 doi:10.20944/preprints201809.0371.v1

Dynamic path in C^4 space-time

D.Mastoridis – 1

Amaliados 17, Athens, Greece, P.O.Box 11523, d.mastoridis@prv.ypeka.gr

K.Kalogirou – 2

Amaliados 17, Athens, Greece, P.O.Box 11523, k.kalogirou@prv.ypeka.gr

keywords:dynamic path;ADM formalism;complex space-time;extended space-time;unified theories;geometric unification

 doi:10.20944/preprints201809.0371.v1

Abstract

After developed the formulation of a "general relativity" in C^4 [?], we proceed with the formulation of a Hamilton-Jacobi equation in C^4. We argue that in this consideration, the usual problems of the ADM formalism, do not exist, due to the complex time as it exists in our consideration. Specifically, we can derive a suitable dispersion relation in order to work with and find a generalised super Hamiltonian

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2018 doi:10.20944/preprints201809.0371.v1

1 Introduction

In the previous papers [?] [?] [?], we have presented a new formulation for a unified physical theory in C^4 space. This formulation has altered many beliefs and strategies existing in the literature of physics. One of the most surprising element, is that we have distinguished two different approaches or pictures; the one that works with the metric tensor G_{ij} as a field and the one that works with $\Omega_i = G_{ij}V^j$ as a field. In the literature of classical quantum gravity the main attempt was to identify the metric tensor as a field and as a consequence we where looking for graviton from the field equations provided by general relativity. In our consideration we have identified the graviton from the geodesic equation and from the field equations provided by an action

$$S_\pi = \int Z\sqrt{g}d\Omega \tag{1}$$

we are seeking for field strengths for the various fields that have appeared, including graviton. Specifically, we will show that the problems existing in the formulation of a classical quantum gravity as the time disappearance in the ADM treatment (in the dispersion relation and the time problem in the commutative relations needed, are solved within our formulation. Of course, the word quantum is used as a relic of the typical language in physics, due to the fact that we showed that quantum theories are just classical theories derived from C^4 after the restriction to the usual 4-d space-time. The main goal of this paper is to formulate a normal Hamiltonian derived from the action S_π after repeating the release of the end-point of the integral, creating a dynamic path length.

2 Dynamic path in C^4

If S_γ is the action that provided us the geodesic equation, where S_γ is the action that comes from

$$ds^2 = G_{ij}dz^i d\bar{z}^j \tag{2}$$

or

$$ds^2 = g_{ij}dx^i dx^j + g_{ij}dy^i dy^j + I_{ij}(dx^i dy^j - dy^j dx^i) \tag{3}$$

and $\mathcal{S}_\gamma$ is the new action that comes from the second exremisation

$$\mathcal{S}_\gamma = \int G^{ij}\frac{\partial S}{\partial z_j}\frac{\partial S^*}{\partial \bar{z}_i} \tag{4}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2018 doi:10.20944/preprints201809.0371.v1

we can make the following formulation; S_γ gives us the geodesic equation or the equation of motion, which means that the "lines" are drawn as geometry dictates. The geometry does not tell us what "it" moves on this lines, but for sure the background was produced and it is produced in a curved space. We have to remember that in S_γ the initial and end points of the integral are fixed. We have released the end point of the integral and we have passed to a complex functional action (for the case described by Eq. (2)) or a real functional (for the case described by Eq. (3)). Afterwards, we put boundary conditions to create a "sphere" or bubble and then $\mathcal{S}_\gamma = \hbar\varphi$ is the field that describes a boson. But, throughout the procedure of releasing the end point, the geodesic equation still holds, which means that "lines" are drawn or the background is already made to receive the $\mathcal{S}_\gamma$ which is actually "moving" in this fixed background or even better, it describes the properties of what it moves. The Poisson brackets or even the associated to them Lie brackets are

$$\{z_i, z_j\} = 0 \qquad \{P_i, P_j\} = 0 \qquad \{z_i, P_j\} = \delta_{ij}$$

and tell us what happens with the geometry itself, while the Poisson brackets

$$\{\mathcal{S}_i, \mathcal{S}_j\} = 0 \qquad \{P_i, P_j\} = 0 \qquad \{\mathcal{S}_i, P_j\} = \delta_{ij}$$

or the Lie brackets

$$[\widehat{\mathcal{S}}_i, \widehat{\mathcal{S}}_j] = 0 \qquad [\widehat{\mathcal{P}}_i, \widehat{\mathcal{P}_j}] = 0 \qquad [\widehat{\mathcal{S}}_i, \widehat{\mathcal{P}_j}] = \delta_{ij}$$

tell us what happens with the fields $\mathcal{S}_\gamma = \hbar\varphi$, that moves in the "lines" where we have set $\mathcal{S}^i = \mathcal{S}(z^i) = \hbar\varphi(z^i)$. In the case that we restrict only to the usual 4-d space-time coordinates and eigenvalue the other coordinates (as we have seen in the previous paper [?]) an "i" will appear in the commutative relations due to the fact that we will have only partial derivatives with respect to x_i, which when they act on the complex $\mathcal{S}_\gamma$ will give us an "i". In the same spirit, we can proceed with the action

$$S_\pi = \int Z\sqrt{G}d\Omega \tag{5}$$

which will tell us about curvatures and field strengths. The fixed points of the integral are actually, hypersurfaces surrounding 4-d complex volumes. We can also define an equivalent integral in the symplectic R^8 surrounding 8-volumes or even for the case of the embedding

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2018 doi:10.20944/preprints201809.0371.v1

of usual 4-d space-time in R^8 or C^4. If we release the end point, we will be able to form a functional action $\mathcal{S}_\pi$. So if we keep fixed the initial hypersurface which is defined by G_{ij} and vary slightly the end point to a $G^{'}_{ij}$, we are making a new value for the extremisation of the integral and by repeating this procedure again and again we create a "dynamic path length" $\mathcal{S}_\pi(G_{ij})$ that connects the given geometries defined by the different Hermitian metric tensors and $\mathcal{S}_\pi(G_{ij})$ is only depended by those Hermitian metric tensors and only. The sliding on the "dynamic path" tell us how we pass from G_{ij} to another $G^{'}_{ij}$ and to another $G^{''}_{ij}$, ..., which eventually inform us how geometry changes "point" by "point" or how geometry is created and destroyed "point" by "point" or even how geometry flows throughout a "Ricci flow" mechanism in C^4 or R^8 or in the embedded R^4. The relations for $\mathcal{S}_\pi$ will be then

$$\frac{\delta \mathcal{S}_\pi}{\delta Z_{ij}} = G^{ij} \tag{6}$$

$$\frac{\delta \mathcal{S}_\pi}{\delta G^{ij}} = Z_{ij} = \Pi_{ij} \tag{7}$$

where the conjugated quantities are G_{ij} and the generalised super momentum Π_{ij} whereZ_{ij} is the Ricci curvature tensor in C^4. Eq. (7) tell us about the rate of change of the action with respect to the "field coordinates" G_{ij}. But, the existence of a complex time or a 2-d time, is saving us by the problems that exist in the ADM formulation. We will not lose time through the "sandwich" procedure. We can define a super momentum π_{ij} and a super energy E_{ij} as

$$\frac{\delta \mathcal{S}_\pi}{\delta G^{ij}} = \pi_{ij}, \tag{8}$$

for $i, j \neq 0$, $i \neq 0$, $j \neq 0$ and

$$\frac{\delta \mathcal{S}_\pi}{\delta G^{ij}} = E_{ij}, \tag{9}$$

for $i, j = 0$, $i = 0$, $j = 0$. This way, we have

$$\delta \mathcal{S}_\pi = \pi_{ij} \delta \overset{6d}{G_{ij}} - E_{ij} \delta \overset{2d}{G_{ij}} \tag{10}$$

The dispersion relation reads us

$$E_{ij} = \mathcal{H} \tag{11}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 September 2018 doi:10.20944/preprints201809.0371.v1

or

$$-\frac{\delta \mathcal{S}_{\pi}}{\overset{2d}{\delta G_{ij}}} = \mathcal{H}\big(\pi_{ij}, \overset{6d}{G_{ij}}\big) \tag{12}$$

and the super Hamilton -Jacobi equation is

$$G_{ijlm} \frac{\delta \mathcal{S}_{\pi}}{\delta G_{ij}} \frac{\delta \mathcal{S}_{\pi}}{\delta G_{lm}} = \Omega^2 \tag{13}$$

where G_{ijlm} stands for

$$G_{ijlm} = \frac{1}{2} G_{ij} Glm - G_{il} G_{jm} \tag{14}$$

and Ω is a function of the the constant D as $\Omega = f(D^4)$. The super Hamilton -Jacobi equation tell us how "wave crests" "propagate" in the space. If we subject Eq. (13) to a new extremisation with respect to Ω we will form a new action Σ_{π} as

$$\Sigma_{\pi} = \int G_{ijlm} \frac{\delta \mathcal{S}_{\pi}}{\delta G_{ij}} \frac{\delta \mathcal{S}_{\pi}}{\delta G_{lm}} \tag{15}$$

The Poisson brackets for the geometry will be

$$\{G_{ij}, G_{lm}\} = 0 \qquad \{\Pi_{ij}, \Pi_{lm}\} = 0 \qquad \{G_{ij}, \Pi^{lm}\} = \delta^{l}_{(i}\delta^{m}_{j)}$$

while the Poisson brackets for the field $\mathcal{S}_{\pi}$ are

$$\{\mathcal{S}^{ij}_{\pi}, \mathcal{S}^{lm}_{\pi}\} = 0 \qquad \{\Pi_{ij}, \Pi_{lm}\} = 0 \qquad \{\mathcal{S}^{ij}_{\pi}, \Pi_{lm}\} = \delta^{i}_{(l}\delta^{j}_{m)}$$

and the Lie brackets

$$[\widehat{\mathcal{S}^{ij}_{\pi}}, \widehat{\mathcal{S}^{lm}_{\pi}}] = 0 \qquad [\widehat{\Pi_{ij}}, \widehat{\Pi_{lm}}] = 0 \qquad [\widehat{\mathcal{S}^{ij}_{\pi}}, \widehat{\Pi_{lm}}] = \delta^{i}_{(l}\delta^{j}_{m)}$$

 doi:10.20944/preprints201809.0371.v1

where we have set $\mathcal{S}_{\pi}^{ij} = \mathcal{S}_{\pi}(G^{ij})$. By extracting $\mathcal{S}_{\pi}$ from Eq. (7), we have the operators

$$\widehat{\Pi_{ij}} = \frac{\delta}{\delta G^{ij}} \qquad \widehat{\pi_{ij}} = \frac{\delta}{\overset{6d}{\delta G_{ij}}} \qquad \widehat{E_{ij}} = \frac{\delta}{\overset{2d}{\delta G_{ij}}}$$

Now, $\mathcal{S}_{\pi}$ can be seen as a physical quantity or describes a physical quantity, which expresses the deviation between two "lines" of C^4 or the "curvature" which is created by those two "lines" . These "lines" represent bosons as we have seen in [?]. The relation

$$\widehat{\Pi_{ij}}\mathcal{S}_{\pi} = \widehat{Z_{ij}}\mathcal{S}_{\pi} = Z\mathcal{S}_{\pi} \tag{16}$$

is the analogue eigenvalue equation.

The Lie algebras with respect to the fields $\mathcal{S}_{\pi}$ or $\mathcal{S}_{\gamma}$ are formed in the space of the solutions of the super H-J and usual H-J respectively. If we want to describe the properties that a boson has, we must look in the space of the solutions of the usual H-J, while the "area that the boson field generates" is described by the properties of the solutions of the super H-J.

3 Conclusion

We have found a suitable dispersion relation, in order to proceed with the definitions of super "energies", "momenta" and Hamiltonians. This way we have the ability to formulate a theory which can describe interactions, compatible with gauge theories. In next paper [?], we will continue with the case of fermions.

4 References

References

[1] D. Mastoridis, K. Kalogirou, *C^4 Space-Time.. a window to new Physics*, www.researchgate.net.

[2] D. Mastoridis, K. Kalogirou, *Field Equations in C^4 space-time*, www.researchgate.net.

[3] D. Mastoridis, K. Kalogirou, *Introduction to Quantum Field Theory in C^4 space-time*, www.researchgate.net.

[4] D. Mastoridis, K. Kalogirou, *Dynamic Path in C^4 space-time*, www.researchgate.net.

[5] D. Mastoridis, K. Kalogirou, *Introduction to Fermionic Structures in C^4 space-time*, www.researchgate.net.