I. THE RIEMANN ZETA FUNCTION AND THE RIEMANN HYPOTHESIS

Prime numbers, the indivisible atoms of arithmetic, seem to be strewn haphazardly along the number line, starting with 2, 3, 5, 7, 11, 13, 17 and continuing without pattern ad infinitum. But in 1859, the great German mathematician Bernhard Riemann hypothesized that the spacing of the primes logically follows from other number-theoretical is formulated by Lagrange multipliers.

The Riemann zeta function takes inputs that can be complex numbers —meaning they have both “real” and “imaginary” components—and yields other numbers as outputs. For certain complex-valued inputs, the function returns an output of zero; these inputs are the “nontrivial zeros” of the Riemann zeta function. Riemann also derived a functional equation, containing the \( \zeta(s) \) function, which is valid for all complex \( s \) plane, except for \( s = 1 \). This analytic continuation of the function is called the Riemann-zeta function. Riemann also derived a functional equation, containing the \( \zeta(s) \) function, which is valid for all complex \( s \) plane, except for \( s = 1 \). This analytic continuation of the function is called the Riemann-zeta function.

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\[
\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots \tag{1}
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Via analytic continuation, one can show that: \( \zeta(-1) = -\frac{1}{12}, \zeta(0) = -\frac{1}{2}, \zeta(1) = \infty \), and \( \zeta(2) = \frac{\pi^2}{6} \).

Bernhard Riemann [1–3], who was the first to apply the tools of complex analysis to this function in Eq.(1), proved that the function defined by the infinite summation (Riemann, 1859) can be analytically continued over the complex plane, except for \( s = 1 \). This analytic continuation of the function is called the Riemann-zeta function.

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So far the statements about the zeros of \( \zeta(s) \) and their locations on the complex plain were simple. However the distribution of the non-trivial zeros holds one of the most intriguing and enigmatic mathematical mysteries of the last century and a half. It is embarrassingly easy to pose Riemann’s conjecture:

**The Riemann Conjecture**  All non-trivial zeros of \( \zeta(s) \) have the form \( s = 1/2 + iy \), where \( y \) is a real number.

In other words all non-trivial zeros lie on the critical line. In 1900 Hilbert nominated the Riemann Hypothesis as the eighth problem on his famous list of compelling problems in mathematics (Hilbert, 1902). Since then not just professional mathematicians but mathematical soldiers of fortune tried and still try, to verify its validity. The stakes are high. Whoever proves or disproves this
II. ASSEMBLING SPRINGS

In mechanics, two or more springs are said to be in series when they are connected end-to-end, so as to act as a single spring. More generally, two or more springs are in series when any external stress applied to the ensemble gets applied to each spring without change of magnitude, and the amount strain (deformation) of the ensemble is the sum of the strains of the individual springs [6].

![Springs in Series](image1)

Equivalent Spring Constant (Series): When putting two springs in their equilibrium positions in series attached at the end to a block and then displacing it from that equilibrium, each of the springs will experience corresponding displacements $d_1$ and $d_2$ for a total displacement of $d_1 + d_2$. For the spring that is equivalent to a system of two springs, in series, whose spring constants are $k_1k_2$. The total equivalent stiffness is given by $\frac{1}{K} = \frac{1}{k_1} + \frac{1}{k_2}$. If the spring system has infinite springs, the total equivalent stiffness is given by

$$\frac{1}{K} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_n} + \cdots = \sum_{n=1}^{\infty} \frac{1}{k_n}. \quad (2)$$

The compressed force has a relation $k_1d_1 = k_2d_2 = \cdots = k_nd_n = \cdots = \text{const.} = \alpha$.

The compressed forces for the in series case is $d = d_1 + d_2 + \cdots = \alpha \sum_{n=1}^{\infty} \frac{1}{k_n}$.

Energy stored for the n-th spring is $E_n = \frac{1}{2}k_n d_n^2$. For the series case, the ratio of energy stored in springs $E_1, E_2, \ldots$ is given by

$$\frac{E_1}{E_2} = \frac{k_1}{k_2} = \frac{d_1}{d_2}.$$

Total energy of the spring is given by

$$E = \sum_{n=1}^{\infty} E_n = \frac{1}{2} \sum_{n=1}^{\infty} k_n d_n^2 = \frac{1}{2} \alpha^2 \sum_{n=1}^{\infty} \frac{1}{k_n} = \frac{1}{2} \alpha^2 \frac{1}{K}. \quad (3)$$

III. THE RIEMANN SPRING IN SERIES

Comparing the two expression Eq.1 and Eq.2, it is easy to notice that they are similar. If the spring system has an infinite springs in series, whose n-th spring has elastic complex modulus $k_n = n^s$, therefore total equivalent modulus $K(s)$ is given by

$$\frac{1}{K(s)} = \sum_{n=1}^{\infty} \frac{1}{k_n} = \sum_{n=1}^{\infty} \frac{1}{n^s} = \zeta(s). \quad (4)$$

where the Riemann Zeta function is $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$. It is clear that the Riemann spring stiffness $K(s)$ is a complex which is different from the physical spring, whose stiffness is real number.

From the previous section, we have following relations for the Riemann spring in series:

The compressed force has a relation $1^sd_1 = 2^sd_2 = \cdots = n^sd_n = \cdots = \alpha$.

The Riemann spring elastic energy for the n-th spring is $E_n = \frac{1}{2} \frac{1}{n^s} d_n^2$.

Total energy of the Riemann spring is given by

$$E = \sum_{n=1}^{\infty} E_n = \frac{1}{2} \sum_{n=1}^{\infty} n^s d_n^2 = \frac{1}{2} \alpha^2 \sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{2} s^2 \zeta(s). \quad (5)$$

which reveals that the total energy of the Riemann spring is proportional to the Riemann Zeta function.
For complex number \( s = x + iy \), thus \( n^s = n^{x+iy} = n^x \exp[(i \ln n)y] = n^x \cos(y \ln n) + i \sin(y \ln n) \), we have \( \Re(n^s) = n^x \cos(y \ln n) \) and \( \Im(n^s) = n^x \sin(y \ln n) \).

The stiffness for the n-th spring is \( k_n = n^s \), therefore, \( \Re(k_n) = n^x \cos(y \ln n) \) and \( \Im(k_n) = n^x \sin(y \ln n) \).

The reciprocal of \( k_n \) is
\[
\frac{1}{k_n} = \frac{1}{n^s} = \frac{1}{n^{x+iy}} = n^{-x}n^{-iy} = n^{-x}\cos(y \ln n) - i \sin(y \ln n).
\]

Thus the total stiffness of the Riemann spring in series is
\[
\frac{1}{K} = \sum_{n=1}^{\infty} \frac{1}{k_n} = \Re\left(\frac{1}{K}\right) + i\Im\left(\frac{1}{K}\right). \tag{7}
\]

where \( \Re\left(\frac{1}{K}\right) = \sum_{n=1}^{\infty} n^{-x} \cos(y \ln n) \) and \( \Im\left(\frac{1}{K}\right) = -\sum_{n=1}^{\infty} n^{-x} \sin(y \ln n) \).

The total energy can be rewritten as
\[
E = \frac{1}{2} \alpha^2 \zeta(s) = \frac{\alpha^2}{2} \sum_{n=1}^{\infty} n^{-x} \cos(y \ln n) - \frac{\alpha^2}{2} \sum_{n=1}^{\infty} n^{-x} \sin(y \ln n) \tag{8}
\]

IV. THE RIEMANN HYPOTHESIS WITHIN THE FRAMEWORK OF CLASSIC SPRING MECHANICS

Using the Riemann spring model, the Riemann conjecture, namely, all non-trivial zeros of \( \zeta(1/2 + iy) \) can be expressed in terms of total stiffness as follows:
\[
\frac{1}{K(1/2 + iy)} = \zeta(1/2 + iy)|_{y=r_k} = 0, \tag{9}
\]
where \( r_k \) is the k-th point that the Riemann Zeta function vanishes. Physically speaking, this condition means that the total Riemann stiffness \( K(1/2 + iy) \) is infinite at points \( r_k \).

Considering the Eq.(9) as a constraint to the Riemann spring, the modified Lagrange \( L \) can be constructed as follows
\[
L = E + \lambda \zeta(1/2 + iy) = \frac{1}{2} \alpha^2 \zeta(1/2 + iy) + \lambda \zeta(1/2 + iy)|_{y=r_k}. \tag{10}
\]

where \( \lambda \) is the Lagrange multiplier, which can determined by the variational \( \delta L = 0 \), namely
\[
\delta L = \frac{1}{2} \alpha^2 \frac{dc}{ds} \zeta(1/2 + iy) + \lambda \frac{dc}{ds} \zeta(1/2 + iy)|_{y=r_k} i\delta y + \delta \lambda \zeta(1/2 + iy)|_{y=r_k} = 0. \tag{11}
\]

which will give following relations \( \zeta(1/2 + iy)|_{y=r_k} = 0 \) and \( \lambda = -\frac{1}{2} \alpha^2 \frac{dc}{ds} \zeta(1/2 + iy)|_{y=r_k} \). Substitute this to Eq.12 will lead a generalized variational as follows
\[
L = \frac{1}{2} \alpha^2 \zeta(1/2 + iy) + \frac{1}{2} \frac{dc}{ds} \zeta(1/2 + iy)|_{y=r_k} \zeta(1/2 + iy)|_{y=r_k}. \tag{12}
\]

V. RIEMANN SPRING-MASS SYSTEM FREQUENCY

If the Riemann spring in series has a central mass \( M \), the spring-mass system will be governed by
\[
M \frac{d^2x}{dt^2} + Kx = f(t). \tag{13}
\]

We can get its frequency \( \omega = \sqrt{\frac{K}{M}} = \frac{1}{\sqrt{2\pi}} \), which is a complex number.

VI. THE RIEMANN SPRING IN PARALLEL

For the Riemann spring in parallel, the equivalent stiffness is given by \( K(s) = n^{-s} \), and displacements \( d_1 = d_2 = \cdots = d_n = \cdots \), forces \( F = F_1 + F_2 + \cdots \), energy relations \( F_1 = k_1, k_2 = \cdots = k_n, \cdots \).

VII. CONCLUSIONS

The Riemann spring proposed here is just analogue to the classic spring in series. All relevant analogue quantities are complex instead of real one. This pure mathematical analogy does not take into account all quantities physical dimensions.


[2] Schumayer, D., and Hutchinson, D.A. W., Physics of the
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