

1 Article

# 2 Artificial neural network trained to predict 3 high-harmonic flux

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11 **Featured Application:** We present a method based on artificial neural networks – which is an  
12 efficient prediction procedure – with the main role to provide quick help for attosecond science  
13 laboratories, aimed to predict the expected outcome of high-order harmonic generation  
14 experiments to be performed with previously unexplored experimental input parameter  
15 combinations.

16 **Abstract:** In this work we present the results obtained with an artificial neural network (ANN)  
17 which we trained to predict the expected output of high-order harmonic generation (HHG)  
18 process, while exploring a multi-dimensional parameter space. We argue on the utility and  
19 efficiency of the ANN model and demonstrate its ability to predict the outcome of HHG  
20 simulations. In this case study we present the results for a loose focusing HHG beamline, where the  
21 changing parameters are: the laser pulse energy, gas pressure, gas cell position relative to focus and  
22 gas cell length. The physical quantity which we predict here using ANN is directly related to the  
23 total harmonic yield in a specified spectral domain (20-40 eV). We discuss the versatility and  
24 adaptability of the presented method.

25 **Keywords:** high-order harmonic generation, 3D non-adiabatic model, simulation, artificial neural  
26 network, prediction  
27

## 28 1. Introduction and motivation

29 High-order harmonic generation (HHG) is a well-established experimental technique for  
30 obtaining coherent radiation in the XUV to soft X-ray spectral domain on the attosecond timescale  
31 [1,2]. The process of HHG is highly nonlinear and has very low efficiency [3]. Despite the fact that  
32 high-harmonic radiation is the result of the highly nonlinear interaction between the atom and the  
33 driving laser pulse, this method is reliable and provides coherent secondary sources in the UV–  
34 XUV–soft-X-ray spectral domain with a table-top setup. The ELI-ALPS pan-European research  
35 infrastructure is also based on HHG sources [4], demonstrating that the increasing research efforts  
36 dedicated to this field are fully justified. Given the advantages of exploring HHG in the emerging  
37 field of Attosecond Science and Technology, considerable research effort is being dedicated to find  
38 ways for increasing high-harmonic efficiency by implementing loose focusing configurations [4,5],  
39 inventing different quasi-phase-matching techniques [6–13], multi-color and/or multi-jet  
40 configurations [11,14–16], but also to extend the available photon energy to reach the water-window  
41 [16–18] region.

42 Numerical approaches are unavoidable in this research for several reasons: (i) often, new  
43 experiments are designed based on predictions resulting from simulations; (ii) interpretation of  
44 existing experimental results cannot be done without additional results from models; (iii) when

45 looking for the best possible outcome within given experimental possibilities, reliable simulation  
46 tools can considerably contribute in finding the best solution.

47 Increasing efficiency and/or finding the optimum photon flux obtainable within a restricted  
48 parameter space is crucial. Experimental scan along many tunable parameters is very time- and  
49 resource consuming, additionally the experimental uncertainties result in variations from one  
50 measurement to the next. Simulation with a complete 3D model seems a more convenient route.  
51 However, as we will show in the following section, scanning along multiple parameters, typically  
52 4-5, implies the execution and interpretation of hundreds to thousands of simulation runs. Since the  
53 amount of raw data and computational effort is very difficult to be managed, we propose an  
54 additional method to extract the needed information from the data pool, and to predict possible  
55 outcomes for parameter combinations which were not included in the original simulation sets.

56 In this respect, artificial neural networks (ANN) are known to be high speed mathematical  
57 models that are capable of solving linear and non-linear multivariate regression problems [19,20],  
58 being generally used for their excellent prediction capabilities [21,22] but also for their capacity to  
59 solve problems such as classification [23,24], pattern recognition [25] or system control [26,27].  
60 Inspired by the human brain, these numerical modelling techniques are able to gain high efficiency  
61 and accuracy in the presence of uncertainties due to their ability to learn from experience [28]. Based  
62 on these properties and also on their effectiveness in processing large amount of data, the ANNs  
63 have applicability in various fields such as medicine [29], engineering [30,31], science technology  
64 [32], nanotechnology [33,34] or physics [35].

65 In this particular paper the physical quantity we want to predict is the total harmonic flux in a  
66 predefined spectral region of the high-harmonic spectrum, which we define as yield:

$$67 \quad Y = \int_{\omega_1}^{\omega_2} \int_0^R [E_h(\omega, r)]^2 r dr d\omega, \quad (1)$$

68 where  $[\omega_1; \omega_2]$  is the spectral domain of interest,  $R$  is the radial extent of the interaction region,  $E_h(\omega, r)$   
69 is the harmonic field in spectral representation.

70 Alternatively, one can identify different physical quantities which we want to predict using  
71 ANN, for example the existence and bandwidth / duration of isolated attosecond pulses. One can  
72 search to maximize the brightness of a single harmonic as function of different experimental  
73 parameters, as it has already been done experimentally [36].

74 We propose here a combined method to predict the photon flux in the 20-40 eV spectral range  
75 while varying four experimental parameters: laser pulse energy, gas pressure, gas cell position and  
76 gas cell length. For this prediction an artificial neural network is designed, trained and tested. The  
77 input data are the results obtained with the 3D non-adiabatic model (see next section) from a  
78 4-dimensional parameter scan [37].

79 The paper has the following structure: in section 2.1 we briefly describe the 3D non-adiabatic  
80 model for laser pulse propagation and high-harmonic generation, while in section 2.2 the artificial  
81 neural network method is presented. In section 3 we present and discuss the results obtained with  
82 the ANN, we test its robustness and reliability. Possible applications are suggested as well, since the  
83 major role of this prediction procedure is to provide a quick help for HHG laboratories: predict the  
84 expected outcome of HHG experiments to be performed with previously unexplored experimental  
85 parameter combinations.

## 86 **2. Theoretical models and numerical methods for high-harmonic generation and artificial neural** 87 **networks**

### 88 *2.1. High-harmonic generation*

89 In order to have sufficient input data for the training of the ANN, extensive simulation of the  
90 complete HHG process has been performed using a 3D non-adiabatic model [38] while the  
91 theoretical basis implemented in the simulation code has been described in [39]. According to the  
92 physical reality, this model has three main parts: (A) solves the propagation equation of the  
93 fundamental laser pulse in the generating medium, which is a noble gas in our case; (B) quantify the

94 response of the individual atoms to the external laser electric field; and (C) solves the propagation  
95 equation of the dipole radiation generated in step (B) within the same medium.

96 (A) The propagation equation for the fundamental laser pulse is the Maxwell wave equation.  
97 This is solved with a self-consistent iterative method in frequency domain and using the paraxial  
98 approximation. The method is implemented in a way that the full electric field (carrier and  
99 envelope) is calculated, we do not use the slowly varying envelope approximation. The interaction  
100 configuration is considered to have cylindrical symmetry being  $z$  the propagation direction, and  $r$   
101 the radial coordinate. The pulse experiences an effective refractive index – function of time and  
102 spatial coordinates – which accounts for the linear refraction and absorption, the  
103 intensity-dependent optical Kerr effect and the dispersion due to the free electrons. In this way the  
104 modifications of the generating pulse during propagation are included: self-phase modulation, pulse  
105 lengthening, intensity clamping, possible self-guiding. The propagation part of the simulation code  
106 is partially parallelized. For detailed mathematical formulation we refer to [38].

107 (B) The elemental interaction of a laser pulse with an atom is treated within the strong-field  
108 approximation and the generated dipole is calculated by solving the Lewenstein integral [40]. This  
109 part of the simulation code has been also parallelized, but it is still the most demanding part of the  
110 calculation in terms of CPU time and memory use.

111 (C) The total harmonic field results from the propagation and coherent addition of the atomic  
112 polarizations calculated in step (B). The wave equation describing the propagation of the harmonic  
113 field is similar to the wave equation for the fundamental, but the source term here is the non-linear  
114 polarization calculated in step (B).

115 This simulation code has several advantages which make it a very useful tool in the design and  
116 interpretation of HHG experiments. When we construct one specific case all the needed input data  
117 are experimentally measurable (or can be estimated). These are information about the fundamental  
118 laser pulse like beam type, pulse energy, duration, CEP, chirp; geometrical arrangement like  
119 aperture, focusing, cell position and length, far field detection configuration; interaction medium  
120 like gas type and pressure profile.

121 One typical full calculation yields a complex set of quantities in the grid points built over the  
122 interaction domain such as: laser pulse shape before and after propagation in time and frequency  
123 representation, peak intensity and ionization fraction, harmonic spectrum in near- and far-field  
124 (spectral domain), harmonic bursts in temporal domain, spatial build-up of selected harmonic  
125 orders. The most important outcome is the harmonic power spectrum which can be directly  
126 compared to the experimentally measured spectra. In addition, the simulation results provide a  
127 detailed insight into the physics of the whole process (laser and harmonic field propagation), at  
128 sub-optical-cycle temporal resolution, while phase-matching conditions can be examined as well.

129 The model is flexible in the sense that it can be modified according to diverse experimental  
130 conditions. It has been extended to be able to treat multiple electric fields (two-color [41–47] and  
131 three- or four-color field coming from a light-field synthesizer [48]) or multiple jets configurations.  
132 At very high laser intensities the atoms become highly charged ions, therefore we implemented also  
133 the multiple ionization scenario [49, 50]. Propagation in a fiber is essentially different from free space  
134 propagation, therefore this scenario has also been recently introduced. In attosecond laboratories  
135 very diverse experimental configurations are tried, for example Airy beams obtained with drilled  
136 mirror – also implemented in one version of the code.

137 The theoretical model implemented in the complete 3D simulation code is suitable for exploring  
138 a multi-dimensional parameter space and searching for optimum based on a given criterion. We can  
139 search for the maximum harmonic yield in a given spectral domain, or can look for the existence of a  
140 single attosecond pulse, or maximum cutoff energy within given parameter ranges. Although, such  
141 a scan along multiple parameters is possible to be performed also experimentally, this process  
142 would require much more resources than usually affordable, therefore it is recommended to  
143 perform a scan with “in silica” experiments.

144 Depending on the complexity of the possible experimental situation to be simulated, one whole  
145 calculation typically takes between few hours to 1-2 days. This estimation is done based on the

146 typical cases implemented so far on a cluster IBM System iDataPlex M4 dx360 (in Data Center of  
147 INCDTIM Cluj-Napoca).

148 When we need to explore an unknown parameter space, and we have to scan along several  
149 experimental parameters, the number of cases to be run is on the order of hundreds to thousands.  
150 Even though, one can afford only sparse sampling along each parameter. Therefore, if we want to  
151 search for an optimum output, for example harmonic yield, this sparse sampling is far not enough  
152 and cannot guarantee that an optimum configuration can be found.

## 153 2.2. Artificial neural networks

154 In an ANN, neurons are the ones responsible for processing the information. Each computing  
155 unit receives a weighted signal from all the previous neighboring neurons, sums the received signals  
156 and its own bias, processes the information via a transfer function and generates the output [22]. The  
157 neurons are organized in input, output and one or several hidden layers. As general feature, for  
158 prediction purposes, the connections between the neurons that belong to the same layer are not  
159 allowed, the information being directed from the input towards the output layer [51]. This kind of  
160 ANN is named feed-forward neural network and, in most cases, it is associated with the  
161 back-propagation learning algorithm which modifies the weights and biases starting from the  
162 output towards the input layer in order to minimize the network error [21].

163 Regarding the HHG process, it is known that the harmonic yield is non-linearly influenced by  
164 several parameters such as the fundamental laser pulse characteristics, focusing geometry, gas type  
165 and pressure which characterize the medium where HHG takes place. For this particular study we  
166 chose as varying parameters the laser pulse energy ( $E$ ), gas pressure ( $p$ ), gas cell position ( $z_i$ ) and gas  
167 cell length ( $L_{med}$ ). Consequently, these four process parameters are chosen to be the ANN inputs,  
168 whereas the harmonic yield is the model output. Based on the 3D non-adiabatic model, a total  
169 number of 453 input-output data sets were obtained and, subsequently, were divided in: i) training  
170 data which represent 77.9% of the total amount of input-output data sets, ii) validation data (15.7%)  
171 and iii) testing data (6.4%). In order to improve the accuracy of the model, all input data are mapped  
172 to the  $[-1,1]$  range.

173 As a result of the complexity of the HHG process, the multilayer feed-forward network is  
174 structured in one input layer, two hidden layers and one output layer, the appropriate number of  
175 neurons in the hidden layers being found by systematically testing the ANN performance. The  
176 information from the hidden and output layers is processed with two different transfer functions:  
177 the hidden layer uses a tansigmoid function, whereas a linear transfer function is applied for the  
178 output layer. Furthermore, the network biases and weights are computed using the backpropagation  
179 algorithm, and the ANN is trained with the Levenberg-Marquardt algorithm.

180 Several networks are trained and the best is chosen such that the ANN performance is  
181 maximized. The performance of the model is statistically evaluated using the Pearson correlation  
182 factor ( $R$ ) and the relative errors ( $RE$ ). Consequently, the best ANN has the highest correlation factor  
183 and the lowest relative errors.

$$184 R = \frac{\sum_{i=1}^n (X_{S_i} - \overline{X_S})(X_{ANN_i} - \overline{X_{ANN}})}{\sqrt{\sum_{i=1}^n (X_{S_i} - \overline{X_S})^2} \sqrt{\sum_{i=1}^n (X_{ANN_i} - \overline{X_{ANN}})^2}} \quad (2)$$

185 where  $X_{S_i}$  is the simulated output value,  $X_{ANN_i}$  is the model predicted output value,  $\overline{X_S}$  and  $\overline{X_{ANN}}$  are  
the arithmetic means of the experimental and respectively, of the predicted output values.

$$186 RE = 100 \frac{X_{ANN_i} - X_{S_i}}{X_{S_i}} \quad (3)$$

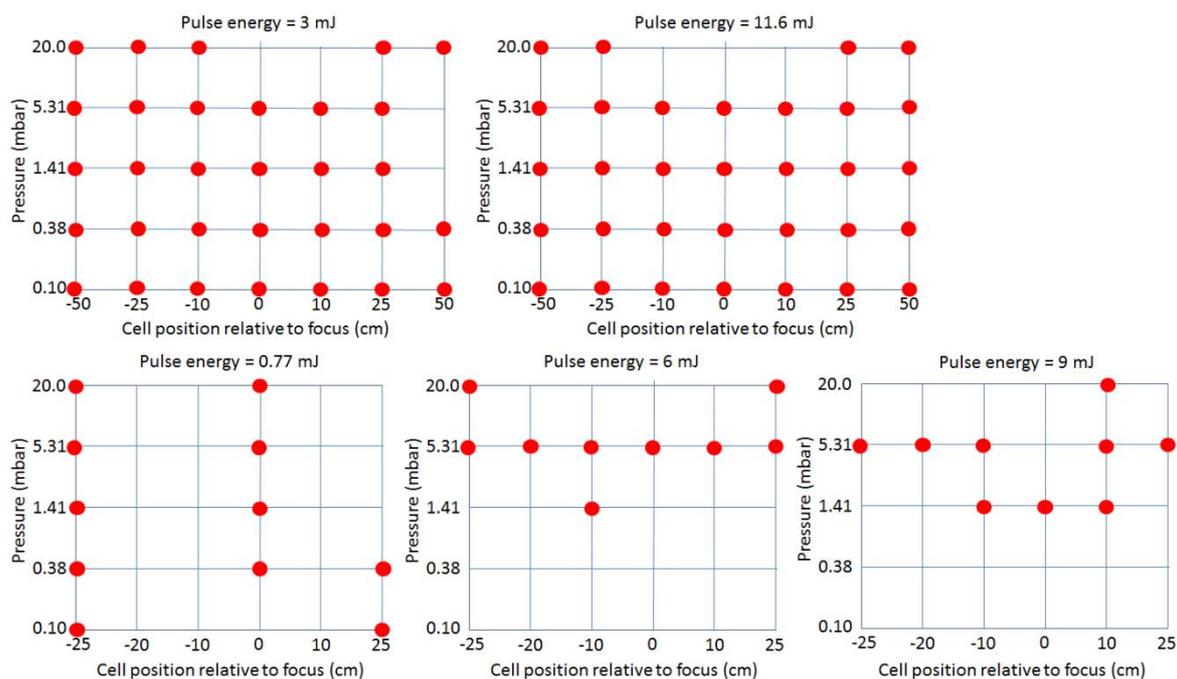
## 187 3. Results and discussion

188 The main goal of this research is to explore a possible loose focusing HHG configuration and  
189 find a combination of input experimental parameters which enable the generation of XUV  
attosecond pulses at high flux. We have built up one scenario which is experimentally feasible in

190 currently operating large attosecond science facilities, for example the ELI-ALPS. All input  
 191 parameters are experimentally measurable, from which we fix all but 4 parameters, these are the  
 192 scanned parameters.

193 Fixed input data are: pulse duration 10 fs, central wavelength 800 nm, focal length 21 m, radius  
 194 ( $1/e^2$  intensity) of the beam at the focusing element is 30 mm, total length of the interaction region is  
 195 20 cm, gas medium is argon. The scanned parameters with the values chosen for the full 3D  
 196 calculations are the following: input laser pulse energy  $E$  (0.77; 3; 6; 9; 11.6) mJ; argon pressure  $p$  (0.1;  
 197 0.38; 1.41; 5.31; 20) mbar; cell entrance position relative to the nominal focus  $z_i$  (-50; -25; -20; -10; 0; 10;  
 198 25; 50) cm, medium length  $L_{med}$  (4; 8; 12; 16; 20) cm. Although the parameters are very sparsely  
 199 spaced, scanning along all these parameters and performing complete 3D simulations, while also to  
 200 individually analyze and interpret the results means an enormous data pool and working time and  
 201 effort, which would be very inefficient. In Figure 1 we represent in a very schematic manner the  
 202 cases for which the complete 3D simulations have been performed, checked, validated and analyzed.

203 The unprecedented power of the ANN comes in exactly at this stage of the research. (1) We set  
 204 the ranges of all the moving parameters; (2) we perform – in a carefully chosen way – a sufficient  
 205 number of complete simulations, such as to have representative points from every part of the  
 206 multi-dimensional parameter space; (3) use these simulation results as inputs for the ANN; (4) train,  
 207 test and validate the ANN as it was presented in section 2.2. The expected results are the estimated  
 208 high-harmonic yields in the whole multi-dimensional parameter space with arbitrarily fine  
 209 resolution along each parameter. We emphasize that the ANN method in the present stage of  
 210 implementation is not yet a parameter optimization method, but an efficient prediction method.  
 211 However, the trained ANN is also suitable to search for optimum, and the maximum yield within  
 212 this 4-dimensional parameter space can be found by the user.



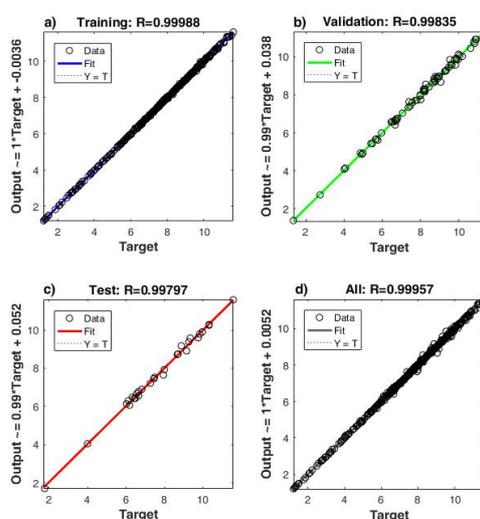
213  
 214 **Figure 1.** Schematic representation of the parameter space where the full 3D calculations have been  
 215 performed. For each pulse energy full 3D calculations were performed for several cell positions  
 216 relative to focus (x axis) and gas pressures (y axis). Red bullets represent the particular cases solved  
 217 by full 3D simulations and used by ANN for training, validation and testing. The fourth scanned  
 218 parameter is the medium length which is included in every simulation with values 4, 8, 12, 16, 20 cm.

219 The results obtained with the complete 3D code are discussed in detail elsewhere [37]. There the  
 220 main result was that the highest harmonic yield around 40 eV can be obtained with the parameter  
 221 combination:  $E = 11.6$  mJ;  $p = 5.31$  mbar;  $z_i = -25$  cm;  $L_{med} = 16$  cm. In this study we also test this result,

222 and present other parameter combinations which guarantee high yield comparable to the one found  
 223 with the multi-parameter scan. We also propose experimentally feasible configurations for existing  
 224 attosecond science laboratories.

### 225 3.1. Testing the ANN against the full 3D simulation results

226 The optimum topology of the ANN is found to have 11 neurons in the first hidden layer and 17  
 227 neurons in the second hidden layer. For this best trained ANN the Pearson coefficient and mean  
 228 relative errors are calculated for the training and validation data sets and its prediction capabilities  
 229 are tested considering a set of data that were not yet seen by the model (i.e. the testing data set). The  
 230 excellent correlation between the experimental results (from 3D simulations, i.e. in silica  
 231 experiments) and the predicted values is demonstrated by the regression plots and the high Pearson  
 232 coefficients (Figure 2).  
 233



234  
 235 **Figure 2.** Plot of ANN calculated  $\log_{10}$ (harmonic yield) against the values obtained with the complete  
 236 3D model for the following data sets: (a) training, (b) validation, (c) testing and (d) overall data.

237 From Figure 2 one can observe that the correlation factor has values greater than 0.99 for all  
 238 data sets: 0.9998 for the training data set, 0.9983 for the validation data set, 0.9979 for the testing data  
 239 and 0.9996 for the overall data set. As a consequence, the best linear fit of the points and the dotted  
 240 line that represents the perfect correlation ( $R=1$ ) overlap, indicating an extremely reduced deviance  
 241 of the ANN predictions from the 3D model data. The high performance of the ANN is also  
 242 demonstrated by the low relative errors which are calculated for the three data sets and are  
 243 presented in Table 1.  
 244

**Table 1.** The mean and maximum values of the relative error

	Mean Relative Error  [%]	Maximum Relative Error  [%]
<b>Training Data Set</b>	0.33	1.80
<b>Validation Data Set</b>	1.04	4.48
<b>Testing Data Set</b>	1.41	3.94

245  
 246 The maximum value of the relative error is 4.48% and is obtained for the validation data set.  
 247 The testing data set has a similar maximum relative error of 3.94 %, whereas the training data set has  
 248 a value of 1.8%. In case of the mean relative errors, the testing data set has the highest relative errors  
 249 among the three data sets, with a fairly reduced value of 1.41%. This is expected, as the input testing  
 250 set of data was processed by the ANN only after the training-validation step, in order to prove its  
 251 prediction capacity for the newly met testing data.

252 As a result of the excellent ANN prediction capabilities (extremely low relative errors and high  
 253 correlation factors), the model was further used to predict the harmonic yield for various input

254 variables. In further applications it is very important to have sufficient flux in a given spectral  
 255 domain, therefore increasing the harmonic yield is of great interest.

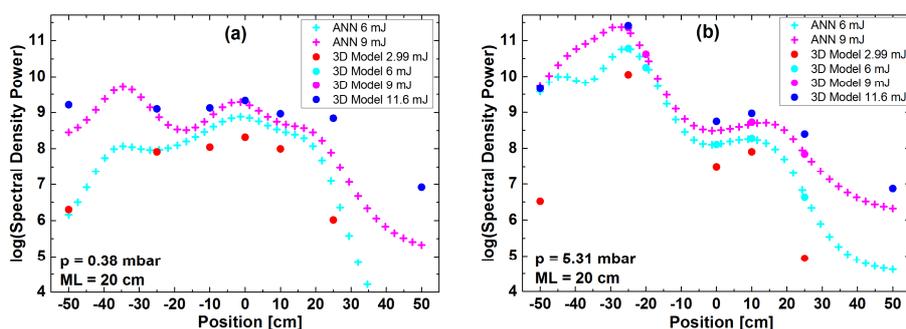
256 We here show that the correctly trained ANN predicts the expected harmonic yield of a possible  
 257 HHG experiment with the parameters changing in the given range. The ANN works best if the new  
 258 parameters take values in the range of those used for training and validation. At first glance it may  
 259 look as a typical interpolation in the multivariable space, however if we consider the complexity of  
 260 the HHG process given by the highly nonlinear nature of the laser-atom interaction, entangled in the  
 261 macroscopic effects like propagation, phase matching, etc., then we can understand that the  
 262 potential of a good prediction method goes far beyond the capabilities of an interpolation.

### 263 3.2. Prediction potential of the ANN

264 The quantity we propose to predict as the main outcome of a virtual HHG experiment is the  
 265 logarithm of the harmonic yield (Eq. (1)) in the specified spectral domain, namely in the 20-40 eV  
 266 region (between harmonic orders H13 – H27). We have chosen this quantity because it can be  
 267 directly related to the total photon flux in the specified spectral domain which is of central interest in  
 268 any attosecond science experiment.

269 One very important parameter in a HHG experiment is the optical density of the generation  
 270 medium. In the usual cases the length of the interaction medium is constant, thus the optical density  
 271 is proportional to the gas pressure. However, since we have as scanning parameter the medium  
 272 length as well, we here use  $p \times L_{med}$  as parameter, a quantity which is proportional to the optical  
 273 density. In the absorption limited HHG regime the harmonic yield (i.e. the total photon number)  
 274 should grow with the square of the optical density [52]. In order to have better insight into the  
 275 physics of how the harmonic yield evolves with the optical density of the medium, we show the  
 276 results as function of the  $\log_{10}(p[\text{mbar}] \times L_{med}[\text{cm}])$ .

277 In Figure 3 we show the prediction capability of the ANN. Dots represent full 3D calculations,  
 278 while the “+” symbols are the ANN predicted yields for two intermediate pulse energies, namely 6  
 279 mJ and 9 mJ. In Figure 3 we represent the expected evolution of the harmonic yield as a function of  
 280 the gas cell position relative to focus for different laser pulse energy values, while the gas pressure  
 281 and medium length are fixed: (a)  $p = 0.38$  mbar and  $L_{med} = 20$  cm; (b)  $p = 5.31$  mbar and  $L_{med} = 20$  cm.  
 282 Comparing the data in (a) and (b), we expect that the ratio of the yields for the same pulse energy  
 283 should be close to the ratio of the square of the pressures (around 200) which is clearly not the case.  
 284 This is because the cell position is a critical factor in every HHG process, influencing the yield via  
 285 propagation of the fields, phase-matching effects and absorption of the harmonics.



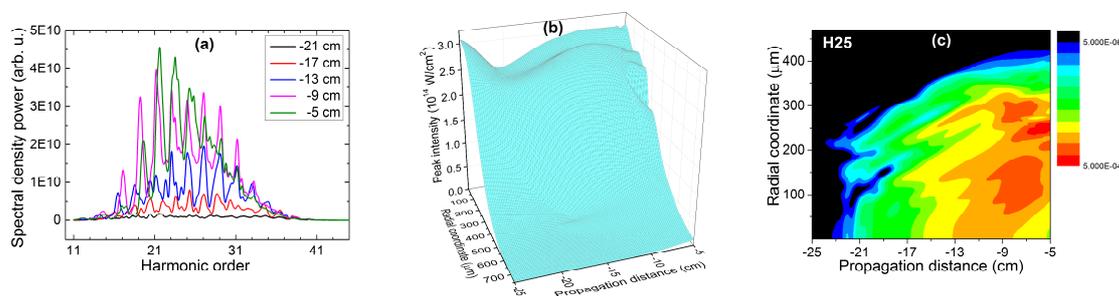
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287 **Figure 3.** Harmonic yield as function of the gas cell position relative to the nominal focus. Dots are  
 288 data obtained from the complete 3D simulations, “+” symbols are the results of the ANN predictions.  
 289 Panel (a) synthesizes the results obtained for different laser pulse energies at the fixed parameter  
 290 values  $p = 0.38$  mbar and  $L_{med} = 20$  cm, i.e. at the cell end. Panel (b) shows the results obtained with  $p =$   
 291  $5.31$  mbar and  $L_{med} = 20$  cm. Colors of the symbols dots and “+” are the same for the same energy. At  
 292 the intermediate energies, 6 and 9 mJ, we have performed merely a few but computationally  
 293 demanding 3D simulations and only with  $p = 5.31$  mbar.

294 The prediction potential of the ANN simulation proves to be excellent, and captures the highly  
 295 nonlinear dependence of the macroscopic harmonic yield on many parameters. Especially important  
 296 conclusion can be drawn from the position-dependence of the yield, namely that very high input  
 297 pulse energy is not in all cases necessary in order to obtain high harmonic efficiency. Harmonic yield  
 298 predicted with  $E = 9$  mJ pulse is almost as high as those obtained from 3D simulations with  $E = 11.6$   
 299 mJ pulse energy. This result can be good news for attosecond science infrastructures where the other  
 300 parameters – including cell position and length – are tunable, but the available laser power is  
 301 limited.

302 Particularly, the ANN simulation was able to reproduce the transition between two different  
 303 regimes of laser pulse propagation and consequently different conditions for HHG. At low pulse  
 304 energy (i.e. starting intensity) and also low pressure conditions we find two comparable maxima of  
 305 conversion efficiency – see data in Figure 3(a) with the cell positions before and after the focus. In  
 306 these mild macroscopic conditions the fundamental pulse does not suffer serious distortions during  
 307 propagation, therefore we find a confirmation of the classical phase-matching theory [53,54]: in the  
 308 converging beam harmonics find favorable phase-matching conditions off-axis; while in the  
 309 diverging beam collinear phase-matching on-axis helps the constructive build-up of the generated  
 310 harmonic field. The situation is different at higher pressure as seen in Figure 3(b). The maximum  
 311 conversion efficiency found in the whole explored multi-dimensional parameter space has been  
 312 found at  $E = 11.6$  mJ,  $p = 5.31$  mbar,  $z_i = -25$  cm,  $L_{med} = 16$  cm. The maximum harmonic yield in Figure  
 313 3(b) ( $p = 5.31$  mbar) is two orders of magnitude higher than the maximum yield in Figure 3(a) ( $p =$   
 314  $0.38$  mbar). The physical reason for this behavior is that the conditions for the formation of a  
 315 self-guided beam propagation are fulfilled. A stable “working intensity” is maintained in a large  
 316 interaction volume (both axially and radially) which creates good phase-matching conditions for a  
 317 range of harmonics [55].

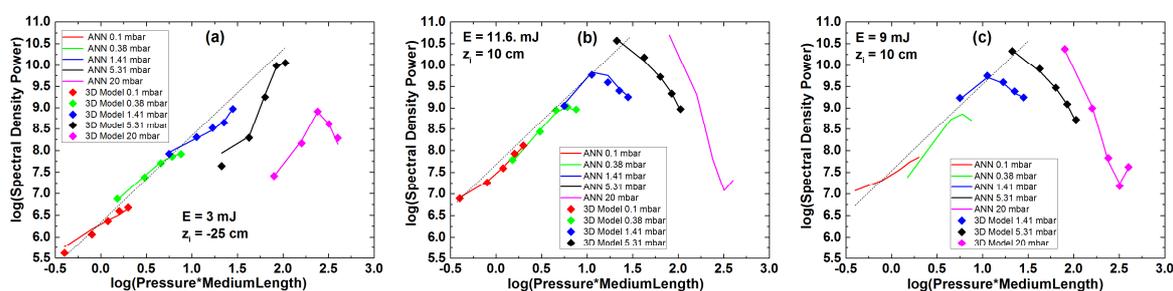
318 The ANN has no knowledge about the physics of HHG, however it has been trained with data  
 319 that contain synthetically the whole nonlinear process. It is highly recommended to perform the  
 320 complete 3D simulation in cases when ANN predicts less intuitive output. In Figure 4 we present the  
 321 main results obtained with the full 3D model for the parameter combination (9 mJ; 5.31 mbar; -25 cm;  
 322 20 cm). The power spectrum in Figure 4(a) confirms that the highest yield is expected towards the  
 323 end of the interaction region. In Figure 4(b) we show the spatial ( $r, z$ ) evolution of the laser pulse’s  
 324 peak intensity, which confirms the formation of a self-guided propagation regime with stable peak  
 325 intensity  $2.5\text{--}3 \cdot 10^{14}$  W/cm<sup>2</sup>. Figure 4(c) shows the spatial ( $r, z$ ) evolution of the harmonic order H25. In  
 326 accordance with the power spectrum, it is confirmed that H25 builds up constructively and attains  
 327 maximum yield towards the end of the interaction region, predominantly off-axis. In correlation  
 328 with the evolution of the fundamental pulse (Figure 4(b)) the spatial region of maximum H25  
 329 intensity is the same where the off-axis refocalization of the fundamental beam happens.



330  
 331 **Figure 4.** Results of the complete 3D simulation performed with the following values of the  
 332 parameters:  $E = 9$  mJ;  $p = 5.31$  mbar;  $z_i = -25$  cm;  $L_{med} = 20$  cm. Other parameters are specified in the  
 333 main text at the beginning of Section 3.

334 In the absorption limited HHG regime the harmonic yield (i.e. the output photon number)  
 335 should grow quadratically with the optical density [52]. In Figure 5 we represent the harmonic yield  
 336 as function of the optical density of the medium. Symbols are data coming from the full 3D  
 337 simulations, lines are the results predicted by the ANN. With dashed line we show as guide to the

338 eye the line with slope = 2. In Figures 5(a) and 5(b) the 3D simulations provide sufficient data and  
 339 they confirm that under low optical density conditions the quadratic growth of the yield is valid.  
 340 Data in Figure 5(c) have been obtained for an intermediate pulse energy  $E = 9$  mJ and the cell  
 341 positioned in the diverging beam at  $z_i = 10$  cm. In this parameter combination there are few 3D  
 342 simulation results available, being a good opportunity to test the accuracy of the ANN against the  
 343 basic physics of the HHG process. Figure 5(c) shows that the ANN works correctly in this respect,  
 344 predicting quadratic growth of the yield in the low density regime. Moreover, the end of the region  
 345 for the absorption limited HHG is also reproduced in a similar manner by the ANN as by the  
 346 complete 3D simulations. It is not possible to show all results, but the main conclusion is that the  
 347 ANN is capable to predict regions in the multi-dimensional parameter space where the absorption  
 348 limited HHG can take place. Furthermore, ANN also predicts for which parameter combinations  
 349 there is a transition from the initially absorption limited HHG to a saturation, then to a drop in  
 350 obtainable harmonic yield. This dynamics is represented for example in Figures 5(b) and 5(c) at  
 351 pressure values 0.38 mbar (green symbols and lines) and 1.41 mbar (blue symbols and lines).



352

353 **Figure 5.** Harmonic yield as function of the optical density of the medium. We represent the  
 354 logarithm of both quantities, the units are arbitrary. Slope = 2 indicates the quadratic growth of the  
 355 harmonic yield with the optical density.

#### 356 4. Conclusions

357 In this work we presented preliminary results for the prediction of the harmonic yield in  
 358 different HHG configurations. The results have been obtained with the combination of a complete  
 359 3D simulation model and an artificial neural network model. The main result of this work is that we  
 360 succeeded to build a complex computational tool which is able to predict the expected outcome of a  
 361 HHG experiment covering conditions when several experimental parameters are varied in a given  
 362 range. Since this is a usual situation in most attosecond science laboratories, we consider our  
 363 achievement being a useful and very practical numerical instrument. We need to perform sufficient  
 364 number of full 3D simulations designed such as to cover (if possible uniformly) the whole explored  
 365 parameter space. The next step is to introduce these results as input into the ANN, train it, validate  
 366 and test. A correctly trained ANN is capable to predict the expected outcome of a possible HHG  
 367 experiment to be performed with parameter combinations that were not implemented in the 3D  
 368 simulations.

369 In general terms: the 3D model for HHG combined with the ANN model provide a smooth scan  
 370 of the parameter space along any variable parameter, and can indicate combinations of possible  
 371 experimental circumstances which are favorable for the targeted outcome.

372 In particular: within the present study we targeted to obtain enhanced harmonic yield in the  
 373 20-40 eV spectral domain. The variable experimental parameters were: input laser pulse energy  $E =$   
 374  $[0.77; 11.6]$  mJ; argon pressure  $p = [0.1; 20]$  mbar; cell entrance position relative to the nominal focus  $z_i =$   
 375  $[-50; 50]$  cm, medium length  $L_{med} = [0; 20]$  cm, while we kept unchanged the pulse duration,  
 376 wavelength, focal length, beam size, gas type.

377 The main message of the obtained results is that a correctly trained ANN fed with reliable input  
 378 data originating from the full 3D calculations is capable to predict the outcome of a HHG experiment  
 379 in the unexplored parts of the multi-dimensional parameter space.

380 In this particular case we only demonstrated the capabilities of the prediction method which  
381 uses as input the results of full 3D simulations taking as the target quantity the harmonic yield.  
382 However, other physical quantities of interest can be defined and predicted if the ANN is modified  
383 accordingly. We can define for example a spectral domain where we search for the existence of  
384 isolated attosecond pulse; if it has satellites, then we can predict the contrast relative to the main  
385 pulse; we can look for the configuration which provides highest brightness for a specified harmonic  
386 order, i.e. high yield combined with low divergence.

387 ANN as a numerical instrument for prediction can use as input data either data originating  
388 from complex simulations (like in our case), or real experimental data (if available), or even a  
389 combination of experimental and simulation results. Obviously in the latter case one has to check for  
390 the agreement between them, and to perform a calibration of the simulated against the experimental  
391 data.

392 We foresee that this prediction method can be extended to become an easy to use optimization  
393 tool, which would mean an essential progress in the way to solve the problem of low conversion  
394 efficiency in HHG.

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397 analysis of the 3D simulations, K. Kovacs; Running and analysis of the ANN simulations, A.M.M. Gherman;  
398 Writing-Original Draft Preparation, K. Kovacs and A.M.M. Gherman; Writing-Review & Editing, K. Kovacs,  
399 A.M.M. Gherman and V. Tosa; Visualization, K. Kovacs and A.M.M. Gherman; All authors discussed the  
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407

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