55
26	54	57	59	54 to 60
27	57	59	61	56 to 60
28	59	61	64	58 to 64
29	61	64	66	63 to 67
30	64	66	69	64 to 72
31	66	69	71	66 to 73
32	69	71	74	68 to 76
33	71	74	76	73 to 75
34	74	76	79	72 to 82
35	76	79	81	79,81
36	79	81	84	78 to 86
37	81	84	86	83 to 87
38	84	86	89	82 to 88
39	86	89	91	87 to 91
40	89	91	94	88 to 96
41	91	94	96	90 to 96
42	94	96	99	92 to 100
43	96	99	101	95 to 99
44	99	101	104	96 to 106
45	101	104	107	99 to 105
46	104	107	109	100 to 110
47	107	109	112	105 to 111
48	109	112	114	106 to 116
49	112	114	117	113,115
50	114	117	120	112 to 126
51	117	120	122	121 to 125
52	120	122	125	120 to 130
53	122	125	128	123 to 135
54	125	128	131	124 to 136
55	128	131	133	133 to 137
56	131	133	136	130 to 138
57	133	136	139	137 to 139
58	136	139	141	134 to 144
59	139	141	144	141 to 143
60	141	144	147	142 to 150
61	144	147	150	145 to 147
62	147	150	152	144 to 154
63	150	152	155	150 to 155
64	152	155	158	148 to 160
65	155	158	161	157 to 159
66	158	161	164	154 to 164
67	161	164	166	163 to 167
68	164	166	169	160 to 172
69	166	169	172	167 to 171
70	169	172	175	166 to 177
71	172	175	178	173 to 176
72	175	178	181	172 to 182
73	178	181	183	177 to 183
74	181	183	186	180 to 186
75	183	186	189	185,187
76	186	189	192	184 to 194
77	189	192	195	188 to 194
78	192	195	198	190 to 198
79	195	198	201	195 to 199
80	198	201	204	194 to 204
81	201	204	207	203 to 205
82	204	207	209	202 to 214
83	207	209	212	207 to 210
84	209	212	215	208 to 210
85	212	215	218	209 to 211
86	215	218	221	218 to 224
87	218	221	224	221 to 223
88	221	224	227	223 to 228
89	224	227	230	225 to 227
90	227	230	233	227 to 234
91	230	233	236	229 to 234
92	233	236	239	232 to 238
93	236	239	242	235 to 239
94	239	242	245	238 to 244
95	242	245	248	241 to 243
96	245	248	251	242 to 250
97	248	251	254	245 to 249
98	251	254	257	248 to 254
99	254	257	260	252 to 255
100	257	260	263	252 to 257

Data has been taken from https://en.wikipedia.org/wiki/Isotope

Table-2: Estimated range of stable mass numbers for Z=93 to 118 with a power factor of 1.19

Z	$(A_s)_{low}$	$(A_s)_{mean}$	$(A_s)_{up}$	Current synthetic isotopes range
101	248	251	254	257 to 260
102	251	254	257	253 to 259
103	254	257	260	254 to 266
104	257	260	263	261 to 267
105	260	263	266	262 to 270
106	263	266	269	265 to 271
107	266	269	271	267 to 278
108	269	271	274	269 to 271
109	271	274	277	274 to 282
110	274	277	280	279 to 281
111	277	280	283	279 to 286
112	280	283	286	277 to 285
113	283	286	289	278 to 290
114	286	289	292	284 to 290
115	289	292	295	287 to 290
116	292	295	298	290 to 294
117	295	298	301	293, 294
118	298	301	304	294,295

8. Nuclear binding energy close to stable mass numbers

Based on the new integrated model proposed by N. Ghahramany et al [25,26],

$$B(Z,N) = \left\{A - \left(\frac{(N^2-Z^2)+\delta(N-Z)}{3Z}+3\right)\right\}\frac{m_n c^2}{\gamma} \quad (14)$$

where, $\gamma =$ Adjusting coefficient $\approx$ (90 to 100).
if $N \neq Z,\ \delta(N-Z)=0$ and if $N=Z,\ \delta(N-Z)=1$.

Readers are encouraged to see references there in [24,25] for derivation part. Point to be noted is that, close to the beta stability line, $\left[\frac{N^2-Z^2}{3Z}\right]$ takes care of the combined effects of coulombic and asymmetric effects. In this context, we would like suggest that,

$$\left.\begin{aligned}\frac{m_n c^2}{\gamma} &\cong \frac{m_n c^2}{(90\text{ to }100)} \cong \text{Constant}\\ &\cong \frac{e_s^2}{8\pi\varepsilon_0\left(G_s m_p/c^2\right)} \cong 10.09\text{MeV}\end{aligned}\right\} \quad (15)$$

Proceeding further, with reference to relation (7), it is also possible to show that, for $Z \cong (40\text{ to }83)$, close to the beta stability line,

$$\left[\frac{N_s^2-Z^2}{Z}\right] \cong kA_s Z \quad (16)$$

$$\left[\frac{N_s^2-Z^2}{3Z}\right] \cong \frac{kA_s Z}{3} \quad (17)$$

Based on the above relations and close to the stable mass numbers of $(Z \approx 5\text{ to }118)$, with a common energy coefficient of 10.06 MeV, we suggest two terms for fitting and understanding nuclear binding energy.

First term helps in **increasing** the binding energy and can be considered as,

$$\text{Term\_1} = A_s \times 10.06\text{ MeV} \quad (18)$$

Second term helps in **decreasing** the binding energy and can be considered as,

$$\text{Term\_2} = \left(\frac{kA_s Z}{2.531}+3.531\right)\times 10.06\text{ MeV} \quad (19)$$

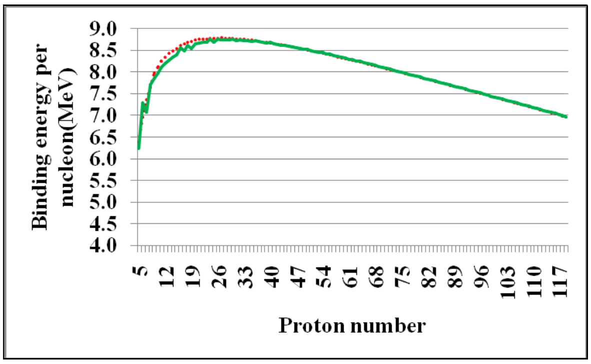
where $\left\{\begin{aligned}&\left(\frac{(m_n-m_p)c^2}{m_e c^2}\right) \cong \ln\left(\frac{1}{\sqrt{k}}\right) \cong 2.531.\\ &3.531 \cong 1+2.531 \cong 1+\ln\left(\frac{1}{\sqrt{k}}\right)\end{aligned}\right.$

Thus, binding energy can be fitted with,

$$B_{A_s} \cong \left\{A_s - \left(\frac{kA_s Z}{2.531}+3.531\right)\right\}\times 10.06\text{ MeV} \quad (20)$$

See the following figure 1. Dotted red curve plotted with relations (7) and (20) can be compared with the green curve plotted with the standard semi empirical mass formula (SEMF) [23,27].

Figure 1: Binding energy per nucleon close to stable mass numbers of Z = 5 to 118



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For medium and heavy atomic nuclides, fit is excellent. It seems that some correction is required for light atoms. See table 3 for the estimated data.

Table 3: Nuclear Binding energy close to stable mass numbers of $Z = 5$ to 118

Proton number	Mass number	Estd. BE (MeV)	SEMF BE (MeV)	Error (MeV)
5	10	63.8	62.3	-1.53
6	12	83.4	87.4	4.01
7	14	102.9	98.8	-4.04
8	16	122.2	123.2	1.03
9	19	151.3	148.9	-2.46
10	21	170.5	167.5	-2.94
11	23	189.5	186.1	-3.35
12	25	208.4	204.7	-3.71
13	27	227.3	223.2	-4.04
14	29	246.0	241.6	-4.35
15	31	264.6	260.0	-4.65
16	34	292.8	290.8	-2.06
17	36	311.2	305.1	-6.18
18	38	329.5	327.2	-2.32
19	40	347.7	341.5	-6.27
20	43	375.4	371.6	-3.84
21	45	393.4	389.6	-3.80
22	47	411.3	407.5	-3.80
23	49	429.1	425.2	-3.85
24	52	456.2	454.6	-1.61
25	54	473.7	468.9	-4.85
26	56	491.2	489.6	-1.61
27	59	517.9	515.2	-2.72
28	61	535.1	532.5	-2.63
29	63	552.3	549.7	-2.61
30	66	578.6	577.9	-0.67
31	68	595.5	592.0	-3.52
32	70	612.3	611.7	-0.60
33	73	638.2	636.6	-1.60
34	75	654.8	653.3	-1.52
35	78	680.4	677.9	-2.56
36	80	696.8	697.0	0.26
37	83	722.2	721.3	-0.84
38	85	738.3	737.6	-0.69
39	88	763.4	761.6	-1.80
40	90	779.3	780.2	0.93
41	93	804.1	803.9	-0.21
42	95	819.7	819.7	0.00
43	98	844.3	843.2	-1.13
44	100	859.7	861.2	1.52
45	103	884.0	884.4	0.38
46	105	899.2	899.8	0.62
47	108	923.2	922.7	-0.49
48	111	947.0	947.6	0.62
49	113	961.9	962.8	0.96
50	116	985.5	987.5	2.03
51	118	1000.1	1000.2	0.16
52	121	1023.4	1024.6	1.22
53	124	1046.5	1046.5	0.05
54	126	1060.8	1063.4	2.62
55	129	1083.6	1085.1	1.47
56	132	1106.3	1108.7	2.38
57	135	1128.9	1130.1	1.17
58	137	1142.7	1144.4	1.73
59	140	1165.0	1165.6	0.58
60	143	1187.1	1188.5	1.42
61	146	1209.1	1209.3	0.23
62	148	1222.4	1225.3	2.91
63	151	1244.1	1245.9	1.77
64	154	1265.6	1268.2	2.56
65	157	1287.0	1288.4	1.41
66	160	1308.3	1310.4	2.16
67	162	1321.0	1322.1	1.14
68	165	1342.0	1343.9	1.94
69	168	1362.8	1363.6	0.86
70	171	1383.4	1385.1	1.64
71	174	1404.0	1404.5	0.58
72	177	1424.3	1425.7	1.34
73	180	1444.5	1444.8	0.30
74	183	1464.6	1465.7	1.06
75	186	1484.5	1484.6	0.06
76	189	1504.3	1505.1	0.82
77	192	1523.9	1523.7	-0.14
78	195	1543.3	1544.0	0.64
79	198	1562.6	1562.4	-0.27
80	201	1581.8	1582.3	0.54
81	204	1600.8	1600.5	-0.33
82	207	1619.7	1620.2	0.51
83	210	1638.4	1638.1	-0.29
84	213	1656.9	1657.5	0.58
85	216	1675.3	1675.2	-0.16
86	219	1693.6	1694.3	0.76
87	222	1711.7	1711.7	0.08
88	225	1729.6	1730.7	1.05
89	228	1747.4	1747.8	0.44
90	231	1765.0	1766.5	1.46
91	234	1782.5	1783.5	0.93
92	238	1807.6	1808.5	0.90
93	241	1824.8	1825.2	0.44
94	244	1841.8	1843.4	1.56
95	247	1858.7	1859.9	1.17
96	250	1875.4	1877.7	2.36
97	254	1899.6	1898.9	-0.71
98	257	1916.0	1916.5	0.54
99	260	1932.2	1932.5	0.34
100	263	1948.3	1949.9	1.66
101	267	1971.7	1971.9	0.16
102	270	1987.5	1989.1	1.56
103	273	2003.1	2004.6	1.54
104	276	2018.6	2021.6	3.02
105	280	2041.3	2041.4	0.17
106	283	2056.4	2058.1	1.74
107	287	2078.7	2079.1	0.36
108	290	2093.5	2095.6	2.01
109	293	2108.2	2110.5	2.30
110	297	2130.0	2131.0	1.01
111	300	2144.3	2145.7	1.42
112	303	2158.5	2161.7	3.26
113	307	2179.7	2181.8	2.08
114	310	2193.6	2197.6	4.02
115	314	2214.4	2216.0	1.53
116	317	2227.9	2231.5	3.59
117	321	2248.4	2250.9	2.53
118	324	2261.6	2266.3	4.69

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9. Nuclear binding energy of isotopes of Z

We are working on understanding and estimating the binding energy of mass numbers above and below the stable mass numbers.

With trial and error, we have developed a third term of the form $\left[\frac{(A_s-A)^2}{A_s}\right]\times 10.06\text{ MeV}$ Using this term, approximately, it is possible to fit the binding energy of isotopes in following way.

$$B_A \cong \left\{\left[A-\left(\frac{kAZ}{2.531}+3.531\right)\right]-\left[\frac{(A_s-A)^2}{A_s}\right]\right\}\times 10.06\text{ MeV} \quad (21)$$

See figure 2 and table 4 for the estimated isotopic binding energy of Z=50. Dashed black curve plotted with relations (7) and (21) can be compared with the green curve plotted with total binding energy of Thomas-Fermi model [26].

For Z=50 and A=100 to 130, with reference to total binding energy of Thomas-Fermi model [26], there is no much more difference in the estimation of binding energy. When $(A>130)$, binding energy seems to be increasing and when $(A>170)$, binding energy seems to be decreasing rapidly. It needs further study and refinement.

See figures 3 to 10 for the estimated isotopic binding energies of Z=22, 32, 42, 52, 62, 72, 92 and 92.

Figure 2: Binding energy of isotopes of Z=50

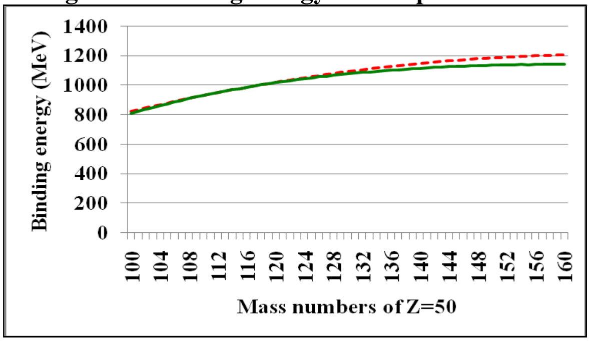

Table 4: Binding energy of isotopes of Z=50

Proton number	Mass number	Estd. BE (MeV)	Total BE (MeV) [26]	Error (MeV)
50	100	822.4	826.0	3.6
50	101	833.9	837.2	3.3
50	102	845.2	850.7	5.4
50	103	856.4	860.7	4.3
50	104	867.3	873.1	5.8
50	105	878.1	882.7	4.6
50	106	888.8	894.6	5.8
50	107	899.2	903.5	4.3
50	108	909.5	914.9	5.5
50	109	919.6	923.5	3.9
50	110	929.5	934.7	5.2
50	111	939.3	942.9	3.6
50	112	948.9	953.5	4.6
50	113	958.3	961.1	2.8
50	114	967.5	971.4	3.9
50	115	976.6	978.7	2.2
50	116	985.5	988.5	3.0
50	117	994.2	995.4	1.3
50	118	1002.7	1004.7	2.0
50	119	1011.1	1011.3	0.2
50	120	1019.3	1020.3	1.1
50	121	1027.3	1026.8	-0.5
50	122	1035.1	1035.5	0.4
50	123	1042.8	1041.5	-1.3
50	124	1050.3	1050.1	-0.2
50	125	1057.6	1055.8	-1.8
50	126	1064.8	1064.1	-0.7
50	127	1071.8	1069.6	-2.2
50	128	1078.6	1077.5	-1.0
50	129	1085.2	1082.8	-2.4
50	130	1091.7	1090.5	-1.2
50	131	1098.0	1095.61	-2.3
50	132	1104.1	1102.6	-1.5
50	133	1110.0	1105.2	-4.8
50	134	1115.8	1109.5	-6.2
50	135	1121.4	1111.4	-9.9
50	136	1126.8	1115.2	-11.6
50	137	1132.0	1116.9	-15.1
50	138	1137.1	1120.5	-16.6
50	139	1142.0	1121.9	-20.1
50	140	1146.7	1125.3	-21.4

See table 5 for the estimated and total binding energies of N = 2Z nuclides starting from Z=20 to 50.

Table 5: Binding energy of N = 2Z nuclides

Proton number	Mass number	Est. BE (Mev)	Exp. BE(Mev) [24,26]	Error (MeV)
20	40	344.6	342.1	-2.6
22	44	380.8	375.5	-5.3
24	48	415.3	411.5	-3.8
26	52	450.7	447.7	-3.0
28	56	484.2	484.0	-0.3
30	60	517.3	515.0	-2.3
32	64	551.6	546.0	-5.6
34	68	583.8	576.3	-7.5
36	72	615.5	606.9	-8.6
38	76	646.8	638.1	-8.7
40	80	677.6	668.4	-9.2
42	84	707.9	700.9	-7.0
44	88	737.8	731.4	-6.4
46	92	767.3	762.1	-5.2
48	96	793.9	793.4	-0.5
50	100	822.4	824.5	2.1

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Figure 3: Binding energy of isotopes of Z=22

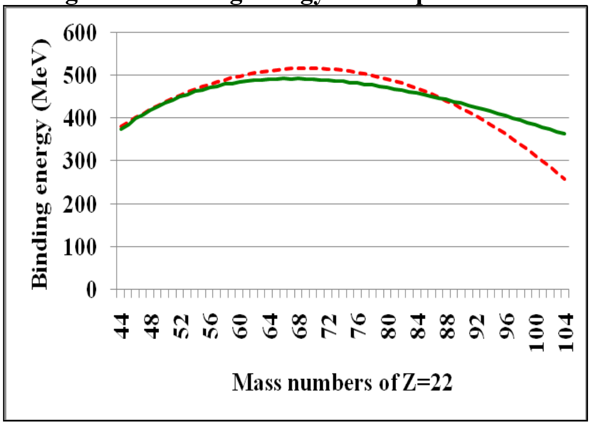


Figure 4: Binding energy of isotopes of Z=32

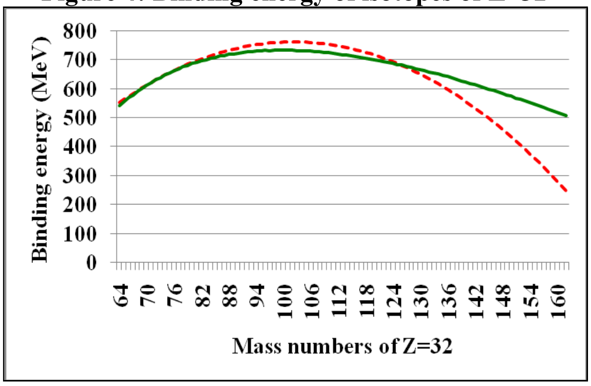


Figure 5: Binding energy of isotopes of Z=42

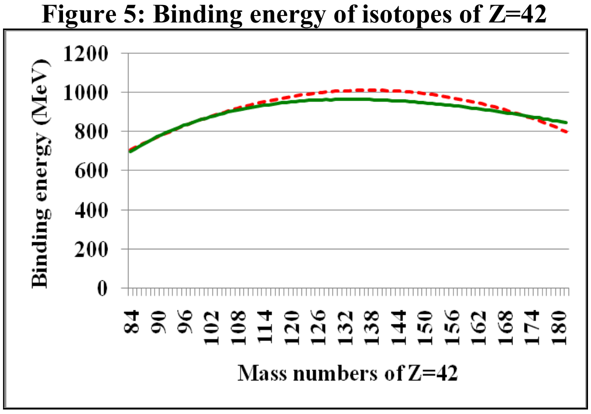


Figure 6: Binding energy of isotopes of Z=52

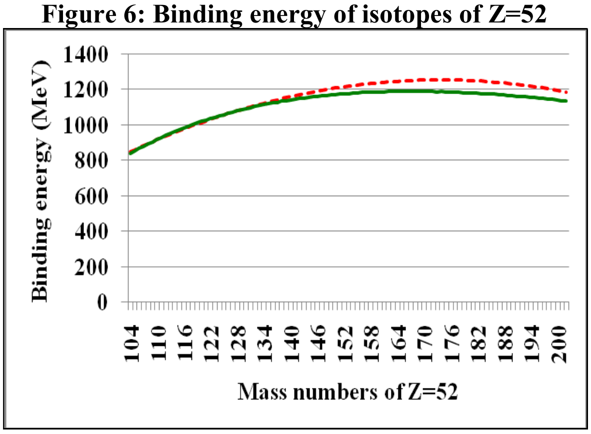


Figure7: Binding energy of isotopes of Z=62

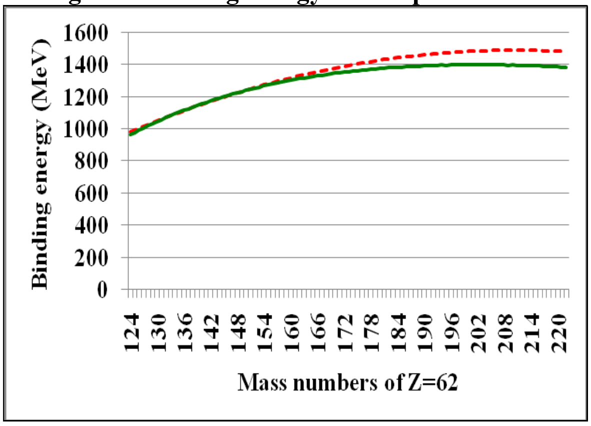


Figure 8: Binding energy of isotopes of Z=72

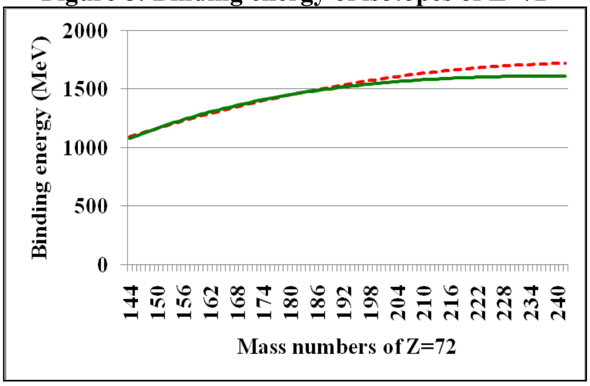


Figure 9: Binding energy of isotopes of Z=82

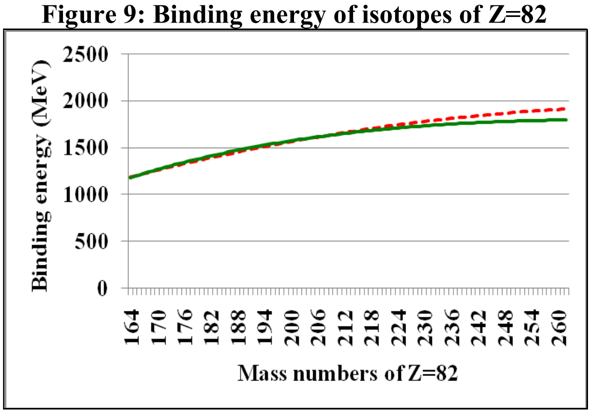
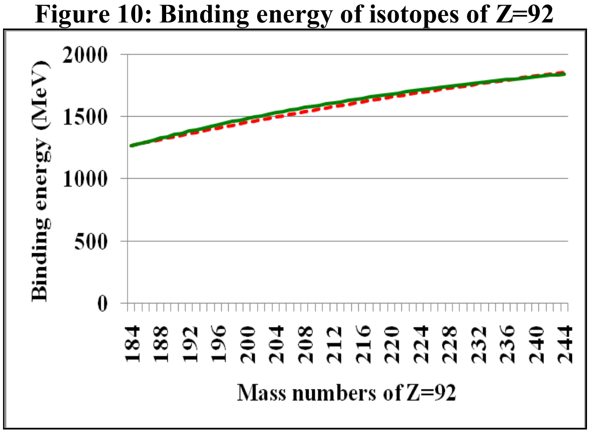


Figure 10: Binding energy of isotopes of Z=92



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10. Understanding the binding energy of light atomic nuclides

It is well established that, in light atomic nuclides, coulombic interaction seems to play a key role in reducing the binding energy. Based on this concept, starting from Z=2 to Z=30, close to stable mass numbers, binding energy can be expressed with the following relation.

$$\begin{aligned} B_{A_s} &\cong \left[A_s - A_s^{\frac{1}{3}}\right](10.06-0.71)\text{ MeV} \\ &\cong \left[A_s - A_s^{\frac{1}{3}}\right]9.35\text{ MeV} \end{aligned} \quad (22)$$

See the following table 6.

Table 6: Binding energy of Z = 2 to 30 based on coulombic correction

Proton number	Mass number	Est. BE (Mev)	SEMF BE (Mev) [22]	Error (MeV)
2	4	22.6	22.0	-0.5
3	6	39.1	26.9	-12.2
4	8	56.1	52.9	-3.2
5	10	73.4	62.3	-11.1
6	12	90.8	87.4	-3.4
7	14	108.4	98.8	-9.6
8	16	126.0	123.2	-2.8
9	19	152.7	148.9	-3.8
10	21	170.6	167.5	-3.0
11	23	188.5	186.1	-2.3
12	25	206.4	204.7	-1.7
13	27	224.4	223.2	-1.2
14	29	242.4	241.6	-0.8
15	31	260.5	260.0	-0.5
16	34	287.6	290.8	3.2
17	36	305.7	305.1	-0.7
18	38	323.9	327.2	3.4
19	40	342.0	341.5	-0.5
20	43	369.3	371.6	2.3
21	45	387.5	389.6	2.1
22	47	405.7	407.5	1.8
23	49	423.9	425.2	1.3
24	52	451.3	454.6	3.3
25	54	469.6	468.9	-0.7
26	56	487.8	489.6	1.8
27	59	515.3	515.2	0.0
28	61	533.5	532.5	-1.0
29	63	551.8	549.7	-2.2
30	66	579.3	577.9	-1.4

11. Understanding magic proton numbers

It may be noted that, the nuclear magic numbers, as we know in stable and naturally occurring nuclei, consist of two different series of numbers. The first series -2, 8, 20 is attributed to the harmonic-oscillator (HO) potential, while the second one - 28, 50, 82 and 126 is due to the spin–orbit (SO) coupling force [28-31]. In this context, our bold idea is that, atoms are exceptionally stable when their nuclear binding energy approaches,

$$B_{A_s} \cong \left[2.531\left(n+\frac{1}{2}\right)\right]^2 10.06\text{ MeV} \quad (23)$$

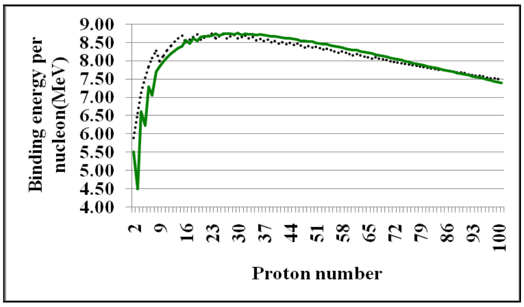
Based on point 5 of section-3, close to stable mass numbers of $Z \approx (2\text{ to }100)$, magnitude of nuclear binding energy can be expressed with a relation of following form.

$$\left.\begin{aligned} B_{A_s} &\approx \left\{\left(Z-\sqrt{\ln(Z)}\right)\frac{e_s^2}{4\pi\varepsilon_0\left(G_s m_p/c^2\right)}\right\} \pm 10.06\text{ MeV} \\ &\approx \left[\left(Z-\sqrt{\ln(Z)}\right) * 20.12\text{ MeV}\right] \pm 10.06\text{ MeV} \end{aligned}\right\} (24)$$

where $A_s \approx 2Z + 0.0063326(Z)^2$

See the following figure 11 for the plotted (dotted) black curve compared with SEMF green curve.

Figure 11: Nuclear Binding energy close to stable mass numbers of Z = 2 to 100



Let M_n be a possible magic proton number. Considering relations (23) and (24), it is possible to develop a relation of the following form having a factor $(1/2)$.

$$\left.\begin{aligned} M_n &\cong \left\{\frac{1}{2}\left[2.531\left(n+\frac{1}{2}\right)\right]^2 + 1\right\} + \Delta \\ &\cong \left[3.203\left(n+\frac{1}{2}\right)^2 + 1\right] + \Delta \end{aligned}\right\} \quad (25)$$

where, after rounding off,

$$\text{if, } \left\{\frac{1}{2}\left[2.531\left(n+\frac{1}{2}\right)\right]^2 + 1\right\} \text{ is Odd, } \Delta = \mp 1$$

$$\text{if, } \left\{\frac{1}{2}\left[2.531\left(n+\frac{1}{2}\right)\right]^2 + 1\right\} \text{ is Even, } \Delta = \mp 2$$

See the following table-7. It is possible to say that,

1) Magic proton numbers 2, (6), (14), 28, 50, 82, 114,.. etc [28-30] can be shown to be n^{th} levels.

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2) Magic proton numbers 2, 8, 20, 40,… can be shown to be $\left(n+\frac{1}{2}\right)$ levels.

Table-7 : To understand the magic proton numbers

$\left(n+\frac{1}{2}\right)$	Round off $\left[3.203\left(n+\frac{1}{2}\right)^2+1\right]$	M_n
0	1	1,2
0.5	2	2,4
1	4	2,4,6
1.5	8	6,8,10
2	14	12,14,16
2.5	21	20,21,22
3	30	28,30,32
3.5	40	38,40,42
4	52	50,52,54
4.5	66	64,66,68
5	81	80,81,82
5.5	98	96,98,100
6	116	114,116,118
6.5	136	134,136,138
7	158	156,158,160
7.5	181	180,181,182
8	206	204,206,208

12. Discussion

1) So far no model could succeed in understanding nuclear binding energy with gravity.
2) So far no model could address or succeed in implementing strong coupling constant in low energy nuclear physics.
3) So far no model could attempt to understand nuclear stability and binding energy with the combined effects of strong nuclear gravity and strong nuclear charge.
4) Understanding nuclear binding energy with a single energy coefficient of magnitude $\frac{e_s^2}{8\pi\varepsilon_0\left(G_s m_p/c^2\right)} \cong 10.09$ MeV is a challenging task and so far, except Ghahramany et al, no one could attempt to do that. It may also be noted that, in Ghahramany's model, energy constant is a variable [32] and in our model energy constant remains same for any nuclide.
5) Estimation of nucleon stability range is simple in our model compared to SEMF and Ghahramany's model. Interesting point to be noted is that, in our model, nucleon stability range or stable mass numbers can be estimated without considering the binding energy formula. We have provided different relations for understanding nucleon stability.
6) Proposed new and result oriented number $k \cong \left(\frac{4\pi\varepsilon_0\hbar^2 m_e c^2}{4e^2 G_s m_p^3}\right) \cong \frac{4\pi\varepsilon_0 G_s m_p m_e}{4e_s^2} \cong 0.0063326$ seems to play a key role in understanding nuclear stability and binding energy vide relations (6), (7), (8), (9), (10), (16) and (20).
7) Proposed first tem is not new and proposed second term $\left[\left(kA_s Z/2.531\right)+3.531\right]\times 10.06$ MeV seems to play an excellent role in fitting and understanding the binding energy of medium and heavy stable nuclides. It can be evidenced form table-3. Correction seems to be required for light atomic nuclides. It needs further study.
8) Proposed third term $\left[\left(A_s-A\right)^2/A_s\right]\times 10.06$ MeV seems to be approximate in fitting and understanding the binding energy of isotopes. We are working on it for its validity and better alternative with respect correct stable mass number of Z. For example, see the following table-8.

Table 8: Binding energy of isotopes of Z = 8, 10 and 20

Proton number	Mass number	Est. BE (Mev)	Total BE (Mev)	Error (MeV)
8	14	100.0	98.7352	-1.25
8	15	111.7	111.9576	0.23
8	16	122.2	127.6211	5.40
8	17	131.4	131.7646	0.32
8	18	139.4	139.8091	0.39
8	19	146.1	143.7665	-2.37
10	17	123.6	112.9107	-10.64
10	18	136.7	132.1432	-4.57
10	19	148.9	143.7827	-5.14
10	20	160.2	160.6521	0.49
10	21	170.5	167.4136	-3.04
10	22	179.8	177.7751	-2.01
10	23	188.2	182.9756	-5.18
10	24	195.6	191.841	-3.72
20	36	297.1	281.3644	-15.69
20	37	309.6	296.1548	-13.50
20	38	321.8	313.1263	-8.65
20	39	333.4	326.4138	-7.03
20	40	344.6	342.0563	-2.58
20	41	355.4	350.4187	-4.94
20	42	365.6	361.9002	-3.72
20	43	375.4	369.8327	-5.58
20	44	384.7	380.9652	-3.77
20	45	393.6	388.3797	-5.21
20	46	402.0	398.7791	-3.20
20	47	409.9	406.0556	-3.84
20	48	417.3	415.9961	-1.35
20	49	424.3	421.1426	-3.19
20	50	430.8	427.495	-3.35

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9) In deuteron, binding energy seems to be proportional to e^2 and in other atomic nuclides, binding energy seems to be proportional to e_s^2.
10) Considering the average of $\left(e^2, e_s^2\right)$ and without considering 0.71 MeV (as there exists only one proton), based on relation (22), binding energies of $1H2$ and $1H3$ nuclides can be estimated as, $\left[2-2^{\frac{1}{3}}\right]5.6 \cong 4.15$ MeV and $\left[3-3^{\frac{1}{3}}\right]5.6 \cong 8.72$ MeV respectively.
11) Considering the average of $\left(e^2, e_s^2\right)$ and considering 0.71 MeV (since there exists two protons), based on relation (22), binding energy of $2He3$ can be estimated as, $\left[3-3^{\frac{1}{3}}\right]4.9 \cong 7.63$ MeV.
12) Coulombic energy coefficient being 0.7 MeV, with reference to $\ln\left(\frac{e^2}{4\pi\varepsilon_0 G_s m_p m_e}\right) \cong 1.515$, volume or surface energy coefficient can be expressed as 1.515*10.09 = 15.3 MeV and asymmetric energy coefficient can be expressed as, 1.515*15.3 = 23.0 MeV. For $(Z \geq 10)$, binding energy can also be estimated with,

$$B_A \cong \left(A - A^{2/3} - 1\right) * 15.3\text{MeV} - \frac{Z^2}{A^{1/3}} * 0.7\text{MeV} - \frac{(A-2Z)^2}{A} * 23.0\text{MeV} \quad (26)$$

13) With advanced research in high energy nuclear physics, hadronic melting points can be understood and bare quarks can be made identifiable.
14) With further research in nuclear astrophysics, it is certainly possible to understand the combined effects of Newtonian gravitational constant and proposed nuclear gravitational constant. Considering the ratio of nuclear gravitational constant and Newtonian gravitational constant, estimated masses of white dwarfs, neutron stars and black holes [33,34], can be fitted approximately. For example,

$$\left.\begin{aligned} M_X &\approx \left(\frac{G_s}{G_N}\right)\sqrt{\frac{e^2}{4\pi\varepsilon_0 G_N}} \approx 0.473 M_\odot \\ M_X &\approx \left(\frac{G_s}{G_N}\right)\sqrt{\frac{e_s^2}{4\pi\varepsilon_0 G_N}} \approx 1.373 M_\odot \\ M_X &\approx \left(\frac{G_s}{G_N}\right)\sqrt{\frac{\hbar c}{G_N}} \approx 5.456 M_\odot \end{aligned}\right\} \quad (27)$$

$$\left.\begin{aligned} M_X &\approx \sqrt{\frac{G_s}{G_N}}\frac{e^2}{4\pi\varepsilon_0 G_N m_p} \approx 0.023 M_\odot \\ M_X &\approx \sqrt{\frac{G_s}{G_N}}\frac{e_s^2}{4\pi\varepsilon_0 G_N m_p} \approx 0.2 M_\odot \\ M_X &\approx \sqrt{\frac{G_s}{G_N}}\left(\frac{\hbar c}{G_N m_n}\right) \approx 3.174 M_\odot \end{aligned}\right\} \quad (28)$$

15) At the moment of a neutron star's birth, the nucleons that compose it have a temperature of around 10^{11} to 10^{12} K [35]. Considering M_X as a critical mass for neutron stars and black holes, corresponding critical temperature can be fitted with,

$$\left.\begin{aligned} T_X &\approx \frac{\hbar c^3}{8\pi k_B G_N \sqrt{M_X M_{pl}}} \\ \text{where, } M_{pl} &\cong \sqrt{\frac{\hbar c}{G_N}} \cong 2.176\times10^{-8}\text{kg} \end{aligned}\right\} \quad (29)$$

16) Quantitatively, Fermi's weak coupling constant [36] and electron rest mass can be fitted with the following relations.

$$G_F \cong \left(\frac{m_e}{m_p}\right)^2 \hbar c R_0^2 \cong \frac{4G_s^2 m_e^2 \hbar}{c^3} \cong 1.4402\times10^{-62}\ \text{J.m}^3 \quad (30)$$

$$m_e \cong \sqrt{\frac{G_F c^3}{4G_s^2\hbar}} \text{ and } \frac{2G_s m_e}{c^2} \cong \sqrt{\frac{G_F}{\hbar c}} \quad (31)$$

17) In a theoretical and verifiable approach, magnitude of the Newtonian gravitational constant can be estimated with nuclear elementary physical constants. For example, with reference to Planck scale, we noticed that [14],

$$\frac{\pi R_0^2}{\pi R_{pl}^2} \cong \frac{G_s^2 m_p^2}{G_N \hbar c} \cong \left(\frac{m_p}{m_e}\right)^{12} \quad (32)$$

$$\text{where, } R_0 \cong \frac{2G_s m_p}{c^2},\ R_{pl} \cong \frac{2G_N M_{pl}}{c^2} \cong 2\sqrt{\frac{G_N\hbar}{c^3}}$$

$$G_N \cong \left(\frac{m_e}{m_p}\right)^{10}\left(\frac{G_F c^2}{4\hbar^2}\right) \cong \left(\frac{m_e}{m_p}\right)^{12}\left(\frac{G_s m_p^2}{\hbar c}\right) G_s \cong (6.66 \text{ to } 6.68)\times10^{-11}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \quad (33)$$

18) Our proposed assumptions seem to ease the way of understanding and refining the basic concepts of final unification.

13. Conclusion

Semi empirical mass formula and Fermi gas model, both, are lagging in implementing the strong coupling constant and gravity in nuclear structure. In this context, understanding and estimating nuclear binding energy with 'strong interaction' and 'unification' concepts seem to be quite interesting and needs a serious consideration at basic level. In this context, relations (6), (7), (9), (10), (11), (20), (21), and (24) can be considered as favorable or supporting tools for our proposed model. With further research, mystery of magic numbers can be understood and a unified model of nuclear binding energy and stability scheme pertaining to high and low energy nuclear physics can be developed.

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