

Article

Optimization of Base Location and Patrol Routes for Unmanned Aerial Vehicles in Border Intelligence, Surveillance and Reconnaissance

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Abstract: The location routing problem of unmanned aerial vehicles (UAV) in border patrol for intelligence, surveillance and reconnaissance is investigated, where the location of UAV base stations and the UAV flying routes for visiting the targets in border area are jointly optimized. The capacity of the base station and the endurance of the UAV are considered. A binary integer programming model is developed to formulate the problem, and two heuristic algorithms combined with local search strategies are designed for solving the problem. The experiment design for simulating the distribution of stations and targets in border is proposed for generating random test instances. Also, an example based on the Sino-Vietnamese border is presented to illustrate the problem and the solution approach. The performance of the two algorithms are analyzed and compared through randomly generated instances.

Keywords: location routing; unmanned aerial vehicle; border patrol; heuristic

1. Introduction

Border patrol is one of military Intelligence, Surveillance and Reconnaissance (ISR) missions. In many ISR missions especially monitoring mission, Unmanned Aerial Vehicles (UAVs) have become a natural choice with the deployment of air surveillance technology [1, 2]. In border patrol, some areas located on or inside the borderline where have the high rates of illegal activities would be focused. Thus, it is feasible to deploy UAVs to detect the targets regularly and ferry the information such as images, videos, sensor data etc. to base stations.

However, patrol mission faces severe challenges with rampant crimes such as smuggling. And an increasingly concern is raised with more diversified forms of crimes. To be able to cope with the complexity and diversity of threats effectively, much higher requirements need to be satisfied to ensure the peace and security near the borderline, such as high patrol efficiency and fast reaction speed. Furthermore, due to the harsh environment and complex geographical conditions, it is dangerous for border guards to patrol manually. And the widespread use of the camera is also difficult with its high cost. Therefore, it is meaningful to apply the unmanned aerial vehicles (UAVs) to the border patrol.

Even though unmanned patrol is of great significance, there are still some challenges. Initially, the capacity constraints of base stations should be taken into account. To make full use of each established station, it is not practicable for the amount of its UAVs to be under a lower limit, while it is also unreasonable to exceed an upper limit with the limitations of military ranks. Moreover, since more than one base station should be established to ensure the mission complement, both the amount and the location of base stations need to be optimized. Meanwhile, the flight paths of UAVs

are also supposed to be jointly planned. These factors are all involved to minimize the total cost, which increases the complexity of the problem.

After considering the above-mentioned factors, this paper provides a solution for border patrol, which is structured as following: Section 2 presents the literature review, and Section 3 illustrates the main assumptions and constructs the optimization model. Section 4 introduces two constructive heuristics and the local search improving strategies. The experimental design and computational results are proposed in Section 5. At last, Section 6 presents the conclusion and future works.

2. Literature Review

In regard to location-routing problem, there are many scholars doing the research after Jacobsen and Madsen [3] integrated the study of locations and routines in 1980. Tuzun et al. [4] promoted that LRP is an NP-hard problem. Afterwards, Wu T H et al. [5] presented a problem named multi-depot LRP, which is solved by a simulated annealing- based decomposition approach. The capacitated location-routing problem is promoted by Prins et al. [6]. Furthermore, Çağrı Koç [7] considered periodic LRP problems with time windows and developed a meta-heuristic algorithm based on large-scale neighbor search. In terms of the solution approach, the exact solutions are initially adopted to solve some small-scale problems, such as Akca [8], Belenguer [9]. At present, heuristics are applied more widely. Nguyen V P [10], Vincent [11] and Ting CJ [12] all made some contribution to the heuristic algorithms. Nevertheless, most of these researches are conducted against the commercial background.

For the routine planning of UAVs, many researches have been carried out. In 2004, Harder et al. [13] put forward the general architecture of UAVs' routine problem and designed the components of the architecture. The constraints of ammunition load are first considered by Shetty et al. [14]. To scheme the attack path in the war, the main idea is distributing the attack targets to different UAVs and visiting targets in the sequence of importance-differentiation. Mufalli et al. [15] also take the limitations of drone payload into account and use the column-generated heuristics for both sensor selection and flight path. Besides, multiple vehicles and time windows are solved [16]. As for the military applications, unmanned combat aerial vehicles are applied to destruct predetermined targets with the constraints of munitions [17]. Moreover, Avellar, Gustavo S. C, et al. [18] present a solution for the problem of using a group of unmanned air vehicles (UAVs) equipped with image sensors to gather intelligence information, which the objective is to minimum time coverage of ground areas. Persistent Intelligence, Surveillance and Reconnaissance (PISR) routing problem are also considered by minimizing the time of delivering the collected data to the control station [2]. Unfortunately, most of these researches are based on the traditional routine problem, with little touch and exploration on the optimization of base location and flight path.

From the current literature, there are a few studies on the location-routing problem for UAVs, with mainly two having investigated it. The first one is that İnci Sarıççek et al. [19] studied border patrols of UAVs in Turkey in 2014, which uses a two-stage solution method, leading to ineffectiveness of integrating the base location into routine planning. The second one is in 2016. Yakıcı [20] studied the base location and path planning under the background of maritime target reconnaissance. However, the constraints of the amount of UAVs are not contained in his study.

In this paper, both capacity constraints on base stations and endurance limitations on UAVs are taken into account in the location-routing problem. And a programming model and two heuristic algorithms are constructed to solve the problem.

3. Model Formulation

In the border patrol mission, there exist a set of potential base stations and a set of determined patrol targets distributed in the border area. Considering the capacity constraints of bases and the endurance limitations of UAVs, the objective of mission planning is to locate the base stations and plan the UAVs' routes in an effort to minimize the overall cost. For starters, this section presents the assumptions, defines the problem and builds the mathematical model.

3.1. Problem Assumptions

- Assume that all UAVs are in the same type, which means that they have same flight endurance and flying speed.
- The flying speed of UAVs is assumed to be constant.
- The UAV must depart from and return to the same base station. The situation of swapping the UAVs between different stations are not considered.
- Each UAV can visit multiple targets, while each target can only be visited by one UAV.
- Patrol cost for each mile is only concerned with UAV's property. Uncertain or unexpected situations, such as weather changes, are not taken into consideration.

3.2. Problem Assumptions

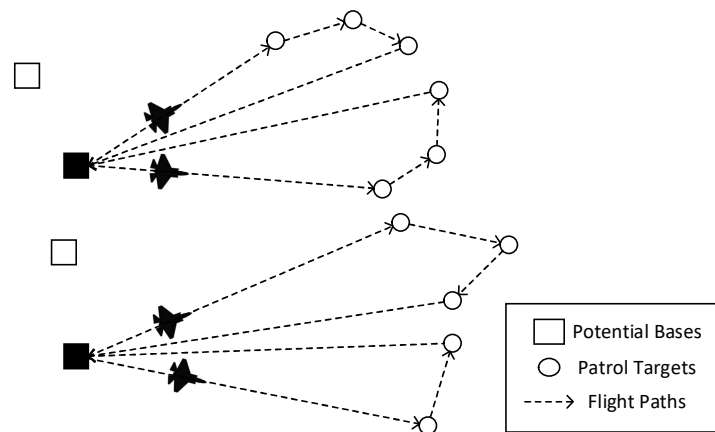


Figure 1. Sketch map for UAV border patrol

In patrol missions, a series of base stations with the corresponding equipment are essential. These stations are established inside the borderline: can be reconstructed with the existing sentry posts for reducing the construction cost, or be built near the border to benefit the UAVs. Thus the set of all potential locations can be constituted as $M = \{1, 2, \dots, m\}$. Once a base station is determined, the equipped facilities, such as takeoff and recovery device, would generate a fixed establishing cost C_i ($i \in M$). Furthermore, under the considerations of economic cost and military establishment, there exist a limitation on the capacity of base stations. For each base station, it is not reasonable for the amount of its UAVs to be under the lower limit a^l which the station would be underutilized, nor to exceed the upper limit a^u with the limitations of military ranks.

Another necessary element in this problem is the set of patrol targets, which are some small areas located on or inside the borderline where have the high rates of illegal activities like smuggling. Since the sensor of the drone detects one area at one time, these areas can be simplified into nodes. Then we get a set $N = \{1, 2, \dots, n\}$. All target points in this set must be detected once in this mission. And the distance d_{ij} between any two targets (or target and base) i, j ($i, j \in M \cup N$) is known. Moreover, UAVs would be consumed some service time to spy on each target, denoted by s_i ($i \in N$).

As for UAVs, $V = \{1, 2, \dots, v\}$ is used to denote the set of all UAVs. Since UAVs require maintenance before and after usage, there is a fixed use cost F_k ($k \in V$). It could be different for each UAV, but with all UAVs in the same type, it is assumed to be constant to simplify the calculation. Furthermore, knowing that UAVs would fly at a constant speed, the flying time d_{ij} from node i to nodes j ($i, j \in M \cup N$) can be given. Besides, the total time containing flying time and service time cannot exceed the UAVs' maximum flight duration D .

The final objective function is formed via the sum of three primary costs: 1) minimize the base establishing cost; 2) accomplish the patrol mission with as few drones as possible; 3) minimize the flight cost of UAVs.

3.3. Mathematical Model

The parameters and variables used in the model are listed as follow.

Sets

- $M = \{1, 2, \dots, m\}$, set of all potential base stations
 $N = \{1, 2, \dots, n\}$, set of all patrol targets
 $V = \{1, 2, \dots, v\}$, set of all UAVs

Parameters

- C_i fixed establishing cost of base i ($i \in M$)
 F_k fixed use cost of UAV k ($k \in V$)
 q unit patrol cost of UAVs per kilometer
 d_{ij} the distance between two targets (or target and base) i, j ($i, j \in M \cup N$)
 t_{ij} the flying time between two targets (or target and base) i, j ($i, j \in M \cup N$)
 s_i detecting time for target i ($i \in N$)
 a^U upper limit of the capacity of bases
 a^L lower limit of the capacity of bases
 D the UAVs' maximum flight duration

Decision Variables

- $x_{ijk} = 1$, if UAV k ($k \in V$) fly from node i to j ($i, j \in M \cup N$); 0 otherwise.
 $y_i = 1$, if a base station is determined to be built on node i ($i \in M$); 0 otherwise.
 z_{ik} auxiliary variable for sub-tour elimination constraints in the route of UAV k ($k \in V$).

$$\min \sum_{i \in M} C_i y_i + \sum_{i \in M \cup N} \sum_{j \in M \cup N} \sum_{k \in V} q d_{ij} x_{ijk} + \sum_{k \in V} F_k \left(\sum_{i \in M} \sum_{j \in N} x_{ijk} \right), \quad (1)$$

St.

$$\sum_{k \in V} \sum_{i \in M \cup N} x_{ijk} = \sum_{k \in V} \sum_{i \in M \cup N} x_{jik}, \quad j \in M \cup N, \quad (2)$$

$$\sum_{k \in V} \sum_{j \in M \cup N} x_{ijk} = 1, \quad i \in N, \quad (3)$$

$$\sum_{i \in M \cup N} \sum_{j \in M \cup N} (t_{ij} + s_i) x_{ijk} \leq D, \quad k \in V, \quad (4)$$

$$\sum_{i \in N} \sum_{j \in M} x_{ijk} \leq 1, \quad k \in V, \quad (5)$$

$$\sum_{j \in N} \sum_{k \in V} x_{ijk} \leq a^U y_i, \quad i \in M, \quad (6)$$

$$\sum_{j \in N} \sum_{k \in V} x_{ijk} \geq a^L y_i, \quad i \in M, \quad (7)$$

$$\sum_{i \in N} x_{ijk} = \sum_{i \in N} x_{jik}, \quad j \in M, \quad k \in V, \quad (8)$$

$$z_{ik} - z_{jk} + |N| x_{ijk} \leq |N| - 1, \quad i, j \in N, \quad k \in V, \quad (9)$$

$$x_{ijk} \in \{0, 1\}, \quad i, j \in M \cup N, \quad k \in V, \quad (10)$$

$$y_i \in \{0, 1\}, \quad i \in M, \quad (11)$$

$$z_{ik} \geq 0, \quad i \in N, \quad k \in V, \quad (12)$$

The objective function (1) minimizes the sum of base establishing cost, UAVs' use cost and its patrol cost. Constraint (2) expresses the limitation in flow conservation. Constraint (3) ensures that each target point must be visited and assigned to only one UAV. Constraint (4) restricts the time elapsed in each UAV's flight, containing both flying time and detecting time. Constraint (5) requires that each UAV can be scheduled at most once in one mission planning. Constraint (6) and (7) define the limitations on the capacity of base station, which the number of equipped UAVs cannot exceed the upper and lower limit. Constraint (8) makes sure that each UAV must turn back to the base

station which it departs from. Sub-tour elimination constrains are expressed in (9). Constraints (10) to (12) declare the variable domains.

4. Algorithms

In this section, two different hydride heuristics are introduced to construct the feasible solution: nearest point searching algorithm or saving algorithm. Then neighbor search serves as an optimization tool to finalize the optimization solution.

4.1. Hydride Heuristics

4.1.1. Heuristic based on clustering and nearest point search (H1)

Heuristic Based on Clustering and Nearest Point Search (H1) utilizes the strategy of Nearest Neighbor. Nearest Neighbor is a well-known constructive search algorithm that is one of the earliest methods proposed for TSP problems [21]. It adopts the principle of selecting the next nearest unvisited node until all nodes have been covered. It runs fast, however, the optimality of the tours it produces highly depends on the layout of the given nodes.

In H1, every target point is allocated to the nearest base station at first. Then for each station which has assigned targets, the path of UAV is arranged one by one until all assigned targets are detected or the number of UAVs reaches the upper limit of the station. If there are unvisited targets, then allocate the remaining points to the second nearest station. At last, there is a check function to find whether there is some stations that the number of UAVs does not reach lower limit. If so, close the station and reallocate the targets.

The corresponding pseudo-code is shown in H1 and the detailed explanation of the heuristic is followed.

Algorithm 1: Heuristic Based on Clustering and Nearest Point Search	
1	Allocate targets: allocate every target to its nearest station
2	Sort base: sort base stations by the number of allocated targets
3	while (unvisited targets) do
4	Select the base with the most targets
5	Detect targets with Nearest Neighbor
6	if (unvisited targets in this base) then
7	Reallocate: allocate the remaining targets to the second nearest station
8	end if
9	end while
10	while (bases with the number of UAVs lower than limit) do
11	Select the base with the least targets and close the station
12	Reallocate: allocate targets of the station to the nearest neighbor station
13	Re-detect targets: add the new targets for the neighbor station
14	end while

As shown in pseudo-code, every target is allocated to its nearest station with the known positions of the potential base stations and target points (Line 1). Then sort the selected base stations (Line 2) and plan the path of UAVs in each station. While detecting the targets, prioritize the base with the most targets (Line 4). As for the details (Line 5), launch one UAV at a time, and choose the nearest unvisited target as the next access point. Judge whether the UAV can return back to the original station after visiting the next point. If so, the UAV travels to the next point and keeps detecting, while turns back to the start if not. Repeat the step until all assigned targets are detected or the number of UAVs reaches the upper limit of the station. At this time, check whether there is assigned targets unvisited after sending out all UAVs (Line 6). If so, allocate the remaining targets to the second nearest station (Line 7).

After all targets have been detected, find whether there are some stations not having enough numbers of UAVs (Line 10). If so, give priority to the base with the least targets and close the station (Line 11). Then allocate targets of the station to the nearest neighbor station (Line 12). After that, re-detect targets for the new neighbor station (Line 13) like what have done in Line 5.

4.1.2. Heuristic based on clustering and CW saving search (H2)

Clarke and Wright proposed the saving algorithm in 1964 [22]. This algorithm provides an easy way to solve the vehicle routing problem, but it neither consider the location problem nor the restriction on the numbers of vehicles.

Similar to H1, heuristic based on clustering and CW saving search (H2) first allocates each target point to its nearest base station and has a check function at last. Different than H1 using Nearest Neighbor, CW saving search algorithm is applied in H2 while planning paths to detect targets.

The corresponding pseudo-code is shown in H2 and the detailed explanation of the heuristic is followed.

Algorithm 2: Heuristic Based on Clustering and CW Saving Search

```

1  Allocate targets: allocate every target to its nearest station
2  Sort base: sort base stations by the number of allocated targets
3  while (unvisited targets) do
4      Select the base with the most targets and calculate the saving matrix
5      Detect targets with CW
6      if (unvisited targets in this base) then
7          Reallocate: allocate the remaining targets to the second nearest station
8      end if
9  end while
10 while (bases with the number of UAVs lower than limit) do
11     Select the base with the least targets and close the station
12     Reallocate: allocate targets of the station to the nearest neighbor station
13     Re-detect targets: add the new targets for the neighbor station
14 end while

```

Since the overall framework of H2 is similar to that of H1, the same process will not be repeated in this part. After selecting the base with the most targets, calculate the corresponding distance matrix and saving matrix (Line 4). In terms of the details (Line 5), launch one UAV at a time. Refer to the saving matrix and merge the target points as many as possible under the limitation of UAVs' flight endurance, which would generate a flight path for a UAV. Repeat the step until all assigned targets are detected or all UAVs have been sent out. Moreover, CW saving search also works in re-detecting targets in Line 13.

4.2. Neighborhood Search Improvement

Since the hydride heuristics are applied to construct the feasible solution, which serves as an initial solution and still has room for improvement, thus neighborhood search is introduced to optimize the solution. The framework of neighborhood search is displayed in the pseudo-code below.

Given an initial solution s , the main process iterates over the parameter i until it reaches the preset value of maximum iterations i_{\max} (Line 1). In each interaction, a solution s' would be generated through the operation of closing base stations, which would be described in section 4.2.1. Then, from Line 5 to Line 16, local search is performed. After initializing the neighborhood list (Line 5), the local search will not end until running all neighborhoods without improvement (Line 7). Neighborhood list would be explored exhaustively every time, which returns the best improvement s' (Line 8). Compared with the current solution s , if s' is better, then s' is the new solution and reinitialize the

neighborhood list, which would restart the counter k (Line 9 - 12). The neighborhoods used in this part are detailed in the sections 4.2.2 to 4.2.4.

Algorithm 3: Neighborhood Search

```

1  Require:  $s, i_{\max}$ 
2   $i \leftarrow 1$ 
3  while  $i \leq i_{\max}$  do
4       $s' \leftarrow \text{CloseBaseStation}(s)$ 
5      Initialize Neighborhood List ( $N$ )
6       $k \leftarrow 1$ 
7      while  $k \leq N$  do
8          Find the best neighbor  $s' \in N(s)$ 
9          if  $s' < s$  then
10              $s \leftarrow s'$ 
11             Reinitialize  $N$ 
12              $k \leftarrow 1$ 
13         else
14              $k \leftarrow k + 1$ 
15         end if
16     end while
17      $i \leftarrow i + 1$ 
18 end while

```

Therefore, combining two heuristics with the neighbor research respectively, we can produce two algorithms and generate two optimal solutions.

4.2.1. Operation of closing the base station

In the feasible solutions, there exists a possibility that some base stations have not fully utilized the UAVs. Therefore, it might be useful to reduce the establishing cost through shutting down some stations. As Figure 2 shows, just two UAVs are launched for both station A and station B, which have not dispatched all UAVs. At this time, we can try to close one of these two stations and find whether these targets can be detected by the UAVs from only one station. If so, then station B can be closed. Furthermore, after closing the base station, the flight paths in station A would be rearranged. To save the calculate time, Nearest Neighbor algorithm is used, which is displayed in Figure 2

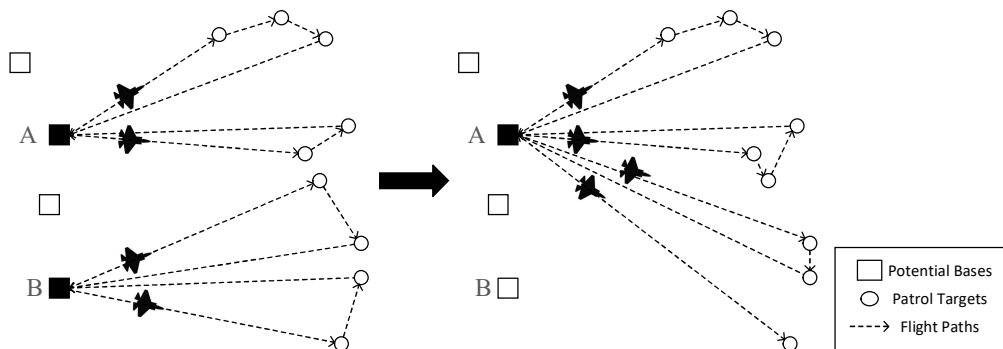


Figure 2. The operation of closing the base station

4.2.2. Neighborhood 1: Opt2

This neighborhood relocates two targets on one flight path in a tentative solution. This operation is mainly meant to reduce the cross-over routes or overlapping routes. In Figure 3, the

initial routine is $1 \rightarrow 2 \rightarrow 3$. Then point 2 and 3 are selected and their positions are swapped to see whether there is a better solution.

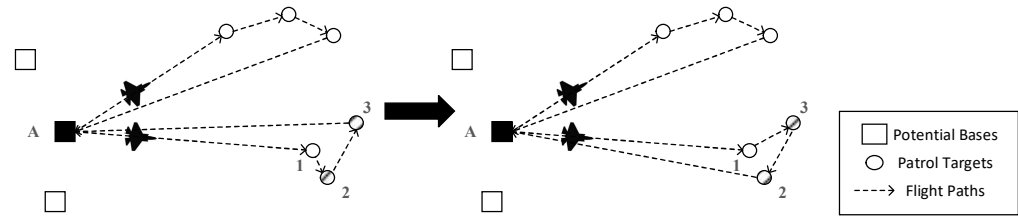


Figure 3. The operation of Opt2

4.2.3. Neighborhood 2: Exchange targets

This neighborhood is set to swap a target with another one which locates on another path in a tentative solution. The two paths can belong to the same station or two adjacent stations. Figure 4 presents the situation that two targets are in different paths of one station. After exchanging the point 3 and 4, the flight distance of both two flight paths would be decreased, which could help lower the flight cost.

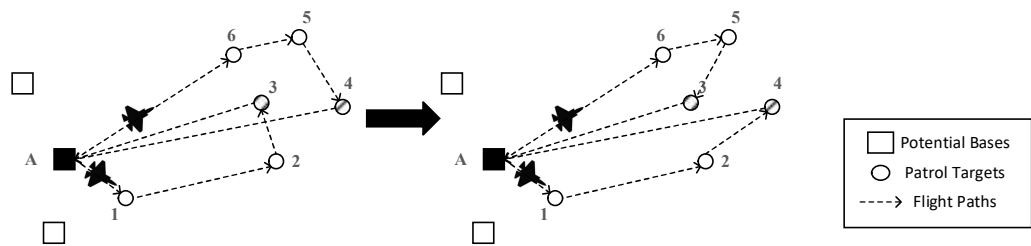


Figure 4. The operation of exchanging targets

4.2.4. Neighborhood 3: Insertion

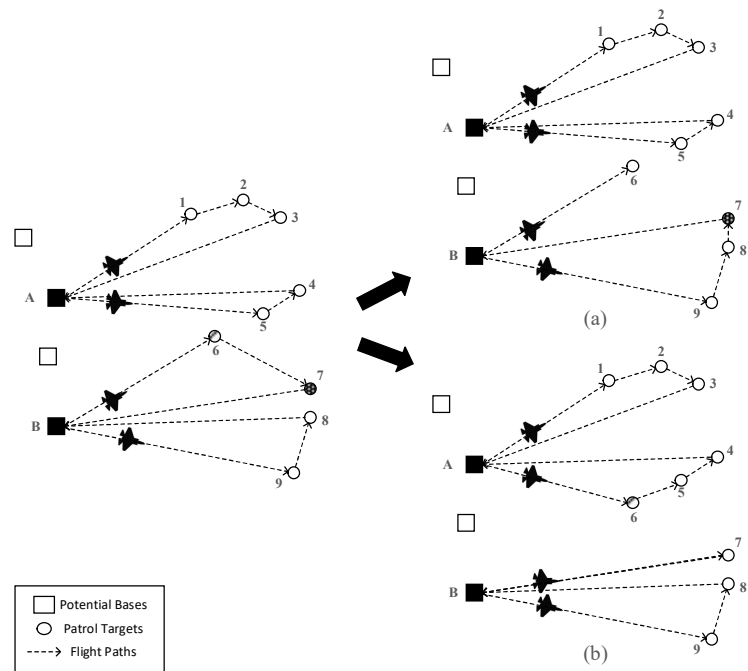


Figure 5. The operation of insertion

This neighborhood removes a target and reinserts it in other position in a tentative solution, which may change the owner base station of the target. Figure 5 (a) illustrates a relatively straightforward move in one station. Additionally, the path represented by Figure 5 (b) relocates the target 6 from station B to station A.

5. Experiment Design and Results

In this section, experiments are designed based on actual characteristics of border patrol and two algorithms are tested with the constructed cases.

5.1 Experiment Design

When designing the experiment, the particularity of border patrol should be taken into account.

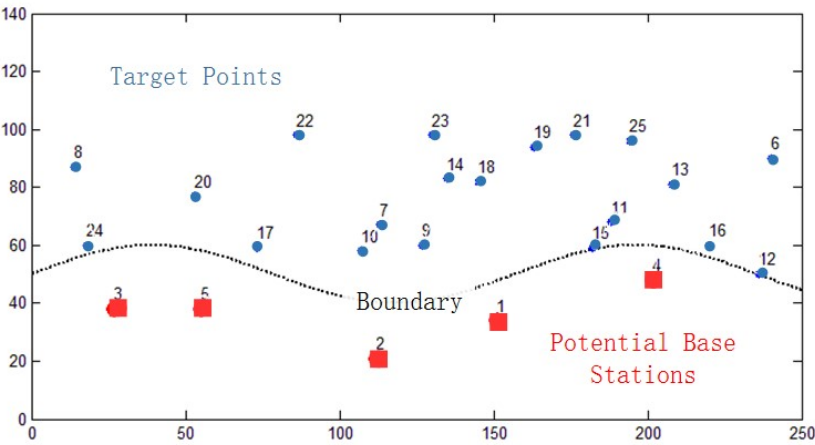


Figure 6. Experiment case generated under limited conditions

Since the detection goal is part of borderline, the mission area should be set as an irregular strip area. Thus, as displayed in Figure 6, it is assumed that the detection area is a rectangle with aspect ratio of 2: 1.

In the mission planning, the target points should be located on the borderline while the base stations are inside the borderline. It means that there would be a boundary between potential bases and target points, which is different from the cases in delivery system. Therefore, a random curve is first generated as a boundary. (Note: This boundary differs from the borderline, the border is in the area of target points.) Then base stations and target points are separately generated on both sides of the boundary, which is the dotted line in Figure 6. Five potential base stations are below the boundary line while twenty target points are above it.

As the target points are abstracted from a small area on the borderline, they should not be too close to each other, otherwise two points could be merged into one node. It is same for the potential stations. It would be impractical to build two stations in a close range. For the purpose of this characteristic, all nodes are generated one by one. Take the base stations as an example, every time a new station is produced, the distance between the station and every existing station would be judged. If the distance is too short, the position would be abandoned and regenerated until there are enough base stations.

The parameters of UAV mainly refer to the public parameters from two typical UAVs which have been applied to border patrol. The detailed parameter is presented in Table I. After some proper randomization, the experiment parameters are generated, such as the flight endurance, patrol speed and so on.

Table 1. Detailed Parameters of two typical UAVs

Parameters	UAV 1	UAV 2
Fuselage	0.7 meters in length 2 meters in wingspan	1.75 meters in length 3.1 meters in wingspan
Body Weight	8.5 kg	15 kg
Payload	1.5 kg	4 kg
Flight Speed	80-120 km / h	maximum: 158 km / h patrol: 29.5 km / h
Flight Altitude	maximum: 3000 meters working: 100-500 meters	maximum: 2048 meters working: 300 meters
Mission Radius	70 km	30 km
Flight Endurance	2.5 hours	≥ 4 hours

5.2 Experiment Result

With the experiment data generated, two algorithms are tested and compared with each other. For the sake of illustration, both H1 and H2 contains the corresponding hydride heuristics and the neighborhood search.

5.2.1 Results of H1 on test case

Take a small-scale case (5 base stations and 20 target points) generated randomly. Figure 7(a) shows the feasible solution given by H1 while the improved solution is provided after neighbor search as Figure 7(b) depicts.

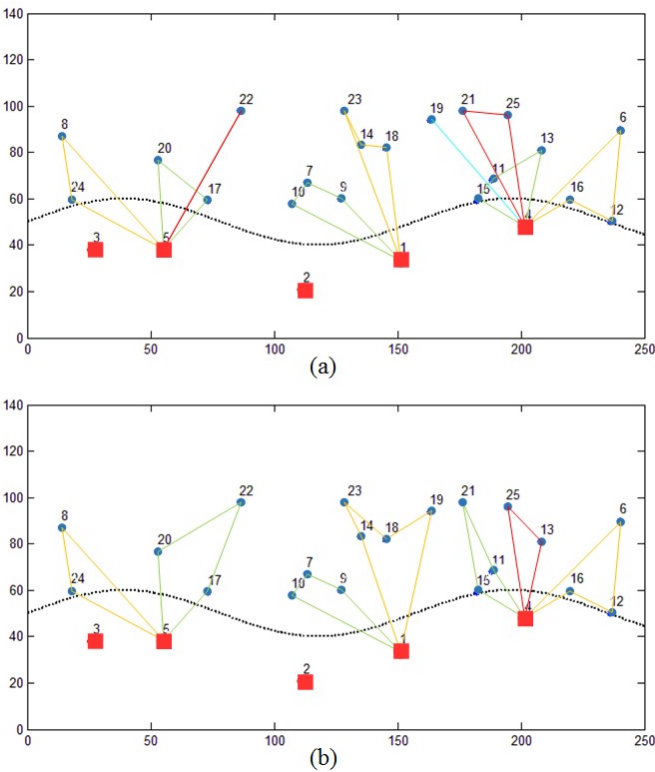


Figure 7. Feasible solution (a) and improved solution (b) given by H1

In Figure 7(a), it is obvious that the performance of neighbor search is flawed. For instance, after the UAV visits Target 17 and 20, the rest energy cannot support it to return to the base if continuing

detecting Target 22. Nevertheless, the route of $5 \rightarrow 17 \rightarrow 22 \rightarrow 20 \rightarrow 5$ satisfies the drone endurance, which has been adjusted in neighborhood search. Besides, several fight paths are merged and the amount of UAVs is decreased after optimization. It can be seen that only 7 UAVs are used in the final solution, which is two less than that of the initial solution. Furthermore, the detecting order of some targets are exchanged, which decreases the UAV's redundant flight. As displayed in Table 2, the overall costs have been decreased from 372992 to 274386 down by 26 percent, proving the feasibility of H1.

Table 2. Detailed comparison between feasible solution and improved solution of H1

Feasible solution			Improved solution		
Base	UAV	Route	Base	UAV	Route
1	1	$1 \rightarrow 9 \rightarrow 7 \rightarrow 10 \rightarrow 1$	1	1	$1 \rightarrow 9 \rightarrow 7 \rightarrow 10 \rightarrow 1$
	2	$1 \rightarrow 18 \rightarrow 14 \rightarrow 23 \rightarrow 1$		2	$1 \rightarrow 14 \rightarrow 23 \rightarrow 18 \rightarrow 19 \rightarrow 1$
4	1	$4 \rightarrow 15 \rightarrow 11 \rightarrow 13 \rightarrow 4$	4	1	$4 \rightarrow 15 \rightarrow 11 \rightarrow 21 \rightarrow 4$
	2	$4 \rightarrow 16 \rightarrow 12 \rightarrow 6 \rightarrow 4$		2	$4 \rightarrow 16 \rightarrow 12 \rightarrow 6 \rightarrow 4$
	3	$4 \rightarrow 25 \rightarrow 21 \rightarrow 4$		3	$4 \rightarrow 13 \rightarrow 25 \rightarrow 4$
	4	$4 \rightarrow 19 \rightarrow 4$			
5	1	$5 \rightarrow 24 \rightarrow 8 \rightarrow 5$	5	1	$5 \rightarrow 24 \rightarrow 8 \rightarrow 5$
	2	$5 \rightarrow 17 \rightarrow 20 \rightarrow 5$		2	$5 \rightarrow 17 \rightarrow 22 \rightarrow 20 \rightarrow 5$
	3	$5 \rightarrow 22 \rightarrow 5$			
Cost: 372992			Cost: 274386		

5.2.1 Results of H2 on test case

The same case is also applied to test H2. The feasible solution given by H2 is presented by Figure 8(a) and the improved in Figure 8(b).

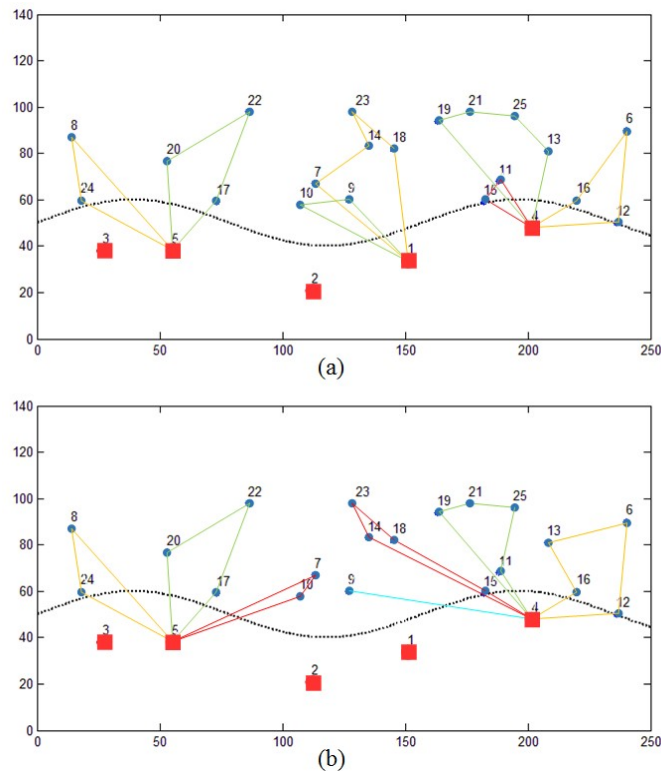


Figure 8. Feasible solution (a) and improved solution (b) given by H2

The conclusion can be drawn that the CW saving algorithm has utilized the flight endurance as far as possible, which is fairly obvious in the feasible solution. Compared with the cost of feasible solution in H1, H2 reduces the cost from 372992 to 336496. After the adjustment of the neighbor search, the structure of the solution has changed a lot. Besides optimizing the routes, the Base station 1 is shut down to reduce the base establishing cost.

The detailed comparison is shown in Table 3. The overall costs have been reduced from 336496 to 189061, dropping by 44%. Therefore, H2 seems to be more accurate than H1 and can solve the problem better.

Table 3. Detailed comparison between feasible solution and improved solution of H2

Feasible solution			Improved solution		
Base	UAV	Route	Base	UAV	Route
1	1	1→9→10→1	4	1	4→11→25→21→19→4
	2	1→18→23→14→7→1		2	4→15→18→23→14→4
4	1	4→13→25→21→19→4		3	4→16→13→6→12→4
	2	4→15→11→4		4	4→9→4
5	3	4→16→6→12→4	5	1	5→24→8→5
	1	5→24→8→5		2	5→17→20→22→5
5	2	5→17→20→22→5		3	5→7→10→5
Cost: 372992			Cost: 274386		

5.2.3 Comparison

During the experiment, three scales of cases have been generated: 5 base stations and 20 target points for small-scale cases, 20 base stations and 50 target points for middle-scale cases, 50 base stations and 100 target points for large-scale cases. The two algorithms are operated in the cases under three different scales. Table 4 shows the results respectively by comparing the cost of feasible solution and improved solution as well as the running time.

It can be seen from the data that H2 can provide a better solution than H1. The cost of H2 is all about 20% lower than the final one of H1 in three scales of cases. However, the accuracy comes at the price of the time. For example, in large-scale cases, H1 spends about 0.1s, while H2 consumes 0.4s more. Besides, it can be proved that neighbor search does optimize the feasible solutions and greatly reduce the overall cost.

Table 4. Algorithm comparison in cases of three scales

Case Scale	Cost of feasible solution	H1 Cost of improved solution	Time /10 ⁻² (s)	Cost of feasible solution	H2 Cost of improved solution	Time /10 ⁻² (s)
5 stations & 20 targets	348882	300435	2.325	336890	238473	3.928
	386308	336416	2.392	362091	263009	3.815
	372992	274386	2.463	336496	189061	3.786
	322293	273432	2.313	287293	239058	3.682
	372327	276420	2.342	325443	179350	3.871
	372439	274322	2.335	336807	238975	5.487
	372369	273543	1.892	337965	240234	3.635
	361322	261822	2.247	348843	250545	3.670

	372538	322555	2.327	336976	237811	3.787
	384262	285167	2.196	336890	237784	3.811
20 stations & 50 targets	842265	692451	3.037	793872	544058	6.950
	731291	631663	3.081	681403	531504	6.895
	792129	592620	4.232	754694	505265	6.627
	730762	631054	3.064	670777	471234	7.285
	730340	630746	3.149	681039	481442	7.002
	856295	656052	3.173	792201	492593	7.867
	807587	608259	3.128	733063	483691	7.419
	804006	654817	3.503	719919	520651	7.432
	828288	628671	3.385	732245	482620	7.494
	804770	655033	2.620	719798	570285	7.242
50 stations & 100 targets	1340095	1028130	1.040	1196218	791338	4.590
	1272888	923026	0.993	1219406	819554	4.819
	1314565	952735	1.061	1143538	843628	4.490
	1396086	946150	1.073	1308837	858909	4.931
	1197487	897623	1.064	1075171	725519	5.414
	1430365	1091920	1.012	1368315	818606	4.525
	1331166	1031168	1.049	1219333	819335	4.508
	1354155	1004220	1.000	1208915	809037	4.550
	1418603	1080944	1.029	1319784	869953	4.446
	1343357	943478	1.023	1256920	807064	4.897

5.3 Example based on the Sino-Vietnamese border

In this section, the Sino-Vietnamese border is used as the example case and solved by the preceding algorithms.



Figure 9. The map of Guangxi Zhuang Autonomous Region (The marked red line is the Guangxi section of the Sino-Vietnamese border)

As is shown in Figure 9, there are about 8 cities and 103 towns bordering on Vietnam in Guangxi Zhuang Autonomous Region. With the development of economy and the implementation of opening-up policy, especially under "Belt and Road" initiative, Guangxi develops an increasingly flourishing border trade as the gateway to the ASEAN. However, smuggling also increases at the same time. Since the border area is mainly delimited by rivers or mountains and has few natural barriers, this part of the border is prone to smuggling which is difficult to monitor. According to the official report, 6726 smuggling cases were seized in Guangxi in 2006. Thus, it is meaningful to apply UAVs on the border patrol, which can improve the patrol efficiency and reaction speed. Therefore, take the Guangxi section of the Sino-Vietnamese border as an example.

As displayed in Figure 10, with the ranging tool of Google Map, the linear distance of this part is about 320 km. Then Paint software is used to discretize this line. Fifty target points are marked on the border and twenty base stations are chosen randomly inside the line. Furthermore, the relative positions of these 70 nodes are obtained via the pixel measurement.



Figure 10. Guangxi section of the Sino-Vietnamese border with the ranging tool

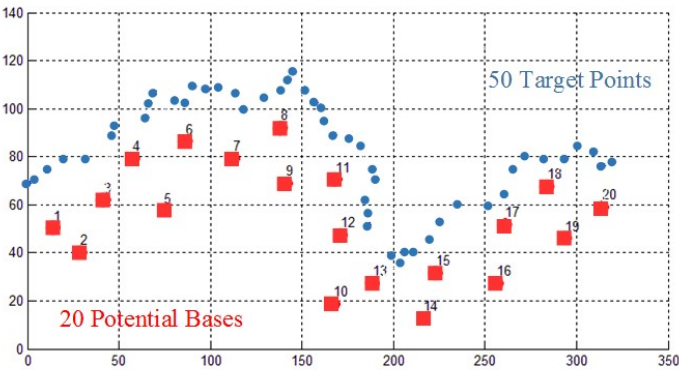


Figure 11. 50 target points and 20 base stations after discretization

It is assumed that all targets and potential bases are located in the area of 320 kilometers multiplied by 160 kilometers. Mapping these nodes into this area, the distribution map looks like Figure 11. The blue dots are target points and the red blocks are potential base stations.

Two algorithms are applied to solve the practical case. The results are shown in Table 5. Since the result of H2 is obviously superior to another one, take the result of H2 as the final solution, in which the cost is 480309 and the calculation takes 0.06840 seconds.

Table 5. Comparison between two results of practical cases

H1			H2		
Cost of feasible solution	Cost of improved solution	Time /10 ⁻² (s)	Cost of feasible solution	Cost of improved solution	Time /10 ⁻² (s)
817492	619800	3.091	837997	480309	6.840

The potential base stations are numbered 1 through 20 when the target points are represented by 21 to 70. Then the detailed bases and flight paths can be listed in Table 6 and the corresponding route map is shown in Figure 12. The final result selects 6 bases and launches 15 UAVs.

Table 6. Detailed bases and flight paths of the final solution

Base	UAV	Route
3	1	3→21→22→3
	2	3→23→24→3
	3	3→25→27→26→3
6	1	6→28→29→30→6
	2	6→31→32→33→34→35→6
	3	6→36→37→6
8	1	8→38→39→40→41→8
	2	8→42→43→44→45→8
11	1	11→46→47→48→49→50→11
	2	11→51→52→53→11
15	1	15→54→55→56→57→15
	2	15→58→59→60→15
18	1	18→61→62→18
	2	18→63→64→65→67→66→18
	3	18→68→70→69→18

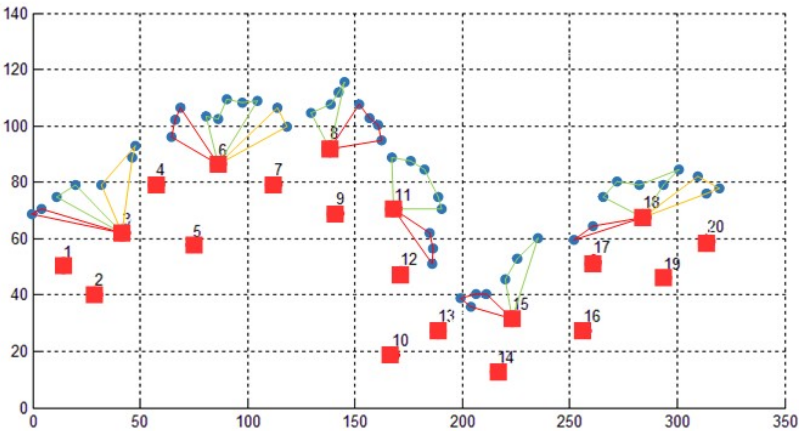


Figure 12. The route map of the final solution

6. Conclusions

In this paper, considering the background of border patrol for ISR, the capacity constraints of bases are involved in the multi-depot location-routing problem. Hence, a modified mathematical model has been presented. As the primary objective of this research is to develop an efficient approach for solving this kind of problem. Thus, two hybrid heuristics with neighbor search are promoted. One is based on clustering and nearest point search, and the other one is based on clustering and CW saving search.

In addition, experiments have been carried out to compare the performance of two algorithms. It can be addicted that neighborhood search plays an optimizing role to the solution. And the heuristic based on CW saving search provides a better solution while consuming more time than the other one. Furthermore, an example based on practical Sino-Vietnamese borderline is proposed. Both algorithms are applied to solve the border patrol on the Sino-Vietnamese borderline and provide the final solution.

Finally, two improvements in solving this kind of problem can be envisaged. First, the current neighbor search can be scaled up to find a global optimal solution. Second, more variants of this kind of problem can be exploited. For example, dynamic detecting could be added.

7. Acknowledgments

The research is supported by the National Natural Science Foundation of China (no. 71771215 and no. 71471174).

References

1. Li, Miao, et al. "Unmanned Aerial Vehicle Scheduling Problem for Traffic Monitoring." *Computers & Industrial Engineering* 122(2018).
2. Manyam, Satyanarayana G., et al. "Multi-UAV routing for persistent intelligence surveillance & reconnaissance missions." *International Conference on Unmanned Aircraft Systems IEEE*, 2017.
3. Jacobsen, S. K., and O. B. G. Madsen. "A comparative study of heuristics for a two-level routing-location problem." *European Journal of Operational Research* 5.6(1980):378-387.
4. Tuzun, Dilek, and Laura I. Burke. "A two-phase tabu search approach to the location routing problem." *European journal of operational research* 116.1 (1999): 87-99.
5. Wu T H, Low C, Bai J W. Heuristic solutions to multi-depot location-routing problems[J]. *Computers & Operations Research*, 2002, 29(10):1393-1415.
6. Prins, Christian, et al. "Solving the capacitated location-routing problem by a cooperative Lagrangean relaxation-granular tabu search heuristic." *Transportation Science* 41.4 (2007): 470-483.
7. Çağrı Koç. A unified-adaptive large neighborhood search metaheuristic for periodic location-routing problems[J]. *Transportation Research Part C Emerging Technologies*, 2016, 68:265–284.
8. Akca Z, Berger R T, Ralphs T K. A Branch-and-Price Algorithm for Combined Location and Routing Problems Under Capacity Restrictions[M]// *Operations Research and Cyber-Infrastructure*. 2009:309--330.
9. Belenguer, José-Manuel, Benavent, Enrique, Prins, Christian, et al. A Branch and Cut method for the Capacitated Location-Routing Problem[J]. *Computers & Operations Research*, 2011, 38(38):931-941.
10. Nguyen V P\, Prins C, Prodhon C. Solving the two-echelon location routing problem by a GRASP reinforced by a learning process and path relinking[J]. *4OR*, 2006, 4(3):221-238.
11. Vincent, F. Yu, et al. "A simulated annealing heuristic for the capacitated location routing problem." *Computers & Industrial Engineering* 58.2 (2010): 288-299.
12. Ting C J, Chen C H. A multiple ant colony optimization algorithm for the capacitated location routing problem[J]. *International Journal of Production Economics*, 2013, 141(1):34-44.
13. Harder R W, Hill R R, Moore J T. A Java™ universal vehicle router for routing unmanned aerial vehicles[J]. *International Transactions in Operational Research*, 2004, 11(3):259-275.
14. Shetty V K, Sudit M, Nagi R. Priority-based assignment and routing of a fleet of unmanned combat aerial vehicles[J]. *Computers & Operations Research*, 2008, 35(6):1813-1828.
15. Mufalli F, Batta R, Nagi R. Simultaneous sensor selection and routing of unmanned aerial vehicles for complex mission plans[J]. *Computers & Operations Research*, 2012, 39(39):2787–2799.
16. Ham, Andy M. "Integrated scheduling of m -truck, m -drone, and m -depot constrained by time-window, drop-pickup, and m -visit using constraint programming." *Transportation Research Part C Emerging Technologies* 91(2018):1-14.
17. Shetty, Vijay K., M. Sudit, and R. Nagi. "Priority-based assignment and routing of a fleet of unmanned combat aerial vehicles." *Computers & Operations Research* 35.6(2008):1813-1828.
18. Avellar, Gustavo S. C, et al. "Multi-UAV Routing for Area Coverage and Remote Sensing with Minimum Time." *Sensors* 15.11(2015):27783-27803.
19. İnci Sarıççek, Akkuş Y. Unmanned Aerial Vehicle hub-location and routing for monitoring geographic borders[J]. *Applied Mathematical Modelling*, 2014, 39(14):3939-3953.
20. Yakıcı E. Solving Location and Routing Problem for UAVs[J]. *Computers & Industrial Engineering*, 2016, 102.
21. Cover, T., and P. Hart. "Nearest neighbor pattern classification." *IEEE Trans.inf.theory* 13.1(1967):21-27.
22. Clarke G, Wright J W. Scheduling of Vehicles from a Central Depot to a Number of Delivery Points[M]. *INFORMS*, 1964.